دفتر :
بحوث عمليات 1

Operation Research 1

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subject to a set of restrictions called constraints. Using mathematical techniques to model, analyze, and solve problem.

Four main phases of operation research

1- Definition of the Problem:

- What are the decision variables
- What is the objective of the study

- What are the specification of the limitations under which the modeled system operates.

2- Model Construction

- Translating the real-world problems into mathematical models

3- Model Validity

- Testing and evaluation of the model. A common method for testing a validity of a model is to compare its performance with some past data available for the actual system.

4- Solution Methodology

- There are wide verities of existing solution algorithms to solve mathematical models yet knowing which one to use might be challenging.

5- Implementing the Solution : Translating the resulting mathematical model into a computer code (i.e, CPLEX, C, C++, etc)

Basic Component of the Model

1- Decision Variables

- Is the unknow to determined from the solution of a model (what dose the model seeks to determine).

2- Objective function • It is the final result desired to be achieved by the system. Decision makers normally care about the value the objective function. A common objective is to minimize the total cost or maximize the organization profit. The objective function expressed as a mathematical function of the system decision variables.

objective function:

$Z = x_1(\text{profit}) + x_2(\text{profit})$

Basic Component of the Model

3- Constraints

• These are the limitation imposed on the variables to satisfy the restriction of the They modeled system. must be expressed as

Can't exceed: $x \leq a$

Can exceed: $x \geq a$

non-negative constraint: $x_1, x_2 \geq 0$

mathematical functions of the system decision variables.

Example 1

- The admission office at Tech wants to determine how many instate and out-ofstate students to accept for next fall's entering freshman class. Tuition for in-state student is $7,600 per year while out-of-state tuition is $22,500 per year. A total of 12,800 in-state and 8,100 out-of-state

freshman have applied for next fall, and Tech does not wa
to accept more than 3,500 students.

Example 1

However, since Tech is a state institution, the s
mandates that it can accept no more than 40% out-of-
students. From past experience, the admissions
knows that 12% of in-state students and 24% of o
state students will drop out during their first year

wants to maximize total tuition while limiting the tot
attrition to 600 first-year students

Decision variables

- Let x1 = Number of in-state students admitted
- Let x2 = Number of out-of-state students admitted

Objective Function Maximize

$$Z= \$7,600X1 + 22,500X2$$

Constraints

X1 + X2 <= 3,500
X2/(X1 + X2)<=0.40
0.12X1 + 0.24X2 <=600
X1,X2 >=0

Mathematical Model

Maximize Z = $7,600X1 + 22,500

X2

Subject to

X1 + X2 <= 3,500
X2/X1 + X2<=0.40
0.12X1 + 0.24X2 <=600
X1,X2 >=0

RHS ⇒ Max يشمل

Right hand Side الاحتمال

Example 2

- The production manager at the Boston Paint Company is preparing production and inventory plans for next year.

- The production manager has the following data concerning the firm.

Quarter	Sales Forecast
1	3,000 units
2	1,800
3	2,400
4	3,500

- Current inventory level = 300 units.
- Current employment level = 600 people.
- Production rate last quarter = 2,400 units (4 units/employee/quarter).
- Inventory carrying cost = $20/unit/quarter
- Hiring cost = $200/employee hired.
- Layoff cost = $200/employee laid off.

يعني بعض المسائل بيعطيك ال profit على طول او بعضها بيعطيك السعر وال cost وانت بتحسب ال profit وبعضها بيعطيك بس السعر وعليه انت بتحسب ال profit
مركزه دقيقه ومنطقي

- Regular time production cost per unit = $320/unit.
- Additional cost of overtime = $60/unit.
- Desired closing inventory level = 100 units (minimum).

Define the decision variables

X_t = Regular production during Quarter t

O_t = Overtime production during Quarter t

I_t = Ending inventory in Quarter t

H_t = Number hired in Quarter t

F_t = Number fired in Quarter t

W_t = Number of employees in Quarter t

S_t = Slack regular production during Quarter t

بطء الانتاج

Objective function (Minimize the sum of inventory holding cost + Hiring & Firing cost + Regular & ovetime production cost)

Minimize:

$20*(I_1 + I_2 + I_3 + I_4) + 200*(H_1 + H_2 + H_3 + H_4) + 200*(F_1 + F_2 + F_3 +$
$F_4)$
$+ 320*(X_1 + X_2 + X_3 + X_4) + 380(O_1 + O_2 + O_3 + O_4)$

<u>Subject to:</u>

$I_0 + X_1 + O_1 - I_1 = 3{,}000$

$I_1 + X_2 + O_2 - I_2 = 1{,}800\ I_2 +$

$X_3 + O_3 - I_3 = 2{,}400\ I_3 + X_4 +$

$O_4 - I_4 = 3{,}500$

I_4 □ 100

$I_0 = 300$

$W_0 = 600$

$4\ W_1 - X_1 - S_1 = 0$

$4\ W_2 - X_2 - S_2 = 0$

$4\ W_3 - X_3 - S_3 = 0$

$4\ W_4 - X_4 - S_4 = 0$

$W_0 - W_1 + H_1 - F_1 = 0$ $W_1 - W_2 + H_2$

$- F_2 = 0$ $W_2 - W_3 + H_3 - F_3 = 0$

$W_3 - W_4 + H_4 - F_4 = 0$

Chapter 2:

Modeling with Linear Programming & sensitivity analysisrt

A lecture by
Dr. Moayad Tanash

Optimal solution

Graphical

Two dision variables

x_1
x_2

LINEAR PROGRAMMING (LP)

-In mathematics, **linear programming (LP)** is a technique for optimization of a linear objective function, subject to linear equality and linear inequality constraints.

