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Continuous change

دفتر :

تصميم نظم صناعية و عناصر الآلات

Design

للطالبة : لين ياسر

اللجنة الأكاديمية لقسم الهندسة الصناعية

2023

Design :-

Ch(3) : Load and stress analysis.

diffnent type of load [concentraited, distributed lateral] , cause stress of diffnent types and magnitude in diffnent location.

Equilibrium and Free-body diagrams :-

balance of force → to prevent translation.
balance of moments → to prevent rotations.

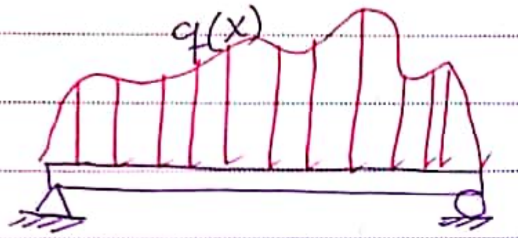
$\Sigma F = Zero$, $\Sigma M = Zero$.

Shear and moment in Beams :-

Shear and moment diagrams are important in locating the critical sections. in a beam.

Shear force and bending moment are related by the equation -

$$V = \frac{dM}{dx}$$ "Shear is the slope of the moment diagram"



when a distributed load q(x) is applied :-

$$q = \frac{dV}{dx} = \frac{d^2M}{dx^2}$$

$$\int_{x_1}^{x_2} q \cdot dx = \int_{V_1}^{V_2} dV = V_2 - V_1$$

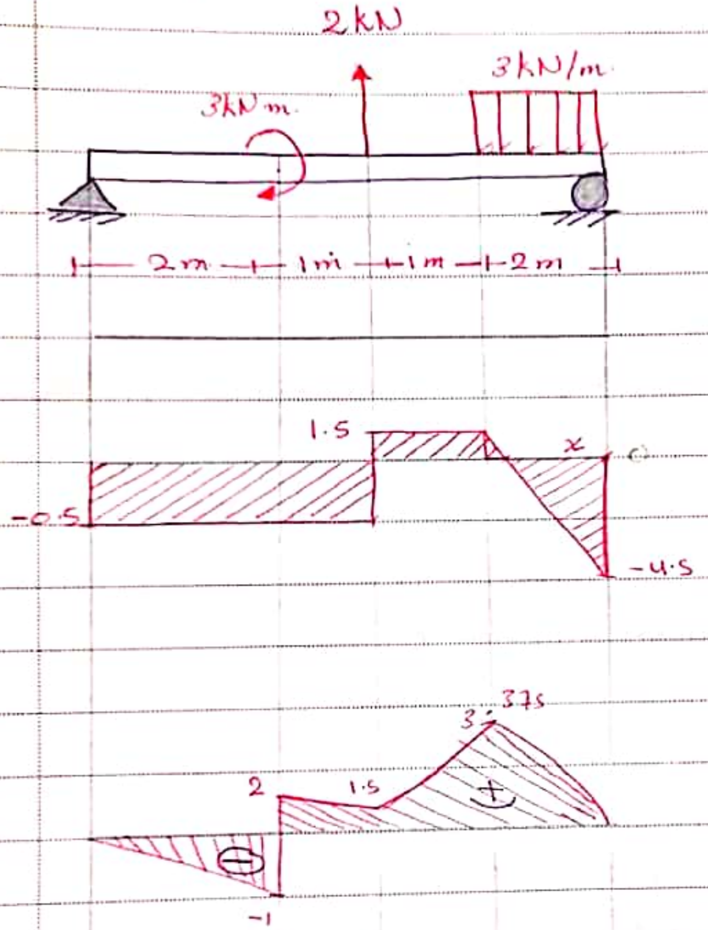
* the change in shear force equal to the area under the loading diagram.

$$\int_{x_1}^{x_2} V \cdot dx = \int_{M_1}^{M_2} dM = M_2 - M_1$$

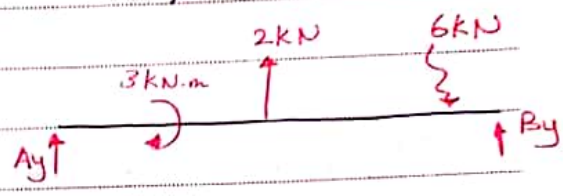
* the change in moment equal to the area under the shear diagram.

NO.

Example :-



F.B.D



$\circlearrowleft \Sigma M_A = 0$

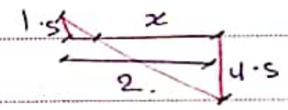
$-3 + 2(3) - 6(5) + B_y(6) = 0$

$\therefore B_y = 4.5$ kN.

$\Sigma F_y = 0$

$A_y + 2 - 6 + 4.5 = 0$

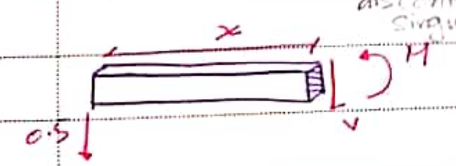
$\therefore A_y = -0.5$ kN ↓

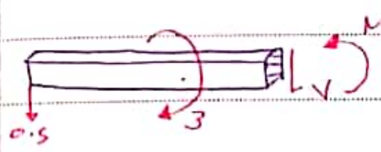


* بالمعادلات :- Sections. نعمل عند حدود الـ load

$0 < x < 2$ $2 < x < 3$

discontinuity singularity





$0 < x < 2$:

$\therefore \Sigma F_y = 0 \Rightarrow V = -0.5$

$\therefore \Sigma M = 0 \Rightarrow M = -0.5x$

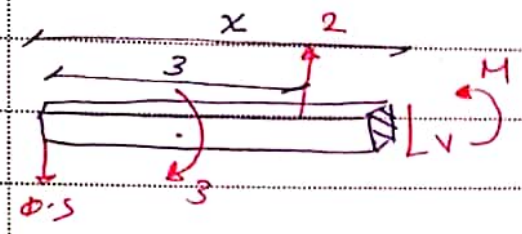
$2 < x < 3$:

$\therefore \Sigma F_y = 0 \Rightarrow V = -0.5$

$\Sigma M = 0 \Rightarrow 0.5(x) - 3 + M = 0$

$\therefore M = 3 - 0.5x.$

$3 < x < 4.$



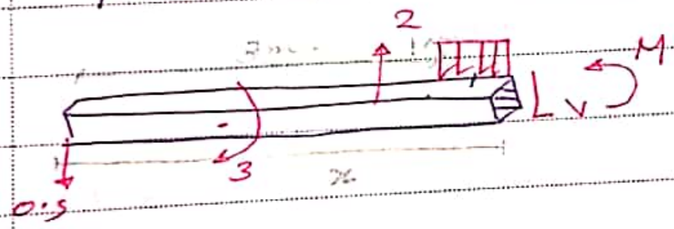
$\therefore \sum F_y = 0 \Rightarrow -0.5 + 2 - V = 0$

$\therefore V = 1.5$

$\therefore \sum M = 0 \Rightarrow M + 0.5(x) - 3 - 2(x-3) = 0$

$\therefore M = 1.5x - 3$

$4 < x < 6$



$\therefore \sum F_y = 0 \Rightarrow -0.5 + 2 - V - 3(x-4)$

$V = -3x + 13.5$

$\therefore \sum M = 0 \Rightarrow M + 3(x-4)\left(\frac{x-4}{2}\right) - 2(x-3)$

$\therefore M = -1.5x^2 + 13.5x - 27.$

+ بدي أتأكد بنشتق معادلة M اذا طلعت معادلة V ✓

NO.

Singularity functions :-

لإيجاد M.D , S.D

loading is not simple , section, integration to obtaining the shear and moment becomes difficult.

Singularity :- mathmatical expresion → that it permits writting analytical expression for shear and moment over a range of discontinuites "no need interval".

* لما يكون عندي discontinuities ممكن نعبر عنها باستخدام ال singularity function
بكتب معادلة وحدة للمومنت مثلاً وبدون ما أقسم.

- A singularity functions of x is written as :-

$$F_n(x) = \langle x - a \rangle^n$$

x : variable.

a : location when discontinuity occurs. الأماكن الي فيها dis.

n : order for singularity function. [integer (+) (-) (0)]

a ← قيمة معينة بتحدد لي المسافة على ال x-axis يلي موجود عليها نوع معين من أنواع discontinuity.

- Evaluation Rules of singularity functions :-

* عند مسافة معينة نحتاج قيمة ال shear أو المومنت

* بعتمد على order (-ve) (+ve) (0)

NO.

بحولها أقواس. — بدي اقارن x مع a

+ve $n > \text{Zero}$	$(x \geq a) \rightarrow$	$F_n(x) = (x-a)^n$.
	$(x < a) \rightarrow$	$F_n(x) = \text{Zero}$.
$n = \text{Zero}$	$(x \geq a) \rightarrow$	$F_n(x) = 1$
	$(x < a) \rightarrow$	$F_n(x) = \text{Zero}$.
-ve $n < \text{Zero}$	$(x = a) \rightarrow$	$F_n(x) = 1$
	$(x \neq a) \rightarrow$	$F_n(x) = \text{Zero}$.

* عندي Beam عليها أنواع مختلفة من ال load فلتب expression لل load
بدي Shear بعمل integration بدي مومنت بعمل كمان integration

- Integration Rule :-

$$n \geq 0 \quad \int \langle x-a \rangle^n dx = \frac{\langle x-a \rangle^{n+1}}{n+1}$$ تكامل عادي

$$n < 0 \quad \int \langle x-a \rangle^n dx = \langle x-a \rangle^{n+1}$$

- Derivation :-

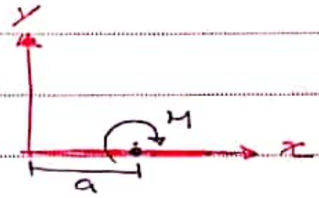
ما بهمنا كثير $\quad n \geq 1 \quad \frac{d}{dx} \langle x-a \rangle^n = n \langle x-a \rangle^{n-1}$

The Singularity functions for the common types of loading are :-

1] Concentrated moment

$$q(x) = M \langle x - a \rangle^{-2}$$

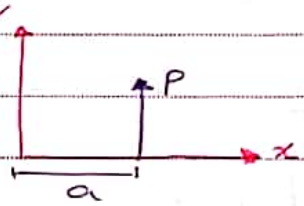
↳ magnitude قيمة (+ve), (-ve)



, order -2.

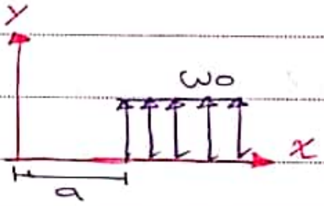
2] Concentrated force.

$$q(x) = P \langle x - a \rangle^{-1}$$



3] uniform load

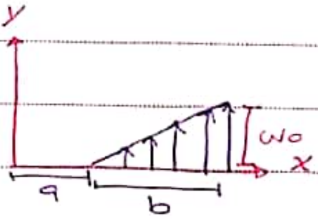
$$q(x) = w_o \langle x - a \rangle^{0}$$



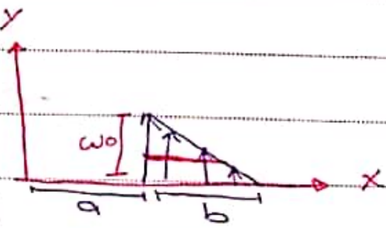
* 4] Ramp

$$q(x) = \frac{w_o}{b} \langle x - a \rangle^{1}$$

↳ slope.



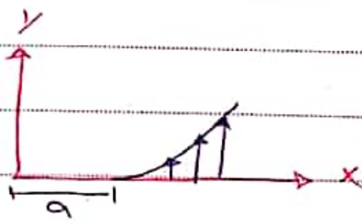
* 5] Inverse Ramp



$$q(x) = w_o \langle x - a \rangle^{0} - \frac{w_o}{b} \langle x - a \rangle^{1}$$

* 6] Parabolic

$$q(x) = \langle x - a \rangle^{2}$$



* ٤، ٥، ٦ ممكن (تكون) magnitude (-ve)

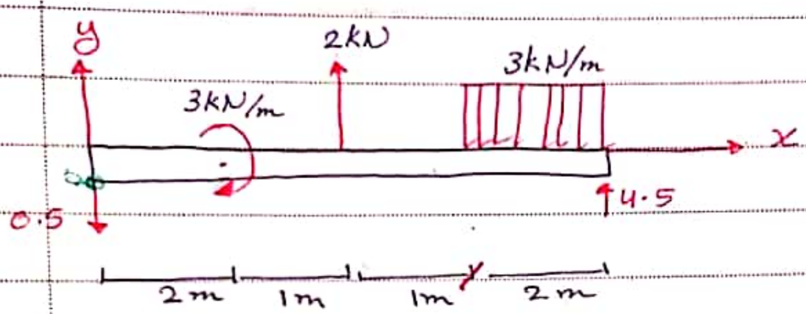
NO.

* بنكتب الـ load equation ← تكامل (1) ← شير ، تكامل (2) ← مومنت
* بالكتاب في معادلة الشير و المومنت مباشرة بدون تكامل.
* الفائدة :- برنامج عل Beam app برسم المومنت والشير

Ex:

evaluate V & M

$x = 4.5\ m.$



Solution :-

معادلة load

$$q(x) = -0.5\langle x-0\rangle^{-1} + 3\langle x-2\rangle^{-2} + 2\langle x-3\rangle^{-1} - 3\langle x-4\rangle^{0} + 4.5\langle x-6\rangle^{-1}$$

تكامل (1) ← V

$$V = \int q\,dx = -0.5\langle x-0\rangle^{0} + 3\langle x-2\rangle^{-1} + 2\langle x-3\rangle^{0} - 3\langle x-4\rangle^{1} + 4.5\langle x-6\rangle^{0}$$

$$V(x=4.5) = -0.5(4.5-0)^{0} + 3(\text{zero})^{-1} + 2(1) - 3(4.5-4) + 4.5(0) = 0\ kN.$$

تكامل (2) ← M

$$M = \int V\,dx = -0.5\langle x\rangle^{1} + 3\langle x-2\rangle^{0} + 2\langle x-3\rangle^{1} - 1.5\langle x-4\rangle^{2} + 4.5\langle x-6\rangle^{1}$$

$$\int 3x = 3\frac{x^2}{2}$$

$$M(x=4.5) = -0.5(4.5-0)^{1} + 3(1) + 2(4.5-3)^{1} - 1.5(4.5-4)^{2} + 4.5(0) = 3.375\ kN.m.$$

NO.

* تكتب كله بعد تقييم Reaction ← R_B , R_A (unknowns)

* بعمل evalution لل Beam قبل ونتيجته (0) وبعده ونتيجته (0)

* في حالة distrubuted ← مسافة (a) يبدأ عند مسافة (a) واستمر (∞)

* Beam عليها distrubuted يبدأ عند مسافة معينة وانتهى قبل نهاية ال Beam

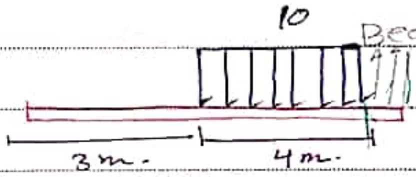
هنا نعرف انه انتهى (ينعكس) ← term off function

It is Not necessary to find the reactions before using the singularity function.

evaluted from the shear and moment equation by evaluting $x<0$ $x<L$, V & $M = 0$ at valu of x

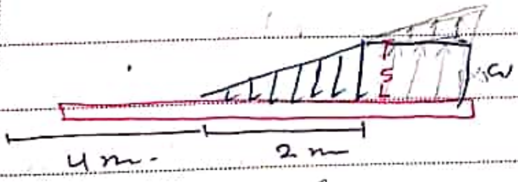
when the distrubuted loading that ends before the end of the beam, need to be turned off

Ex:



عشان ال Beam ما [illegible] لل ∞ بعكس عشانه ومد نهايته

$$q(x) = -10 \langle x-3 \rangle^0 + 10 \langle x-7 \rangle^0$$

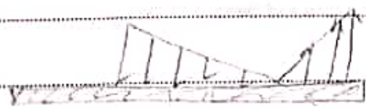


$$q(x) = \frac{-5}{2} \langle x-4 \rangle^1 + \frac{5}{2} \langle x-6 \rangle^1 + 5 \langle x-6 \rangle^0$$

+ عشان [illegible] لل [illegible] عند a ← uniform, Ramp

+ لو في inverse Ramp

بعكس a ← Ramp [illegible]



Stress :-

is the term used to define the intensity and direction of the internal forces acting at a given point on a particular plane

$\sigma = \frac{P}{A}$

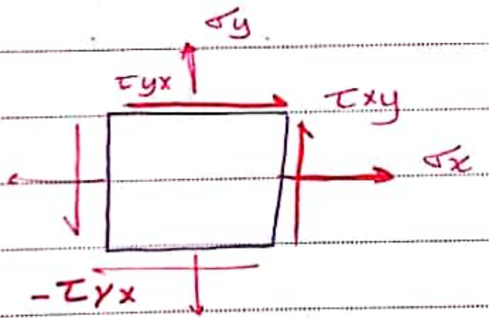
avg., as force acting over an area.

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta P}{\Delta A} = \frac{dP}{dA}$$

Stress at a point on a cross - section.

+ Normal stress , Shear stress (tangitial).

Cartesian. Stress - Components . 3D



σ_x : on the (x) face in the (x) direction.

τ_{yz} : on the (y) face in the (z) direction.

(+ve) stress : surface (+) , dir (+)
surface (-) , dir (-)

(-ve) stress : surface (+) , dir (-)
surface (-) , dir (+) .

Tension +ve , comp. (-ve)

FIVE APPLE

NO.

