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TURBO
Continuous change

دفتر :

بحوث عمليات(1)

Operation Research (1)

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اللجنة الأكاديمية لقسم الهندسة الصناعية

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Model Development

- A mathematical expression that describes the problem's objective is referred to as the **objective function**. For example, the profit equation $P=10x$ would be an objective function for a firm attempting to maximize profit. A production capacity constraint would be necessary if, for instant, 5 hours are required to produce each unit and only 40 hours are available per week. Let x indicate the number of units produced each week. The production time constraint is given by $5x \leq 40$
- The value of $5x$ is the total time required to produce x units; the symbol $\leq$ indicates that the production time required must be less than or equal to the 40 hours available.
- The decision problem or question is the following; How many units of the product should be scheduled each week to maximize profit? A complete mathematical model for this simple production problem is

Maximize $P=10x$	objective function
Subject to (s.t.)	
$5x \leq 40$	first constraint
$x \geq 0$	second constraint

- The $x \geq 0$ constraint requires the production quantity x to be greater than or equal to zero, which simply recognizes the fact that it is not possible to manufacture a negative number of units. The optimal solution to this model can be easily calculated and is given $x=8$, with an associated profit of \$80. **This model is an example of a linear programming model.**

No. CH 1

Linear programm

Model → constraints / objective Fun / variables يتكون من

profit equation $P = 10x$ → For example

* profit Maximize / Minimize اذا في

objective Fun. لـه تسمى

$x \geq 0$ → non negativity constraint → model ثابت لأي

Question 1: A manufacturer Produces three products Radio, Calculator, Tv. profit For each product:

15$ For each unit of Radio

5$ " " " of Calculater

20$ " " " of TV

اول خطوة

① تعريف variables

② تحديد objective Fun

③ constraints

production data are as follows:

No.

	Per unit		
Products	Material (Kgs)	Labor (Hours)	Machining (Hours)
Radio	2	1	1
Calculator	1	0.5	0.75
TV	10	3	3
Total available per week	500	700	350

The manufacturer wishes to establish the weekly production plan which maximizes the profit. Formulate as a Linear programming model.

Sol:

X_1 = number of Radios produced weekly

X_2 = number of calculators produced weekly

X_3 = number of TV's produced weekly

$$\text{Max } Z = 15X_1 + 5X_2 + 20X_3$$

S.t

$$2X_1 + 1X_2 + 10X_3 \leq 500$$

$$X_1 + 0.5X_2 + 3X_3 \leq 700$$

$$X_1 + 0.75X_2 + 3X_3 \leq 350$$

$$X_1, X_2, X_3 \geq 0$$

No.

ex: profit per unit

Max $Z = 7x_1 + 6x_2$

s.t.

$x_1 + x_2 \le 4$

$x_1 + 2x_1 \le 6$

$x_1, x_2 \ge 0$

نستخدمها لما يكون

Solve: متغيرين فقط

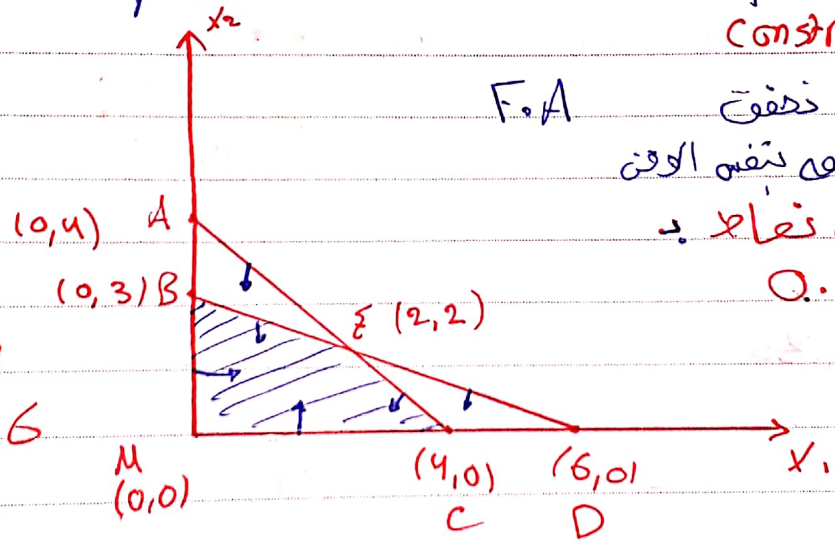
Use Graphical Method to solve LPM : Graphical خطوات

① رسم constraints

② تحديد area F.A

جميع الشروط المجتمعه بنفس الوقت

③ جدول وتعويض نقاط بـ O.F



$x_1 + x_2 = 4$

$x_1 + 2x_2 = 6$

F.A: BECM

optimal Max النقاط التي تحدد (corners)

Corners	$Z = 7x_1 + 6x_2$
(0,0)	\$0
(0,3)	\$18
(2,2)	\$26
(4,0)	\$28

optimal sol

$x_1 = 4$ units

$x_2 = 0$ units

$Z = \$28$ max profit

36
24
12 3/36
12

ex

$-5x_2 = -12$

$\text{Max } z = 10x_1 + 2x_2$

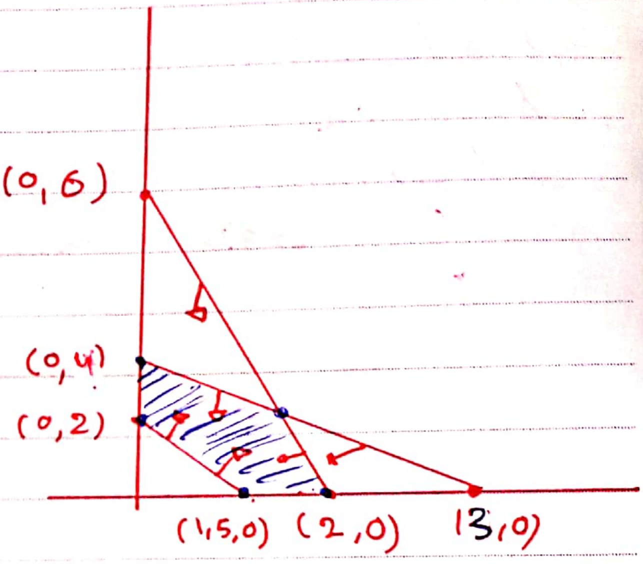
s.t

$12x_1 + 4x_2 \le 24$

$4x_1 + 3x_2 \le 12$

$4x_1 + 3x_2 \ge 6$

$x_1, x_2 \ge 0$



Corners	$z = 10x_1 + 2x_2$
(0,2)	\$4
(0,4)	\$ 8
(1.5,0)	\$ 150
(2,0)	\$ 20
$(\frac{9}{5}, \frac{12}{5})$	\$ 22.8

