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TURBO
Continuous change

دفتر :
ديناميكا
Dynamics

للطالب :بشار منير

اللجنة الأكاديمية لقسم الهندسة الصناعية

2023

* Dynamics.

⁂ CH 12: Kinematics of a particle.

12.1 :-

- Kinematics:- Describes the motion of point, bodies, system of bodies without consideration of the causes of motion.

- Kinetics :- concerned with relationship between the motion of bodies and It's causes.

- particle:-
  - it has a mass but negligible Size and Shape.
  - Bodies of finite size in which the motion is charactrized by the motion of It's mass center.

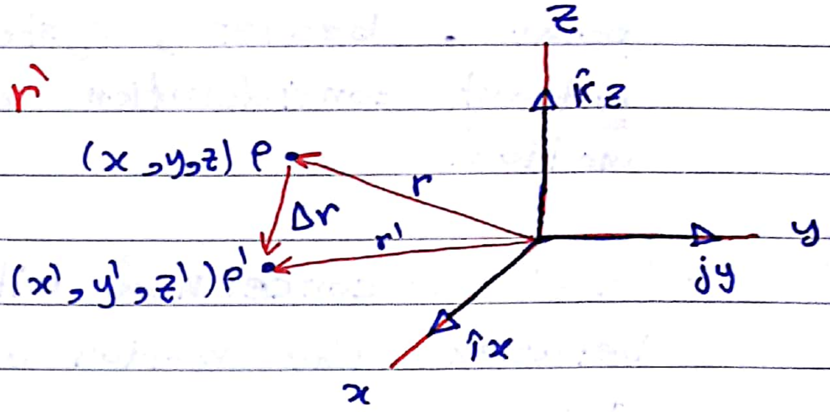
12.2 :- Rectilinear Kinmetics continous motion.

- Rectilinear :- The path of motion is a stright line.

- continous motion :- The position function $S(t)$; consists of one equ.
- Erratic motion:- The position fun. $S(t)$; consists of multiple equ.

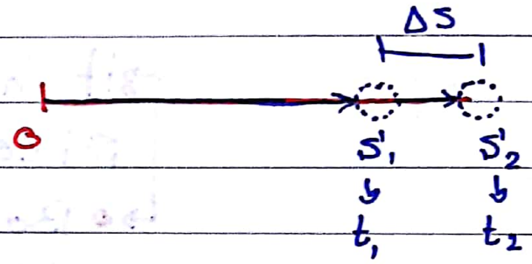
- position:- the position of particale can be represented by the position vector.

\# $\Delta r = r - r'$



\# $\Delta s = s_2 - s_1$

$= s_f - s_i$



\# -ve S indicates a position in the opposite direction to the origin.

\# average velocity, $v_{avg} = \frac{\Delta s}{\Delta t} = \frac{s_f - s_i}{t_f - t_i}$

when $\Delta t \to 0$ the avg. velocity become instaneous velocity.

$$\boxed{v = \frac{ds}{dt}}$$

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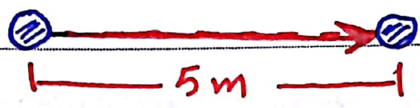
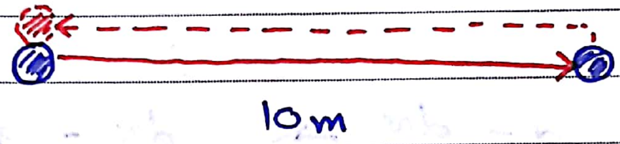
Speed ; The magnitude of the velocity. It has no sense of direction! (non -ve)

Distance travelled ; S_T :- the Length of the actual path traveled.



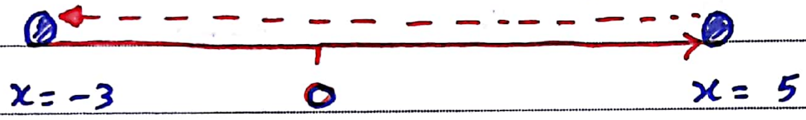
$\Delta s = 5m$

$S_T = 5m$



Δs = Zero

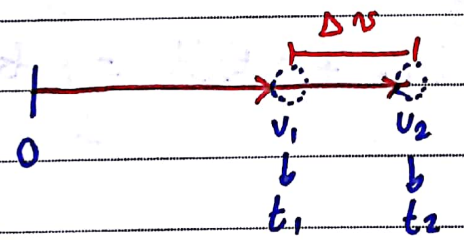
$S_T = 20m$



$\Delta s = -3m$

$S_T = 13m$

Acceleration :-



avg. acc ; $a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$

$\Delta t \rightarrow 0$; become instanten. acceleration.

$$a = dv/dt = d^2s/dt^2$$

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$$\left.\begin{aligned} v &= \frac{ds}{dt} \\ a &= \frac{dv}{dt} \end{aligned}\right\}$$ two independant eq.

$$dt = \frac{ds}{v} = \frac{dv}{a} \Rightarrow \boxed{v\,dv = a\,ds}$$

For constant acceleration & intial velocity v_0 s_0 @ $t=0$

$$a = \frac{dv}{dt} \Rightarrow \int_{v_0}^{v} dv = a_c \int_{t_0=0}^{t} dt$$

$$\Rightarrow v - v_0 = a_c t$$

1st eq. * $\underline{v = v_0 + a_c t}$

$$v = \frac{ds}{dt} \Rightarrow \int_0^t (v_0 + a_c t)\,dt = \int_{s_0}^{s} ds$$

$$\Rightarrow v_0 t + \tfrac{1}{2} a_c t^2 = s - s_0$$

2nd eq * $\Rightarrow$ $\underline{s = s_0 + v_0 t + 1/2\, a_c t^2}$

$$\int_{s_0}^{s} a\,ds = \int_{v_0}^{v} v\,dv$$

$$a_c\,(s - s_0) = \tfrac{1}{2}\,(v^2 - v_0^2)$$

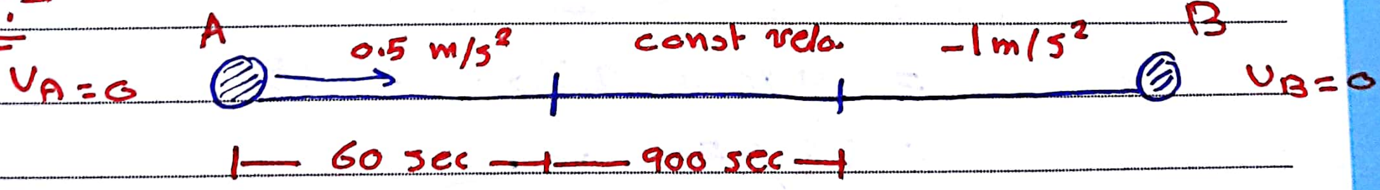
3^{ed} eq * $v^2 = v_0^2 + 2\,a_c\,(s - s_0)$

Ex:- P 12.19 :-

A train starts from rest at station A and accelerate at $0.5\ m/s^2$; for 60 sec. Afterward it travels with a const velocity for 15 min. It then decelerate at $1\ m/s^2$ until it brought to rest at station B.

- Determine the distance between the stations.