Note, $\tau_{xy} = \tau_{yx}$

$\tau_{yz} = \tau_{zy}$

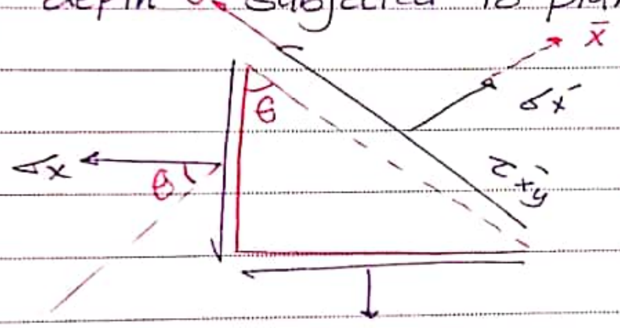
$\tau_{xz} = \tau_{zx}$

2-D

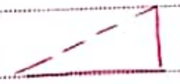
when the stresses on one of the surface is (Zero) the state stresses is called plane stress and stresses components, σ_x, σ_y & τ_{xy}

Mohr's Circle for Plane Stress :-

Consider a wedge shaped element of unit depth subjected to plane stresses.



$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta \cos\theta$$



$$\sigma_{y'} = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy} \sin\theta \cos\theta .$$

$$\tau_{x'y'} = -(\sigma_x - \sigma_y)\sin\theta \cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

استخدام المتطابقات

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

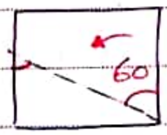
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

NO.

* That equation called transformation equations.

* Can be used to find the σ & τ in desired direction.

* To defined the angle θ measured from +ve x-axis



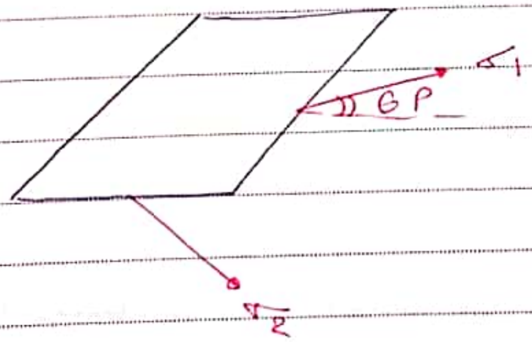
θ: C.C.W

θ: C.W (-)

- principal stresses.

[1] max. Normal stresses.

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



τ = Zero

$$\frac{d\sigma_x}{d\theta} = 0 \Rightarrow \tan 2\theta_P = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

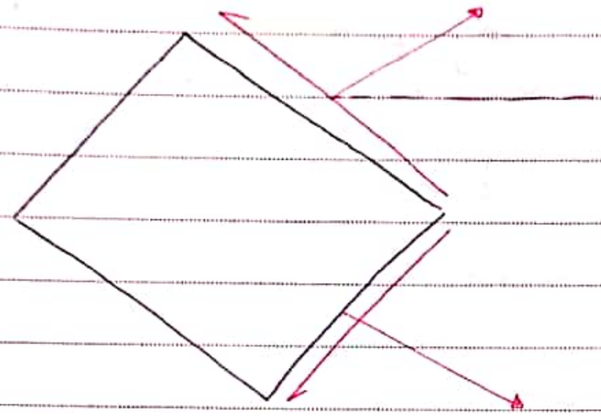
$\rightarrow \theta_{P_1}, \theta_{P_2}$

جا	كل
ظا	جتا

نختار منهم حسب الربع.

[2] Max. Sherr Stresses.

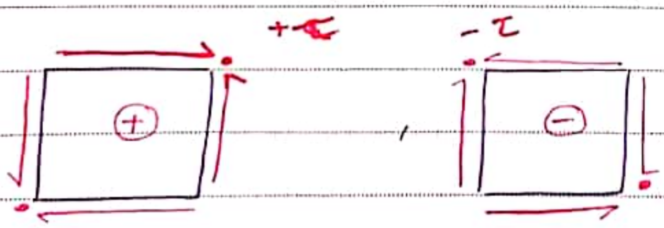
$$\tau_{max} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



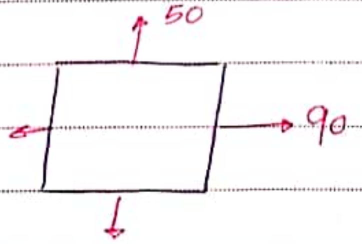
$$\frac{d\tau_{xy}}{d\theta} = 0 \Rightarrow \tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)}{2\tau_{xy}}$$

$\rightarrow \theta_{s_1}, \theta_{s_2}$.

$$\sigma = \sigma_x' = \sigma_y' \ avg = \frac{\sigma_x + \sigma_y}{2}$$



Notes:

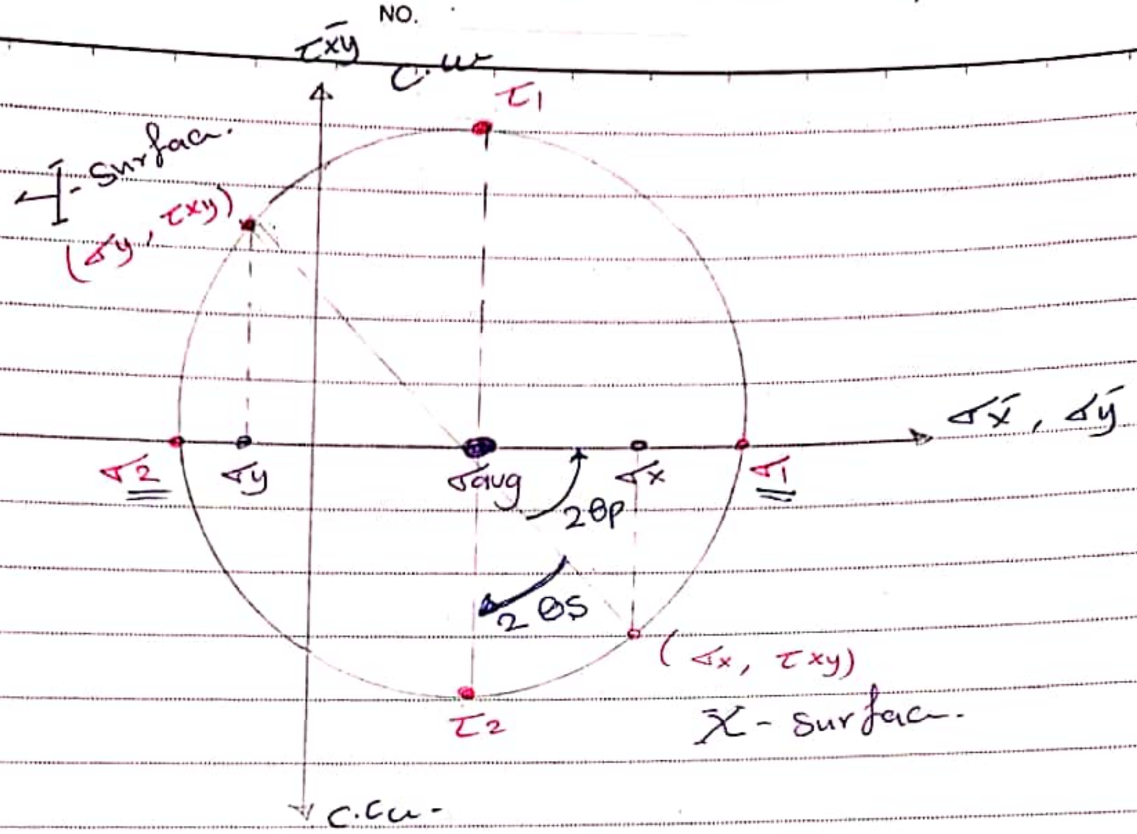


$\sigma_{max} = 90$ MPa

τ = Zero.

$\theta_{P_1} = 0$, $\theta_s = 45°$

$\theta_{P_2} = 90$.



$\sigma_1, \sigma_2 = \sigma_{avg} \pm R$

$\tau_{max} = \pm R$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

C.CW $(2\theta_p)$ σ_x بنا ندور لنصل +

C.W $(2\theta_s)$ τ // // +

Example:

$\sigma_x = 9$ Mpa , $\sigma_y = 19$ Mpa , $\tau_{xy} = 8$ Mpa.

9

$$R = \sqrt{\left(\frac{9-19}{2}\right)^2 + 8^2} = 9.43 \text{ MP.}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{9+19}{2} = 14 \text{ Mpa.}$$

$\sigma_1 = 14 + 9.43 = 23.43 \text{ Mpa}$

$\sigma_2 = 14 - 9.43 = 4.57 \text{ Mpa}$

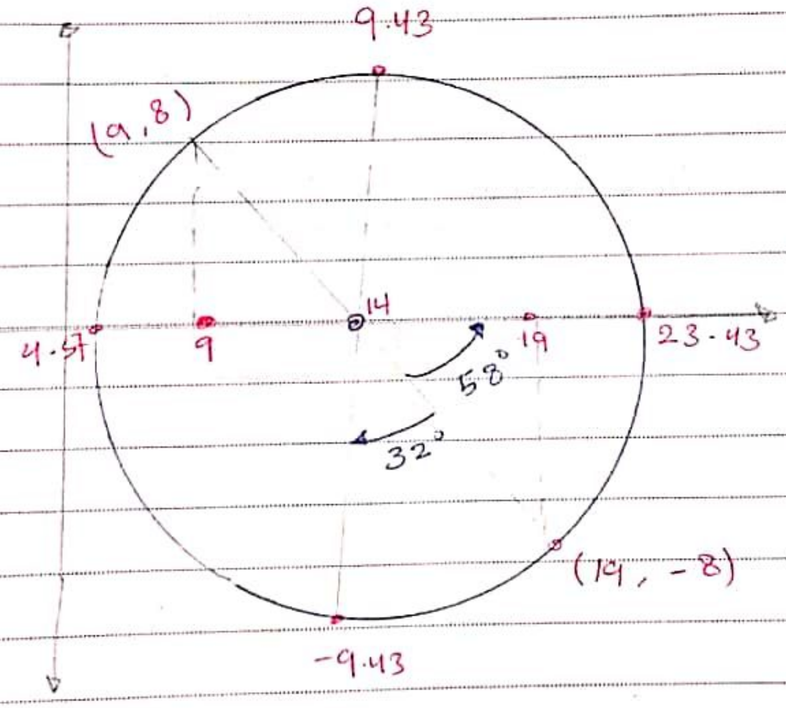
$\sigma_{center} = \frac{\sigma_x + \sigma_y}{2} = 14 \text{ Mpa}$

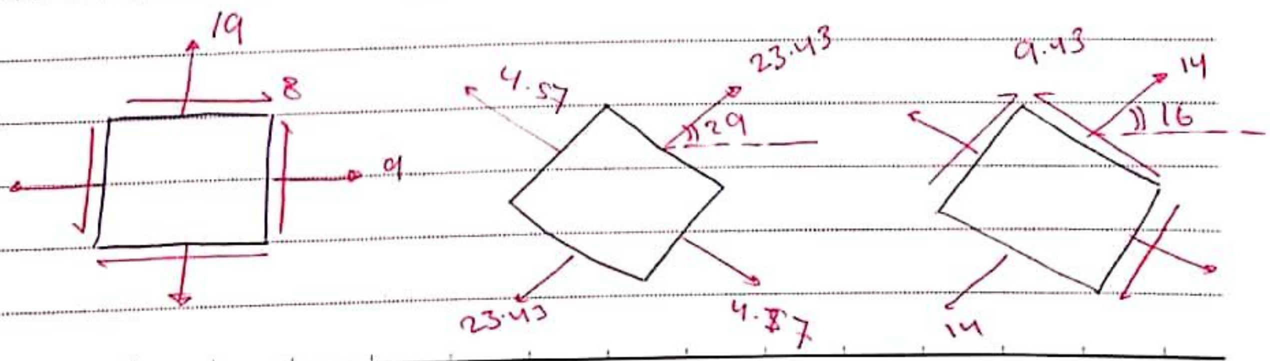
$\tau_{max} = \pm 9.43$

$2\theta_p = \tan^{-1}\left(\frac{\tau_{xy}}{\sigma_x - \sigma_y}\right) \Rightarrow \tan^{-1}\left(\frac{(2)8}{14-9}\right) = 58^\circ$ $2\theta_p$

$\theta_p = 29^\circ$

$2\theta_s = 90 - 58 = 32$ $\quad \theta_s = 16^\circ$





principle ⇒ σ_1 , σ_2

stress ⇒ NO. σ_x , σ_y

[b] what is the state of stress when the axes are rotated 30° C.C.W

$30(2) = 60$ ⇒ circle

$\sigma_x = 14 + 9.43 \cos(180 - 58 - 60) = 18.43 \text{ Mpa}$

$\sigma_y = 14 - 9.43 \cos(180 - 58 - 60) = 9.57 \text{ Mpa}$.

$\tau_{xy} = 9.43 \sin(180 - 58 - 60) = 8.33 \text{ Mpa}$.

static ترانيز

- General Three-dimensional stress :-

$\sigma_1 > \sigma_2 > \sigma_3$.

(I) σ_1 , σ_2 : واحدة موجبة وواحدة سالبة

$\tau_{max} = R$.

(II) σ_1 , σ_2 : موجبات

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

(III) σ_1 , σ_2 سالب

$$\tau_{max} = \frac{\sigma_1 - 0}{2}$$

Strain :-

1] non-dimensional.

2] measure of the deformation resulting from the stress acting upon the solid material.

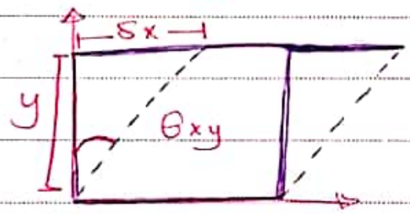
Normal strain :- measure the change in length.

$$\varepsilon_x = \frac{\delta_x}{L_x} \quad , \quad \varepsilon_y = \frac{\delta_y}{L_y}$$

Shear strain :- measure the angular direction

$$\gamma_{xy} = \frac{\delta x}{y} = \tan \theta_{yx} \simeq \theta_{xy}$$

↳ for small strain.



* In the elastic region under uniaxial stress condition or pure shear stress condition, the stress, strain

$\sigma = E \varepsilon$

$\nu = -\dfrac{\text{lateral strain}}{\text{axial strain}}$,

$\tau = G \gamma$, $G = \dfrac{E}{2(1+\nu)}$

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} - \nu \frac{\sigma_z}{E}$$

$$\varepsilon_y = -\nu \frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \nu \frac{\sigma_z}{E}$$

$$\varepsilon_z = -\nu \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} + \frac{\sigma_z}{E}$$

$\frac{bh^3}{36}$

$I_y = b^3h/36$

NO.

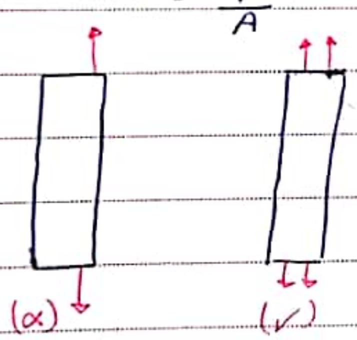
* for biaxial or triaxial state, these relations are not valid and the generalized hook's law is used to relate stress and strain.

uniformly distributed stress :-

The assumption of uniformly distributed stress is often made in design when loading is simple such as pure tension, comp. or shear

* for tension or compression.

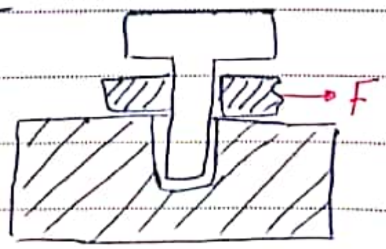
$\sigma = \frac{P}{A}$



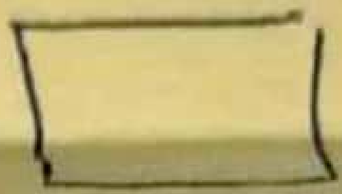
((The load should be Centroidal or Symmetric))

* for shear.

$\tau = \frac{P}{A} = \frac{V}{A}$



$A = bh$



$I_x = \frac{bh^3}{12}$

$I_y = \frac{b^3 h}{12}$

$A = \frac{\pi D^2}{4}$

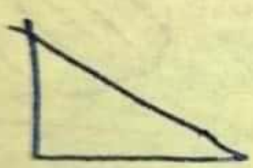
$I_x, I_y = \frac{\pi D^4}{64}$

$J = \frac{\pi}{32} d^4$

$$A = \frac{\pi}{4}(D^2 - d^2)$$

$$I_x = I_y = \frac{\pi}{64}(D^4 - d^4)$$

$$J = \frac{\pi}{32}(d_2^4 - d_1^4)$$



$$A = \frac{1}{2} h b$$

$$I_x = \frac{b h^3}{36}$$

$$I_y = b^3 h / 36$$

$\frac{P}{A}$

((The load should be centroi

or Symmetric))

NO.

Normal stress in Beams :-

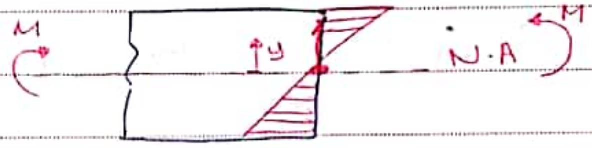
Beam bending stress equation " flexure formula" is developed under the following assumptions:-

① beam straight, long and having constant cross section with an axis of symmetry in the plan of bending

② material is isotropic, homogenous, linearly elastic

③ Beam subjected to pure bending moment (No axial, Shear, torsion)

$$\sigma = \frac{-My}{I}$$



y: height from the neutral axis.
I: moment of inertia about z axis.

+ maximum bending stress $\sigma = \frac{Mc}{I}$

$\sigma = \frac{M}{Z}$, $Z = \frac{I}{c}$ called the section modulus.