$12x_1 + 4x_2 = 24$

$4x_1 + 3x_2 = 12$

Optimal sol

$x_1 = 2$ units

$x_2 = 0$ units

$z = 20\$$ max profit

ex

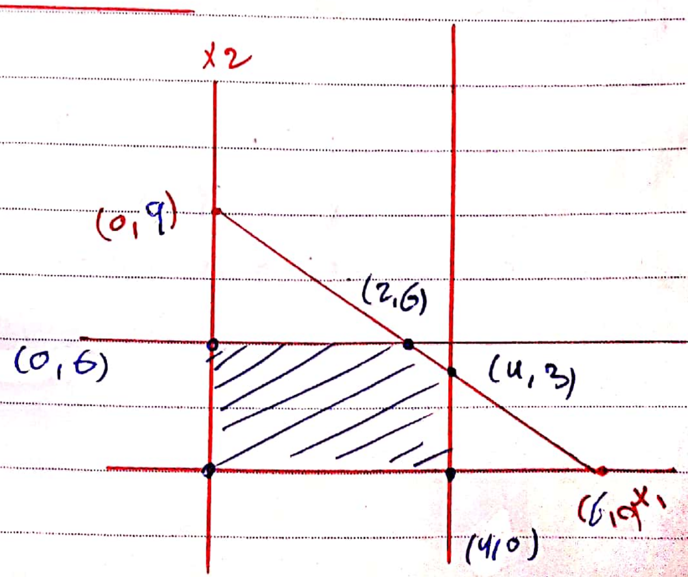
$\text{Max } z = 3x_1 + 5x_2$

$x_1 \le 4$

$2x_2 \le 12$

$3x_1 + 2x_2 \le 18$

$x_1, x_2 \ge 0$

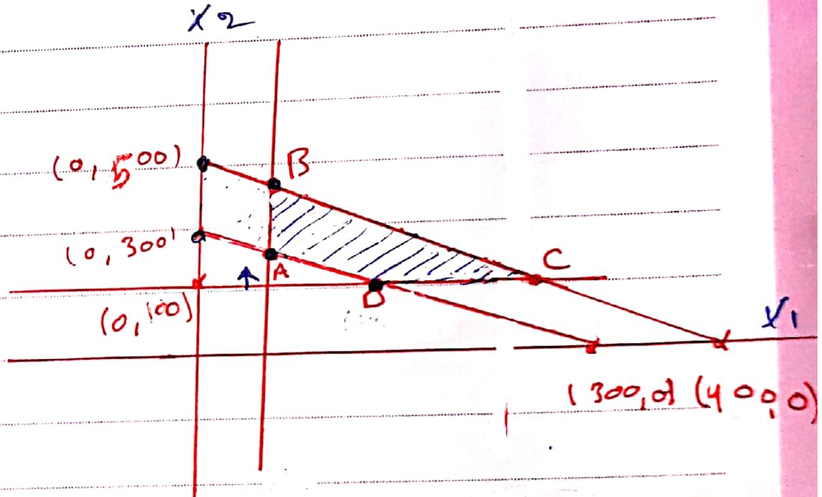


No.

Corners	$Z = 3x_1 + 5x_2$
(0,0)	\$ 0
(0,6)	\$ 30
(2,6)	\$ 36
(4,3)	\$ 27
(4,0)	\$12

optimal sol
$x_1 = 2$ units
$x_2 = 6$ "
$Z = \$36$

ex:
Min $Z = 24x_1 + 28x_2$
s.t
$5x_1 + 4x_2 \le 2000$
$x_1 \ge 80$
$x_1 + x_2 \ge 300$
$x_2 \ge 100$
$x_1, x_2 \ge 0$



Corners	$Z = 24x_1 + 28x_2$
D (200,100)	\$7600
C (320,100)	\$ 10480
A (80,220)	\$ 8080
B (80,400)	\$13120