Sol:-



Stag I)

$$s = s_0 + v_0 t + \tfrac{1}{2} a_c t^2$$

$$s = 0 + 0 + \tfrac{1}{2}(0.5)(60)^2$$

$$s = 900\ m$$

stage II

$$S = S_0 + v_0 t + \tfrac{1}{2} a_c t^2$$

$$S = 0 + \underline{30}(900) + 0$$

$$\underline{S = 27000 \text{ m}}$$

$v_0 \Rightarrow$

$$v = v_0 + a_c t$$

$$v = 0 + 0.5(60)$$

عوضنا فوق. ↑ $\underline{v = 30 \text{ m/s} = v_0}$

stage III

$$S = S_0 + v_0 t + 0.5\, a_c t^2$$

$$S = 0 + 30(t) + 0.5(-1)(t^2)$$

$$S = 30(30) + 0.5(-1)(30)^2$$

$$\underline{S = 450 \text{ m}}$$

$t \Rightarrow$

$$v = v_0 + a_c t$$

$$0 = 30 + (-1)(t)$$

عوضنا فوق ↑ $\underline{t = 30 \text{ sec}}$

$$D = \text{I} + \text{II} + \text{III}$$

$$= 900 + 27000 + 450$$

$$= 28{,}350 \text{ m.}$$

Ex. P.12.09 :

The acceleration of the particale as it moves along a straight line is given by $a = (2t-1)\ m/s^2$; IF $s = 1m$ $v = 2\ m/s$; when $t = $ zero .

- Determine the paticale's position ; velocity when $t = 6$ sec.

- Also ; determine the total distance the particle moves during this time ~~past~~ period.

Sol:-

$$a = \frac{dv}{dt} \Rightarrow \int_{v_0}^{v} dv = \int_{t_0}^{t} a\, dt$$

$$v - v_0 = \int_{t_0}^{t} 2t-1\ dt$$

$$v - v_0 = t^2 - t \Big|_{\cancel{t_0}\, 0}^{t}$$

$$v - 2 = t^2 - t$$

$$v = t^2 - t + 2 \longrightarrow *$$

$$v = 36 - 6 + 2$$

$$v = 32\ m/s$$

$$v = \frac{ds}{dt} \Rightarrow \int_{s_0}^{s} ds = \int_{t_0}^{t} v\, dt$$

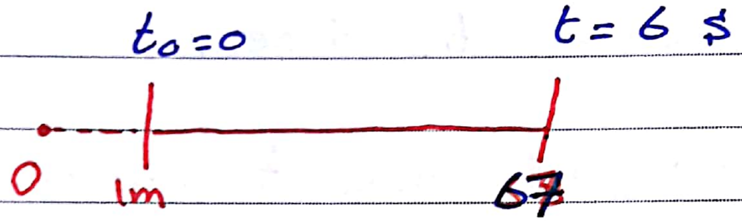
$$s - 1 = \int_{0}^{t} t^2 - t + 2\ dt \Rightarrow$$ ع ل

$$\frac{t^3}{3} - \frac{t^2}{2} + 2t = s - 1$$

$$s = 67 \text{ m}$$

$* \longrightarrow \quad v = t^2 - t + 2 \neq 0$

$\Rightarrow$ It's not have a root.



$$\underline{\underline{S_T = 66 \text{ m}}}$$

*Ex. P12.26 :- ((H.W))

$*a = (2t - 9)$; $S_o = 1\,m$

$v_o = 10\ m/s$

when $t = 9$ sec. Det. position ; velocity and total distance?!

Sol: $a = dv/dt \rightarrow \int_{v_o}^{v} dv = \int_{t_o}^{t} a\,dt \rightarrow \int_{10}^{v} dv = \int_{0}^{t} (2t-9)\,dt$

$\boxed{v = t^2 - 9t + 10}$ — * eq. 1

at $t = 9 \rightarrow v_{1_9} = 9^2 - 9(9) + 10 = 10\ m/s$ — 1

$v = ds/dt \rightarrow \int_{S_o}^{s} ds = \int_{t_o}^{t} v\,dt \rightarrow \int_{1}^{s} ds = \int_{0}^{t} (t^2 - 9t + 10)\,dt$

$\boxed{s = 1/3\,t^3 - 9/2\,t^2 + 10t + 1}$ — * eq. 2

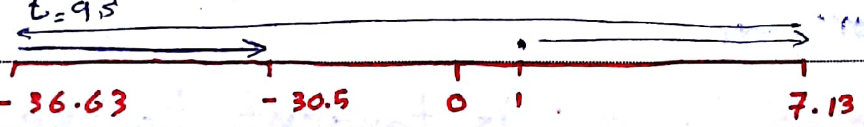
By eq. 1 :- $t^2 - 9t + 10 = 0$ من الدوال

$t_1 = 1.298$ $t_2 = 7.701$ $t_3 = 9$

$\Rightarrow s_{1_{t=1.298s}} = 1/3(1.298)^3 - 9/2(1.298)^2 + 10(1.298) + 1 = \underline{7.13\ m}$

$s_{1_{t=7.701s}} = 1/3(7.701)^3 - 9/2(7.701)^2 + 10(7.701) + 1 = \underline{-36.63\ m}$

$s_{1_{t=9s}} = 1/3(9)^3 - 9/2(9)^2 + 10(9) + 1 = -30.5\ \underline{m}$ — 2



$S_T = 6.13 + 7.13 + 36.63 + 6.13 = 56.01$ — 3

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1^{st} : 15/3 3-04

12.4 : General curvilinear Motion.

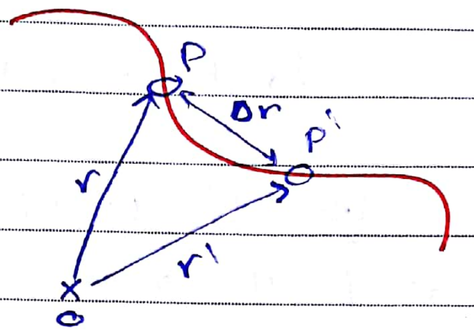
12.5 : Rectangular components (x, y, z)

• 12.4

* a partical moves along a curved path undergoes curvilinear motion.

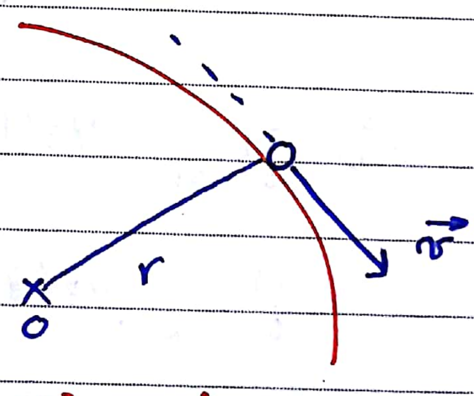
* position

$\Delta r = r' - r$



* velocity $\vec{v} = \frac{d\vec{r}}{dt}$

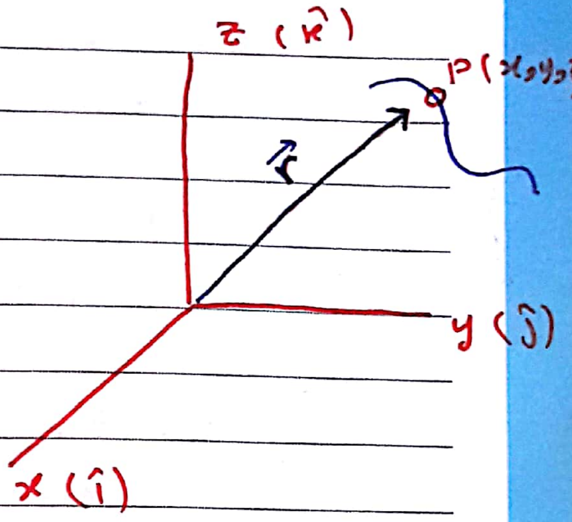
* The velocity vector is always tangent to the path of motion.