-Linear programming determines the way to achiev the best outcome (such as maximum profit or lowes cost) in a given mathematical model and given som list of requirements represented as linear equations

TWO-VARIABLE LP MODEL

Example 2.1-1 (The Reddy Mikks Company)

Reddy Mikks produces both interior and exterior paints from two raw materials M1 and M2

	Tons of raw material per ton of		
	Exterior paint	Interior paint	Maximum daily availability (tons)
Raw material M1	6	4	24
Raw material M2	1	2	6
Profit per ton ($1000)	5	4	

-Daily demand for interior paint cannot exceed that of exterior paint by more than 1 ton

-Maximum daily demand of interior paint is 2 tons

-Reddy Mikks wants to determine the optimum product mix of interior and exterior paints that maximizes the total daily profit

Solution:

Let

$X_1 =$ tons produced daily of exterior paint

$X_2 =$ tons produced daily of interior paint Let z represent the total daily profit (in thousands of dollars)

Objective:

??

Solution:

Let

X_1 = tons produced daily of exterior paint

X_2 = tons produced daily of interior paint Let z

represent the total daily profit (in thousands of dollars)

Objective:

Maximize $Z = 5X_1 + 4X_2$

(Usage of a raw material by both paints) $\leq$ (Maximum raw material availability)

Usage of raw material M1 per day = $6X_1 + 4X_2$ tons

Usage of raw material M2 per day = $1X_1 + 2X_2$ tons

- daily availability of raw material M1 is 24 tons
- daily availability of raw material M2 is 6 tons

Restrictions:

$6X_1 + 4X_2 \leq 24$ (raw material M1)

$X_1 + 2X_2 \leq 6$ (raw material M2)

- Difference between daily demand of interior (X_2) and exterior (X_1) paints does not exceed 1 ton,

so $X_2 - X_1 \leq 1$

- Maximum daily demand of interior paint is 2 tons, so $x_2 \leq 2$

- Variables x_1 and x_2 cannot assume negative values, so $x_1 \geq 0$, $x_2 \geq 0$

Complete Reddy Mikks model:

Maximize $z = 5x1 + 4x2$ (total daily profit)

subject to

$6x1 + 4x2 \leq 24$ (raw material M1) $x1 + 2x2 \leq 6$ (raw material M2)

$x2 - x1 \leq 1$ $x2 \leq 2$

$x1 \geq 0$

$x2 \geq 0$

- Objective and the constraints are all linear functions in this example.7

Properties of the LP model:

Linearity implies that the LP must satisfy three basic properties:

التناسب

1) Proportionality:

- Contribution of each decision variable in both the objective function and constraints to be directly proportional to the value of the variable

2) Additivity:

-Total contribution of all the variables in the objective function and in the constraints to be the direct sum of the individual contributions of each variable

Properties of the LP model:

3) Certainty:

- All the objective and constraint coefficients of the LP model are deterministic (known constants)
- LP coefficients are average-value approximations of the probabilistic distributions
- If standard deviations of these distributions are sufficiently small , then the approximation is acceptable

- Large standard deviations can be accounted for directly by using stochastic LP algorithms or indirectly by applying sensitivity analysis to the optimum solution

Feasible Solutions for Linear Programs:

FEASIBLE SOLUTION

INFEASIBLE SOLUTION

Feasible Solutions for Linear Programs:

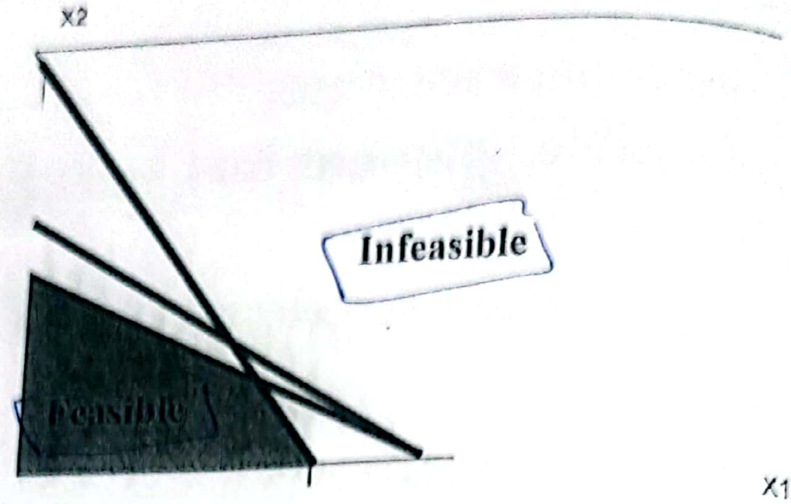
- The set of all points that satisfy all the constraints of the model is called

FEASIBLE SOLUTION

Otherwise, the solution is

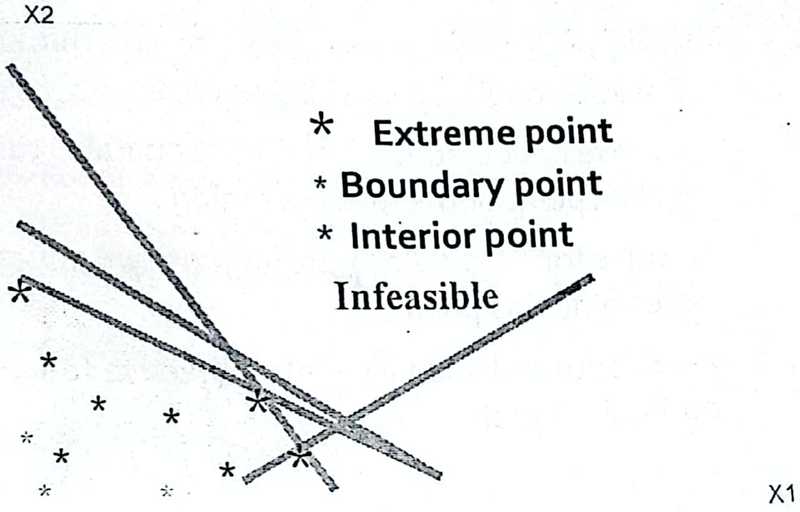
هو قيم x_1 و x_2 الي يحققوا ال constrains في نفس الوقت

INFEASIBLE SOLUTION



Type of feasible points

- Interior point: satisfies all constraint but non with equality.
- Boundary points: satisfies all constraints, at least one with equal
- Extreme point: satisfies all constraints, two with equality.