+ use (Z) when your designing based on stress
+ use (I) when your designing based on deflection

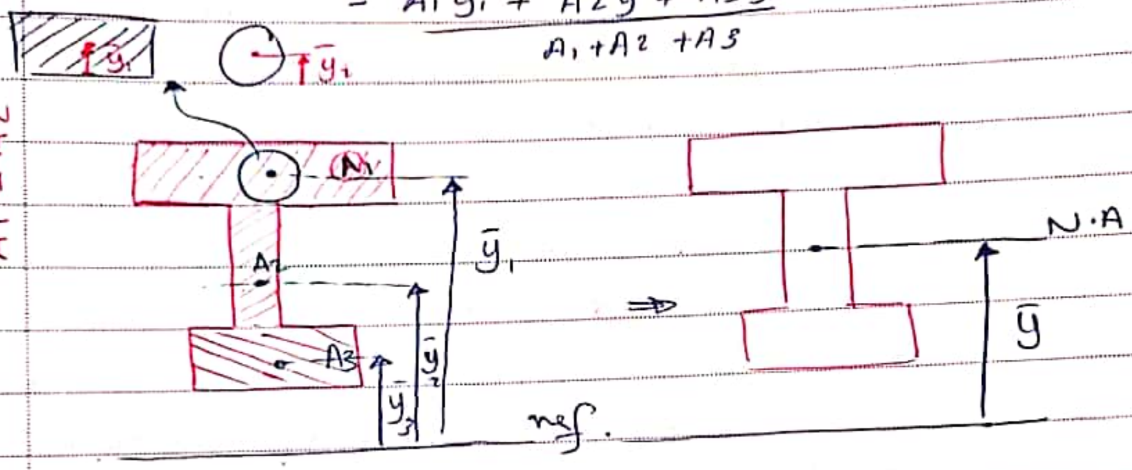
NO.

(I) Centroid : $\bar{y}_c$ (لكي) $= \dfrac{\sum y_i A_i}{\sum A_i}$

$$= \frac{A_1\bar{y}_1 + A_2\bar{y} + A_3\bar{y}_3}{A_1 + A_2 + A_3}$$

وحاجة دائرة مفرغة

$$\bar{y} = \frac{A_1\bar{y}_1 - A_2\bar{y}_2}{A_1 - A_2}$$



(II) Second moment of inertia.

$$I_{N.A} = I_1 + I_2 + I_3.$$

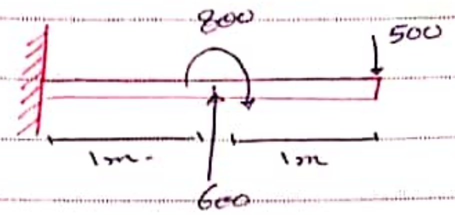
when the axis is not passing through the Centroid of an area. we use the parrallel axis theorm.

$$I_x = \bar{I}_x + A\bar{y}^2$$

(III) Use $\sigma_{max} = \dfrac{My}{I}$

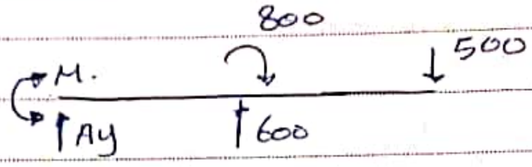
NO.

Ex:



max. bending stress not exceed 250 MPa

Sol: F.b.d.

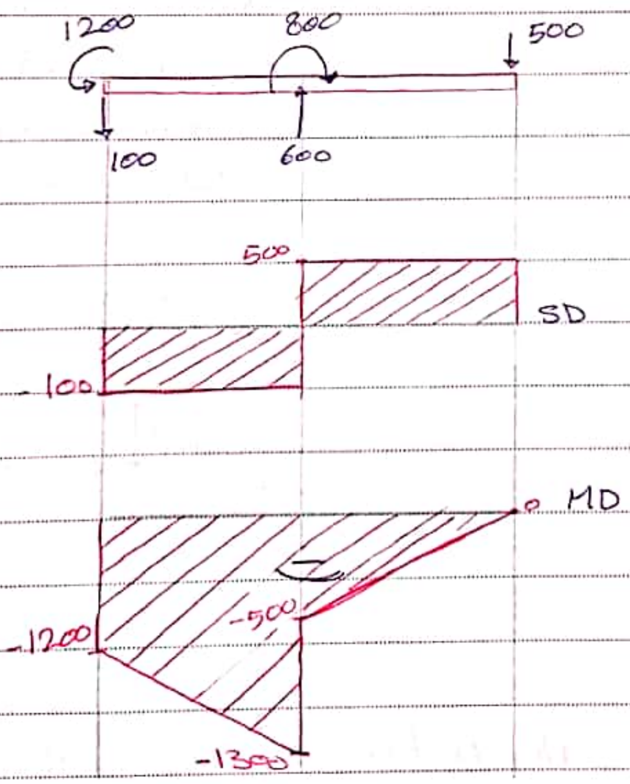


$\Sigma F_y = 0$

$600 + A_y - 500 = 0 \quad \therefore A_y = -100 \downarrow N$

$\circlearrowleft + \quad \Sigma M_A = 0$

$-M + 800 - 600(1) + 500(2) = 0 \quad \therefore M = 1200 \ N \cdot m$



$\sigma_{max} = \frac{M_{max}}{Z} =$

$\frac{1300 (N \cdot m) \times 1000 \left(\frac{mm}{m}\right)}{250 \left(\frac{N}{mm^2}\right)}$

$Z = 5200 \ mm^3 = 5.2 c$

with $Z_{2\text{-}2} \geq 5.2 \ cm^3$

from Table A-7

a: 102 mm.

b: 51 mm

$Z_{2\text{-}2}$: $8.16 \ cm^3$.

NO.

Shear stress for beams in bending:

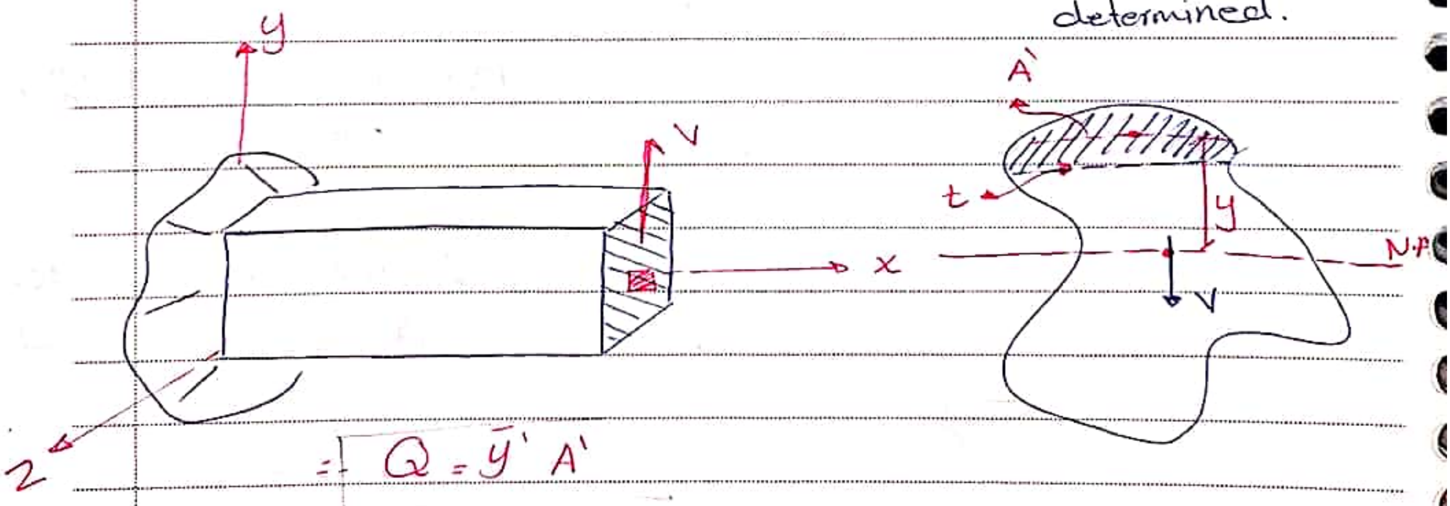
Beams are subjected to both shear force and bending moment.

beam bending stress equation developed based on.
1) holds reasonably accurate with the presence of shear forces. كل مع وجود قوة القص

$\tau = \frac{VQ}{It}$

V: internal shear force.
Q: first moment of intria
I: second mont of intria
t: is the width at the point when τ is determined.

sec.

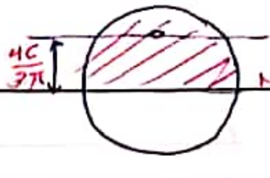


: $Q = \bar{y}' A'$

A': is the area of the portion of the section above or below the point when τ is determnd.

$I_{N.A} = \frac{\pi}{4} c^4$

$\bar{y} = \frac{4c}{3\pi}$

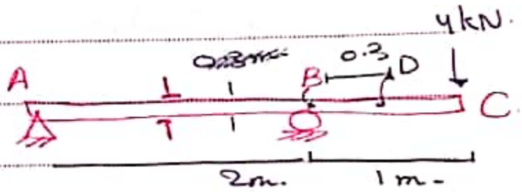


$I = \frac{bh^3}{12}$

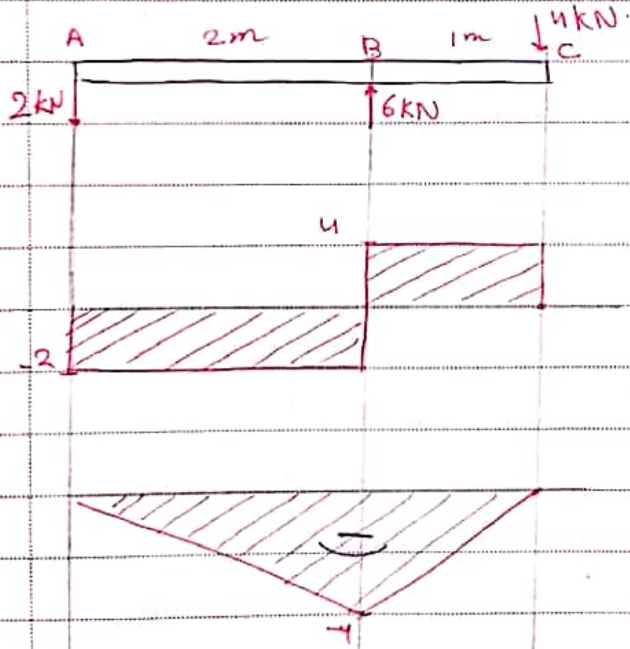
y': distance to the centroid of the area A' measured from the neutral axis of the beam.

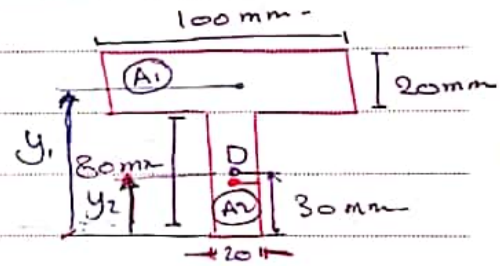
يكون الـ Shear max بالـ center لأن Q: max وتكون Zero عند الطرف

Ex: Simply supported beam. 4kN load.



F.B.D :-





$$\bar{y} = \frac{90(20)(100) + 40(80)(20)}{(20)(100) + (80)(20)}$$

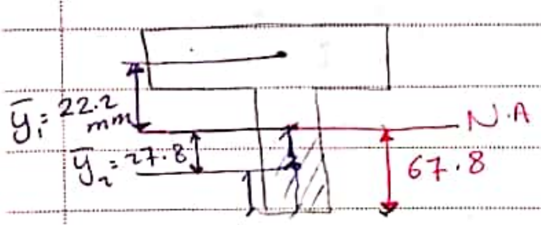
$\bar{y} = 67.8$ mm.

$I_{N.A} = I_1 + I_2$.

$I_1 = \frac{1}{12}(100)(20)^3 + 20(100)(22.2)^2$

$I_2 = \frac{1}{12}(20)(80)^3 + (20)(80)(27.8)^2$

$I_1 + I_2 = 3.142 \times 10^6$ mm^4.



NO.

[a] max. bending stress at point B.

$$\sigma = \frac{Mc}{I} = \frac{(4\times10^6)(67.8)}{3.142\times10^6} = 86.31 \text{ MPa}$$

[b] max. shear force bet. B and C. & max. shear on nutral axis :-

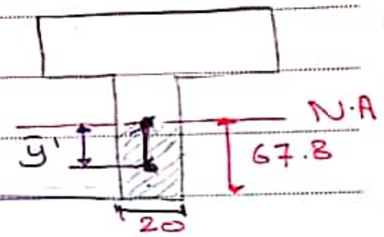
$$Q_{N.A} = \bar{y}' \bar{A}'$$

$$= 33.9\ (20 \times 67.8)$$

$$= 45968 \text{ mm}^3.$$

Area of center new N.A

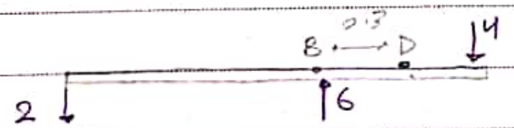
$33.9 = \frac{67.8}{2}$



$$\therefore \tau_{max} = \frac{VQ}{tI} = \frac{(4\times10^3)(45968)}{(20)(3.142\times10^6)} = 2.93 \text{ MPa}$$

t, I

[c] stress at "D"



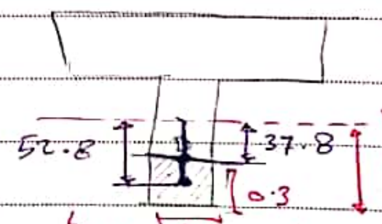
$V_D = 4$ kN. ΣM

$M_D = -2.8$ kN.m.

$$Q_D = \bar{y}_D A'_D$$

$$= 52.8\ (20)(30)$$

$$= 31680 \text{ mm}^3.$$

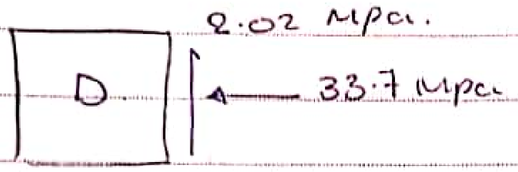


$$\sigma_{max} = -\frac{Mc}{I} = -\frac{(-2.8\times10^6)(-37.8)}{(3.142\times10^6)}$$

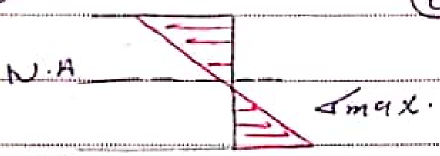
$$= -33.7 \text{ MPa}$$

NO.

$$\tau_D = \frac{VQ}{tI} = \frac{(4\times10^3)(31680)}{(3.142\times10^6)(20)} = 2.02\ Mpa.$$



ⓐ τ توزيع



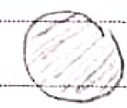
ⓑ τ توزيع

Shear Stress is standerd section Beams :.

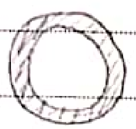
we look at max stress to ensure the safty of the element.

[1] Rectangular $\tau_{max} = \frac{3V}{2A}$

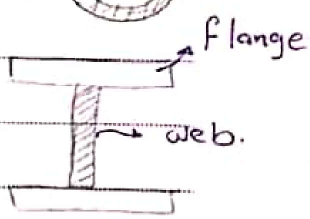
[2] Circular $\tau_{max} = \frac{4V}{3A}$



[3] Hollow-round (thin welled) $\tau_{max} = \frac{2V}{A}$



[4] I-beam-(thin welled) $\tau_{max} = \frac{V}{A_{web}}$



NO.

Torsion :

moment vector is colliear with the axis of an element it is called torque.

Circular shaft :

angle of twist. $\theta = \frac{TL}{GJ}$ $G :- \frac{E}{2(1+\nu)}$

J :- Polar moment of intria.

$\frac{\pi}{32} d^4$ → hollow → $\frac{\pi}{32}(d_2^4 - d_1^4)$

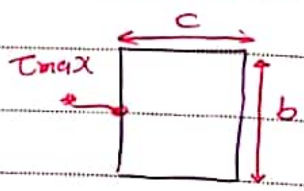
 

Shear strain at outer surface. $\gamma_{max} = \frac{r\theta}{L}$

Shear stress at outer surface $\tau_{max} = \frac{Tr}{J}$

* For rectangular cross-section. maximum shear stress is found as.

$$\tau_{max} = \frac{T}{bc^2}\left(3 + \frac{1.8}{b/c}\right)$$



b: longest said.

* I machin design applications , usually the torque is not given , but rather the transmitted Power and rotational speed

To find Torque $H = T\omega$ $(2\pi f)$

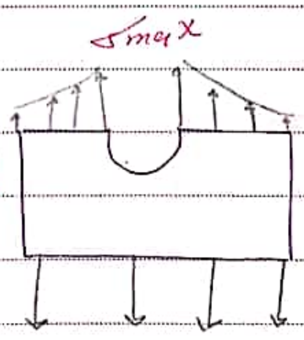
H → Power (watt)
T → torque (N.m)
ω → angular velocity (rad/s)

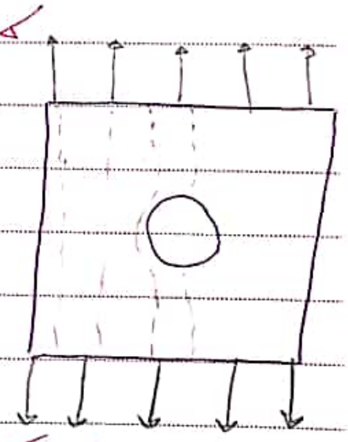
Torsion of closed - thin wall tubes ∝

Stress Concentration.