* acceleration $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2 s}{dt^2}$

* acceleration ; is tangent to the hodograph.

12.5



$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$$

magnitud

velocity $\Rightarrow$ $\vec{v} = \dfrac{d}{dt}(x\hat{i} + y\hat{j} + z\hat{k})$

$$\vec{v} = i\frac{dx}{dt} + \underbrace{\cancel{x\frac{di}{dt}}}_{zero} + j\frac{dy}{dt} + \underbrace{\cancel{y\frac{dj}{dt}}}_{zero} + k\frac{dz}{dt} + \underbrace{\cancel{z\frac{dk}{dt}}}_{zero}$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

$$= \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}k$$

$$\boxed{|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}}$$

acceleration $\Rightarrow$ $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

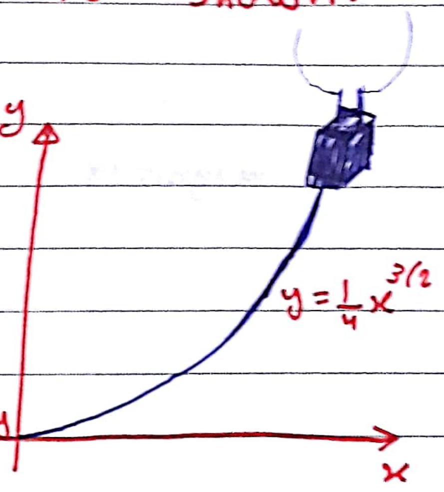
$$= \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$$

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

Ex:- An opject travels from the origin along a curved path as shown:-

- If the horizon velocity ($v_x = 8t$ m/s)

- Det. the magnitud and direction of position velocity and acceleration @ ($t = 2$ sec)



Sol:

$y = \frac{1}{4} x^{3/2} = \frac{1}{4} (16)^{3/2}$ —*

$= 16$ m/s —*

$x = \int 8t = 4t^2$

$= 16$ —*

$\dot{y} = \frac{1}{4} * \frac{3}{2} x^{1/2} * \dot{x}$

$= \frac{3}{8} (x^{1/2} * \dot{x})$

$= \frac{3}{8} (16^{1/2} * 16)$ —*

$= 24$ m/s —*

$\dot{x} = 8t = 16$ —*

$\ddot{x} = 8$ —*

$\ddot{y} = \frac{3}{8} (\frac{1}{2} x^{-1/2} . \dot{x} * \dot{x} + x^{1/2} . \ddot{x})$

$= \frac{3}{8} (\frac{1}{2} 16^{-1/2} * 16 * 16 + 16^{1/2} . 8)$ — *

$= 24$ m/s^2 —*

$\Rightarrow |r| = 22.6$; $\theta = 45°$

$\Rightarrow |v| = 28.8$ m/s ; $\theta = 56.3°$

$\Rightarrow |a| = 25.3$ m/s^2 ; $\theta = 71.1°$

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((H.W)) Ex:- A box slides down the slope described by the eq. $y = 0.05x^2$ m when

$v_x = -3$ m/s $\quad a_x = -1.5$ m/s^2

- at $x = 5$ m ; find the y components of velocity & acceleration @ $x = 5$ m.

Sol:-

$x = 5$ m $\qquad y = 0.05\, x^2 = 0.05(5)^2 = 1.25$ m

$\dot{x} = -3$ m/s $\qquad \dot{y} = 0.05(2)\, x \cdot \dot{x} = 0.05(2)(5)(-3)$

$\qquad\qquad\qquad\qquad = -1.5$ m/s

$\ddot{x} = -1.5$ m/s^2

$\ddot{y} = 0.1\left(\dot{x}\dot{x} + x\ddot{x}\right)$

$= 0.1\left((-3)^2 + (5)(-1.5)\right)$

$= 0.15$ m/s^2

* 12.6 :- Motion of a projectile :-

* projectile :- An object in free fall, subject to gravity and air resistance.

* Air resistance is ignored ⇒ const. acc.

$-g = 9.81\ m/s$ — S.I units.

$= 32.2\ ft/s^2$ — FPI system.

* Recall :- $v = v_0 + a_c t$

$s = s_0 + v_0 t + 1/2\, a_c t^2$

$v^2 = v_0^2 + 2 a_c (s - s_0)$

* — Rull in x :- * $v_x = v_0$; const. throughout the flight

Range :- * $x = x_0 + v_0 t$

* $s = s_0 + v_0 t$

* @ peak v_y = zero ; peak height : Max ~~...~~ vertical disp.

* A projectile horizontal motion is unacc.

a_x = zero

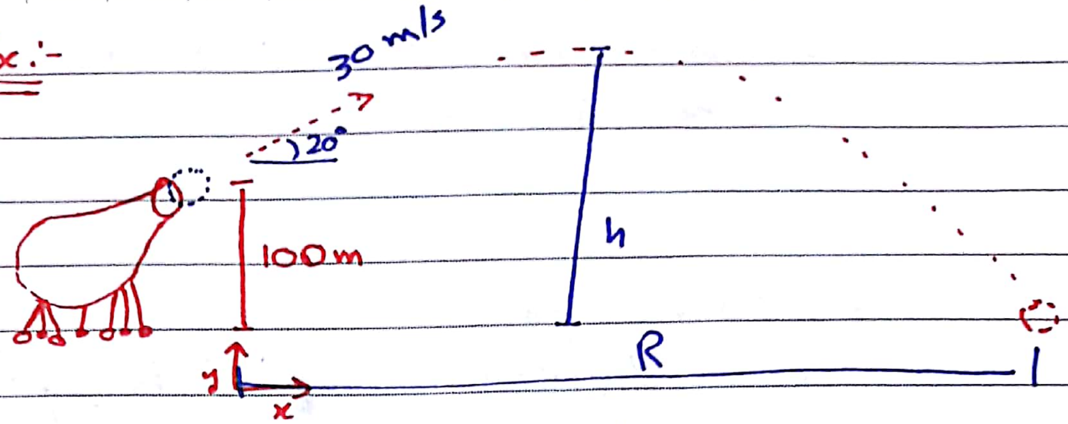
* — Rull in y :-

1 — $v_y = v_{0y} - g t$

2 — $v_y^2 = v_{0y}^2 - 2g(y_f - y_0)$

3 — $y_f = y_0 + v_{0y} t - 1/2\, g t^2$

Ex:-



A projectile is launched from a cliff 100 m above level ground with a launch angle of 20° above the horizon. → with a launch velo. 30 m/s