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* Optimal Solution :-

* If a linear programming has a unique optimal solution , then one of the extreme point is optimal.

* Summery of graphical solution procedure

1- graph constraint to find the feasible point

2- set objective function equal to an arbitrary value so that lin passes through the feasible region.

3- move the objective function line parallel to itself until it touche the last point of the feasible region .

4- solve for X_1 and X_2 by solving the two equation that intersect t determine this point

5- substitute these value into objective function to determine it optimal solution.

Complete Reddy Mikks model:

Maximize $z = 5X_1 + 4X_2$ (total daily profit)

subject to

$6X_1 + 4X_2 \le 24$ (raw material M_1)

$X_1 + 2X_2 \le 6$ (raw material M_2)

$X_2 - X_1 \le 1$

$X_2 \le 2$ $X_1 \ge 0$

$X_2 \ge 0$

2.2.1 Solution of a Maximization model

Step 1:

Example 2.2-1 (Reddy Mikks model)

1) Determination of the feasible solution space:

- Find the coordinates for all the 6 equations of the restrictions (only take the equality sign)

$6X_1 + 4X_2 \leq 24$ 1

$X_1 + 2X_2 \leq 6$ 2

$X_2 - X_1 \leq 1$ 3

$X_2 \leq 2$ 4

$X_1 \geq 0$ 5

$X_2 \geq 0$ 6

- Change all equations to equality signs

$6X_1 + 4X_2 = 24$ 1

$X_1 + 2X_2 = 6$ $2X_2 - X_1 = 13$ X_2

$= 2$ $4X_1 = 0$ $5X_2 = 0$

6

0

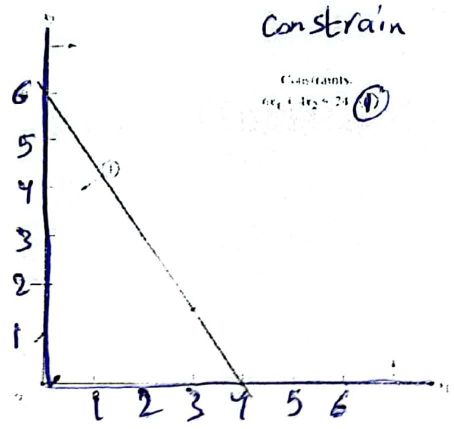
0

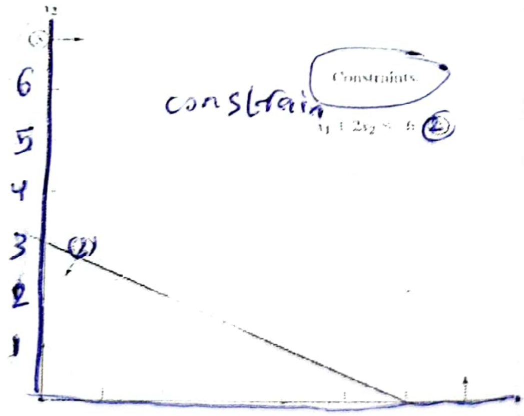
0

0

0

- Plot graphs of $x_1 = 0$ and $x_2 = 0$ - Plc





coordinates of the equation

Assume $x1 = 0$ → $x2 = 6$

Assume $x2 = 0$ → $x1 = 4$

- Plot graph of $x1 + 2x2 = 6$ by using the coordinates of the equation

Assume $x1 = 0$ → $x2 = 3$ Assume $x2 = 0$ $x1 = 6$

- Plot graph of $x_2 - x_1 = 1$ by using the coordinates of the equation ($x1=-1$, $x2=0$) and

($x1=0, x2=1$)

- Plot graph of $x_2 = 2$ by using the coordinates of the equati

Pinding constrains

extreme point

Slope = 4/5

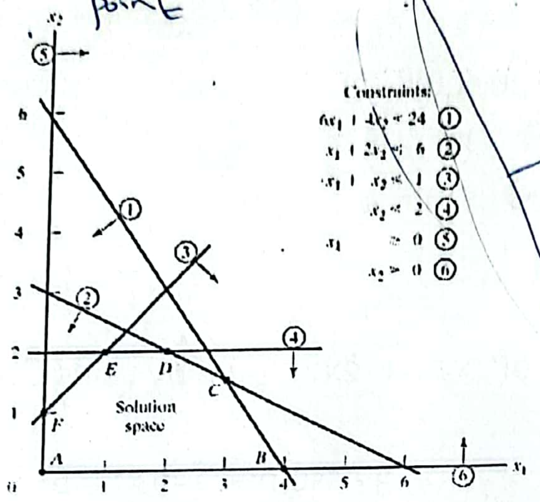


Figure 2.1
Feasible space of the Reddy Mikks model.

- Now include the inequality of all the 6 equations
- Inequality divides the (x1, x2) plane into two half spaces, one on each side of the graphed line
- Only one of these two halves satisfies the inequality
- To determine the correct side, choose (0,0) as a reference point
- If (0,0) coordinate satisfies the inequality, then the side in which (0,0) coordinate lies is the feasible half-space, otherwise the other side is

- If the graph line happens to pass through the origin (0,0) , then any other point can be used to find the feasible half-space

Step 2:

2) Determination of the optimum solution from among all the feasible points in the solution space:

- After finding out all the feasible half-spaces of all the 6 equations, feasible space is obtained by the line segments joining all the corner points A, B, C, D ,E and F

- Any point within or on the boundary of the solution space ABCDEF is feasible as it satisfies all

the constraints

- Feasible space ABCDEF consists of infinite number of feasible points

- To find optimum solution identify the direction in which the maximum profit increases , that is $z = 5x1 + 4x2$

- Assign random increasing values to z , z = 10 and z = 15

$5x1 + 4x2 = 10$

$5x1 + 4x2 = 15$

- Plot graphs of above two equations

- Thus in this way the optimum solution occurs at corner point C which is the point in the solution space

- Any further increase in z that is beyond corner point C will put points outside the boundaries of ABCDEF feasible space

- Values of X_1 and X_2 associated with optimum corner point C are determined by solving the equations and

$6x_1 + 4x_2 = 24$ $\qquad$ $x_1 + 2x_2 = 6$

- $X_1 = 3$ and $X_2 = 1.5$ with $z = 5 \times 3 + 4 \times 1.5 = 21$

- So daily product mix of 3 tons of exterior paint and 1.5 tons of interior paint produces the daily profit of $21,000.