$x_1 + x_2 = 300$
$-x_2 = -100$

Optimal sol
$x_1 = 200$ units
$x_2 = 100$ "
$Z = \$7600$

Special cases

1. Alternative optimal

لما يكون احد constraints عن ضرب/قسمت الـ objective Fun برقم يعطي

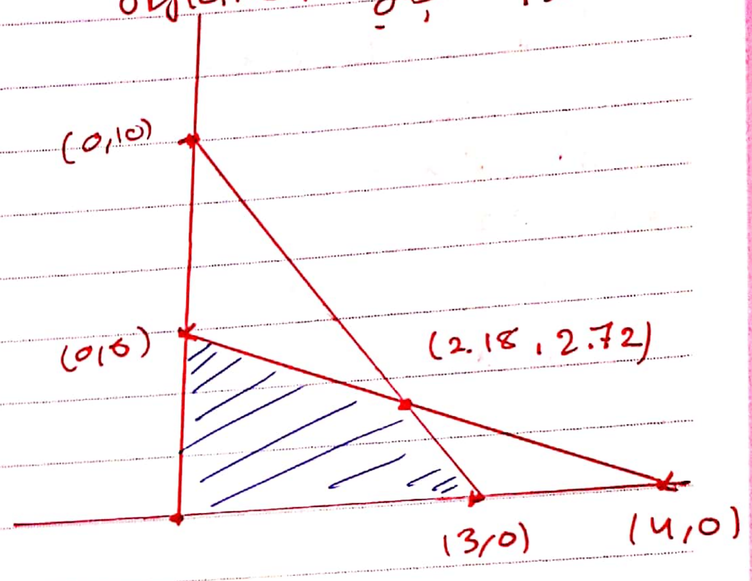
Max $Z = 3X_1 + 2X_2$

s.t

$\div 2 \rightarrow 6X_1 + 4X_2 \leq 24$

$6X_2 + 3X_2 \leq 30$

$X_1, X_2 \geq 0$



Corners	$Z = 3X_1 + 2X_1$
(0,6)	\$12
(3,0)	\$9
(0,0)	\$0
(2.18, 2.72)	\$12

optimal Sol

1. $X_1 = 0$ unit
   $X_2 = 6$ units
   $Z = \$12$

2. $X_1 = 2.18$ units
   $X_2 = 2.72$ units
   $Z = \$12$

use Graphical

Max $Z = 2X_1 + 3X_2$

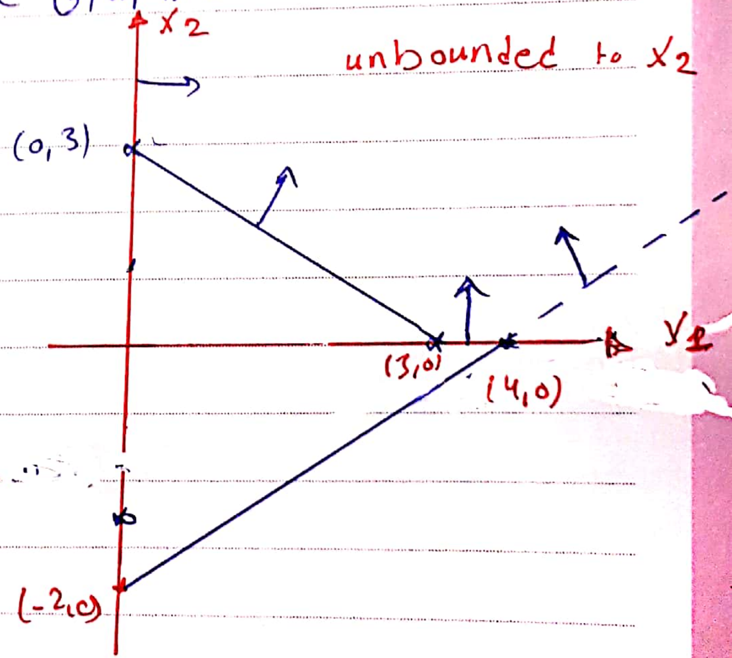
s.t

$X_1 + X_2 \geq 3$

$X_1 - 2X_2 \leq 4$

$X_1, X_2 \geq 0$

unbounded to X_2



2. ↗ unbounded solution

Area open باتجاه X_1 او X_2 ...

No.

3. Infeasible problems

لا يتم فيها تحقيق الشروط

$$\text{Max } Z = 4x_1 + 3x_2$$

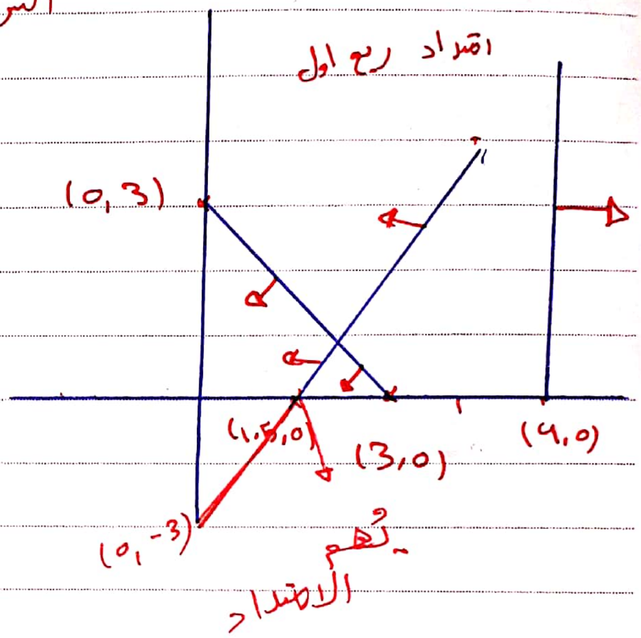
s.t

$$x_1 + x_2 \le 3$$

$$2x_1 - x_2 \le 3$$

$$x_1 \ge 4$$

$$x_1, x_2 \ge 0$$



4. Degeneracy - Redundant constraint.

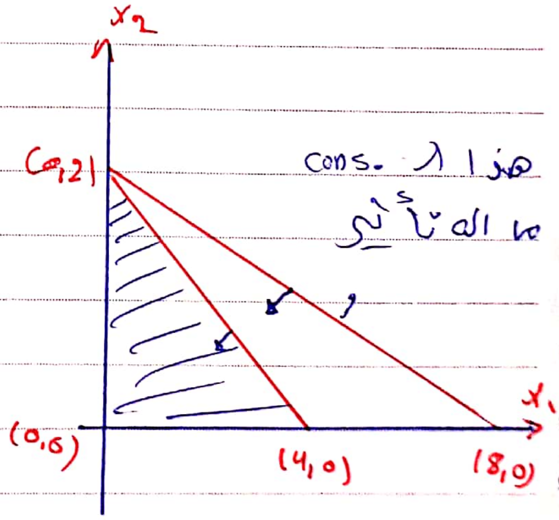
$$\text{Max } Z = 3x_1 + 9x_2$$

s.t

$$x_1 + 4x_2 \le 8$$

$$x_1 + 2x_2 \le 4$$

$$x_1, x_2 \ge 0$$



Sensitivity Analysis
(Post - optimality)

profit per unit

Max $Z = 5x_1 + 7x_2$

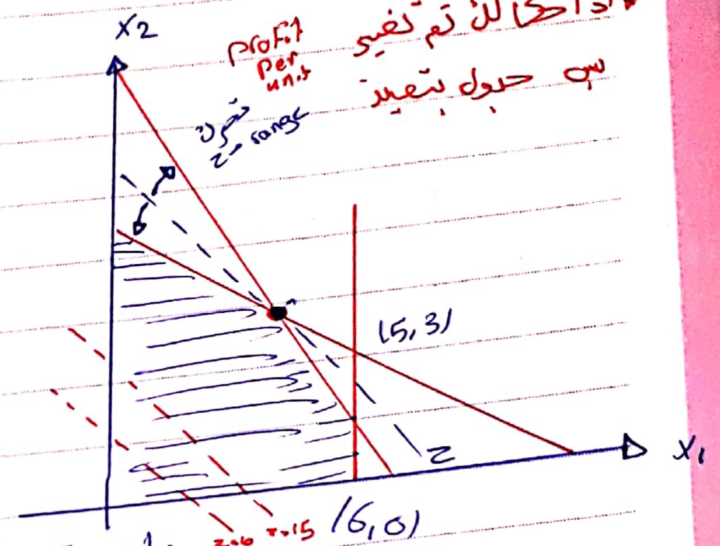
s.t

$x_1 \le 6$

$2x_1 + 3x_2 \le 19$

Binding Constraints → $x_1 + x_2 \le 8$

$x_1 , x_2 \ge 0$



* اذا حكالك تم تغيير بس حدود بتعيد

فرض

$10 = 5x_1 + 7x_2$

$15 = 5x_1 + 7x_2$

حتى اعرف الاتجاه زيادة

$x_1 = 5$ units

$x_2 = 3$ "