- Det. projetile Range (R)
  " peak height (h)

Sol:- $y_o = 100\,m$ $\qquad x_o = zero$

$y_f = zero$ $\qquad x_f = R = ?!$

$v_{oy} = 30 \sin 20$ $\qquad v_{ox} = 30 \cos 20$

to find t | $x - x_o = (v_o)_x t \rightarrow t = ?!$

$R - 0 = 30 \cos 20 * 5.68 = 160.1\ m$

$y_f = y_o + v_{oy} t - 1/2\, g t^2$

$0 = 100 + 30 \sin 20\, t - 1/2 (9.81) t^2$

$t = 5.68\ s$

$v_y = (v_o)_y - gt$

$0 = 30 \sin 20 - 9.81 * t \Rightarrow t = 1.05\ s$

$y = y_o + v_{oy} t - 1/2\, g t^2$

$y = 100 + 30 \sin 30 (1.05) - 1/2 * 9.81 * (1.05)^2$

$h = y_{max} = 105.4\ m$

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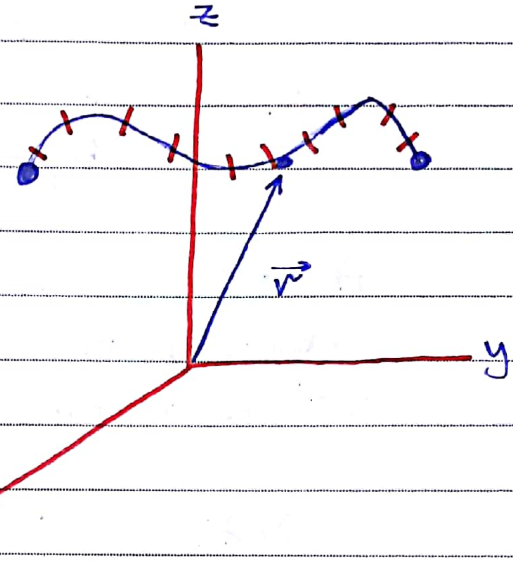
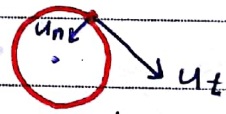
12.7 :- Curvilinear Motion: Normal and tangential components.

※ 3D curved path can be divived into small segment to curves with equal Length.



※ when the segments get small enough each one of them appraches an arc which is a segment of a circle.

※ Velocity. ※



- the velocity has only one component tangent to the path.

$$\vec{v} = v\, u_t$$

※ acceleration. It has two component

① ↳ Tangential component, (a_t): describes the chang in speed.

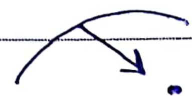
$a_t = \frac{dv}{dt} = \dot{v}$; If constant speed $a_t =$ zero.

↳ ② Normal componant : describes the direction of the velocity.

$$a_n = \frac{v^2}{\rho} \quad ; \quad \rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{\left| d^2y/dx^2 \right|}$$

ρ : radius of curviture.

- if ρ is (∞) the a_n is zero.
- a_n : always points to the center of curve.



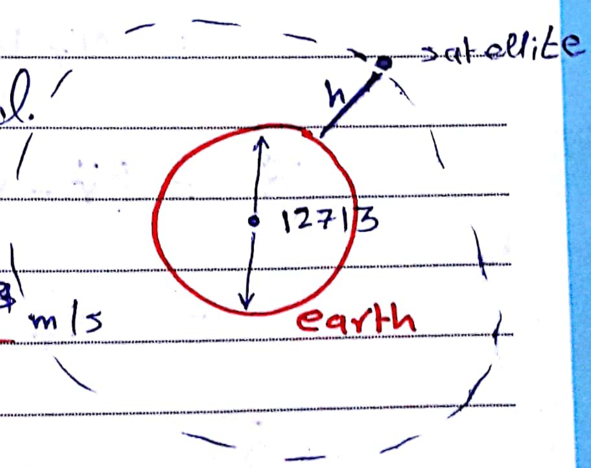
Ex:- P (12.119).

The satellite, s travels around the earth in acirvular path with a constant speed of 20 Mm/h. IF the acc. is 2.5 m/s². Determine the altitude, (h) Assume the earth's to be 12713 Km.

Sol/

- $a_t = 0 \rightarrow$ constant speed.

$a = a_n = 2.5$



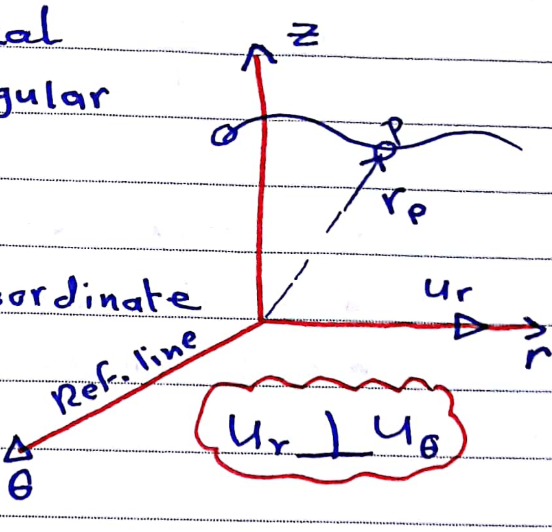
$v = 20 \times 10^{+6} \times \frac{1}{3600} = 5.56 \times 10^{3}$ m/s

$a_n = \frac{v^2}{\rho}$

$$2.5 = \frac{(5.56 \times 10^{3})^2}{\left(h + \frac{12713 \times 10^3}{2}\right)} \Rightarrow h = 5.99 \times 10^6 \text{ m}$$

12.8 :- Curvilinear Motion
Cylindrical Components:-

* — z-coordinate is identical to that used for rectangular coordinate.



* — r-coordinate : Radial coordinate
It's unit vector u_r

* — θ-coordinate : Transrese coordinate it is unit vector u_θ.

$$\vec{r_p} = r u_r + z u_z \longrightarrow \text{position.}$$

$$\vec{v} = \dot{r} u_r + (r\dot{\theta}) u_\theta + \dot{z} u_z \rightarrow \text{velocity}$$
$$= v_r u_r + v_\theta u_\theta + v_z u_z$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2) u_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta}) u_\theta + \ddot{z} u_z$$

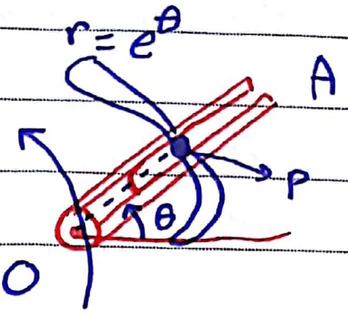
$$= a_r u_r + a_\theta u_\theta + a_z u_z \rightarrow \text{acc.}$$

$\dot{\theta}$: Angle velocity rad / sec.
$\ddot{\theta}$: " acc. rad / sec^2.
a_r: Radial acc.
a_θ: Transrese acc.

ex:- ((سنوات))

peg, P driven by forked link OA along the path $r = e^\theta$ where r is in meters.