* higher stress near the discontinuity , any tube of discontinuity (hole, fillet , Notch, inclusion) serve as a stress raiser , when it increases the stress in the vicinity of the discontinuity

Stress concentration factor (K_t) is used to relate the actual maximum stress at the discontinuity to nominal stress without the discontinuity.



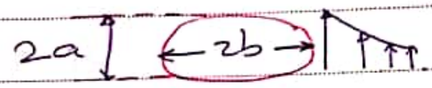


$$K_t = \frac{\text{actual maximum stress}}{\text{nominal stress}} = \frac{\sigma_{max}}{\sigma_0}$$

* Stress Concentration factor are Independent of the material properties, It depends only on the type of discontinuity and the geometry.

+ theoretical stress concentration factor is elliptical hole in an infinite plate loaded in tension.

$$k_t = 1 + \frac{2b}{a}$$



- if the hole is circular $(a=b)$ in an infinit plate then $k_t = 3$.

Stress concentration ration factors are very diffecult to find using theoretical analysis.

usually they are found experimentally using Photoelasticity or finite element analysis and they are usually presented in charts for diffrent geometric and loading configurations in specialized books Table A-13 & A-14.

+ دفتر المادة :

NO.

* When using stress concentration factors from Charts, how Kt is defined " with respect to stress in the net area to total area."

$$\sigma_0 = \frac{P}{wt} \quad \text{or} \quad \sigma = \frac{P}{(w-d)t}$$

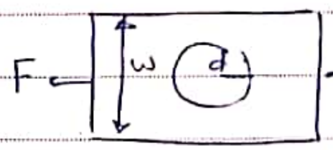
wt → total area. $(w-d)t$ → Net area.

t: thickness

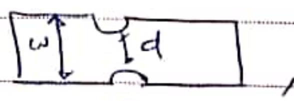
Note:-

* brittle material it is very imp. to consider the stress Concentrtion because rupture will initate then and the entire part will fail

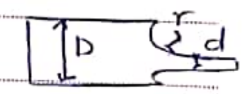
* ductile material stress Concentrtion usually not considered because the material will yield at high stress location and this relieve the stress Concentrtion.



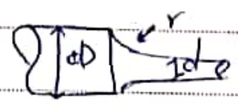
$$\sigma = k \frac{P}{(w-d)t}$$



$$\sigma = k \frac{P}{wdt}$$



$$\sigma = k \frac{P}{dt}$$



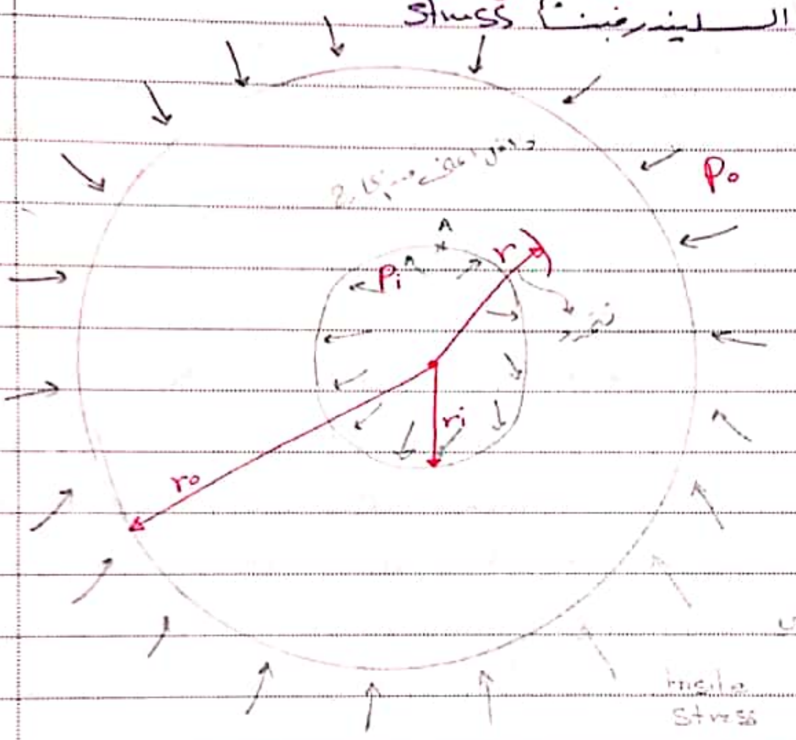
$$\sigma = k \frac{P}{\frac{\pi}{4} d^2}$$

NO.

Stress in pressurized Cylinders :-

pressurized cylinders include pressure vessels hydraulic or pneumatic cylinders, gun barrels and pipes carrying high pressure fluids.

الفرق بالضغط داخل وخارج السلندر ينشأ stress



Stress develop in both the radial and tangential directions.

* الضغط يأثر بشكل عمودي stress في الاتجاه العاكس radial

* الضغط الداخلي أكبر من الخارجي الـ cylinder تتمدد tangential

tangential $\sigma_t = \dfrac{P_i r_i^2 - P_o r_o^2 - r_i^2 r_o^2 (P_o - P_i)/r^2}{r_o^2 - r_i^2}$

$r = r_i$ النقطة يلي بدي أطلع الـ stress عندها على السطح الداخلي

$r = r_o$ الخارجي

$r =$ —— عند مسافة معينة

radial $\sigma_r = \dfrac{P_i r_i^2 - P_o r_o^2 + r_i^2 r_o^2 (P_o - P_i)/r^2}{r_o^2 - r_i^2}$

General Case: في الداخل والخارج

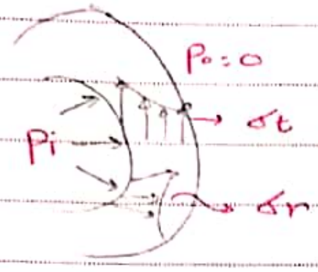
FIVE APPLE

refrance pressure

external pressure equals = Zero $P_o = 0$

+ سطح خارجي (الخارجي) الاغوصغير

+ المواد القصفوتة عليها غير الطبال مثلا



$$\therefore \sigma_t = \frac{P_i r_i^2}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

$$\sigma_r = \frac{P_i r_i^2}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r^2}\right)$$

* radial componant : (comp) Stress عليها المادة , بضغط pressure

Stress ماكسيمم على السطح الداخلي تحت قيمة pressure

$\sigma_r \; max = -P_i$ → compressive

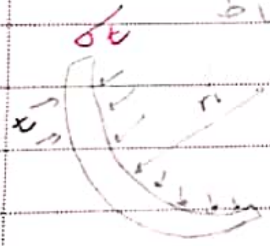
$r = r_i$

* tangential componant : الاسطوانة بتعيد (+) داخلي pressure

تقل النقاط tension قيمة tensial غير متساوي على كل النقاط (Stress)

الماكسيمم تكون على inner surface (قيمته بوصل) Zero عند outer surface.

عشان المحيط الخارجي رح يتمدد



* Close ended الاطراف مغلقة مثل جرة الغاز ,

end مش عليه بتأثر (axial) longitudinal تكون Stress

under tensial Stress in axial direction.

$$\sigma_L = \frac{P_i r_i^2}{r_o^2 - r_i^2} \quad \text{(uniform stress)}$$

Thin - walled cylinders :-

thickness is small compared to the radius

* إذا الـ thickness أصغر من radius بـ (20) مرة $\left(t \le \frac{r_i}{20}\right)$
بنكون الـ thin-wall

* نفرض الـ radial stress $\sigma_r = 0$ لأنه الـ thickness غير كافي ليعطي support ما في أي مقاومة بصير كرنشر

* بنحاول نتمدد في ten. stress — (open) tangential stress

* الـ thick قليل الفرق بالـ stress
بالمقطع الداخلي والخارجي قليل فنفرضه متساوي ∴ uniform

$$\sigma_t = \frac{P d_i}{2t}$$

* الكتاب بيشمل غير هنا بحسب (stress خارجي) diameter + t

(Closed)-ended $\sigma_L = \frac{P d_i}{4t}$

* بالإضافة للـ tangential

longitudinal (axial) < tangential و الـ tangential هي نفسها الـ (Hoop stress) σ_H

⇓

* لأنها أكبر رح يأثر عليه طريقة الكسر crack في خط مستقيم
زي مطرة الثلج 

Temperature effects :-

بسبب تغير درجة الحرارة لل Component أثناء عملها بتولد Stresses

* لو في مادة أثرت عليها بدرجة حرارة رح تتمدد وما رح تكون في Stress

بس لو حددتها ب fixed رح يكون في stress 

free :- $\epsilon_x = \epsilon_y = \epsilon_z = \alpha(\Delta T)$

α: coeffcien of thermal expansion.

* نوع المادة ، درجة الحرارة

2- plate :- restrained in any direction.

(one-direction) منعته باتجاه واحد

$$\sigma = -\epsilon E = -\alpha(\Delta T)E$$

(−) → Comp.

(both direction) منعته باتجاهين

$$\sigma = -\frac{\alpha(\Delta T)E}{1-\nu}$$

* مثل سكة حديد.

* Thermal stresses usually ~~subjected~~ occur during welding or any restraind member subjected to temp. Change during operation.

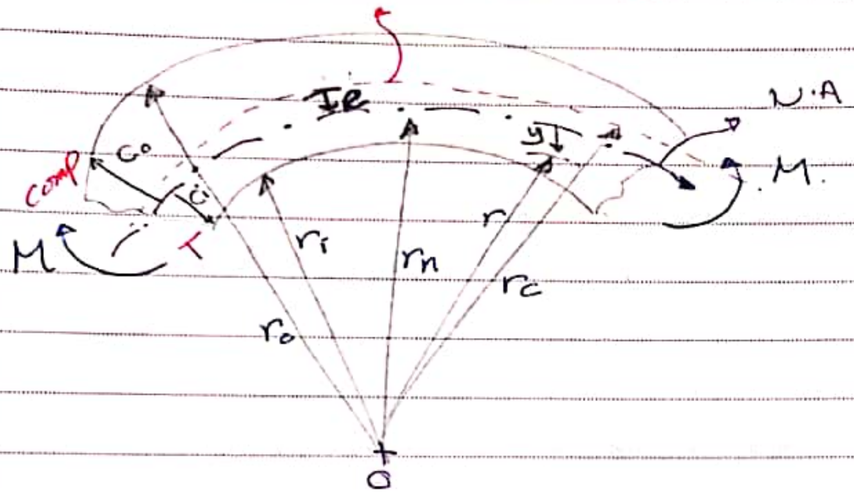
Scanned by CamScanner

N.A = C

NO.

Curved Beams in Bending :- stress (axial)

Centroidal - axis.



* ال Curved و Stright الفرق بينهم بال (Nutral axis) : نعمل بين جهة ten / comp

① بعيد عن ال Centroid → Centroid.

② Stress doess not vary linearly from the neutral - axis " hyperbolic stress distrubution"

r_i :- مسافة داخلية

r_o :- مسافة خارجية

r_n :- N A مسافة

r_c :- centroid مسافة

C_i :- مسافة بين السطح الداخلي وال N.A

C_o :- مسافة بين السطح الخارجي وال N.A

e : eccentricity, مسافة بين ال N.A وال Centroid

y :- (+) N.A مسافة مقيسة من

(+ve) باتجاه ال curve

M :- decreases the curvature (+ve)

increase (-ve)

r :- مسافة من ال center لحد النقطة يلي بدي احسب عليها .

FIVE APPLE

1] location of Neutral-axis, found as :-

$$r_n = \frac{A}{\int \frac{dA}{r}}$$ A: cross-sectional area

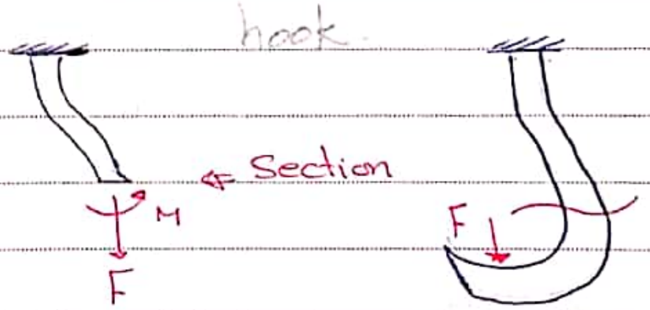
2] stress at any distance "y" from the neutral :-

$$\sigma = \frac{My}{Ae(r_n - y)}$$ $e = r_c - r_n$. not linearly.

3] Maximum tension and comp. stress :-

inner $y = c_i$, outer $y = c_o$

$$\sigma_i = \frac{Mc_i}{Ae\,r_i}, \quad \sigma_o = -\frac{Mc_o}{Ae\,r_o}$$



$\sigma = \frac{F}{A}$ ← tension force, Force بوضعت Max عند ابعد نقطة من *

* بنحسب قيمة المومنت Fd ، d مسافة للـ centroid

* Approximate Calculations :-

$$e \simeq \frac{I}{Arc}$$ ← الهدف منه نحدد قيمة e

← قريبة من الواقع اذا كان الـ curvature قليل

FIVE APPLE

بقارن r مع depth ← r > depth ✓

NO.

2) $\sigma = \frac{Ms}{I} \cdot \frac{r_c}{r}$

S :- 50 mm , inner surface $r = r_i = 300$ mm.
من Centroid للسطح الداخلي

$I = \frac{1}{12} bh^3 = \frac{1}{12}(18)(100)^3 = 1.5 * 10^6 \ mm^4$

$M = 22000\ (350) = 7.7 * 10^6$ N.mm.

$\therefore \sigma_i = \frac{7.7*10^6\,(50)}{1.5*10^6} \cdot \frac{350}{300} = 299.4$ MPa.

Safty — افضل الحسابات

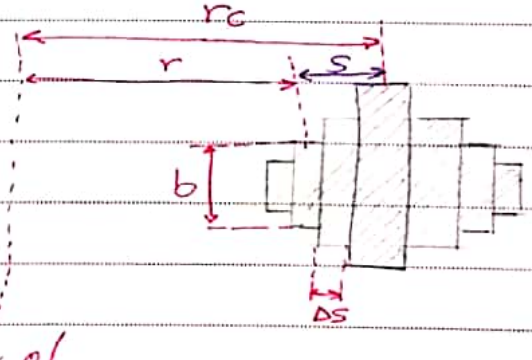
$\text{Error} = \frac{299.4 - 282.8}{282.8} * 100\% = 5.9\%$

قطاع r أكبر

* Numerical Calculations :-

rn. صعب غير Non-Regular و كالـ Curvature

⇒ discrediting the cross-sectional area.
"dividing the area into rectangulares of small thickness"



ΔS : thickness

b : width — لكل element

rc : من Center الـ Curvature لـ Centroid

S :- من Centroid لـ Center الـ

Center of Curvature — ازاي اهمام

r :- من Center الـ element لـ Curvature

كل ما قل الـ thickness أقل أدق

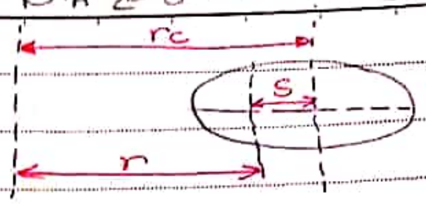
FIVE APPLE

NO.

مو. كاصة نتعامل مع N.A بتعني C

نعني y

$$\sigma = \frac{Ms}{I} \cdot \frac{r_c}{r}$$

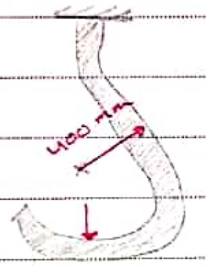


S:- distance from the centroidal axis.

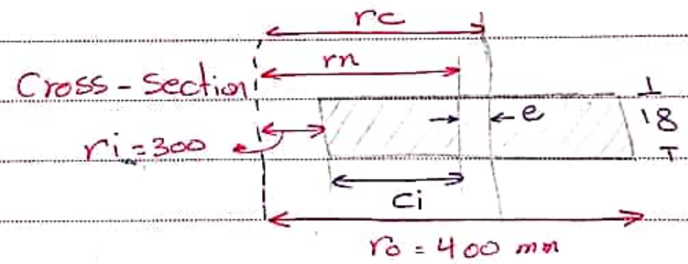
Example 3-15

outer radius $r_o = 400$ mm. — كبرنة فرض امل Curvature

* Find the maximum bending tensile stress at the inner surface using : [1] Exact error
[2] Approximate Calculations and Compute the error



F=22 kN.



[1] $r_n = \frac{h}{\ln\frac{r_o}{r_i}} = \frac{100}{\ln(400/300)} = 347.6$ mm.

$r_c = 300 + \frac{100}{2} = 350$ mm.

$e = r_c - r_n = 2.4$ mm. — يتغير مع Curvature

$A = 100\,(18) = 1800$ mm^2

inner surface $y = c_i = r_n - r_i$

$$\sigma = \frac{My}{Ae(r_n - y)} = \frac{22000\,(350)(47.6)}{1800\,(2.4)(300)} = 282.8 \text{ MPa.}$$

FIVE APPLE

NO.

Chapter "4" :- deflection and stiffness.

Stress analysis is done to ensure that machine elements will not fail due to stress levels exceeding the allowable values.

* Spring Rates :- "Stiffness"

* علاقة بين ال load, deflection

* element [bar, shaft, beam] بتغير حسب طبيعة ال(load analysis)

* في linear relashion بين deflection و ال load يعني بيتغير deflection

$$k = \frac{\text{Loading}}{\text{deflection}}$$

Loading → axial, lateral, torque

deflection → axial, bending, twisting.

* قسم deflection الى تبعا لل load twisting → torque
moment → bending.

* Tension, Compression and Torsion :-

(1) For bar ⇒ $\delta = \frac{FL}{AE}$

(2) spring rate ⇒ $k = \frac{AE}{L}$

(3) round ~~beam~~ shaft subjected to torque $\theta = \frac{TL}{GJ}$.