$Z = \$ 46$

* اذا حكالك تم تغيير طرف ايمن لاحد constraint مطر تغير الحل

طريقه اخرى لايجاد optimal ← فرض نقاط حتى يصير هوا من عند نقطة optimal

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ex: solve

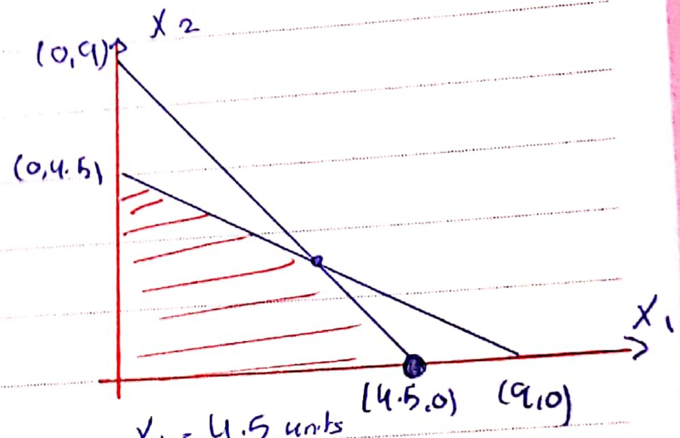
Max $Z = 3x_1 + x_2$

s.t

$2x_1 + x_2 \le 9$

$x_1 + 2x_2 \le 9$

$x_1 , x_2 \ge 0$



$x_1 = 4.5$ units

$x_2 = 0$ "

$Z = \$$ ____

IF the optimal sol is on the Line then its Binding → عند تعويضها طرف ايمن = ايسر

L.H.S = R.H.S

(4.5,0)

* $2x_1 + x_2 = 9$

$2(4.5) + 0 = 9$ $\quad 9 = 9$ ✓

* $x_1 + 2x_2 = 9$

$4.5 + 2(0) < 9$ not Binding

example

Max $z = 3x_1 + 5x_2$

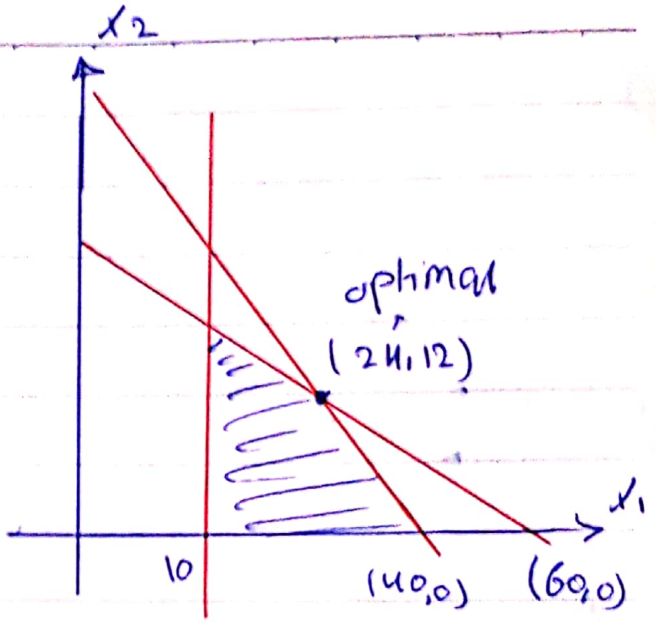
s.t

$x_1 + 3x_2 \le 60$

$3x_1 + 4x_2 \le 120$

$x_1 \ge 10$

$x_1, x_2 \ge 0$



① $x_1 + 3x_2 = 60$

$(24) + 3(12) = 60$ ✓ Binding

② $3(24) + 4(12) = 120$ ✓

$3x_1 + 4x_2 = 120$

ايجاد Range of optimality مهم ← الورقة الجاية

نفس اول سؤال تحت عنوان sensitivity

Max $z = 5x_1 + 7x_2$

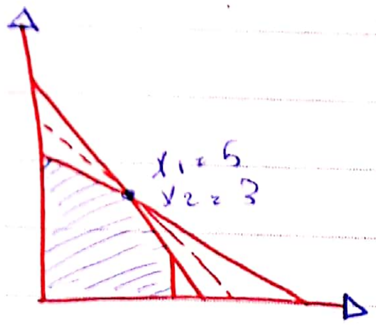
s.t

$x_1 \le 6$

$2x_1 + 3x_2 \ge 19$

$x_1 + x_2 \le 8$

$x_1, x_2 \ge 0$



- The limits of a range of optimality found by changing the slope of the objective function line within the limits of the slopes of the binding constraint lines

No.

- Slope of an objective Function line.
Max $c_1 x_1 + c_2 x_2$ is $-\frac{c_1}{c_2}$

- slope of a constraint
$a_1 x_1 + a_2 x_2 = b$ $\quad -\frac{a_1}{a_2}$

Range of optimality For C_1: coffecient 1 (5)

باعتبار c_2 ثابت

Binding
$x_1 + x_2 = 8$ slope -1
$2x_1 + 3x_2 = 19$ slope $\frac{-2}{3}$

* Find Range of C_1 (with C_2 staying 7) constant
s.t the objective Fun line slope lies between that of the two Binding

$$-1 \le \frac{-C_1}{7} \le \frac{-2}{3} \qquad 7 \ge C_1 \ge \frac{14}{3}$$

$$\frac{14}{3} \le C_1 \le 7$$

- Range of optimality For C_2 (C_1 constant) 5

$$-1 \le \frac{-5}{C_2} \le \frac{-2}{3} \qquad 5 \le C_2 \le \frac{15}{2}$$

No.

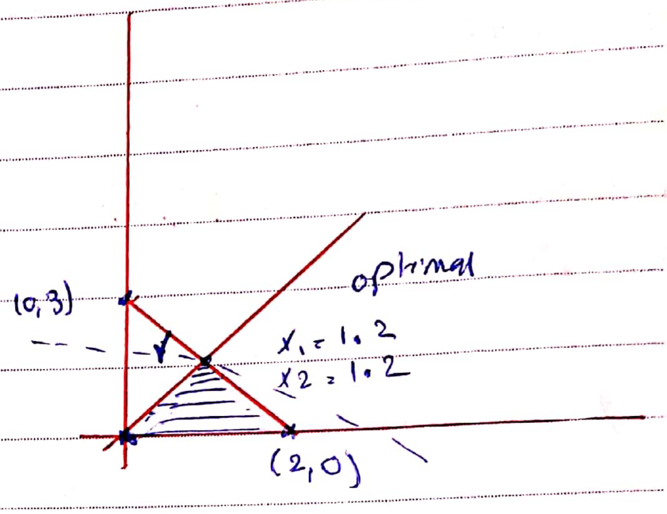
Max $z = 2x_1 + 3x_2$

s.t

$3x_1 + 2x_2 \le 6$

$x_1 = x_2$ $\quad -x_1 + x_2 \le 0$

$x_1, x_2 \ge 0$



$3x_1 + 2x_2 = 6$

slope $\frac{-3}{2}$

$-x_1 + x_2 = 0$ $\quad$ slope 1

$\frac{-3}{2} \le -\frac{C_1}{3} \le 1$ $\qquad$ $\frac{-3}{2} \le -\frac{2}{C_2} \le 1$

$-3 \le C_1 \le \frac{9}{2}$ $\qquad$ $-2 \le C_2 \le \frac{4}{3}$

CONSTRUCTION OF THE LP MODEL

This section illustrate the basic elements of an LP model by using a simple **two – variable** example. The results provide concrete ideas for the solution and interpretation of the general LP problem.

