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ملخص :

بحوث عمليات (1)

Operation Research (1)

Second

للطالب : علاء الدرديسي

اللجنة الأكاديمية لقسم الهندسة الصناعية

2023



Matrix form

of simplex method

X : n variables n constraints

C : objective function n constraints

A : constraint matrix n variables n constraints

b : constraint RHS

C_B : objective function coefficients of basic variables

B : basic matrix

$C_B B^{-1} A - C$: reduced cost

Z	X		RHS	
	C_B	$B^{-1} A - C$	$C_B B^{-1} b'$	
X_B	$B^{-1} A$	$B^{-1} b'$		

$$Max\ Z = 5x_1 + 4x_2 \quad Max\ Z = 5x_1 + 4x_2 - M A_3$$

S.T

$$2x_1 + x_2 \leq 20$$

$$2x_1 + x_2 + S_1 = 20$$

$$x_1 + x_2 \leq 18$$

$$\approx) x_1 + x_2 + S_2 = 18$$

$$x_1 + 2x_2 \geq 12$$

$$x_1 + 2x_2 - S_3 + A_3 = 12$$

$$X = [x_1 \ x_2 \ S_1 \ S_2 \ S_3 \ A_3]$$

$$C = (5 \ 4 \ 0 \ 0 \ 0 \ -M)$$

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 & -1 & 1 \end{bmatrix}$$

$$b = \begin{bmatrix} 20 \\ 18 \\ 12 \end{bmatrix}$$

Step 1: Find the basic Variables

basic Variables $\Leftarrow$ constraints like x like

$$B = S_1 \quad S_2 \quad A_3$$

basic Variables

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

basic Variables
Matrix A

$$B^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B^{-1}A = \begin{bmatrix} 2 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 & -1 & 1 \end{bmatrix}$$

$$B^{-1}b = \begin{bmatrix} 20 \\ 18 \\ 12 \end{bmatrix}$$



Subject = SI Date = 20/11/2020

$$C_B = [0 \ 0 \ 0 \ -M]$$

موضوع
تاریخ

$$C_B B^{-1} A = [-M \ -2M \ 0 \ 0 \ M \ -M]$$

$$C_B B^{-1} b = [-12M]$$

$$C_B B^{-1} A - C = [-M-4 \ -2M-4 \ 0 \ 0 \ M \ 0]$$

	x_1	x_2	s_1	s_2	s_3	A_3	RHS
Z	$C_B B^{-1} A - C$						$C_B B^{-1} b$
s_1	0	0	1	0	1	1	
s_2	1	1	0	0	1	1	
A_3							

iteration 1 is optimized ✖

Subject	Day	Date						
	x_1	x_2	s_1	s_2	s_3	A_3	RHS	
2	$-M-5$	$-2M-4$	0	0	M	0	$-12M$	
s_1	2	1	1	0	0	0	20	
s_2	1	1	0	1	0	0	18	
A_3	1	2	0	0	-1	1	12	

بال 2-row في هذه السوال وأحد سالب هو $-2M-4$
 A_3 : leaving variable و x_2 entering variable

نبدأ من أول وجدده

$$B = s_1 \quad s_2 \quad x_2$$

$$B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

من ال blue

التي فوق بالشئ

$$B^{-1} = \begin{bmatrix} 1 & 0 & -0.5 \\ 0 & 1 & -0.5 \\ 0 & 0 & 0.5 \end{bmatrix}$$

$$B^{-1} A = \begin{bmatrix} \frac{3}{2} & 0 & 1 & 0 & 0.5 & -0.5 \\ 0.5 & 0 & 0 & 1 & 0.5 & -0.5 \\ 0.5 & 1 & 0 & 0 & -0.5 & 0.5 \end{bmatrix}$$

$$B^{-1}b' = \begin{bmatrix} 14 \\ 12 \\ 6 \end{bmatrix}$$

على اول C
 ملاحظنا
 وبالترتيب

$$C_B = [0 \quad 0 \quad 4]$$

$$C_B B^{-1}A = [2 \quad 4 \quad 0 \quad 0 \quad -2 \quad 2]$$

$$C_B B^{-1}b' = 24$$

$$C_B B^{-1}A - C = [-3 \quad 0 \quad 0 \quad 0 \quad -2 \quad M+2]$$

	x_1	x_2	s_1	s_2	s_3	A_3	RHS
2	-3	0	0	0	-2	$M+2$	24
s_1	$\frac{3}{2}$	0	1	0	0.5	-0.5	14
s_2	0.5	0	0	1	0.5	-0.5	12
x_2	0.5	1	0	0	-0.5	0.5	6

x_1 : entering S_1 : leaving

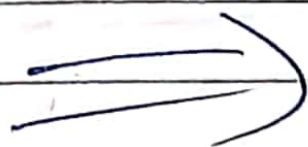
$$B = [X_1 \quad S_2 \quad X_2] = A^{-1}b$$

(entering S_1 A is possible)

$$B = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \quad B = \begin{bmatrix} \frac{2}{3} & 0 & -\frac{1}{3} \\ -\frac{1}{3} & 1 & -\frac{1}{3} \\ -\frac{1}{3} & 0 & \frac{2}{3} \end{bmatrix}$$

$$B^{-1}A = \begin{bmatrix} 1 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & -\frac{1}{3} & 1 & \frac{1}{3} & -\frac{1}{3} \\ 0 & 1 & -\frac{1}{3} & 0 & -\frac{2}{3} & \frac{2}{3} \end{bmatrix}$$

$$B^{-1}b = \begin{bmatrix} \frac{28}{3} \\ \frac{22}{3} \\ \frac{4}{3} \end{bmatrix}$$



entering S_1 is

Subject

X_1, S_2, X_2

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$$C_B = [5 \ 0 \ 4]$$

$$C_B B^{-1} A = [5 \ 4 \ 2 \ 0 \ 2 \ -1 \ M+1]$$

$$C_B B^{-1} b = 52$$

$$C_B B^{-1} A - C = [0 \ 0 \ 2 \ 0 \ -1 \ M+1]$$

Optimal Solution

Optimal Solution

	X_1	X_2	S_1	S_2	S_3	A_i	RHS
Z	0	0	2	0	-1	$M+1$	52
X_1							$28/3$
S_1							$22/3$
X_2							$4/3$

S_3 : increasing & S_2 : leaving

$$B = X_1 \quad S_3 \quad X_2 \quad \left[B, B^{-1} \text{ of } \text{mod } A \text{ is} \right]$$

$$B = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix} \quad B^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 3 & -1 \\ -1 & 2 & 0 \end{bmatrix}$$

$$B^{-1} A = \begin{bmatrix} 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 3 & 1 & -1 \\ 0 & 1 & -1 & 2 & 0 & 0 \end{bmatrix}$$

$$B^{-1} b = \begin{bmatrix} 2 \\ 22 \\ 16 \end{bmatrix} \quad \left. \vphantom{\begin{bmatrix} 2 \\ 22 \\ 16 \end{bmatrix}} \right\} C_B = \begin{bmatrix} 5 & 0 & 4 \end{bmatrix}$$

من الأرقام السابقة

$$C_B B^{-1} A = \begin{bmatrix} 4 & 5 & 1 & 3 & 0 & 0 \end{bmatrix}$$

$$C_B B^{-1} A - C = \begin{bmatrix} 0 & 0 & 1 & 3 & 0 & 0 \end{bmatrix}$$

$$C_B B^{-1} b = 74$$

$$\text{optimal } z = 74$$

$$X_1 = 2, \quad S_3 = 22, \quad X_2 = 16$$