(4) spring Constant ⇒ $k = \frac{GJ}{L}$

NO.

I lacatin for centroid

$$rc = \frac{\Sigma r b \Delta s}{\Sigma b \Delta s}$$

$$e = \frac{\Sigma \frac{s}{rc-s} b \Delta s}{\Sigma \frac{b \Delta s}{rc-s}} \qquad rc - s = r$$

$rn = rc - e.$ وطبعاً عادي.

FIVE APPLE

لا يكون عنا load واحد.

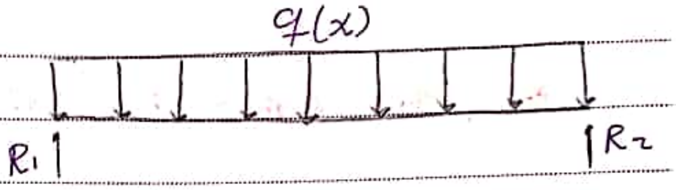
* Deflection due to bending :-

deflection في beam قيمتها كبيرة.

Method of integration :-

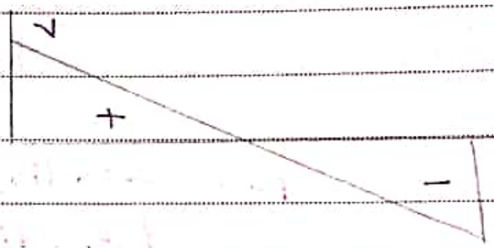
Load intensity :-

$$\frac{q}{EI} = \frac{d^4y}{dx^4}$$



Shear force :-

$$\frac{V}{EI} = \frac{d^3y}{dx^3}$$

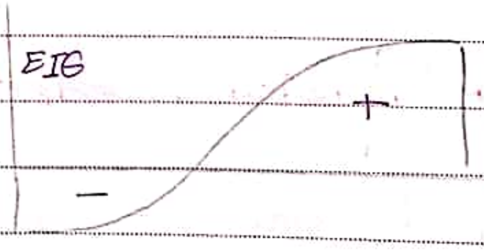


Moment :-

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

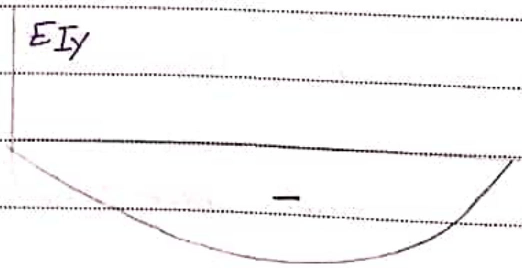
Slope :-

$$\theta = \frac{dy}{dx}$$



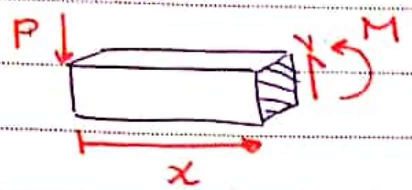
Deflection :-

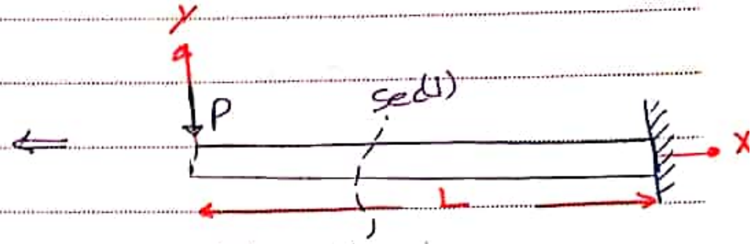
$$y = f(x)$$



Example :- Find the deflection, Slope and location and value of maximum slope.

Section.





$M + Px = 0 \Rightarrow M = -Px$

$\Rightarrow$ moment $\dfrac{d^2y}{dx^2} = \dfrac{M}{EI} = \dfrac{-Px}{EI}$

slope $\Rightarrow$ Integration $\dfrac{dy}{dx} = \dfrac{-Px^2}{2EI} + C_1$

deflection $\Rightarrow$ Integration $y = \dfrac{-Px^3}{6EI} + C_1 x + C_2$.

Boundry Conditions :- Fixed support

Slope $\dfrac{dy}{dx} = 0$ $\quad x = L$

x: support

deflection $y = 0$ $\quad x = L$

$\therefore \quad 0 = \dfrac{-PL^2}{2EI} + C_1 \Rightarrow C_1 = \dfrac{PL^2}{2EI}$

$\therefore \quad 0 = \dfrac{-PL^3}{6EI} + \dfrac{PL^2}{2EI} x + C_2 \Rightarrow \dfrac{-PL^3}{3EI}$

Thus :- $y = \frac{P}{6EI}(-x^3 + 3L^2x - 2L^3)$

Slope ⇒ $\frac{dy}{dx} = \frac{P}{2EI}(L^2 - x^2)$

maximum Slope :- عند ال load

بنعوض معادلة ال Slope وبساويها بالصفر .

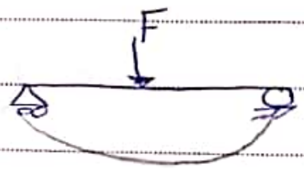
$\frac{dy}{dx}\left(\frac{dy}{dx}\right) = Zero \rightarrow x = 0$

$\left(\frac{dy}{dx}\right)_{x=0} = \frac{PL^2}{2EI}$

* لو طلب max-deflection :- بعوض zero في معادلة deflection

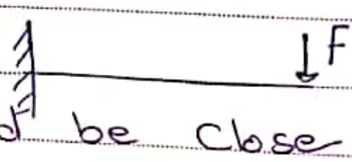
Note :-

1] Simply support beam.



max - deflection at the load.
min - deflection at the end
max - Slope at the end.
min - Slop = Zero when deflection is max.

2] Cantilever beam



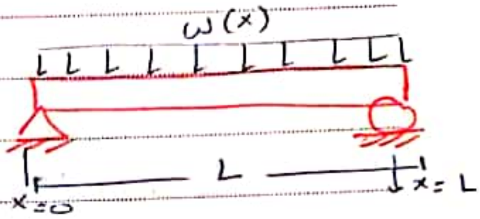
max - deflection when the load be close with free end

Slope = Zero at fixed Point.

NO.

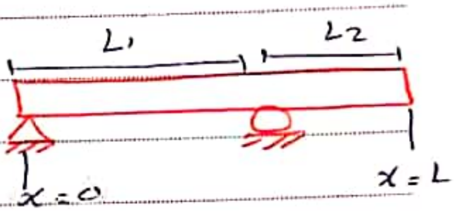
Boundry conditions

[1] Simply supported beam :-



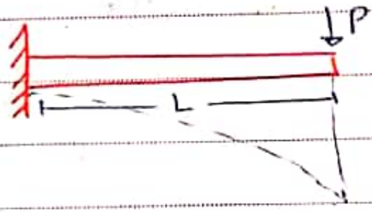
$y(0) = \text{Zero}$, $y(L) = \text{Zero}$

[2] over-hanging beam :-

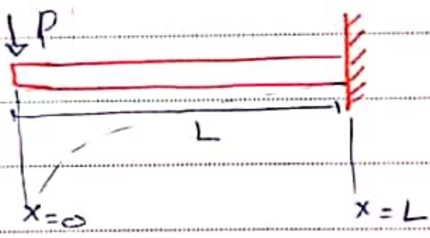


$y(0) = \text{Zero}$, $y(L) = \text{Zero}$.

[3] Cantilever supported beam :-



$y(0) = \text{Zero}$, $y'(0) = \text{Zero}$



$y(L) = 0$, $y'(L) = \text{Zero}$

لأن Curvature بيلف من (L)

NO.

* Beam deflection by Superposition :-

لما يكون عنا multiple load

1) Resolve :- بنفصل كل نوع من انواع ال load لوحده.

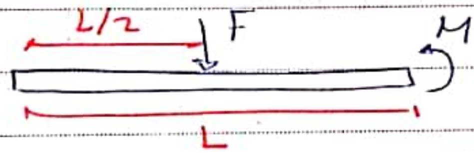
2) Find :- بوجد ال deflection الناتج عن كل نوع من أنواع load

3) Add :- نجمع ال load ← total deflection

(a) علاقة بين load و deflection ← linear

(b) أي نوع من أنواع ال load لوحده ما بيسبب deflection كبير

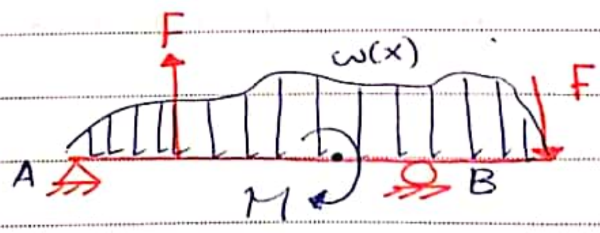
مثلاً :- لو كان عندي



* إذا كان ال deflection الناتج من المومنت large موقع ال force ما بعود عند L/2 بتغير أقرب

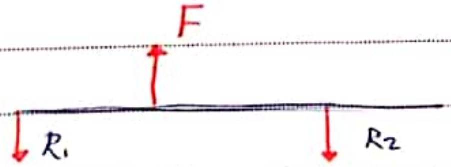
* لانه أي نوع من انواع load كان ما بعطي (large deflection)

Example :-



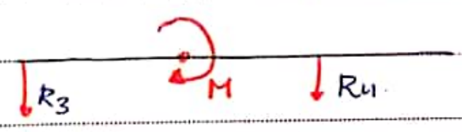
* مرة باخذ ال force ومرة المومنت ونجمع deflection الناتج عنهم

2: Cases :



Case (6)

table

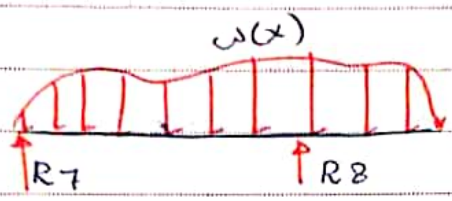


Case (8)

$R_A = R_1 + R_3 + R_5 + R_7$, $R_B = R_2 + R_4 + R_6 + R_8$.

NO.

Case(10)



non-uniform

مش موجود ب Table

يعمل integration.

$y = f(x) \Rightarrow \int w(x) \cdot dx$

عمل تكامل 2 مرات.

* Beam deflection by singularity functions :-

- Used to write an expression for loading over a range of discontinuities.

- The loading intensity function $q(x)$ can be integrated 4 times to obtain the deflection eq. $y(x)$

Ch(5) تقف

Scanned by CamScanner

NO.

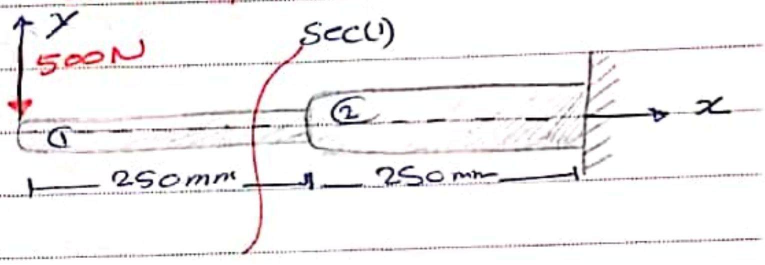
C: is the cross-section correction factor :-

$C = 1.2$ (rectangular)
$C = 1.11$ (circular)
$C = 2$ (thin-walled circular).

Ex: find the strain energy in beam (cantilever) squar cross-section.

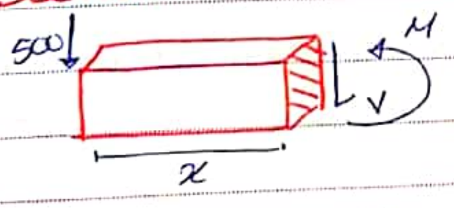
$E = 210$ Gpa, $G = 75$ Gpa.

$I_1 = 4 \times 10^5 \, mm^4$
$I_2 = 8 \times 10^5 \, mm^4$
$A_1 = 2.2 \times 10^3 \, mm^2$
$A_2 = 3.1 \times 10^3 \, mm^2$.



* تحت تأثير ال load في ال deformation ال work يلي ال اشتغلة ال force
بتخزن داخل ال beam على شكل energy.

Section.



$V = 500 \, N$, $M = 500x$ function.

Strain energy from moment :- (diffrent cross-section)

$$U_m = U_{m1} + U_{m2} = \int_0^{250} \frac{M^2}{2EI_1} dx + \int_{250}^{500} \frac{M^2}{2EI_2} dx.$$

$$= \frac{1}{2E} \int_0^{250} \frac{25 \times 10^4 x^2}{4 \times 10^5} dx + \int_{250}^{500} \frac{25 \times 10^4 x^2}{8 \times 10^5}$$

FIVE APPLE

NO.

$$= \frac{1}{2(210 \times 10^3)} \left[\frac{25x^3}{120} \Big|_0^{250} + \frac{25x^3}{240} \Big|_{250}^{500} \right]$$

$U_m = 34.877 \text{ mJ}.$

Strain energy from Shear. (diffrent cross-section)

Constant

$$U_V = U_{V1} + U_{V2} = \frac{C V^2 L_1}{2GA} + \frac{C V^2 L_2}{2GA_2}$$

$$= \frac{1.2(500)^2}{2(75 \times 10^3)} \left[\frac{250}{2.2 \times 10^3} + \frac{250}{3.1 \times 10^3} \right]$$

$U_V = 0.389 \text{ mJ}.$

Total strain $U_{Total} = 34.877 + 0.389$

$= 35.266 \text{ mJ}.$

✓ * قيمة Um اكبر من deflection ... الشد

* Castigliano's Theorem :-

- one of energy methods (based on strain energy)
- Used for solving a wide range of deflection

* نجيب energy expression ناتجة من ال load
* نحسب deflection عند اي نقطة بأثر عليها force في نفس اتجاه هذه ال force

(نحسب نقطة عليها Force) $\delta_i = \frac{\partial U}{\partial F_i}$ → نحسب قيمة deflection باتجاه ال force يلي بنعي

FIVE APPLE

Force → translation.
moment → rotation.

NO.

* Theory applies to both linear and rotational deflection

* δ_i : displacement of the point of application of the F_i in the direction of F_i

or

$$\theta_i = \frac{\partial u}{\partial M_i} \qquad \theta \rightarrow \text{radians.}$$

* θ_i :- rotational displacement of moment M_i in the direction of M_i

* لو بدنا نحسب deflection عند نقطة ما عليها force : نروح نحط Force مش موجودة

! عمليا (dummy load) ← (fictious) عند النقطة يلي بدي احسب عندها Q.

1. dummy load Q.
2. Total Strain. "including the dummy load" + all load.
3. deflection.

↙

$$\delta_i = \frac{\partial u}{\partial F_i} \quad \text{or if a dummy load is used} \quad \delta = \frac{\partial u}{\partial Q_i}$$

Q_i = zero = ← I

* deflection (+) → Same direction of the load. *

* deflection (−) → opposite direction " "

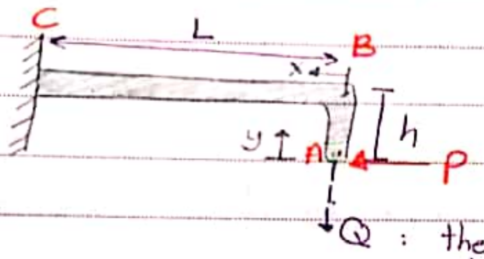
✓ بـ : partial derivative ip بأخذ بعدين بعمل Integration

$I_{circular} = \frac{\pi d^4}{64}$, $A_{circular} = \frac{\pi d^2}{4}$

NO.

Ex: A: horizontal force.

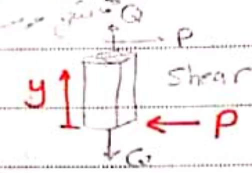
BC → L , BA → h. Find the vertical deflection ?! at A



Q : there is no vertical load acting at point A , we add a dummy load.

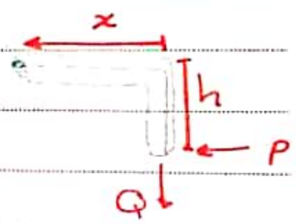
* Strain energy → deformation from the load.

- Bending in AB ① $M_{AB} = Py$



② axial load in AB = Q, shear = P ③ axial load in CB = P, shear = Q

- Bending in BC ④ $M_{CB} = Ph + Qx$



Strain energy : variables :- constant

$$U = \int_0^h \frac{P^2 y^2}{2EI}\,dy + \int_0^L \frac{(Ph+Qx)^2}{2EI}\,dx + \frac{Q^2 h}{2EA} + \frac{P^2 L}{2EA}$$

The vertical deflection at "A" is :- $\frac{2x(Ph+Qx)}{2EI}$

$$\delta_A = \left[\frac{\partial U}{\partial Q}\right]_{Q=0} = \int_0^L \frac{x(Ph+Qx)\,dx}{EI} + \frac{Qh}{EA}$$

$Phx + Qx^2 \Rightarrow \frac{Phx^2}{2} + \frac{Qx^3}{3}$

$$= \frac{PhL^2}{2EI} + \frac{QL^3}{3EI} + \frac{Qh}{EA}$$

$Q = 0 \Rightarrow \frac{PhL^2}{2EI}$ (vertical at A).

FIVE APPLE

NO.