Example

Reddy Mikks produces both interior and exterior paints from two raw materials, *M1* and *M2*. The following table provides the basic data of the problem:

	Tons of raw material per ton of		Maximum daily Availability (tons)
	Exterior paint	Interior paint	
	x_1	x_2	
Raw material, *M1*	6	4	24
Raw material, *M2*	1	2	6
Profit per ton($1000)	5	4	

A market survey restricts the maximum daily demand of interior paint to 2 tons. Additionally, the daily demand of interior paint cannot exceed that of exterior paint by more than 1 ton. **Reddy Mikks wants to determine the optimum (best) product mix of interior and exterior paints that maximizes the total daily profit.** optimal

$6x_1 + 4x_2 \le 24$

$x_1 + 2x_2 \le 6$

$x_2 \le 2$

$x_2 - x_1 \le 1$

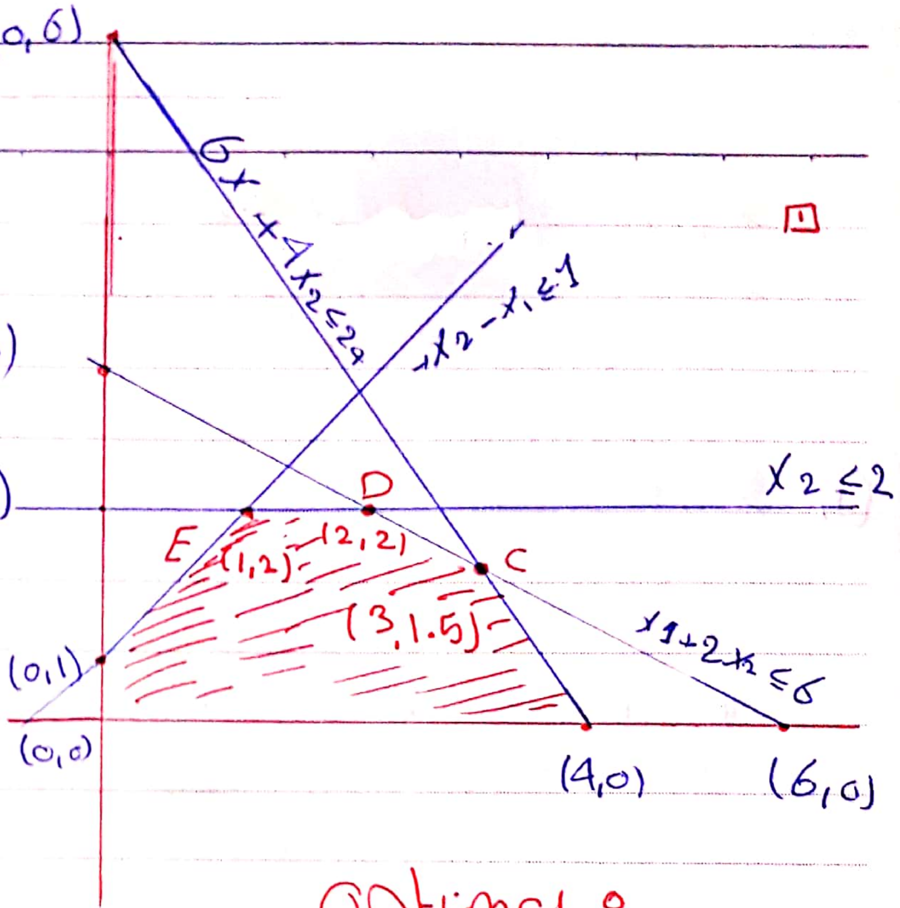
$x_1, x_2 \geq 0$

$\max Z = 5x_1 + 4x_2$

any LP model :

1. Decision variables, we seeks to determine
2. objective (goal) we aim to optimize
3. Constraints that we need to satisfy

1



D:

$x_2 = 2$

$x_1+2x_2=6$

(2,2)

E

$-x_1+x_2=1$

$x_2=2$

(1,2)

Optimal:

$x_1 = 3$ units

$x_2 = 1.5$ "

$Z = \$21,000$

profit per unit

* ممكن يجي السؤال ويعطي الحل ويغير بغير جدول بس

Range of optimality

slope

$x_1+2x_2=6$ $\quad -\frac{1}{2}$

$6x_1+4x_2=24$ slope $-\frac{6}{4}$

$$-\frac{1}{2} \le -\frac{c_1}{4} \le -\frac{3}{2}$$

Scanned with CamScanner

x_1 : type 1
x_2 : type 2
h_1 : labor time type 1
h_2 : labor time type 2

2–10 A company produces two types of cowboy hats. Each hat of the first type requires twice as much labor time as does each hat of the second type. If all hats are of the second type only, the company can produce a total of 500 hats a day. The market limits daily sales of the first and second types to 150 and 200 hats. Assume that the profit per hat is \$8 for type 1 and \$5 for type 2. Determine the number of hats of each type to produce to maximize profit.