$\dot{\theta}, \ddot{\theta}$



when $\theta = \pi/4$ rad, the link has an angular velocity $\dot{\theta} = 2$ rad/s and an angular acc $\ddot{\theta} = 4$ rad/s^2

Determine the radial and transverse compon. of the peg's acc @ this instant.

$a_r = ?!$ $\quad a_\theta = ?!$

$r = e^\theta = e^{\pi/4} = 2.193$ m —— *

$\dot{r} = e^\theta . \dot{\theta} = e^{\pi/4} * 2 = 4.38$ m/s —— *

$\ddot{r} = e^\theta . \dot{\theta}\,\dot{\theta} + e^\theta \ddot{\theta} = e^{\pi/4}(2)^2 + e^{\pi/4}(4)$

$= 17.54$ m/s^2 —— *

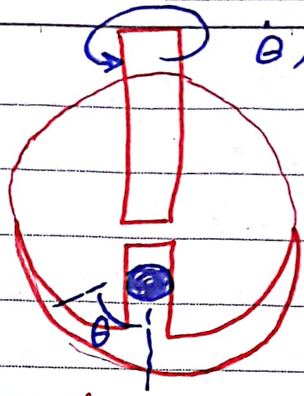
$a = a_r u_r + a_\theta u_\theta$

$= (\ddot{r} - r\dot{\theta}^2) u_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta}) u_\theta$

$= (17.54 - 2.193 * (2)^2)\, u_r + (2.193 * 4 + 2(2)(4.38)) u_\theta$

$= \underline{8.768\, u_r} + \underline{26.31\, u_\theta}$

ex: F.12.34



$\theta = 4t^{3/2}$ rad

$r = 0.1\, t^3$ m

velocity and acc at $t = 1.5$ s

magntude ← بطلب بيكون velocity لما يعطيني

$\theta = 4t^{3/2}$ rad $= 4(1.5)^{3/2} = 7.34$ — $\cancel{*}$

$\dot{\theta} = 4 \times \frac{3}{2} t^{1/2} = 6(1.5)^{1/2} = 7.34$ — $\cancel{*}$

$\ddot{\theta} = 3t^{-1/2} = 3(1.5)^{-1/2} = 2.45$ — $\cancel{*}$

$r = 0.1(1.5)^3 = 0.3375$ — $\cancel{*}$

$\dot{r} = 0.3t^2 = 0.3(1.5)^2 = 0.675$ — $\cancel{*}$

$\ddot{r} = 0.6t = 0.6(1.5) = 0.9$ — $\cancel{*}$

→ velocity.

$\vec{v} = \dot{r}\, u_r + (r\dot{\theta})\, u_\theta$

$= (0.675)\, u_r + (2.477)\, u_\theta$

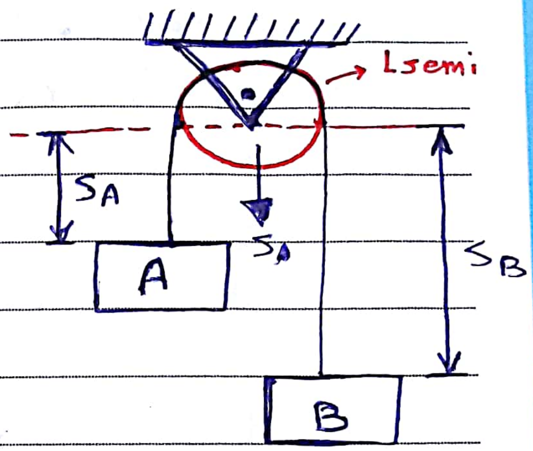
velocity $= \sqrt{(0.675)^2 + (2.477)^2} = \underline{2.57 \text{ m/s}}$

→ acc. $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\, u_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\, u_\theta$

$\vec{a} = (-17.28)\, u_r + (10.73)\, u_\theta$

12.8/ Absolate Dependent Motion Analysis:-

ex → two objects connected by anon-elastic rope wrapped apully.



- $L_{total} = S_A + L_{semi} + S_B$

الإشتقاق بالنسبة للزمن.

$0 = v_A + 0 + v_B$

$\Rightarrow \boxed{v_A = -v_B}$

- Mathmatical analysis:-

Step 1: set up coordinate ('s') along the direction motion from a fixed point or fixed datum line.

datum:- aline that any point on it represents aposition of zero.

step 2: Repesent the positions of the opjects S_A and S_B are both variable with time.

Step [3]: Recognise the const Length.

Step [4]: find the depending relation between the position variables.

$$L_T = S_A + S_B + L_{semi}$$

Step [5]: Differntiate the entire eq wrt. time and the relation in velocity.

$$\frac{dL_T}{dt} = \frac{dS_A}{dt} + \frac{dS_B}{dt} + \frac{dL_{semi}}{dt}$$

$$0 = v_A + v_B + 0$$

$$\boxed{v_A = -v_B}$$

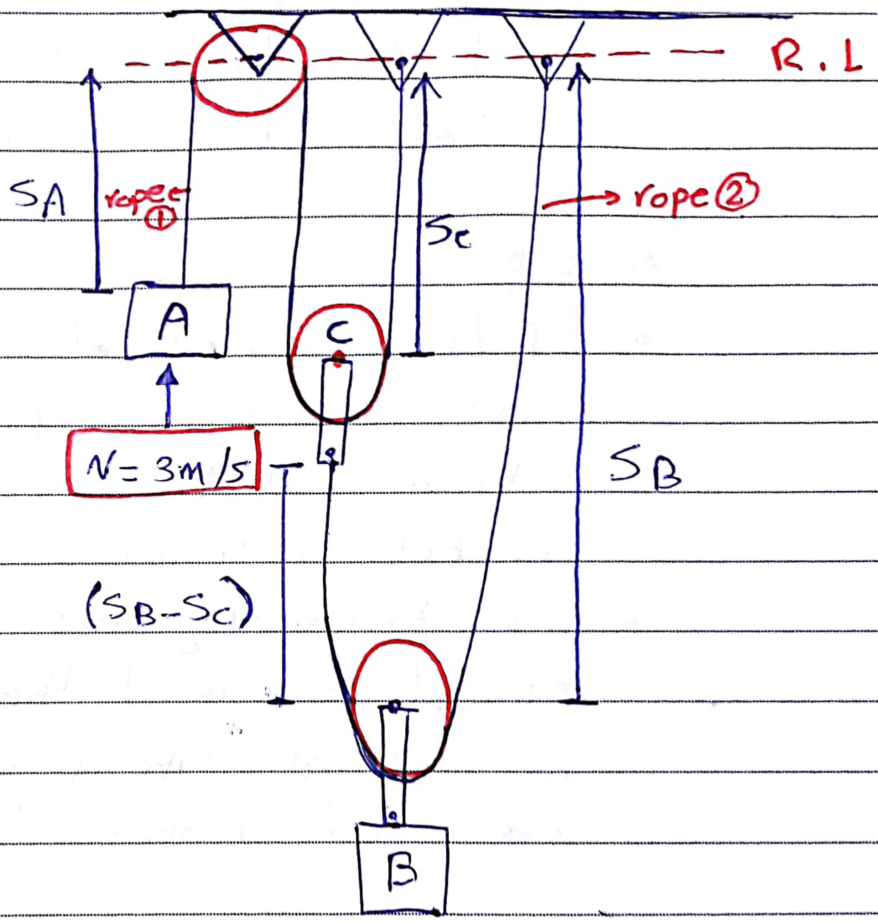
Step [6]: repeat the diffentiation and extract the relation in acceleration.

$$0 = \frac{dv_A}{dt} + \frac{dv_B}{dt} + 0$$

$$0 = a_A + a_B$$

$$\boxed{a_A = -a_B}$$

Ex:- for the system in th fig. of pullys, the velocity of object A is 3 m/s Detrmine the velocity of object B.