* Expression for finding deflection " Castigliano's theorm "

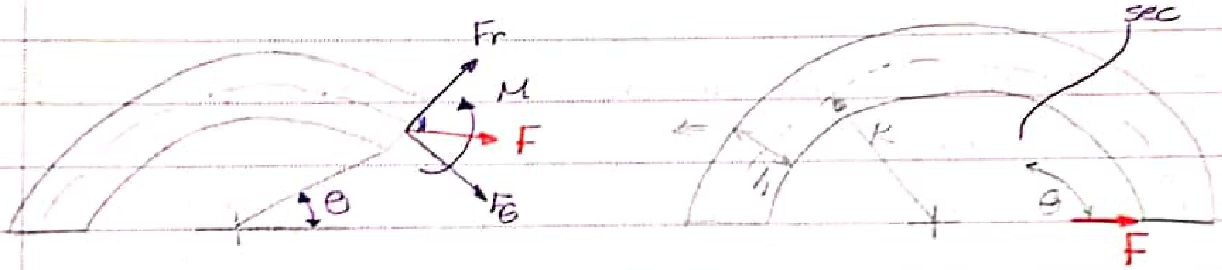
$\delta_i = \frac{\partial U}{\partial F_i} = \int \frac{1}{AE} \left(F \frac{\partial F}{\partial F_i} \right) dx$:- tension, Comp. (axial)

$\delta_i = \frac{\partial U}{\partial F_i} = \int \frac{1}{EI} \left(M \frac{\partial M}{\partial F_i} \right) dx$:- Bending ✓

$\theta_i = \frac{\partial U}{\partial T_i} = \int \frac{1}{GJ} \left(T \frac{\partial T}{\partial T_i} \right) dx$:- Torsion.

* لما نستخدم هذه المعادلات أو نوجد (total strain) ونجيبه بعدها بالتفاضل الجزئي

X * Defluction of Curved beam :-



Shear $F_r = F \cos\theta$

axial $F_\theta = F \sin\theta$

Bending moment $M = F R \sin\theta$

Strain energy due to bending moment

$$U_1 = \int \frac{M^2}{2AeE} d\theta$$ Curvature ✓

$e = R - r_n$.

approximated.

$$U_1 = \int \frac{M^2 R}{2EI} d\theta$$ for $\frac{R}{h} > 10$ قليل ✓

Axial force :- $U_2 = \int \frac{F_\theta^2 R}{2AE} d\theta$

Moment produced by the axial force (F_θ) :-

يتحاول تربيع curvature, إتجاهها عكسي

$U_3 = (-) \int \frac{M F_\theta}{AE} d\theta$

deflection in the direction opposite to the force.

Shear Force F_r :- $U_4 = \int \frac{c F_r^2 R}{2GA} d\theta$

* deflection in the direction of the force "F" is found as (Total)

$$\delta = \frac{\partial U}{\partial F} = \frac{\partial U_1}{\partial F} + \frac{\partial U_2}{\partial F} + \frac{\partial U_3}{\partial F} + \frac{\partial U_4}{\partial F}$$

* Integration between 0 to π

axial shear

$$\delta = \frac{\pi F R^2}{2AeE} - \frac{\pi F R}{2AE} + \frac{\pi c F R}{2AG}$$

→ R^2 قيمة عالية جدا

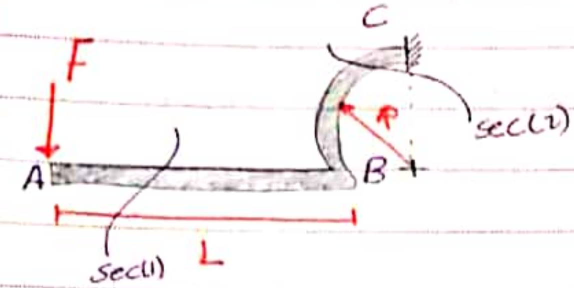
$$\delta \cong \frac{\pi F R^2}{2AeE} \rightarrow \text{Curvature عالي}$$

Ex:- Find the veritical deflection at A

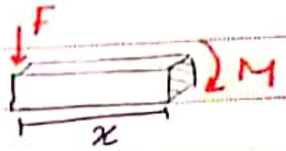
$R/h > 10$

شمالي

NO.

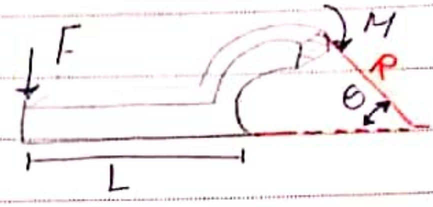


Sec(1)



$M_{AB} = Fx$ شيرعمل

Sec(2)



طريقة التكامل كاملة

$M_{BC} = FL + F(R - R\cos\theta)$
$= FL + FR(1 - \cos\theta)$.

$$U_{BC} = \int_0^L \frac{M_{AB}^2}{2EI}\,dx + \int_0^{\pi/2} \frac{M_{BC}^2}{2EI}\,d\theta$$

approximate solution :-

$$\delta = \frac{\partial U}{\partial F} = \int_0^L (Fx)\frac{x}{EI}\,dx + \int_0^{\pi/2} FL + FR(1-\cos\theta)\,\frac{(L+R(1-\cos\theta))}{EI}\,R\,d\theta$$

$$\delta = \frac{F}{12EI}\left(4L^3 + 3R\left(2\pi L^2 + 4(\pi-2)LR + (3\pi-8)R^2\right)\right)$$

✗ Statically indeterminate problems :-

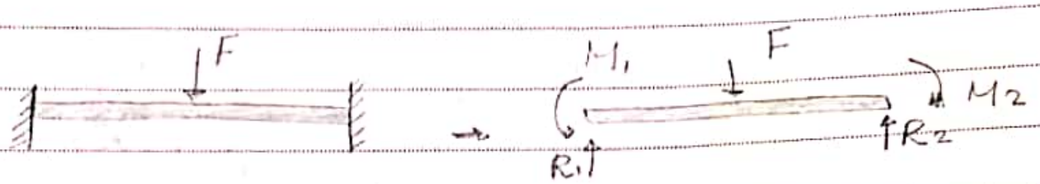
number of unknown is more than the number of available equations.

* بيكون عندي "redundant supports" هي ال support يلي ممكن نستغنى عنها لغيرها وحافظ ع ال static equilibrium

NO.

degree of indeterminacy & Same number of additional equation is need to solve the Problem.

* عدد ال reaction بكي تطلب اضافي



* الreadundant ← عشان M_2, R_2 مع (cantilivar-beam)

← " " M_2, M_1 " (Simpfy-beam)

Redundant reactions:-

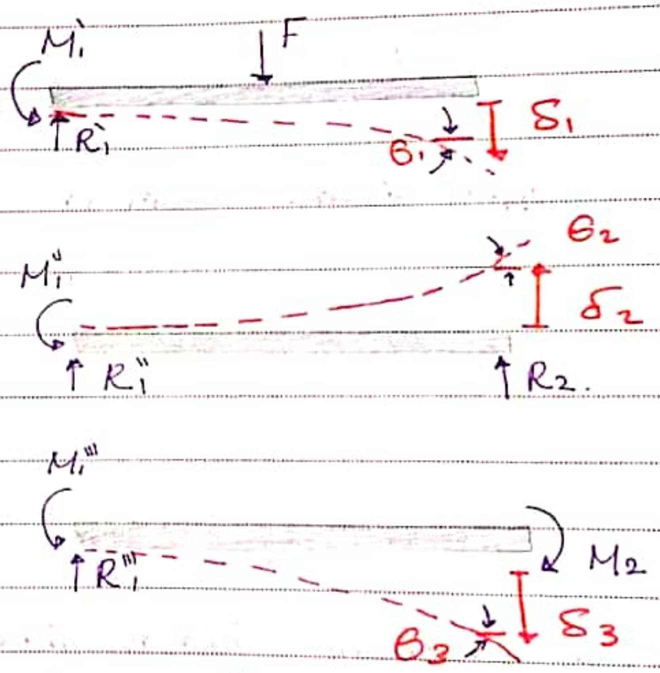
R_2 & M_2.

$\delta_1 + \delta_2 + \delta_3 = 0$

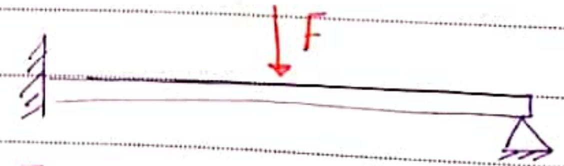
$\theta_1 + \theta_2 + \theta_3 = 0$

$M_1 = M_1' + M_1'' + M_1'''$

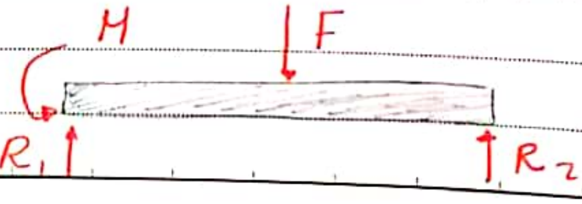
$R_1 = R_1' + R_1'' + R_1'''$



Ex: determine the reaction of the beam using superposition



F.B.D



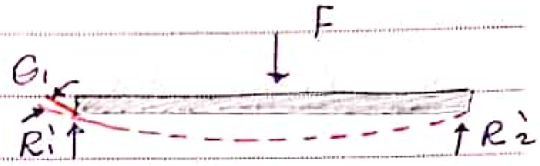
+ بدون تأثير ال Redundant
+ مع تأثيره.

NO.

+ علشان ال Redundant لو (R2) تصير Cantilever أو (M1) تصير Simply

Choosing the reaction M_1 to be the redundant reaction :-

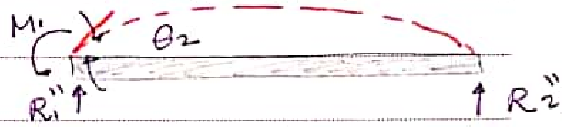
Table A-9 Case(5) (تأثير ال Force)



$$\theta_1 = \left[\frac{dy}{dx}\right]_{x=0} = \left[\frac{F}{48EI}(4x^2 - 3L^2)\right]_{x=0}$$

$$\theta_1 = \frac{4Fx^2}{48EI} - \frac{3FL^2}{48EI}$$

(تأثير العزم)



اشتقاق

$$\theta_1 = \left.\frac{16Fx}{48EI}\right|_{x=0} - \frac{3FL^2}{48EI} = (-)\frac{FL^2}{16EI}$$

deflection تحت

Table A-9 Case(8) a= Zero. معادلتين بنفعوا

$$\theta_2 = \left[\frac{dy}{dx}\right]_{x=0} = \left[\frac{M_1}{6EIL}(x^2 + 2L^2)\right]_{x=0} = (+)\frac{M_1 L}{3EI}$$

نعكس اتجاه العزم بتاع ال case

C.C.W.

$$\theta_1 + \theta_2 = 0 \Rightarrow \frac{M_1 L}{3EI} - \frac{FL^2}{16EI} = \text{Zero}.$$

$$M_1 = \frac{3FL}{16}$$

Static equations :-

Case (5, 8) من

$$R_1 = \frac{F}{2} + \frac{M_1}{L} = \frac{11F}{16}$$

$$R_2 = \frac{F}{2} - \frac{M_1}{L} = 5F/16$$

Mos ⇒ $n = \frac{S_{ys}}{\tau_{max}}$ $L = \left(\frac{L}{k}\right)_1 \sim \sqrt{\frac{I}{A}}$, $P_{cr} = P\,n\,d$,

NO. $k = d/4$

* Compression members :-

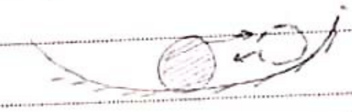
* تسمية ال members تعتمد على نوع ال load ← tension (bar)
Comp. (columns)

* اهمال ال failur بسبب buckling

+ buckling :- lateral deflection ينتج عن axial compressive load
يعبر عنه elastic unstability

* Three states of equilibrium :-

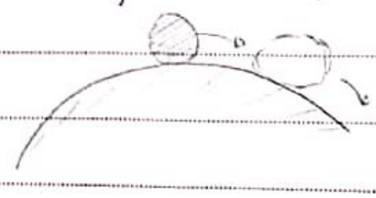
* Stable equilibrium : when member is moved it tend to turn to original equilibrium position.



* Neutral equilibrium : when the member is moved from its equilibrium position, it is still in equilibrium at the displaced position.

multiple equilibrium

Buckling * Unstable equilibrium : when the member is moved from its equilibrium position, it is becomes imbalance and accelerates away from its equilibrium position.



* متى نعرف ال load يلي خلى ال columns unstable يلزمني Pcr

لـ تقسيم ال ... على حسب ال geometry ال column ← short / long (intermediate)

axial load ← central / eccentric

- Long Columns with Central Loading :- bending بسبب

Critical buckling load بتأثر :

(1) Geometry :- length (buckling) Column كل ما زاد الطول كل احتمال الـ

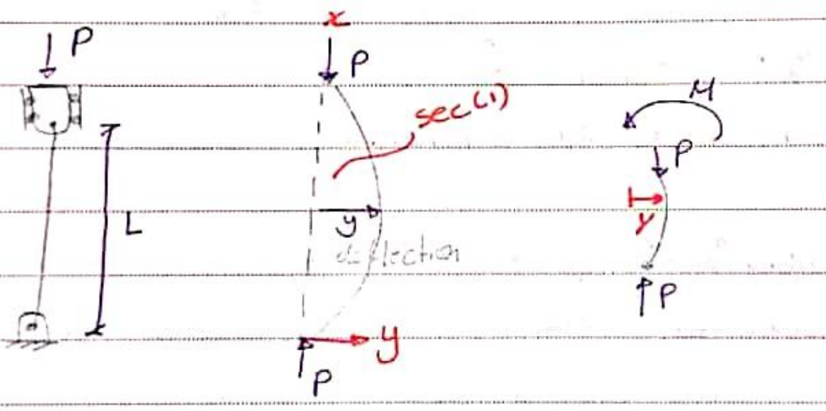
Cross section كل ما كبر يزيد تحمل الـ load (buckling) أمن

(2) End condition :- Pinned - pinned , fixed - fixed

fixed - pinned, fixed - free

(3) The column material "E" بأثر في قيمة

buckling مباشر على (Critical buckling load)



bending moment developed in the Column $M = -PY$

Using the beam governing equation :-

$$\frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{-P}{EI}y \qquad \therefore \frac{d^2y}{dx^2} + \frac{Py}{EI} = \text{Zero}$$

(homogenous second order differential equation)

- general Solution :- $y = A\sin\sqrt{\frac{P}{EI}}\,x + B\cos\sqrt{\frac{P}{EI}}\,x$

- equation Can be evaluated using the boundary - condition

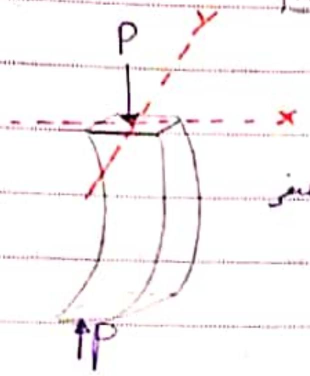
pin - pin بمعرفة الـ

NO.

buckling mode.

$\boxed{n=1}$ First-mode ال mod الوحيد المنطقي ال *

-- n=2 ما بنطلع عليهم لأنه non-Reylastic.



* يصير buckling على هذا ال column لأنه تكون قيمة ال I أصغر ما عليه. بعوض (Ix) مش (x) تكون دائماً سهل

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E A k^2 / L^2}{A} = \frac{\pi^2 E}{(L/k)^2}$$

$I = A k^2$

A: cross-section.

k: radius of gyration.

L/k : slenderness ratio. (درجة الرفع)

① "non-daimentional ratio"

→ كل ما قلت النسبة آكبر ال column ارفع

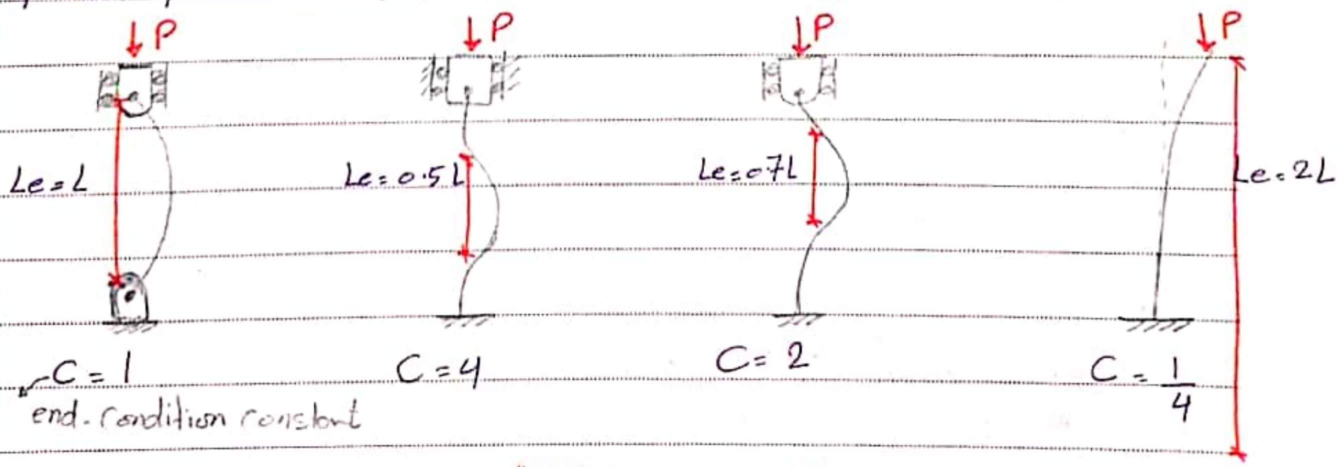
② تعبر عن cross-section

This ratio (L/k) using to classfiy columns in to length catogires.

* بنقارن الطول مع ال cross-section

* نقسمهم. بحصل من ال non-damentional ratio تعتمد الطول

pinned-pinned | fixed-fixed | fixed-pinned | fixed-free.