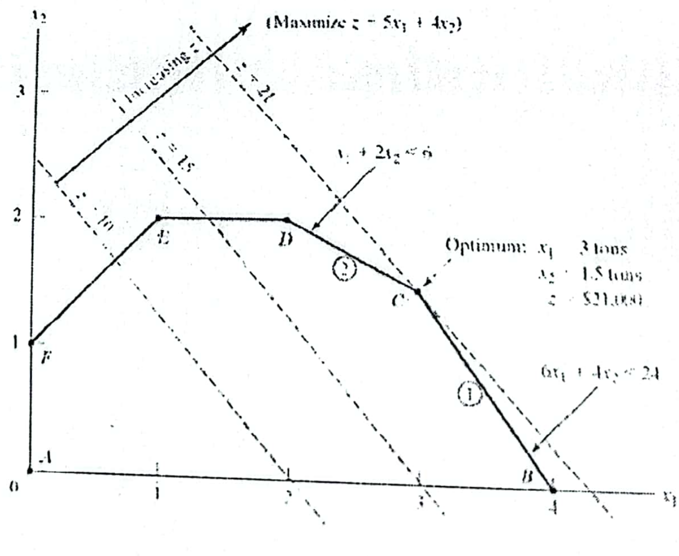


Figure 2.2
Optimum solution of the Reddy Mikks model.

- Important characteristic of the optimum LP solution is that it is always associated with a corner point of the solution space (where two lines intersect)
- This is even true if the objective function happens to be parallel to a constraint
- For example if the objective function is,

$z = 6x_1 + 4x_2$

- The above equation is parallel to constraint of equation
- So optimum occurs at either corner point B or corner point C when parallel

- Actually any point on the line segment BC will be an alternative optimum
- Line segment BC is totally defined by the corner points B and C

Corner points	(X_1, X_2)	$Z = 600\,X_1 + 400\,X_2$
A	(0, 40)	16000 B
(12,4)	8800	
C	(22,0)	13200

- In 12 days all the three types of bottles (Coca-cola, Fanta, Thumps-up) are produced by plant at Coimbatore
- In 4 days all the three types of bottles (Coca-cola, Fanta,

Thumps-up) are produced by plant at Chennai

- So minimum production cost is 8800 units to meet the market demand of all the three types of bottles (Coca-cola, Fanta,

Thumps-up) to be produced in April

- Since optimum LP solution is always associated with a corner point of the solution space, so optimum solution can be found by enumerating all the corner points as below:-

	Corner point	(X_1,X_2)	Z
A	(0,0)	0	
B	(4,0)	20	
C	(3,1.5)	21 (optimum solution)	
D	(2,2)	18	
E	(1,2)	13	
F	(0,1)	4	

- As number of constraints and variables increases , the number of corn points also increases

2.2.2 Solution of a Minimization model

Example 2.2-3

- Firm or industry has two bottling plants - One plant located at Coimbatore and other plant located at Chennai

- Each plant produces three types of drinks Coca-cola , Fanta and Thumps-up

Number of bottles produced per day by plant at

	Coimbatore	Chennai
Coca-cola	15,000	15,000
Fanta	30,000	10,000
Thumps-up	20,000	50,000
Cost per day (in any unit)	600	400

-Market survey indicates that during the month of April there will be a demand of 200,000 bottles of Coca-cola , 400,000 bottles of Fanta , and 440,000 bottles of Thumps-up

Solution:

- For how many days each plant be run in April so as to minimize the production cost , while still meeting the market demand?

Let X_1 = number of days to produce all the three types of bottles by plant at Coimbatore X_2 = number of days to produce all the three types of bottles by plant at Chennai

Objective:

Minimize $z = 600\,X_1 + 400\,X_2$

Constraint:

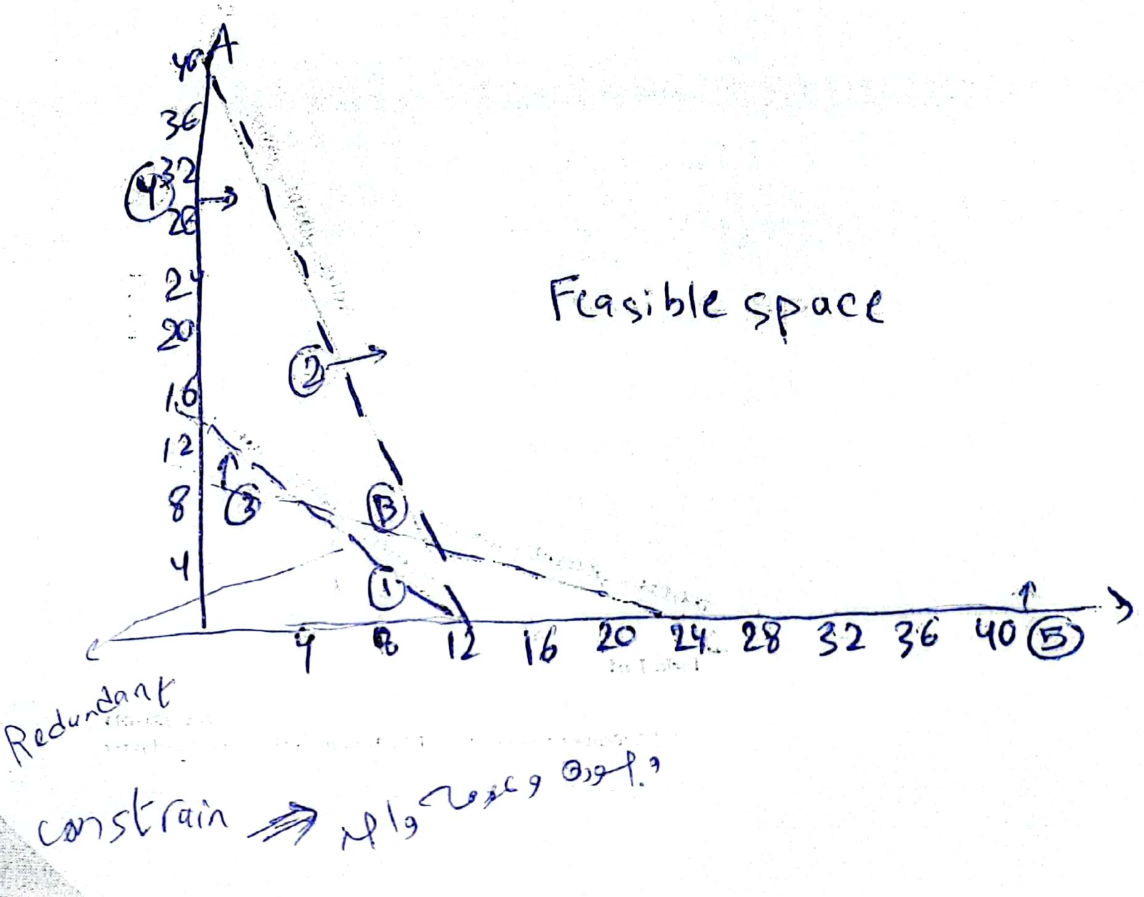
$15{,}000\,X_1 + 15{,}000\,X_2 \geq 200{,}000$