Sol:

$$L_{t_1} = S_A + S_c + S_c$$

$$0 = v_A + 2v_C$$

$$\boxed{\frac{-v_A}{2} = v_c}$$

$$L_{t_2} = S_B - S_c + S_B$$

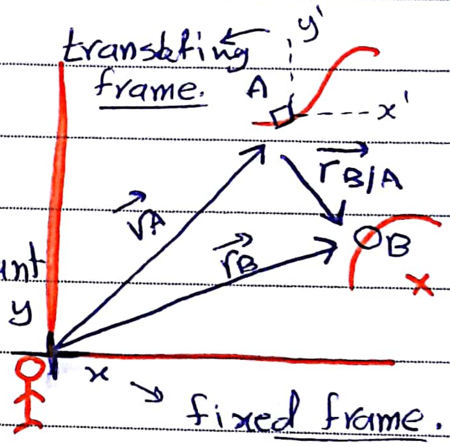
$$0 = 2S_B - S_c$$

$$S_B = \frac{S_c}{2} \rightarrow \frac{\frac{-v_A}{2}}{2} = \frac{-v_A}{4} = \boxed{\frac{-3}{4}\ m/s}$$ ans.

※ H.w 1 ※ 14th Edition:-

P. 12.16	127	198	and fun 12-23
28	136	201	
85	138	205	
92	169	218	
107	173	219	
120	182	228.	

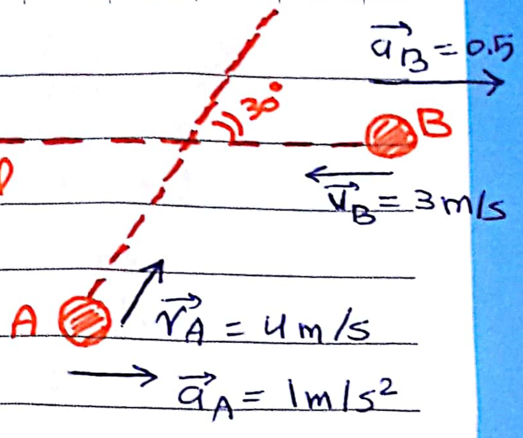
※ 12.9



- If the two particles A and B undergo independant motion to their relative motion.
- Using translating axis ~~attached~~ attached to one of the particles $\vec{r}_A$ and $\vec{r}_B$ are absolute position. (they are oboserved from a fixed ref frame.)

- position vector. $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$
- Relative velocity. $\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$
- Relative acc. $\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$

ex:- ⓐ the shown, object A's speed is incresing at $1 m/s^2$ object B speed is decresing at $0.5 m/s^2$



• Determine the magnitude of velocity and the acc. of B w.r.t A. $\vec{v}_{B/A} = ?!$ $\vec{a}_{B/A} = ?!$

SOL:-

$\vec{v}_B = -3 m/s\ i$

$\vec{v}_A = 4 \cos 30\ i + 4 \sin 30\ j$

$\vec{v}_{B/A} = (-3\ i) - (3.46\ i + 2\ j)$

$= -6.46\ i - 2\ j$

$|\vec{v}_{B/A}| = \sqrt{6.46^2 + 2^2}$

$= 6.76\ m/s$ ——— *

$\vec{a}_B = 0.5\ i$

$\vec{a}_A = 1\ i$

$\vec{a}_{B/A} = (0.5 - 1)\ i = -0.5\ i$

$|\vec{a}_{B/A}| = 0.5\ m/s^2$

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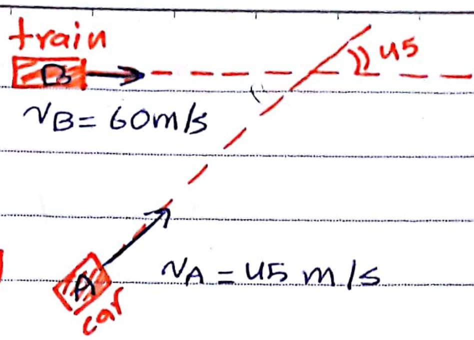
ex: 12.25



Determine the mag. and Direction of velocity of the train relative to car.

$$\vec{v}_{B/A} = (60\,i) - (45\cos 45\, i + 45 \sin 45\, j)$$

$$= 28.18\, i - 31.81\, j$$

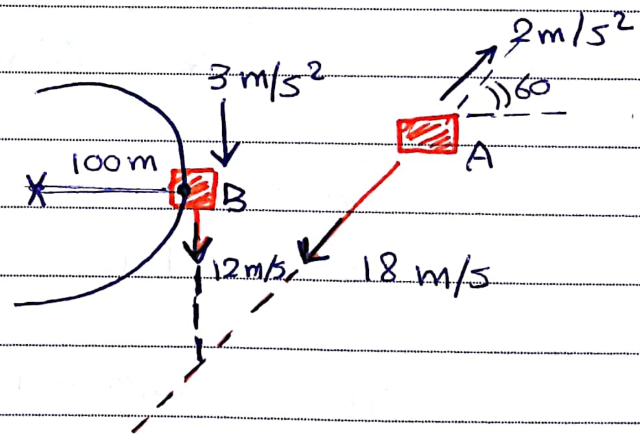
$$|\vec{v}_{B/A}| = \sqrt{28.18^2 + 31.81^2} = 42.5 \text{ —*}$$

$$\theta = \tan^{-1}(-31.81/28.18) = -48.46^\circ$$

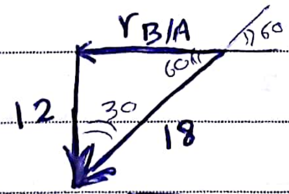
$$= 311.53^\circ$$

---

ex. 12.27.