Max $z = 8x_1 + 5x_2$

$x_1 \le 150$

$x_2 \le 200$

$\frac{h_1}{h_2} = 2$

$h_1 = 2h_2$

$x_1, x_2 \ge 0$

قسمة h_2 على

$h_1 x_1 + h_2 x_2 = 500 h_2$

$2x_1 + x_2 = 500$

$A = x_1$
$B = x_2$

2. A company produces two products, *A* and *B*. The sales volume for *A* is at least 80% of the total sales of both *A* and *B*. However, the company cannot sell more than 100 unit of *A* per day. Both products use one raw material whose maximum daily availability is limited to 240 lb a day. The usage rates of the raw material are 2 lb per unit of *A* and 4 lb per unit of *B*. The unit prices for *A* and *B* are \$20 and \$50, respectively.

solve

(a) Determine the optimal product mix for the company.

max $z = 20x_1 + 50x_2$

$x_1 \le 100$

$\frac{x_1}{x_1 + x_2} \ge 0.8$

$2x_1 + 4x_2 \le 240$

$x_1, x_2 \ge 0$

8. Dean's Furniture Company assembles from precut lumber two types of kitchen cabinets: regular and deluxe. The regular cabinets are painted white, and the deluxe ones are varnished. Both the painting and the varnishing occur in one department. The daily capacity of the assembly department can produce a maximum of 200 regular cabinets and 150 deluxe ones. Varnishing a deluxe unit takes twice as much time as painting a regular one. If the painting/varnishing department is dedicated to the deluxe units only, it can complete 180 units daily. The company estimates that the profits per unit for the regular and deluxe cabinets are \$100 and \$140, respectively.

نفس فكرة السؤال الأول

(a) Formulate the problem as a linear program and find the optimal production schedule per day.
(b) Suppose that because of competition, the profits per unit of the regular and deluxe units must be reduced to \$80 and \$110, respectively. Use sensitivity analysis to determine whether or not the optimum solution in (a) remains unchanged.

No. CH 3

simplex عن Min / Max . 2-10 ضروري حل سؤال

Simplex Method: 2 variables and more

① Standard Form → تحويل الاكبر والاصغر الى اشارة يساوي

ex

$6X_1 + 3X_2 - 2X_3 \leq 50$

* add a slack variable → ما يكون $\leq$ اصغر متوقع

$6X_1 + 3X_2 - 2X_3 + X_4 = 50$

لا يوجد له تأثير على objective fun

actual amount

$X_4 = 50 - (6X_1 + 3X_2 - 2X_3)$

X_4: measures the unused amount 50 - actual amount

ex

$4X_1 + 2X_2 + 3X_3 \geq 15$ → ما يكون $\geq$ اكبر

* subtract a surplus variable

$4X_1 + 2X_2 + 3X_3 - X_4 = 15$

$X_4 = (4X_1 + 2X_2 + 3X_3) - 15$

ex:

Max $Z = 80X_1 + 120X_2$

s.t

$2X_1 + 4X_2 \leq 80$

$3X_1 + X_2 \leq 60$

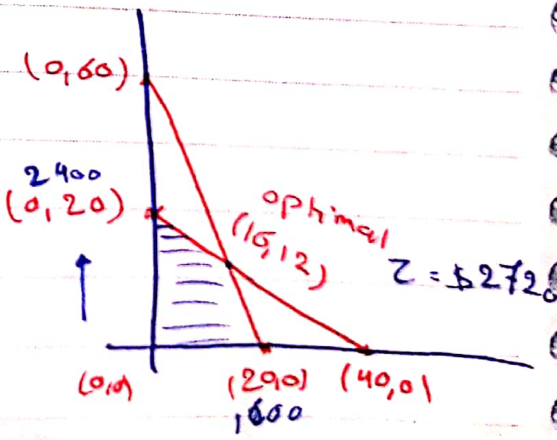
$X_1, X_2 \geq 0$

$2X_1 + 4X_2 = 60$

$-12X_1 - 4X_2 = -240$

$-10X_1 = -160$

$X_1 = 16$



حله طريقة Graphical

Simplex Method → عدد مجاهيل اكثر من معادلات

حله simplex

$$\text{Max } z = 80x_1 + 120x_2 + 0x_3 + 0x_4$$

s.t

$$2x_1 + 4x_2 + x_3 + 0x_4 = 80$$
$$3x_1 + x_2 + 0x_3 + x_4 = 60$$
$$x_1, x_2, x_3, x_4 \geq 0$$
$$Z - 80x_1 - 120x_2 = 0$$

Most negative entering

	x_1	x_2	x_3	x_4	R.H.S	ratio
Z	-80	$\boxed{-120}$	0	0		
x_3	2	4	1	0	80	20
x_4	3	1	0	1	60	60
	-20	0	30	0	2400	
x_2	$\frac{1}{2}$	1 (pivot)	$\frac{1}{4}$	0	20	
x_4	2.5	0	$-\frac{1}{4}$	1	40	
	0	0	28	8	2720	
x_2	0	1	3/10	-1/5	12	
x_1	1	0	-1/10	$\frac{2}{5}$	16	

ratio اقل leaving

120 قسمة على 4 ويضرب

÷ 2.5

ضرب pivot -1 وجمع للصف الثاني

$20 \cdot -\frac{1}{2}$

ممكن ماتكون مرتبة / اكتشاف I_3 3x3

نقل معلومات الى الجدول

$x_1, x_2 = 0$ z

(1) لكل معادلة لازم Identity

خلي عدد متغيرات $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(2) لكل معادلة z row و x_3, x_4

zero ← coff

(3)

Max ← entering Most negative

min ← entering Most positive

less ratio ← leaving

optimal

$x_1 = 16$

$x_2 = 12$

$x_3 = 0$

$x_4 = 0$

$Z = 2720$

* كل ايتريشن يجب التأكد اذا optimal او لا

لما انو المسألة Max في z row negative ← not optimal

No.

طريقة

linear Algebra

$2X_1 + 4X_2 + X_3 = 80$

$3X_1 + X_2 + X_4 = 66$

فرض نقاط

① $X_1 = X_2 = 0$

$X_3 = 60$

$X_4 = 60$

$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 60 \\ 60 \end{bmatrix}$

② $X_1 = X_3 = 0$

$X_2 = 20$

$X_4 = 46$

$\begin{bmatrix} 0 \\ 20 \\ 0 \\ 40 \end{bmatrix}$

③ $X_1 = X_4 = 0$

$X_2 = 60$

$X_3 = -160$

$\begin{bmatrix} 0 \\ 60 \\ -160 \\ 0 \end{bmatrix}$

④ $X_3 = X_4 = 0$

$X_1 = 16$

$X_2 = 12$

عدد مجاهيل $\leftarrow n!$

$\frac{n!}{m!(n-m)!} = \frac{4!}{2!(4-2)!} = 6$

عدد معادلات $\leftarrow m!$

⑤ $X_2 = X_3 = 0$

$X_1 = 40$

$X_4 = -60$

⑥ $X_2 = X_4 = 0$

$X_1 = 20$

$X_3 = 40$

No. Solve

Max $Z = 12X_1 + 8X_2$ Simplex

s.t.