$30{,}000\, X_1 + 10{,}000\, X_2 \geq 400{,}000$

$20{,}000\, X_1 + 50{,}000\, X_2 \geq 440{,}000$

$X_1 \geq 0$

$X_2 \geq 0$



Chapter 3: The Simplex Method and Sensit Analysis

A lectu

Dr. Moayad T

Simplex Method

- Most real-world LP problems have more than two decision variables.

- Graphical solution procedure can not be used to solve such problems.

instead .
- A simplex method can be used to find the optimal solution

Simplex Method

- The simplex method provides an algorithm which is based on the fundamental theorem of linear programming. This states that "the optimal solution to a linear programming problem if it exists, always occurs at one of the corner points of the feasible solution space."

Simplex Method

It consists of:

- (i) Having a trial basic feasible solution to constraint equation,
- (ii) Testing whether it is an optimal solution
- (iii) Improving the first trial solution by repeating th process till an optimal solution is obtained

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Computational Procedure of Simplex Method:

Convert each inequality constraint in an LP formulatio into an equation .

- replace <= constraints to equations by adding sla variables.
- Example

- $6x_1+4x_2 \leq 24$ can be written as
- $6x_1+4x_2+S_1=24$, $S_1 \geq 0$

Computational Procedure of Simplex Method:

- Replace $\geq$ constraints to equations by adding surplus variables.
- Example:

- $x_1+x_2 \geq 800$ can be written as
- $x_1+x_2-R_2=800$, $R_2 \geq 0$

Computational Procedure of Simplex Method:

- Replace $\geq$ constraints to equations by adding surplus variables.
- Example:
- $x_1+x_2 \geq 800$ can be written as

- $x_1+x_2-S_2=800,\ S_2\geq 0$
- $X_1+x_2=800+S_2$

Optimality Conditions

- The entering variable in a maximization (minimizatio
problem is the non basic variable having the mo
negative
(positive) coefficient in the objective function

- The optimal solution is reached at a given iteration where a
z-row coefficients of the nonbasic variables are nonnegati
(nonpositive)

- $Z-5X_1-4X_2$

Feasibility Conditions

- The For both maximization and minimization problems, th
leaving variable is the basic variable associated with

smallest nonnegative ratio (with strictly positive nominator)

X1 X2

- X1--- 5 6=24 ➔ 24/6=4
- S2----X1+X2=5 ➔ 5/1 =5

* لما نريد ال Disison variables عن 3 نفحص Simplex algorithm.

* ال entering variable هو اعلى قيمة سالبة في حالة LP max. والعكس صحيح.

* ال Leaving variable ممنوع يكون سالب او ∞.

Simplex Method

- **Step 1.** determine a starting basic feasible solution
- **Step 2.** Select an entering variable using the optimality conditions. Stop if there is no entering variable; the last solution is optimal. Else, go to step 3.
- **Step 3.** select leaving variable using the feasibility conditions.

• **Step 4.** Determine the new basic solution by using the appropriate Gauss-Jordan computation. Go to step 2

Simplex Method

• Gauss-Jordan row operation:

1- Pivot row a- replace the leaving variable in the basic column with the entering variable

b- new pivot row = Current pivot row / pivot element

2- All other rows including z

New row = current row – pivot column coefficient

* new pivot r

Example: Reddy Mikks model

subject to:

Maximize $z = 5x_1 + 4x_2$ (total daily profit)

$6x_1 + 4x_2 \leq 24$ (raw material M1)

$x_1 + 2x_2 \leq 6$ (raw material M2)

X2 - X1 ≤1

X2 ≤2 X1 ≥

0

X2 ≥0

- replace <= constraints to equations by adding slack variables and add them to the objective function .