Determine the velocity and acc. of B w.r.t A.



$$r_{B/A} = \sqrt{18^2 + 12^2 - 2 * 18 * 12 \cos 30}$$

$$= 9.69 \text{ m/s —*}$$

$$a_A = 2\cos 60\, i + 2 \sin 60\, j$$

$$= 1\, i + \sqrt{3}\, j$$

$$a_B = -\left(\frac{12^2}{100}\right) i - 3\, j$$

$$= -1.44\, i - 3\, j$$

$$\vec{a}_{B/A} = -2.44\, i - 4.73$$

$$|\vec{a}_{B/A}| = 5.32 \text{ m/s}^2$$

H.W *

*12-16 $v_0 = 6\ m/s$ $a = -1.5\ v^{1/2}\ m/s^2$

$s_f = ?!$ (stop) $t_f = ?!$ (stop) $\rightarrow v_f = 0\ m/s$

Sol:- $v\,dv = a\,ds \rightarrow ds = \frac{v}{a}\,dv$

$ds = \frac{v}{-1.5v^{1/2}}\,dv \rightarrow \int_0^s ds = \int_6^0 \frac{-1}{1.5}\,v^{1/2}\,dv$

$S - 0 = -2/1.5(3)\ v^{3/2}\Big|_6^0$

$\boxed{S} = 0 - -2/1.5(3)\ (6)^{3/2} = 6.53$ —— *

$a = \frac{dv}{dt} \rightarrow \int_0^t dt = \int_6^0 \frac{-1}{1.5}\,v^{-1/2}\,dv$

$t - 0 = -2/1.5\ v^{1/2}\Big|_6^0$

$\boxed{t} = 0 - -2/1.5\ (6)^{1/2} = 3.265$ —— *

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$\ast$ $\boxed{12-28}$ $v_{0A} = 0$ $\quad a_A = 6t - 3$
$v_{0B} = 0$ $\quad a_B = 12t^2 - 8$ $\quad \Big] \rightarrow s = 0$

$t = 4 \rightarrow$ dist between A & B ?!
total dist A & B ?!

SOL:- $a_A = \dfrac{dv}{dt} \rightarrow dv = a\,dt \rightarrow \int_0^v dv = \int_0^t 6t - 3\,dt$

$v - 0 = 3t^2 - 3t \Big|_0^t \rightarrow \boxed{v = 3t^2 - 3t}$

$v = \dfrac{ds}{dt} \rightarrow ds = v\,dt \rightarrow \int_0^s ds = \int_0^t 3t^2 - 3t\,dt$

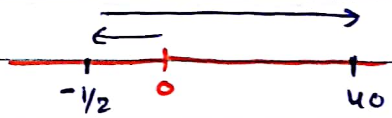
$s - 0 = t^3 - \dfrac{3}{2}t^2 \Big|_0^t \rightarrow \boxed{s = t^3 - \frac{3}{2}t^2}$

$\boxed{s_{1\,4s}} = (4)^3 - \frac{3}{2}(4)^2 = 40\text{ m}$ —— $\ast$

← نطلع الجذور $v = 3t^2 - 3t = 0$ $\quad 3t(t-1) = 0$ $\quad \boxed{t=0}$ $\boxed{t=1}$

$s_{1\,1s} = (1)^3 - \frac{3}{2}(1)^2 = -1/2$ —— $\ast$

$(s_{total})_A = 0.5 + 0.5 + 40$
$= \underline{41\text{ m}}$ ans



$a_B = dv/dt \rightarrow dv = a\,dt \rightarrow \int_0^v dv = \int_0^t 12t^2 - 8$

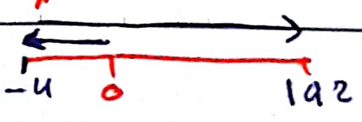
$v - 0 = 4t^3 - 8t \Big|_0^t \rightarrow \boxed{v = 4t^3 - 8t}$

← نطلع الجذور $v = 0 = 4t^3 - 8t \rightarrow 4t(t^2 - 2) = 0$ $\quad \boxed{t=0}$ $\boxed{t=\sqrt{2}}$

$v = ds/dt \rightarrow ds = v\,dt \rightarrow \int_0^s ds = \int_0^t 4t^3 - 8t\,dt$

$s = t^4 - 4t^2$

$s_{1\,0} = 0$; $s_{1\,\sqrt{2}} = -4\text{ m}$; $\boxed{s_{1\,4}} = 192\text{m}$ —$\ast$



$\boxed{\text{dist betw} = 192 - 40 = 152\text{m}}$ ans $\quad \boxed{(s_{tot})_B = 200\text{m}}$ ans

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12-85| $x = 2t^2$ $y = .04t^3$

distance from point A ?! $v = ?!$ $a = ?!$

Sol:-

$x_{10} = 2(10)^2 = 200 m$

$y_{10} = \cancel{2} .04(10)^3 = 40 m$

$d = \sqrt{200^2 + 40^2} = 204 m.$ → ans

$\dot{x}_{10} = 4t = 4(10) = 40 m/s$

$\dot{y}_{10} = .12t^2 = .12(10)^2 = 12 m/s$

$v = \sqrt{40^2 + 12^2} = 41.8 m/s$ → ans

$\ddot{x}_{10} = 4 m/s^2$

$\ddot{y}_{10} = .24t = .24(10) = 2.4 m/s^2$

$a = \sqrt{4^2 + 2.4^2} = 4.66 m/s^2$ → ans

12 – 92

$(v_0)_x = 80\cos 30 = 69.28$ ft/s

$(v_0)_y = 80\sin 30 = 40$ ft/s

$g = 32.2\ ft/s^2$

$x = ?!$ $-y = ?!$

SOL:

$* \text{ — } y = -0.04x^2 \Rightarrow \boxed{-y = .04x^2}$

→

$x = x_0 + (v_0)_x t$

$x = 0 + 69.28t$

$t = \frac{1}{69.28}x \text{ — } *$

$y = y_0 + (v_0)_y t - .5gt^2$

$-y = 0 + 40t - .5(32.21)t^2$

$.04x^2 = 40 * \frac{1}{69.28}\cancel{x} - .5(32.21)\left(\frac{1}{69.28}\right)^2 x^2$

$.04335$

$x^2 - .577x = 0$

$\boxed{x = 13.3\ ft}$ $x = 0$ → ans

$y = -0.04x^2$

$= -0.04(13.3)^2 = 7.075\ ft$ → ans

12-107/ $v_A = 80\ ft/s$ $g = 32.2\ ft/s^2$

$x_0 = 0$ $x = 35\,ft$ $(v_0)_x = 80 \cos\theta$

$y_0 = 0$ $y = -20\ ft$ $(v_0)_y = -80 \sin\theta$

$\theta = ?!$

SOL:-

$x = x_0 + v_{0x} t \rightarrow 35 = (80 \cos\theta)\, t$

$\rightarrow t = \dfrac{35}{80 \cos\theta}$

$y = y_0 + v_{0y} t - .5\,(32.2) t^2$

$-20 = 0 + (-80 \sin\theta)\, t - .5\,(32.2)\, t^2$

$-20 = 0 - \cancel{80} \sin\theta \left(\dfrac{35}{\cancel{80} \cos\theta}\right) - .5\,(32.2)\left(\dfrac{35}{80} \cdot \dfrac{1}{\cos\theta}\right)^2$

$\left(+20 = +35 \sin\theta \cdot \dfrac{1}{\cos\theta} + 3.081 \cdot \dfrac{1}{\cos^2\theta}\right) \times \cos^2\theta$

$20 \cos^2\theta = 35 \sin\theta \cos\theta + 3.081$

$\left(20 \cos^2\theta = 35 \left(\tfrac{1}{2} \sin 2\theta\right) + 3.081\right) \div -20$; $\cos^2 = 1 - \sin^2$

$\sin^2\theta = -.875 \sin 2\theta + .8459$

$\theta = 24.9$; $\theta = 85.2$

12-120

$a_t = .5\, e^{t}$ m/s^2

$s = 18\,m$

velocity ?! acc ?!