$5x_1 + 2X_2 \leq 150$

$2x_1 + 3X_2 \leq 100$

$4x_1 + 2x_2 \leq 80$

$x_1, X_2 \geq 0$

Max $Z = 12X_1 + 8X_2$

$5x_1 + 2X_2 + X_3 = 150$

$2X_1 + 3X_2 + X_4 = 100$

$4X_1 + 2X_2 + X_5 = 80$

$X_1, X_2, X_3, X_4, X_5 \geq 0$

$Z - 12X_1 - 8X_2 = 0$

No.

	X_1 ↓	X_2	X_3	X_4	X_5	R.H.S	ratio
Z	-12	-8	0	0	0	0	
X_3	5	2	1	0	0	150	30
X_4	2	3	0	1	0	100	50
→ X_5	4	2	0	0	1	80	20 min ratio
	0	-2	0	0	3	240	
X_3	0	-1/2	1	0	-5/4	50	- لا يجوز قسمة سالب أو صفر
X_4	0	2	0	1	-1/2	60	30
pivot → X_1	1	1/2	0	0	1/4	20	40
	0	0	0	1	5/2	300	
X_3	0	0	1	1/4	-11/8	65	
pivot X_2	0	1	0	1/2	-1/4	30	
X_1	1	0	0	-1/4	3/8	5	

optimal sol

$X_1 = 5$

$X_2 = 30$

$X_3 = 65$ $\quad Z = \$300$

$X_4 = 0$

$X_5 = 0$

$$X_3 = 150 - (5X_1 + 2X_2)$$
$$= 150 - (5(5) + 2(30))$$
actual 85

$= \underline{65}$

تمام

No.

Big M (penalty techniques) ← نستخدم عند وجود اكبر او يساوي

$$\min Z = 4x_1 + x_2$$

s.t

$$3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

ماذا لو مدرس ضرب ومط بدل Min ← Max ماذا يحدث ؟؟

solve simplex

$$\min Z = 4x_1 + x_2 + MR_1 + MR_2 \quad + M(R_1 + R_2)$$

Max طرح / Min اضافة

s.t

هل ,شكل عدم وجود Identity

$$3x_1 + x_2 + R_1 = 3$$
$$4x_1 + 3x_2 - x_3 + R_2 = 6$$
$$x_1 + 2x_2 + x_4 =$$

to create Identity

	x_1	x_2	x_3	x_4	R.H.S	ratio
Z	-4	-1	0	0		
	3	1	0	0		
	4	3	-1	0		
	1	2	0	1		

لا يوجد Identity

Scanned with CamScanner

No

	x_1	x_2	x_3	x_4	R_1	R_2	RHS
Z	-4	-1	0	0	-M	-M	0
	3	1	0	0	1	0	3
	4	3	-1	0	0	1	6
	1	2	0	1	0	0	4

احياناً يجيب سؤال Basic variables معاملاتهم ب Zrow مش zero

مشكلة Identity اتحلت لكن

حل المشكلة :

$$R_1 = 3 - 3x_1 - x_2$$

$$R_2 = 6 - 4x_1 - 3x_2 + x_3$$

$$R_1 + R_2 = 9 - 7x_1 - 4x_2 + x_3$$

$$Z = 4x_1 + x_2 + M(9 - 7x_1 - 4x_2 + x_3)$$

$$Z + (7M - 4)x_1 + (4M - 1)x_2 - Mx_3 = 9M$$

No.

Most positive

	x_1	x_2	x_3	x_4	R_1	R_2	R.H.S	ration
z	$7M-4$	$4M-1$	$-M$	0	0	0	$9M$	
R_1	3	1	0	0	1	0	3	3 → Min ratio
R_2	4	3	-1	0	0	1	6	1.5
x_4	1	2	0	1	0	0	4	4
	0	$5/3M+1/3$	$-M$	0	$-\frac{7}{3}M+11/3$	0	$2M+4$	
x_1	1	1/3	0	0	1/3	0	1	3
R_2	0	5/3	-1	0	-4/3	1	2	6/5
x_4	0	5/3	0	1	-1/3	0	3	9/5
	0	0	1/5	0	$-M+8/5$	$-M-1/5$	18/5	
x_1	1	0	1/5	0	3/5	-1/5	3/5	3
x_2	0	1	-3/5	0	-4/5	3/5	6/5	—
x_4	0	0	1	1	1	-1	1	1
	0	0	0	-1/5	$7/5-M$	$-M$	17/5	→ z row as negative
x_1	1	0	0	-1/5	2/5	0	2/5	
x_2	0	1	0	3/5	-1/5	0	9/5	
x_3	0	0	1	1	1	-1	1	

optimal sol

$x_1 = 2/5$ units

$x_2 = 9/5$ "

$z = 17/5$

Scanned with CamScanner

Two phase Method

$\min \ Z = 4X_1 + X_2$

s.t

$3X_1 + X_2 = 3$

$4X_1 + 3X_2 \geq 6$

$X_1 + 2X_2 \leq 4$

$X_1, X_2 \geq 0$

$\min \ Z = 4X_1 + X_2$ phase 1 دائماً بيكون min problem

s.t

$3X_1 + X_2 + R_1 = 3$

$4X_1 + 3X_2 - X_3 + R_2 = 6$

$X_1 + 2X_2 + X_4 = 4$

$X_1, X_2, X_3, X_4, R_1, R_2 \geq 0$

$\min \ r = \sum R$

$R_1 = 3 - 3X_1 - X_2$

$R_2 = 6 - 4X_1 - 3X_2 + X_3$

$r = R_1 + R_2 = 9 - 7X_1 - 4X_2 + X_3$

$r - 9 + 7X_1 + 4X_3 - X_3 = 0$

$r + 7X_1 + 4X_2 - X_3 = 9$

No.