Maximize $z = 5X1 + 4X2 + 0S_1 + 0S_2 + 0S_3 + 0S_4$

Subject to:

$6X1 + 4X2 + S_1 = 24$

$X1 + 2X2 + S_2 = 6$

$X2 - X1 + S_3 = 1$

$X2 + S_4 = 2$

$X1, X2, S_1, S_2, S_3, S_4 \geq 0$

the variables S_1, S_2, S_3 and S_4

Are the slacks associated with the respective constraints. Next, we write the objective equation as

Z-5x1-4x2-0=0

Itteration#1

The starting simplex tableau is

	x1	x2	S1	S2	S3	S4	RHS	ratio
Z	-5	-4	0	0	0	0	0	
S1	6	4	1	0	0	0	24	4
S2	1	2	0	1	0	0	6	6
S3	-1	1	0	0	1	0	1	-1
S4	0	1	0	0	0	1	2	Infinity

Note that,

- X1 and x2 are Non basic variables (zero)
- S1,S2,S3 and S4 are basic variables (non zero)

Itteration#1

- The entering variable corresponds to the most negative coefficient in the objective function.
- In this example x1 has the most negative coefficient (-5)

Itteration#1

- To determine the leaving variable from the basic solution, we compute the ratio between the RHS and the coefficient of the x_1 in that constraint
- The basic variable with the lowest ration is the one who leaves the bases

* ال slack هم ال variables الي ما بأثروا على objective function.
* نقاط تلاقي ال slack قيمتهم بتكون (1) لانه identical matrix.
* الي تحت ال Basic variables لازم يعطيني identical matrix
بمعناه انه نقاط تلاقي ال slacks مع بعض بتكون قيمتهم 1 والباقي صفر.
* ال Ratio هي النسبة بين ال RHS (Right hand side).
الي هي قيمة ال Z (objective function) و ال entering v.

Itteration#1

X_1 will entry the base instead of S_1
The swapping process is based on Gauss-Jordan row operations. It is identify the entering variable column as the pivot column and the leaving variable raw as the pivot row. The intersection between the pivot column and the pivot raw is called the pivot element

non-basic variables, Reduce cost variables

	x1 (pivot column)	x2	s1	s2	s3	s4	RHS	ratio	
Z	-5	-4	0	0	0	0	0		
S1	6 (pivot element)	4	1	0	0	0	24	24/6 =4	minimum (pivot row)
S2	1	2	0	1	0	0	6	6/1 = 6	
S3	-1	1	0	0	1	0	1	-1/1 =-1	ignore
S4	0	1	0	0	0	1	2	2/0=infinity	ignore

Basic variables

القيمة الاكبر من ال non-basic v. بتكون هي ال entering v. بتكون pivot column.

Itteration#1

Next

1- divide the pivot row by the pivot element.

2- replace the leaving variable in the basic column with the entering variable.

-5X1-4X2=0

X1+2/3X2+1/6S1

	x1	x2	s1	s2	s3	s4	RHS	ratio
Z	-5	-4	0	0	0	0	0	
x1	6/6=1	4/6=2/3	1/6	0	0	0	24	
S2								
S3								
S4								

اللي هو معامل x1 اللي هو entering variable

- For other non pivot rows:

Itteration#1

New row = current row – (its pivot column coefficient)*(new pivot row)

Gauss-Jordan elmenatib

-5X1-4X2=0

(X1+2/3X2+1/6S1=4)*-5

-5X1-4X2=0

-5X1-10/3X2+5/6S1=-20

0X1-2/3X2+5/6S1=20

	x1	x2	s1	s2	s3	s4	RHS	ratio
Z	0	-2/3	5/6	0	0	0	20	
x1	1	2/3	1/6	0	0	0	4	
S2								
S3								
S4								

Current z row - x1 row Multiply by -5

	x1	x2	s1	s2	s3	s4	RHS	ratio
Z	0	-2/3	5/6	0	0	0	20	
x1	1	2/3	1/6	0	0	0	4	
S2	0	1 1/3	-1/6	1	0	0	2	
S3								
S4								

Current z row - x1 row Multiply by

Current s2 row - x1 row Multiply by

Itteration#1

	x1	x2	s1	s2	s3	s4	RHS	ratio
Z	0	- 2/3	5/6	0	0	0	20	
x1	1	2/3	1/6	0	0	0	4	
S2	0	1 1/3	- 1/6	1	0	0	2	
S3	0	1 2/3	1/6	0	1	0	5	
S4								

Current z row - x1 row Multiply by -5

Current s2 row - x1 row Multiply by 1

Current s3 row - x1 row Multiply by -1

	x1	x2	s1	s2	s3	s4	RHS	ratio
Z	0	- 2/3	5/6	0	0	0	20	
x1	1	2/3	1/6	0	0	0	4	
S2	0	1 1/3	- 1/6	1	0	0	2	
S3	0	1 2/3	1/6	0	1	0	5	
S4	0	1	0	0	0	1	2	

Current z row - x1 row Multiply by -5

Current s2 row - x1 row Multiply by 1

Current s3 row - x1 row Multiply by -1

Current s4 row - x1 row Multiply by 0

The new basic solution now is (x1=4, s2=2, s3=5 and s4=2)

The new objective value is z=20.

$$Z= 5x_1+4x_2+0S_1+0S_2+0S_3+0S_4$$

New z = Old z+ (5*4+4*0+0*0+0*2+0*5+0*2)

Itteration#2

X2 will entry the base and S2 leaves the base

	x1	x2	s1	s2	s3	s4	RHS	ratio
Z	0	- 2/3	5/6	0	0	0	20	
x1	1	2/3	1/6	0	0	0	4	6
S2	0	1 1/3	- 1/6	1	0	0	2	1 1/2
S3	0	1 2/3	1/6	0	1	0	5	3
S4	0	1	0	0	0	1	2	2

minimum

Itteration#2

poivt column

	x1	x2	s1	s2	s3	s4	RHS	ratio	
Z	0	- 2/3	5/6	0	0	0	20		
x1	1	2/3	1/6	0	0	0	4	6	
S2	0	1 1/3	- 1/6	1	0	0	2	1 1/2	minimum (piovt row)
S3	0	1 2/3	1/6	0	1	0	5	3	
S4	0	1	0	0	0	1	2	2	

	x1	x2	s1	s2	s3	s4	RHS	ratio	
Z	0	0	3/4	1/2	0	0	21		Current z row - x2 row Multiply by -2/3
x1	1	-0	1/4	- 1/2	0	0	3		Current x1 row - x2 row Multiply by 2/3
x2	0	1	- 1/8	3/4	0	0	1 1/2		
S3	0	-0	3/8	-1 1/4	1	0	2 1/2		Current s3 row - x2 row Multiply by 1 2/3
S4	0	-0	1/8	- 3/4	0	1	1/2		Current s4 row - x2 row Multiply by 1

- Based on the optimality conditions, non of the z-row coeffici associated with the nonbasic variables s1 and s2 are negati

optimal . Hence the last tableau is

$X_1=3$

$X_2 = 3/2$

$Z= 21$

- The optimal solution for the above example is

Chapter 3: The Simplex Method and Sensitivity Analysis

الى هنا مادة ال
First

A lecture by:
Dr. Moayad Tanash

Special Cases in Simplex Method

- Degeneracy
- Alternative Optimal
- Unbounded solutions
- Infeasible solutions → conflict between constrains.