C = 1 | C = 4 | C = 2 | C = $\frac{1}{4}$

end-condition constant

$$P_{cr} = \frac{\pi^2 E I}{L_e^2}$$ "effective length" or $$P_{cr} = C\frac{\pi^2 E I}{L^2}$$

1) $y = \text{zero} \rightarrow x = 0$

2) $y = \text{zero} \rightarrow x = L$

$$0 = A\sin(0) + B\cos(0) \quad \therefore B = 0.$$

$$0 = A\sin\sqrt{\frac{P}{EI}}L + 0\cos\sqrt{\frac{P}{EI}}L \quad \therefore 0 = A\sin\sqrt{\frac{P}{EI}}L.$$

A = 0 مرفوضة لانه مافي buckling sin = 0

$$\sin\sqrt{\frac{P}{EI}}L = 0 \Rightarrow \sqrt{\frac{P}{EI}}L = n\pi$$

$n = 1, 2, 3, \ldots$

$$P = \frac{n^2\pi^2 EI}{L^2}$$

* Smallest "p" for n = 1
The first mode of buckling

* critical load "Pcr" with pinned ends

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

Euler formula-

buckling بالدخل = material (E) المعامل الـ geometry

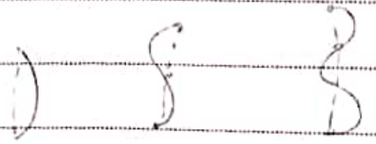
(E) بالقانون ما بيعطي load ← deflection

2: columns, steel + AL

+ عند نفس القيمة من ال strain بعمل buckling

+ تعمل load بتختلف قيمتها للـ strain الطول حسب stiffness (E)

$$y = A\sin\frac{\pi x}{L}$$



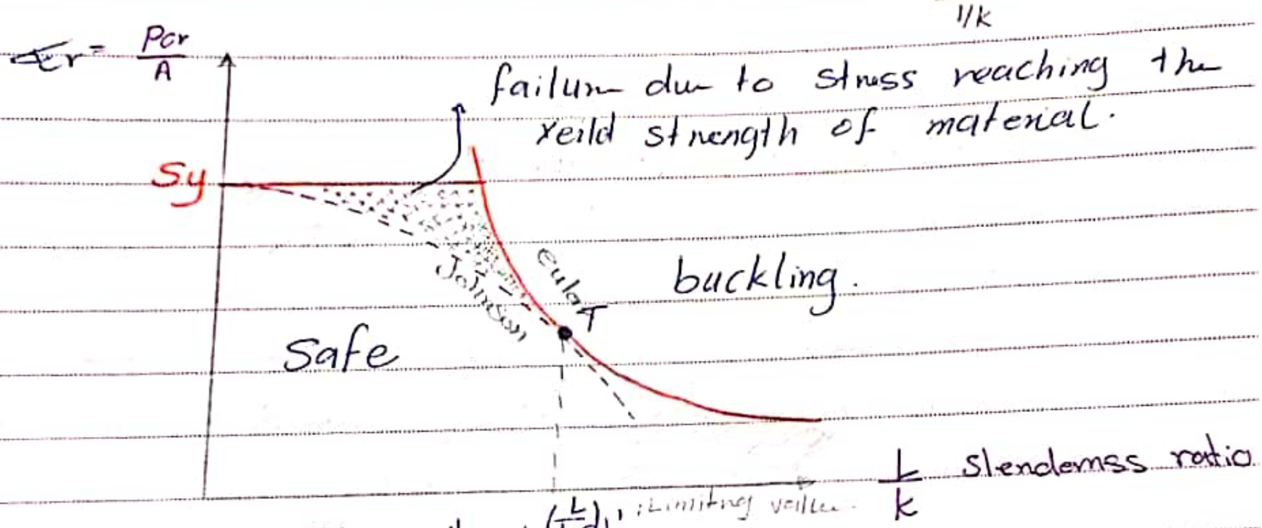
n=1 n=2 n=3

عدد الأمواج

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{C\pi^2 E}{(L/k)^2}$$

Table (4-2) (theoretical value) ← C = 4 , C = 1 ..... * قيم C بكل استخدام

* تأثير الاطوال على critical value
1/k



Euler critical stress.

* load جبيرة قسمة كل A طريقة الـ stress في الـ curve ← (safe)
(critical) ← " " على
(buckling) ← " " خارج

* كسر بيه داهم سبب buckling مش بسبب yeilding

* Prabola curve ⇒ بيمر Sy , (T): tangent point

* T. point بيفصل بين الـ case اللي هيعمل عليها Euler واللي ما يطبعش

* متى نعرف long , short column ?!

$$\left(\frac{L}{k}\right)_1 = \sqrt{\frac{2\pi^2 C E}{S_y}}$$

NO.

Euler formula :- slenderness ratios larger than $(L/k)_1$

Long Columns $\left(\frac{L}{k}\right) > \left(\frac{L}{k}\right)_1$

Short Columns $\left(\frac{L}{k}\right) < \left(\frac{L}{k}\right)_1$ → Johnson formula- fit between (T) (Sy)

* The Johnson formula predicts the criticals stress as :-

short $\sigma_{cr} = \frac{P_{cr}}{A} = S_y - \frac{S_y^2}{4\pi^2 CE}\left(\frac{L}{k}\right)^2$

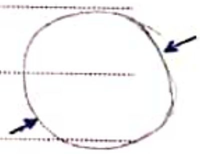
$\sigma_{cr} = S_y - \frac{S_y^2}{4\pi^2 E}\left(\frac{L_e}{k}\right)^2$ → effective length-

Ex:- Column with fixed-free , made of AL , $E = 72$ Gpa $S_y = 97$ Mpa , cross-section area 600 mm², $L = 2.5$ m.

Find the critical buckling load for

1 Solid round bar 2 Solid squar bar.

Sol:- Fixed- Free $C = 1/4$



1 round :- $A = \frac{\pi}{4} d^2 \Rightarrow d = \sqrt{\frac{4(600)}{\pi}} = 27.64$ mm.

- moment of intrai $I = \frac{\pi}{64} d^4 = \frac{\pi}{64}(27.64)^4 = 28650$ mm⁴.

- Radius gyration $k = \sqrt{\frac{I}{A}} = \sqrt{\frac{28650}{600}} = 6.91$ mm.

- Slendness ratio $\left(\frac{L}{k}\right) = \frac{2500}{6.91} = 361.8$.

- Limiting value: $\left(\frac{L}{k}\right)_1 = \sqrt{\frac{2\pi^2(0.25)(72\times10^9)}{97\times10^6}}$

$= 60.52$

- $361.8 > 60.52 \Rightarrow$ Long Column : Euler formula.

$\therefore P_{cr} = \frac{C\pi^2 EI}{L^2} = \frac{(0.25)\pi^2(72\times10^9)(28.65\times10^{-7})}{2.5^2} = 814.4\ N$

2] Squar : $A = b^2 \rightarrow b = \sqrt{600} = 24\ mm$.

- moment of inertia $I = \frac{b^4}{12} = \frac{(24.5)^4}{12} = 30,000\ mm^4$

- raduis gyration. $k = \sqrt{\frac{I}{A}} = \sqrt{\frac{30,000}{600}} = 7.07\ mm$

- Slendrness ratio $\left(\frac{L}{k}\right) = \frac{2500}{7.07} = 353.6$

- Limiting value $\left(\frac{L}{k}\right)_1 = 60.25$

- $353.6 > 60.25 \Rightarrow$ Long colum : Euler formla

$\therefore P_{cr} = 852.7\ N$.

* المربع بتحمل load اكبر
* اربع ال load بدون ما نزيد الوزن نفرغ الشكل hollow
(ازيد ال I)

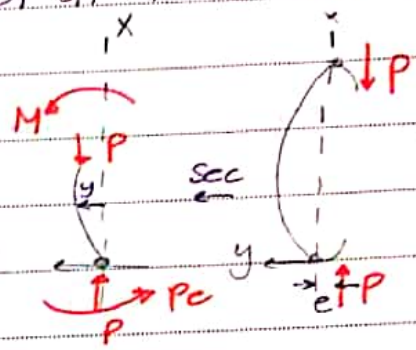
. الدائرة المفرغ احسن من المربع ∴ جدران الدائرة curved تقاوم ال Buckling
جدران المربع مسطحة (اضعف)

FIVE APPLE

* Columns with Eccentric Loading :-

- most applications the load is not at the Centroid.

- the distance between the Centroid axis and the Point of the load application is called eccentricity "e"



- Moment ⇒ $M = -P(e+y)$
- $\frac{d^2y}{dx^2} = \frac{M}{EI}$ ∴ $\frac{d^2y}{dx^2} + \frac{P}{EI} y = \frac{-Pe}{EI}$

second Linear non-differential equation.

- Boundry Condition. ⇒ max. deflection - $x = L/2$

$$\delta = y_{max} = e\left[\sec\left(\frac{L}{2}\sqrt{\frac{P}{EI}}\right) - 1\right]$$

⇒ max. moment at mid span.

$$m_{max} = -P(e+\delta) = -Pe \sec\left(\frac{L}{2}\sqrt{\frac{P}{EI}}\right)$$

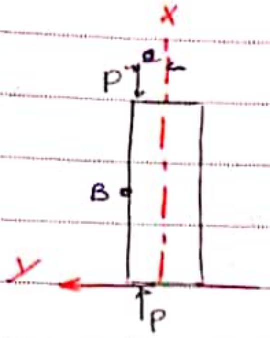
- Compressive Stress at mid-span. (two component) ↙ $\frac{P}{A}$ ↘ moment

$$\sigma c_{max.} = \frac{P}{A} - \frac{Mc}{I} = \frac{P}{A} - \frac{Mc}{Ak^2}$$

c: distance from the neutral axis to the outer surface.

NO.

* Short members :- (اهماله) bending قليل ← تتعامل بال +



$$\sigma_c = \frac{P}{A} + \frac{My}{I}$$

دي اي مسافة

(u.s.s) $= \frac{P}{A} + \frac{(Pe)y}{I}\frac{A}{A} = \frac{P}{A}\left(1 + \frac{ey}{k^2}\right)$

- max. compressive stress at point B $y = c$

أكبر قيمة عند السطح اللي برا ، مسافة من الـ centroid لـ outer surface

$$\sigma_c = \frac{P}{A}\left(1 + \frac{ec}{k^2}\right)$$ ✓ # short.

- max. stress value S_y , for the critical load per

- To distinguish short columns (struts) from long columns with eccentric loading the slenderness ratio is used:-

Limiting value $\left(\frac{L}{k}\right)_2 = 0.282\sqrt{\frac{AE}{P}}$ ①

$\frac{L}{k} > \left(\frac{L}{k}\right)_2 \Rightarrow$ Long column "secant formula"

$\frac{L}{k} < \left(\frac{L}{k}\right)_2 \Rightarrow$ strut (short)

NO.

عوضنا فيها

M_{max} $\quad \sigma_c = \frac{P}{A}\left[1 + \frac{ec}{k^2}\sec\left(\frac{L}{2k}\sqrt{\frac{P}{EI}}\right)\right]$ — $\frac{ec}{k^2}$ → eccentricity ratio

- Secant equation :-

(4-50) $\quad \frac{P}{A} = \frac{A\,\sigma_{yc}}{1 + \left(\frac{ec}{k^2}\right)\sec\left(\frac{L_e}{2k}\sqrt{\frac{P}{EI}}\right)}$

σ_{yc} → yeild strength

L_e → effective Length

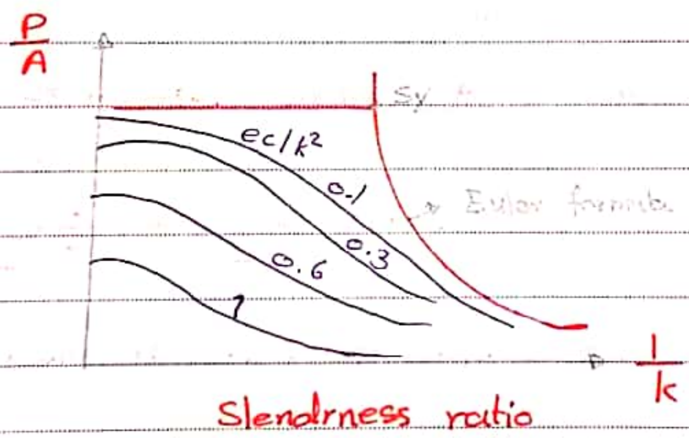
→ $\frac{C\pi^2 EI}{L^2}$ طلع معي جواب

(4) منه

$P = \frac{}{\sqrt{4}}$ (5)

كم مابقدر أوصل

* First iteration the critical buckling Load obtained from Euler formula "P_{cr}"



+ load decreases with increasing eccentriticity ratio.

load عالي

* eccentricity ratio بتعبر عن مقدار ال eccentricity عالية ↑ بعيدة عن ال center

قليل load قريبة " "

zero load بال center

NO.

Chapter "5" failures Resulting from static Loading

* F.S = strength/stress
* T / C ⇄ بقدر احسبه إذا ال stress
* Load traxial ⇒ Shear "شي" ⇐ F.S مابقدر احسب

Static Load :- constant magnitude, location, direction.

ductile :- * yeilding ⇒ failuer (مش معناته المادة انكسرت) *
(ببعتبرها)
* yeilding : permenant change in the geometry
* بأثر على ال function والاداء فبعتبره failuer لانه بغير بالاداء تبع المادة
* $F.S = \frac{\text{Strength}_{yeild}}{\text{stress}}$

Brittle :- * مابصير في yeilding على الغالب
* بينفصل مباشرة مع التحميل
* $F.S = \frac{\text{strength}_{ult.}}{\text{stress}}$ (ultimate)

static strength :- * بتأكد انه ال Componant بتحمل بدون ما ينهار
ال Static-strength تعتبر كافي بتحمل ال load بدون failuer

* عشان نتأكد بنصنع prototype بنعرضه لنفس ال load الواقعي.
بنعتمد على experment مش على الحسابات فقط. لكن ال cost بتكون عالية.
كاك بنعمل phesical-test قبل ما نصنع منتج كامل أو بكميات

* الهدف من failure-theorise نربط بين uniaxial, traiaxial, (Biaxial)
(ten / Comp) (Shear → Craiteria)
عشان نحسب F.S

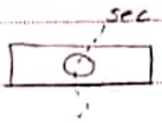
$$n = \frac{\text{Strength}}{\text{Stress}}$$

NO.

Stress Concentration :-

* لو عنا مادة نشد عليها ten. أخذنا section رح يكون الـ stress {"uniform distrubution"}

* لكن لو كان عنا holle (dis contnute)



رح يغير الـ stress concentration

Ductile:

* رح يصير في plastic deformation في منطقة معينة
* ما رح يوصل لـ σ_{max}
* local deformation لا يؤثر على كل geomtry
* بالتالي بنهمل حساب الـ stress concentration.

$k_t = 1$

Brittle

* أي منطقة تجاوزت الـ ultm يصير fracture ويصيب كل المادة
* σ_{max} تجاوزت الـ (ULton) ← crack ويكسر المادة
* بنحسب stress concentration

$k_t = \frac{\sigma_{max}}{\sigma_{nomored}}$

* في exception لـ Brittle إلي بكون فيها عدد الـ micro discontnuties كبير مثل cast iron الفرق بين steel كمية الكربون ← بتتجمع عند grain-boundys مناطق grain-boundy إلي بتجمع عليه الكربون تشكل نقاط ضعف هي (cracks) تسبب stress concentration، تأثير micro discontnudies أكثر من (macro) holes

∴ إذا تأثير الـ macro (hole) ما بناخده (بنهمل الـ stress-constu)

بشكل عام نأخذ تأثير الـ stress concntraton إلا إذا كان فيها عدد كبير من micro

* **Yield criteria (ductile) :-**

1. maximum shear stress MSS , "Tresca Criteria."
2. Distortion energy DE , "von mises Criteria"
3. Ductile Coulomb - Mohr DCM.

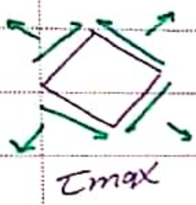
NO.

* (Brittle). fracture criteria :-

1. maximum Normal Stress MNS
2. Brittle Coulomb Mohr BCM and its modification.

Ductile

① maximum shear stress theory :-



* Came from the observation that for ductile during tension test
* fracture surface occures at 45° angle.
* زاوية ال 45 ...

* Max. Shear state of stress Biaxial 2- or triaxial
failure ...
... Shear-stress ... tensile ...
Yeilding

* τ_{max} at Yielding $= \frac{S_y}{2}$. yeild strength / 2

* principal oriantition 3D :- mohr circal

Radius ← $\tau_{max} = \frac{(\sigma_1 - \sigma_3)}{2}$

* MSS theory, yielding occurs when :-

$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} \geq \frac{S_y}{2}$ or $\sigma_1 - \sigma_3 \geq S_y$

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NO.