	x_1	x_2	x_3	x_4	R_1	R_2	R.H.S	
r	7	4	-1	0	0	0	9	
R_1	3	1	0	0	1	0	3	1
R_2	4	3	-1	0	0	1	6	1.5
x_4	1	2	0	1	0	0	4	4
r	0	5/3	-1	0	-7/3	0	2	
x_1	1	1/3	0	0	1/3	0	1	3
R_2	0	5/3	-1	0	-4/3	1	2	6/5
x_4	0	5/3	0	1	-1/3	0	3	9/5
r	0	0	0	0	-1	-1	0	
x_1	1	0	1/5	0	3/5	-1/5	3/5	
x_2	0	1	-3/5	0	-4/5	3/5	6/5	
x_4	0	0	1	1	1	-1	1	

end of phase

* كله صافي r row Positive

Basic

zero row

phase 2

$$Z = 4x_1 + x_2$$

ارجاع للاصل ← min

$$x_1 + \frac{1}{5}x_3 = \frac{3}{5}$$

اخر row operation

موقع قانون

$$x_2 - \frac{3}{5}x_3 = \frac{6}{5}$$

$$Z = 4\left(\frac{3}{5} - \frac{1}{5}x_3\right) + \frac{6}{5} + \frac{3}{5}x_2$$

$$Z + \frac{1}{5}x_3 = \frac{18}{5}$$

	x_1	x_2	x_3	x_4	R.H	
z	0	0	4/5	0	18/5	
x_1	1	0	1/5	0	3/5	3
x_2	0	1	-3/5	0	6/5	—
x_4	0	0	1	1	1	1
z	0	0	0	-1/5	17/5	
x_1	1	0	0	-1/5	2/5	
x_2	0	1	0	3/5	9/5	
x_3	0	0	1	1	1	

optimal

$x_1 = 2/5$

$x_2 = 9/5$

$z = 17/5$ \$

Max $z = 2x_1 + 5x_2$

s.t

$3x_1 + 2x_2 \geq 6$

$2x_1 + x_2 \leq$

$x_1, x_2 \geq 0$

$3x_1 + 2x_2 - x_3 + R_1 = 6$

$2x_1 + x_2 + x_4 = 2$

$x_1, x_2, R_1, x_3, x_4 \geq 0$

No.

max r = R

Special Cases

$R_1 = 6 + X_3 - 2X_2 - 3X_1$

$r - X_3 + 2X_2 + 3X_1 = 6$

	X_1	X_2	X_3	X_4	R	R.H.S	
r	3	2	-1	0	0	6	
R	3	2	-1	0	1	6	2
X_4	2	1	0	1	0	2	1
r	0	1/2	-1	-3/2	0	3	
R	0	1/2	-1	-3/2	1	3	6
X_1	1	1/2	0	1/2	0	1	2
r	-1	0	-1	-2	0	2	
R	-1	0	-1	-2	1	2	
X_2	2	1	0	1	0	2	

end of phase

*special case

كل شروط ذ متحققة لكن

(1) Infeasible → لان R.H.S لا يساوي zero

No.

ex:

$$\text{Max } z = \underline{3}x_1 + \underline{2}x_2$$

مضاعفات $\underline{6}x_1 + \underline{4}x_2 \le 24$

$$10x_1 + 3x_2 \le 36$$

$$x_1, x_2 \ge 0$$

Solve: Don't use Graphical

standerd Form $\text{Max } z = 3x_1 + 2x_2$

$$\text{s.t } 6x_1 + 4x_2 + x_3 = 24$$

$$10x_1 + 3x_2 + x_4 = 30$$

$$x_1, x_2, x_3, x_4 \ge 0$$

$$Z - 3x_1 - 2x_2 = 0$$

	x_1	x_2	x_3	x_4	R.H.S	R
Z	-3	-2	0	0	0	
x_3	6	4	1	0	24	4
x_4	10	3	0	1	30	3
Z	0	-11/10	0	3/10	9	
x_3	0	22/10	1	-6/10	6	30/11
x_1	1	3/10	0	1/10	3	10
Z	0	0	11/22	0	12	وجود صفر غريب
x_2	0	1	10/22	-6/10	30/11	يدل على alternative
x_1	1	0	-3/22	2/11	24/11	
Z	0	0	11/22	0	12	
x_2	60/4	1	21/44	0	6	
x_4	11/2	0	17/20	1	12	

Altrenative case

No.

optimal $Z = 125$

$x_1 = 24/11$ $\quad x_2 = 30/11$

$x_2 = 6$ $\quad x_4 = 12$

Max $Z = 3x_1 + 9x_2$

s.t

$x_1 + 4x_2 \leq 8$

$x_1 + 2x_2 \leq 4$

$x_1, x_2 \geq 0$

$\longrightarrow$ standered Form

$Z = 3x_1 + 9x_2$

$x_1 + 4x_2 + x_3 = 8$

$x_1 + 2x_2 + x_4 = 4$

$x_1, x_2, x_3, x_4 \geq 0$

$Z - 3x_1 - 9x_2 = 0$

	x_1	x_2	x_3	x_4	RH.S	ratio
Z	-3	-9	0	0	0	
x_3	1	4	1	0	8	2 $\rightarrow$ tie
x_4	1	2	0	1	4	2
Z	-3/4	0	9/4	0	18	
x_2	1/4	1	1/4	0	2	
x_4	1/2	0	-1/2	1	0	
Z	0	0	3/2	3/2	18	
x_2	0	1	1/2	-1/2	2	
x_1	1	0	-1	2	0	

اي ابتريشن فيها tie
نتوقع بعده optimal
next

Redundant Case

$$\text{Max } Z = 2X_1 + X_2$$

s.t

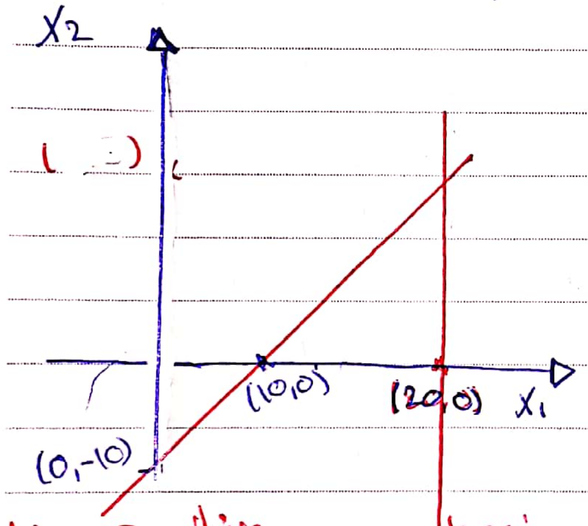
$$X_1 - X_2 \le 10$$
$$2X_1 \le 40$$
$$X_1, X_2 \ge 0$$

$\rightarrow$

$$\max Z = 2X_1 + X_2$$

s.t

$$X_1 - X_2 + X_3 = 10$$
$$2X_1 + X_4 = 40$$
$$X_1, X_2, X_3, X_4 \ge 0$$
$$Z - 2X_1 - X_2 = 0$$



unbounded X_2
Graphical

اي اشترك اذا كان معاملات non basic variable
تكون Non positive
zero -
* unbounded case

	X_1	X_2	X_3	X_4	R.H.S
Z	-2	-1	0	0	0
X_3	1	-1	1	0	10
X_4	2	0	0	1	40

smile...