$x_1 + x_2 \le 500$

$x_1 + x_2 \ge 2000$

Degeneracy

- The simplex algorithm starts at a corner point and moves to an adjacent corner point by increasing the value of a non-basic variable with a positive cost coefficient.
- Typically, the entering variable xs does increase in value, and the objective value z improves.
- It is possible that xs does not increase at all. This situation can occur when one of the RHS coefficients is 0.

- In this case, the objective value and solution does not change, but there is an exiting variable. This situation is called degeneracy.

Example : Degeneracy

Maximize z=3x1+9x2

Subject to:

X1+4x2<=8

X1+2x2<=4

X1,x2>=0

Example : Degeneracy

	x1	x2	S1	S2	RHS	Ratio
Z	-3	-9	0	0	0	
S1	1	4	1	0	8	2
S2	1	2	0	1	4	2

	x1	x2	S1	S2	RHS	Ratio
Z	-0.75	0	2.25	0	18	
x2	0.25	1	0.25	0	2	8
S1	0.5	0	-0.5	1	0	0

	x1	x2	S1	S2	RHS	Ratio
Z	0	0	1.5	1.5	18	
x2	0	1	0.5	-0.5	2	
x1	1	0	-1	2	0	

Example : Degeneracy

- Degeneracy implications :

1- Cycling phenomena

2- Each iteration leads to the same objective function value

Alternative optimal solution

Subject to

When the objective function is parallel to a nonredundant binding constraint, the objective function can assume the same optimal value at more than one solution point, thus giving rise to alternative optimal.

Example: Alternative optimal solution

Maximize z=2x1+4x2

* في حال كان ال slope
لل objective function موازي
نفسه
لو واحدة من ال constrains
يكون موازي الها.

x1+2x2<=5 x1+x2<=4 X1,x2>=0

ويكون عنده multi optimal sol.
هنتكلم عنه كمان

Example: Alternative optimal solution

	x1	x2	S1	S2	RHS	Ratio
Z	-2	-4	0	0	0	
S1	1	2	1	0	5	2.5
S2	1	1	0	1	4	4

	x1	x2	S1	S2	RHS	Ratio
Z	0	0	2	0	10	
x2	0.5	1	0.5	0	2.5	5
S1	0.5	0	-0.5	1	1.5	3

	x1	x2	S1	S2	RHS	Ratio
Z	0	0	2	0	10	
x2	0	1	1	-1	1	
x1	1	0	-1	2	3	

$0 \le \alpha \le 1$

$x_1 = \alpha x_1'' + (1-\alpha) x_1'$

Example: Alternative optimal solution

- The simplex method determine only the two corner points B (0,2.5 and C (3,1).

Mathematically, we can determine all the points (x1,x2) on that lir segment as a nonnegative weighted average points B and C, thus al points on the line segment BC are given by:

$$\hat{x}_1 = \alpha(0) + (1-\alpha)(3) = 3 - 3\alpha$$
$$\hat{x}_2 = \alpha(\tfrac{5}{2}) + (1-\alpha)(1) = 1 + \tfrac{3}{2}\alpha \quad 0 \le \alpha \le 1$$

when α = 0 → (x1,x2) = (3,1) and when α = 1 → (x1,x2) = (0,2.5)

Unbounded solution

- In some models, the value of the variables may be increased indefinitely without violating any of the constraints meaning that the solution space in at least one variable.

- Unbounded points to the possibility that the model is poorly constructed . • Most likely irregularity in such models is that one or more nonredundant constraints have not been counted for and the, parameters (constants) of some constraints may not have been estimated correctly.

Example: Unbounded solution

Maximize $z = 2x_1+x_2$

Subject to

$x_1-x_2 \le 10$ / $2x_1 \le 40$

$x_1,x_2 \ge 0$

	x1	x2	S1	S2	RHS	Ratio
Z	-2	-1	0	0	0	
S1	1	-1	1	0	10	
S2	2	0	0	1	40	

Example: Unbounded solution

Infeasible solution

Maximize z = 3X1+2X2

Subject to

2X1+X2<=2

3X1+4X2>=12

X1,X2>=0

Example: Infeasible solution

	x1	x2	S1	S2	R1	RHS	ratio
z	-3	-2	0	0	100	0	
S1	2	1	1	0	0	2	
R1	3	4	0	1	1	12	

	x1	x2	S1	S2	R1	RHS	ratio
z	-303	-402	0	100	0	-1200	
S1	2	1	1	0	0	2	2
R1	3	4	0	-1	1	12	3

	x1	x2	S1	S2	R1	RHS	ratio
z	501	0	402	100	0	-396	
x2	2	1	1	0	0	2	
R1	-5	0	-4	-1	1	4	

Chapter 3: The Simplex Method and Sensitivity Analysis

A lecture by:
Dr. Moayad Tanash

M Method

The M-method is a version of the Simplex algorithm that first finds a basic feasible solution by adding " artificial variables to the problem. The objective function of the original LP must, of course, be modified to ensure that the artificial variables are all 0 at the conclusion of the simplex algorithm

M-Method Steps

1.Modify the constraints so that the RHS of each constraint is nonnegative (This requires that each constraint with a negative RHS be multiplied by - 1.

Remember that if you multiply an inequality by any negative number, the direction of the inequality is reversed!). After modification, identify each constraint as <= or = constraint.