* one of principle stress is zero : "plane stress"

- Case 1

$\sigma_A \geq \sigma_B \geq 0$: (+ve) تنتمي أكبر من الصفر

$\sigma_1 = \sigma_A$, $\sigma_3 = zero$

∴ yielding occurs $\sigma_A \geq S_Y$

تعبير failuer →

- Case 2

$\sigma_A \geq 0 \geq \sigma_B$: σ_A (+ve) , σ_B (-ve)

$\sigma_1 = \sigma_A$, $\sigma_3 = \sigma_B$

∴ yielding occurs $\sigma_A - \sigma_B \geq S_Y$

- Case 3

$0 \geq \sigma_A \geq \sigma_B$ (-ve) تنتمي أقل من الصفر

$\sigma_1 = 0$, $\sigma_3 = \sigma_B$

∴ yielding occurs $\sigma_B \leq -S_Y$

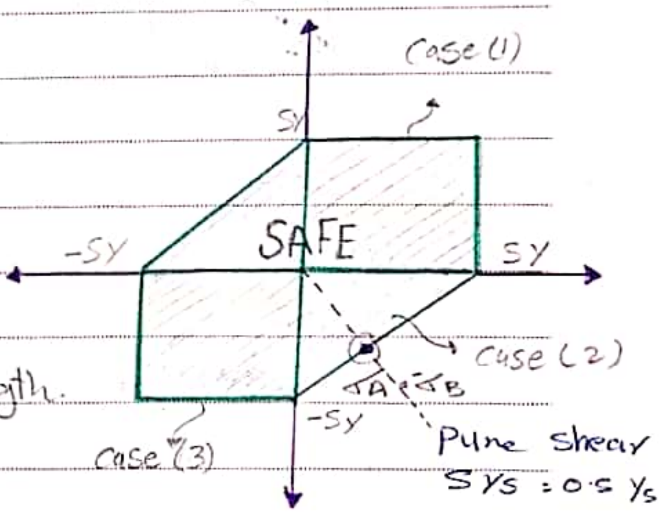
inside ⇒ SaFE F.S > 1

on line ⇒ critical F.S = 1

out side ⇒ fail F.S < 1

S_{Ys} : Yield shear torsion strength.

$= 0.5\ S_Y$



Failure "Yielding"

(2) Distrortion Energy theory :-

distortion energy = 0

مسببة كل تجربة مع قطعة في ظل

* ductile material Subjected to hydrostatic stress ($\sigma_1 = \sigma_2 = \sigma_3$)
  ↳ stress متساوي comp.
  ↳ متساوي في كل الاتجاهات

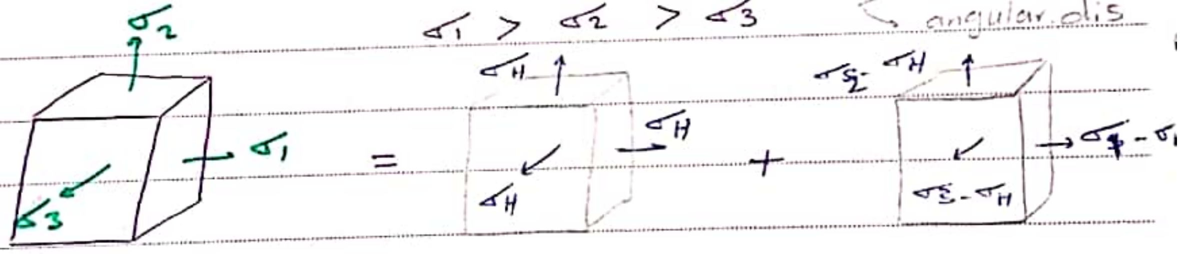
* Stress >> Yeld strength (without yielding)

* قطعة تم وضعت ل stress متساوي في كل الاتجاهات غير Volume

* Yielding و faliure مرتبط مع angular distortion مش مع volume change

* بنلاقي distrotion strain energy per unit volume وانقارنها مع ال tension عند ال Yield

$\sigma_1 > \sigma_2 > \sigma_3$ ↳ v. change, angular dis



Triaxial stress = Hydrostatic Component "Pure Volume change" + Distortion Component "Pure distortion No V. Change"

* $\sigma_H = \sigma_{avg} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$

* strain energy: $U = \frac{1}{2}(\epsilon_1 \sigma_1 + \epsilon_2 \sigma_2 + \epsilon_3 \sigma_3)$

NO.

* Hook's Law :- stress بنعوض Strain عشان نتخلص من ال

$$U = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

* Strain energy due to Hydrostatic energy :-

$\sigma_H = \sigma_1 + \sigma_2 + \sigma_3 / 3$

$$U_v = \frac{3\sigma_H^2}{2E}(1-2\nu)$$

⇓

$$U_v = \frac{1-2\nu}{6E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2\sigma_1\sigma_2 + 2\sigma_2\sigma_3 + 2\sigma_3\sigma_1\right]$$

⇓

* distortion strain energy

$$U_{d_1} = U - U_v = \frac{1+\nu}{3E}\left[\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}{2}\right]$$

$U_{d_1} > U_{d_2}$ distortion

Tensile test :- $\sigma_1 = S_y$, $\sigma_2 = 0 = \sigma_3$

$$U_{d_2} = \frac{1+\nu}{3E} S_y^2$$ distortion energy at yielding

The DE theory

$$\sqrt{\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}{2}} \geq S_y$$

Von mises stress (σ') = $\sqrt{\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}{2}}$

$$\sigma' \geq S_y$$

stress → principle MSS → principle

DE → معادلة

NO.

plane stress σ_A, σ_B only

$$\sigma' = \sqrt{\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2}$$

r = Sy DE : الشغي) MSS :

outside : failure inside : SAFE

Hydrostatic stress Line $\sigma_1 = \sigma_2 = \sigma_3$

- $S_{ys} = 0.577\, S_y$ more accurate من MSS

3D
$$\sigma' = \sqrt{\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)\right] / 2}$$

plane stress $\sigma' = \sqrt{\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}}$

Failure Theory (فيديوهات)

NO.

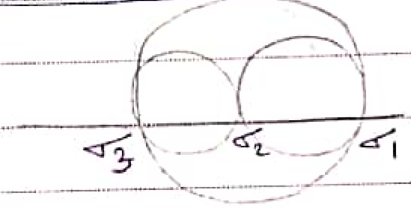
* ductile (Yielding) →
- MSS
- DE
- DCM.

* Brittle (failure). →
- MNS
- BCM.

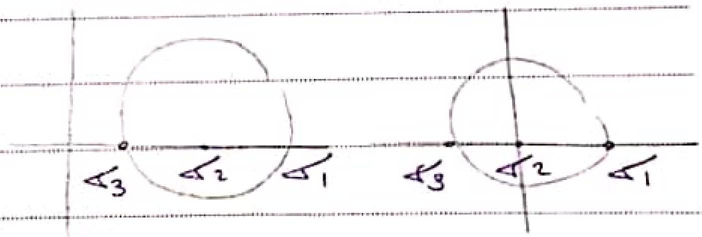
MSS → $\sigma_1 - \sigma_3 \geq S_y$ (prencipl stress) ∴ $F.S = \frac{S_Y}{\sigma_1 - \sigma_3}$

$S_{ys} = 0.5\, S_y$ Pure shear.

3D Moh. circle

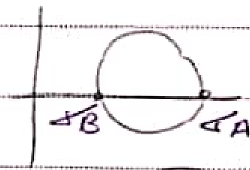


2D Moh. circle



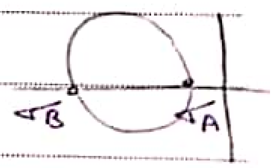
* plane Stress 2D :- σ_A, σ_B, $\sigma_A > \sigma_B$

→ σ_A, σ_B ⊕ $\sigma_A \geq S_Y$

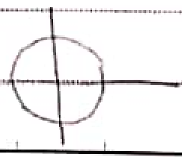


→ both ⊖ $- \sigma_B \geq S_Y$

$\sigma_B \leq -S_Y$



→ ⊕ ⊖ $\sigma_A - \sigma_B \geq S_Y$



uniaxial NO. Purshear.

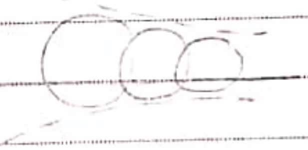
$\therefore n = \frac{S_y}{\sigma_1 - \sigma_3}$, $n = \frac{S_{ys}}{\tau_{max}}$

DE $\sigma' \geq S_y$, $S_{ys} = 0.577 S_y$

Prenciple 5-12 , 5-13. $n = \frac{S_y}{\sigma'}$

Non-principle 5-14 , 5-15.

DCM < S_{yt}, S_{yc} (Comp) (ten) غير متماثلة ductile مادة *



(Comp) (ten) لكل case درسوا

$\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}} \geq 1$ $n = 1 / \left(\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}}\right)$

$S_{ys} = \frac{S_{yt} * S_{yc}}{S_{yt} + S_{yc}}$

* اذا B لـ Syt , Syc متساويين نرجع MSS

FIVE APPLE

NO.

MSS , DS ⇒ تفصيلية σ_B only, σ_A only } uniaxial, $\sigma_A = \sigma_B$ $\frac{\text{Strength}}{\text{Stress}}$

Brittle :- Sut , Suc

MNS $\sigma_1 \geq S_{ut}$, $\sigma_3 \leq -S_{uc}$

في (n) نأخذ الأصغر.

BCM   ~~$\frac{S_{uc} - S_{ut}}{S_{uc} S_{ut}} \sigma_A - \frac{\sigma_B}{S_{uc}} \geq 1$~~ خطأ

~~$n = 1 / \left(\frac{S_{uc} - S_{ut}}{S_{uc} S_{ut}} \sigma_A - \frac{\sigma_B}{S_{uc}} \right)$~~

$\frac{\sigma_1}{S_{ut}} - \frac{\sigma_3}{S_{uc}} \geq 1$

↳ Mod. I
- → σ_A ✓, σ_B ✓ (+ve) $\sigma_A \geq S_{ut}$
- → σ_A, σ_B (−ve) $\sigma_B \leq -S_{uc}$.
- → ⊕ ⊖ $\left| \frac{\sigma_B}{\sigma_A} \right|$ (−ve) ≤ 1. $\sigma_A \geq S_{ut}$

  > 1 $\left(\frac{S_{uc} - S_{ut}}{S_{uc} S_{ut}} \right) \sigma_A - \frac{\sigma_B}{S_{uc}}$

* طريقة اول :- ① موضوع Load → Static → (ϵ_f) Sy , Su
Loading

↳ ductile 0.05 اكبر

Brittle. 0.05 اقل

* ductile :- Stress analysis σ_1 , σ_2 , σ_3

→ falin creteria.

* Brittle : Stress analysis σ_1 , σ_2 , σ_3.

→ K_t

→ falin creteria.

Chapter [6]: fatigue failure Resulting from Variable Loading

* machine subjected to varying or fluctuating stress
* فجأة sudden failure [illegible] component [illegible]
* due to movement

Linear elastic fracture :- (three stages)

[1] development of one or more microcracks (2-5 grain) due to cyclic plastic deformation.

[2] Cracks progresses from micro cracks to larger cracks (macro), keep growing making smooth. surface : smooth, flat, loading direction عمودي على

[3] occurs during the final stress cycle, material can n't support Load ("sudden fracture")

* fatigue cracks usually initiate at location with high stresses "dis continuities" [hole, Notch, scrach, etc.]

* fatigue crack also initiate at surface having rough surface finish due to the presence to tensile residual stress

NO.

* all Parts Subjected to fatigue Loading are heat treated and polished to inc. fatigue life.

Fatigue Life methods :-

* ال Component بتتعرض لـ loading (ten, comp) بيصير فيها failure

* الهدف نحسب عدد cycle يلي تتحملها الـ Component تحت تأثير قيمة معينة من Loading

* (fatigue life) تعتمد Loading, Condition, geometry

* Three diff. method :-
  1. Stress Life method
  2. Strain Life method
  3. Linear elastic fracture

* The fatigue life is usually classified according to the number of cycles :-

- Low cycle fatigue $(1 \leq N < 1000)$
- High cycle fatigue $(N > 10^3)$
  - → finite life $10^3 - 10^6$
  - → infinite life $> 10^6$

→ * ما بصير Low fatigue إلا إذا تجاوز Yield strength
* ignore fatigue effect
* use static failure analysis

NO.

Strain - Life method :- applicable to Low cycle fatigue

* Component صار فيها fatigue-failure عند عدد قليل من cycle
* زي الـ عمل bending لـ الـ
* Component لما (ten) تجاوزنا Yield بـ زاد Yield strength تبع (ten)
(comp) ″ ″ ″ بـ قل

لو كملنا Yield comp صار عند قيمة أقل

Yield strength in the reversed direction will be smaller than its initial value

* Cold-work في جهة (comp) Sy↑ خلينا Sy↓ (ten)، بالحالة هاي بصير فيها hardened في (comp) بصير softened بـ (ten) والعكس

* رجعنا Load لـ zero، ومشينا ندخل في (ten) الـ Yield اللي صار أقل من Yield، ومشي [كمية كبيرة من plastic deformation] hardened، في (ten)، softened في (comp) بـ Yield صار على قيمة أقل

* مع عدد الـ cycle الـ Strain بزيد مقدار معين كل ما أكثر (الحالة)

Linear elastic fracture machines method :- applicable to high cycle fatigue

* Stress-Concentration (hole)، بنسبيًا حتى بصير Crack، حتى IS size الـ hole تبع بنعتبر انها في crack
I
معروف.

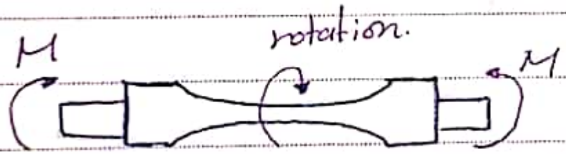
* بنحسب الفترة تبعت Stage ② , final critical cracke
بكم باخد فترة حتى ال Crack يكبر لـ الكسر

* ادور method 2 كيف نستخدمها

Stress - Life method :-

* Component بيتعرض لـ Stress بتتكرر بسبب قيمة معينة لـ stress
كسر سريع بسبب ↓ Stress والعكس
* بنربط Stress مع expected Life

- Rotating fatigue experiment :- "to obtained Stress-life
(تجربة لعب المقاومة)



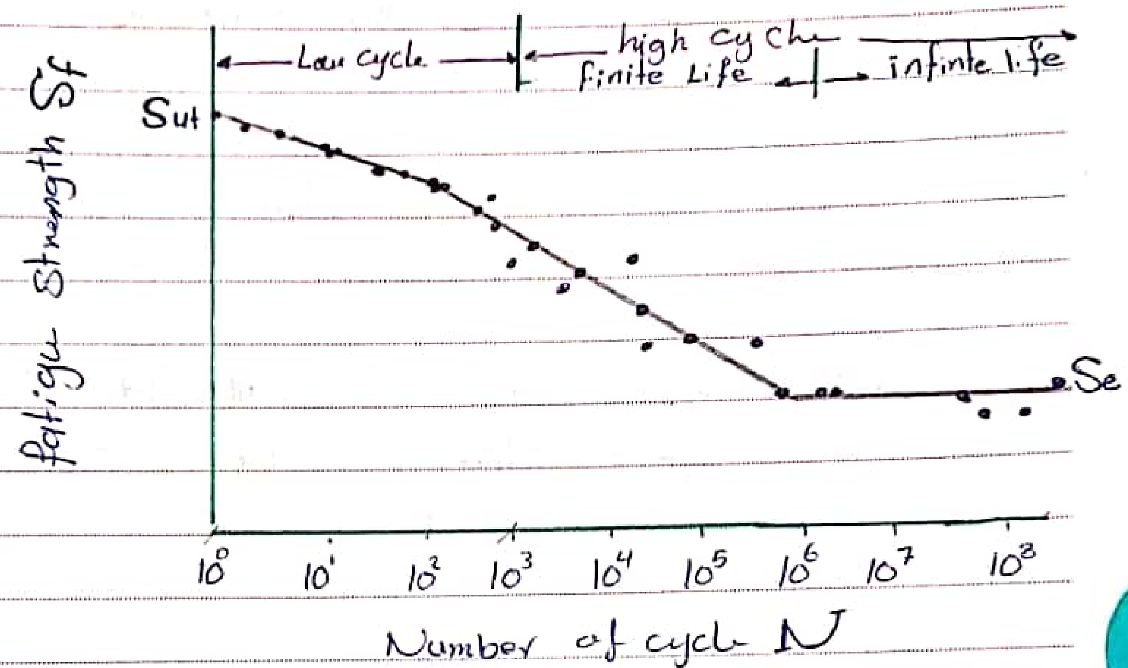
* نتيجة (M) بعمل bending M
$\sigma = \frac{Mc}{I}$, ten , comp

* الكرسي هون Section الأصغر (بالنص)
* لطبقوا موضوع لمعرفة قيمة ال Stress
* المتوسط، كل ما تتكرر العينة بنشوف عدد ال cycle وقيمة stress
يلي طبقناها

- S-N diagram :-

NO.

S-N diagram for Steel (Log-Log scale)



* المنحنى يلي في diagram بعني اي cycle بتتحمل اي Stress (عينة)

* اول قيمة بتحملها يلي بتكون الصينة بتكسر في طول Ultimate Yield (تعرفها)

"failure in half a cycle" ult (tensile test)

* كل ما قللت ال Stress كل ما طول بعيش الصينة بتعيش أكثر

* لا نوصل لقيمة معينة مليون 10^6 — infinite (never fail)

* قيمة ال Stress لو كنت أقل منها عمر العينة تكون infinite — endurance Limite

Stress > Se → يتكسر finite (fatigue strength)

Stress < Se → infinite (fatigue - Life)

* لعمل design قيمة ال Stress لازم تكون اقل من endurance Limit

* حساب factor-safety قيمة ال strength يلي بتقارن معها endurance

NO.

* عشان كل مواد مثل Steel وتيتانيوم titanium فالهم endurance-limit

* مثلاً (AL) مافي endurance: الكل تفرد قيمة عند 500×10^6 cycle
بنعتبر endurance

* في علاقة واضحة بين Se و Sut لـ Steel علاقة طردية
بنعمل tensial بنجيب Sut و Se من خلال rotating test

$$S_e' = \begin{cases} 0.5\, S_{ut}, & S_{ut} \le 1400\ \text{Mpa} \\ 700\ \text{Mpa}, & S_{ut} > 1400\ \text{Mpa} \end{cases}$$

(S_e') ← عشان نعرفها experimently

* هادا الحكي عند قيمة 1400 Sut بعد قيمة 1400 لـ Se ثابتة عند 700