2.Convert each inequality constraint to standard form (If constraint i is a < constraint, we add a slack variable Si; and if constraint i is a > constraint, we subtract an excess variable Ri).

3.Add an artificial variable ai to the constraints identified as > or = constraints at the end of Step 1. Also add the sign restriction ai > 0.

$x_1 \geq 10$, $x_1 \leq 10$ ⇒ $x_1 = 10$

max ⇒ $z - MR_1$

min ⇒ $z + MR_2$

M-Method Steps

- 4. If the LP is a max problem, add (for each artificial variable) -MRi to the objective function where M denote a very large positive number.
- 5. If the LP is a min problem, add (for each artificial variable) MRi to the objective function.
- 6. Solve the transformed problem by the simplex . Since each artificial variable will be in the starting basis, all artificial variables must be eliminated from row 0 before beginning the simplex.
- Now (In choosing the entering variable, remember that M is a very large positive number!). use the
- objective function coefficients to approximate the value of M

If all artificial variables are equal to zero in the optimal solution, we have found the optimal solution to the original problem. If any artificial variables are positive in the optimal solution, the original problem is infeasible!!!

Example (M-Method Steps)

Minimize z=4x1+x2

Subject to

3x1+x2=2

4x1+3x2>=6

x1+2x2<=4

x1,x2>=0

* مافي في الـRHS قيم سالبة معناه انا ضمنت الـ feasability

* وجود الاشارات ستجبرني انه يكون الحل على Big M method

Example (M-Method Steps)

To convert the constraint to equations, use x3 as a surplus in the second constraint and x4 as a slack in the third constraint. Thus

Minimize $z=4x1+x2$

Subject to

$3x1 + x2 = 3$

$4x1 + 3x2 - x3 = 6$

$x1 + 2x2 + x4 = 4$

$x1, x2, x3, x4 \geq 0$

* تذكري لما تضرب في سالب انك بتقلب الاشارات

* ولما الا عندي mix بين الاشارات في انا محتاجة Big M method

M-Method Steps

The third equation has its slack variable, x4, but the first and second equations do not. Thus, we add the artificial variables R1 and R2 in the first two equations and penalize them in the objective function with MR1 +
MR2 (because we are minimizing). The resulting LP becomes

Subject to

Minimize $z = 4x1 + x2 + MR1 + MR2$ $+0x_3 + 0x_4$

3x1 + x2 + R1 =

3 4x1 + 3x2 - x3 +

R2 = 6 x1 + 2x2 +

x4 = 4 x1, x2, x3,

x4, R1, R2 ≥0

M-Method Steps

- From the standpoint of solving the problem using the computer, M must be a numerical value.
- What value for M should we use ?
- The objective function coefficients 4 and ,1 for x1,x2, respectively. Thus, it seems reasonable to use 100 for M

M-Method Steps

Using M=100, the starting simplex tableau is

	x1	x2	x3	R1	R2	x4	RHS	Ratio
Z	-4	-1	0	-100	-100	0	0	
R1	3	1	0	1	0	0	3	
R2	4	3	-1	0	1	0	6	
x4	1	2	0	0	0	1	4	

M

	x1	x2	x3	R1	R2	x4	RHS	Ratio
z	696	399	-100	0	0	0	900	
R1	3	1	0	1	0	0	3	
R2	4	3	-1	0	1	0	6	
x4	1	2	0	0	0	1	4	

-Method Steps

This tableau is ready for us to apply the Simplex method.

M-Method Steps

- Before proceeding with Simplex method computations, we need to make the *z-row* consistent with the rest of the tableau.
- X1=x2=x3=0, which yield the starting basic solution S1=3, S2=6 and S3=4.
- The new z-row can be computed as :

New z-row =old z-row+100*(S1-row)+100(S2-row)

M-Method Steps

	x1	x2	x3	R1	R2	x4	RHS	Ratio	
Z	696	399	-100	0	0	0	900		
R1	3	1	0	1	0	0	3	1	mini
R2	4	3	-1	0	1	0	6	1.5	
x4	1	2	0	0	0	1	4	4	

Iteration #1

	x1	x2	x3	S1	S2	x4	RHS	Ratio	
Z	0	167	-100	-232	0	0	204		(Z-row)- (x1-row)*696
x1	1	0.333333	0	0.333333	0	0	1		(x1-row)/3
R2	0	1.666667	-1	-1.33333	1	0	2		(S2-row)-(x1-row)*4
x4	0	1.666667	0	-0.33333	0	1	3		(S3-row)-(x1-row)*1

167 سالبة

min infeasable solution

2 or 3 digits

$z = 4x_1 + 5x_2 + 5$ RHS

$z - 4x_1 - 5x_2 = 5$

M-Method Steps

	x1	x2	x3	S1	S2	X4	RHS	Ratio
z	0	0	0.2	-98.4	-100.2	0	3.6	
x1	1	0	0.19998	0.599973	-0.19998	0	0.60004	3.0005
x2	0	1	-0.6	-0.8	0.6	0	1.2	-2
x4	0	0	1	1	-1	1	1	1 min!

Iteration #3

	x1	x2	x3	S1	S2	x4	RHS	Ratio	Note
z	0	0	0	-98.6	-100	-0.2	3.4		(z-row)-(x3-row)*0.2
x1	1	0	-1.98E-05	0.399974	1.98E-05	-0.2	0.40004		(x1-row)-(x3-row)*0.1998
x2	0	1	0	-0.2	0	0.6	1.8		(x2-row)-(x3-row)*-0.6
x3	0	0	1	1	-1	1	1		(x3-row)/1

Two-Phase method

phase I

Put the problem in equation form and add the necessary artificial variables to the constraints (exactly as in the M-method) to secure a starting basic solution. Next, find a basic solution of the resulting equations that always minimizes the sum of the artificial variables, regardless of whether the LP is maximization or minimization. If the minimum value of the sum is positive, the LP problem has no feasible solution. Otherwise, proceed to Phase II.

phase II.

Use the feasible solution from Phase I as a starting basic feasible solution for the original problem.