Sol:-

$a_t = .5\, e^{t}$ $a_t = \frac{dv}{dt}$

$\rightarrow dv = a_t\, dt$ $\rightarrow \int_0^{v} dv = \int_0^{v} .5\, e^{t}\, .dt$

$\rightarrow v = .5\,(e^{t} - 1)$

$\rightarrow v = \frac{ds}{dt}$ $\rightarrow \int_0^{18} ds = \int_0^{t} v\, dt$

$18 = .5\,(e^{t} - t - 1)$ $\rightarrow$ $\boxed{t = 3.7664}$

$v_1 \Big|_{3.7664} = .5\,(e^{t} - 1) = 19.85$ m/s ans.

$a_{t_1} \Big|_{3.7664} = .5\,(e^{t} - t - 1) = 20.35\ m/s^2$

$a_n = (19.85)^2/30 = 13.14\ m/s^2$

$a = \sqrt{20.35^2 + 13.14^2} = 24.2\ m/s^2$ ans

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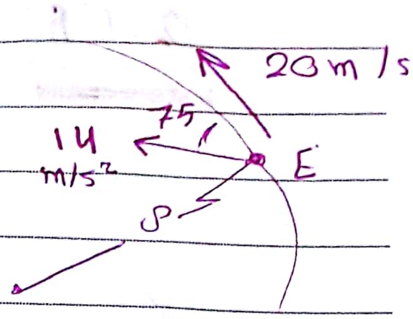
12-127



$a_E = 14\ m/s$ $\quad v_E = 20\ m/s$

$a_t = ?!$ $\quad \rho = ?!$

Sol:-

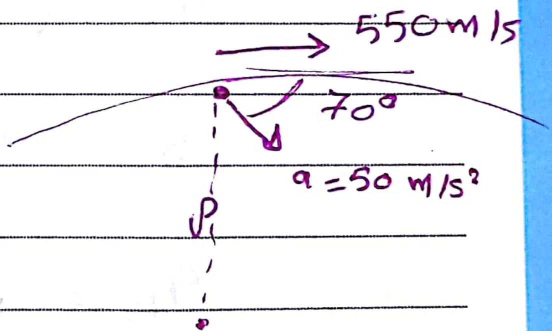
$a_t = 14 \cos 75 = 3.62\ m/s^2$ —— * ans

$a_n = 14 \sin 75 = 13.529\ m/s^2$

$a_n = (v_t)^2/\rho \Rightarrow \rho = 20^2/13.52$

$\rho = 29.59\ m$ —— * ans.

* 12-136



$v = 550\ m/s$ $\quad a = 50\ m/s^2$

$a_t = ?!$ $\quad \rho = ?!$

Sol:- $a_t = 50 \cos 70 = 17.10\ m/s^2$ —— * ans

$a_n = 50 \sin 70 = 46.98\ m/s^2$

$a_n = v^2/\rho \rightarrow \rho = v^2/a_n$

$\rho = 550^2/46.98 = 6438.91$

$= 6.44\ KM$

12-138

$v_0 = 40$ m/s

$a = -0.05s$ m/s^2

$v = ?!$ $a = ?!$ in B ?!

$$\int_{40}^{v} v\,dv = \int_0^s a\,ds$$

$$\frac{v^2}{2}\Big|_{40}^{v} = \int_0^s -0.05s\,ds$$

$$v^2 - 800 = -0.025 s^2$$

$$v^2 - 800 = -0.025 s^2$$

$$\sqrt{\frac{v^2}{2}} = \sqrt{800 - 0.025 s^2}$$

$$v = \sqrt{1600 - 0.05 s^2}$$

$$S = 150 * \frac{\pi}{3} = 50\pi \text{ m}$$

$$v_B = \sqrt{1600 - 0.05(50\pi)^2}$$

$$v_B = 19.139 \text{ m/s}$$

$$a_{t_B} = -0.05(50\pi) = -2.5\pi \text{ m/s}^2$$

$$a_n = (19.139)^2/150 = 2.442 \text{ m/s}^2$$

$$a = \sqrt{(2.5\pi)^2 + (2.442)^2} = 8.224 \text{ m/s}^2$$

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12-182

$\theta = 30 \qquad r = 4\sec\theta$

$\dot{\theta} = 2 \qquad \dot{r} = 4\sec\theta\tan\theta \cdot \dot{\theta}$

$\ddot{\theta} = 3 \qquad \ddot{r} =$

find mag. $\vec{v} = ?!$ & $\vec{a} = ?!$

SOL:-

$r = 4\sec\theta = 4.618 \text{ ft}$

$\dot{r} = 4\sec(30)\tan(30)(2) = 5.33 \text{ ft/s}$

$\ddot{r} = 4\left[\sec\theta\tan\theta \cdot \ddot{\theta} + \dot{\theta}\left(\sec\theta\tan\theta\,\dot{\theta}\,\tan\theta + \sec\theta\sec^2\theta\,\dot{\theta}\right)\right]$

$= 4\left[\sec\theta\tan\theta\,\ddot{\theta} + \dot{\theta}^2\left(\sec\theta\tan^2\theta + \sec^3\theta\right)\right]$

$\ddot{r} = 38.79 \text{ ft/s}^2$

$\vec{v} = (\dot{r})u_r + (r\dot{\theta})u_\theta$

$= (5.33)u_r + (9.236)u_\theta$

$\vec{a} = (\ddot{r} - r\dot{\theta}^2)u_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})u_\theta$

$= (20.318)u_r + (35.174)u_\theta$

$|\vec{v}| = \sqrt{(5.33)^2 + (9.236)^2} = 10.66$ — ans.

$|\vec{a}| = \sqrt{(20.318)^2 + (35.174)^2} = 40.6$ — ans

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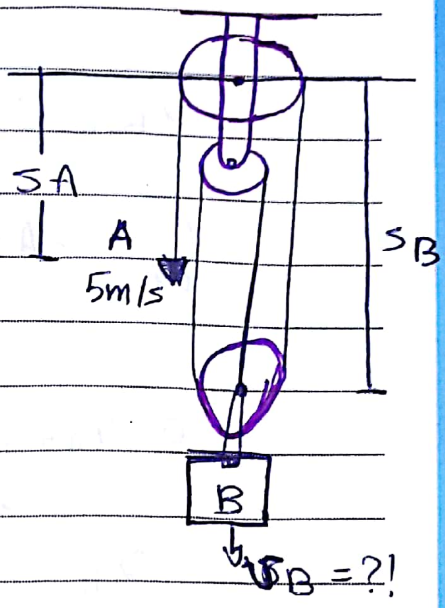
12 – 198 $v_A = 5 m/s$

$v_B = ?!$



Sol:- $S_T = S_A + 3S_B$

$0 = v_A + 3 v_B$

$0 = 5 + 3 v_B$

$v_B = -5/3 = -1.667$

$$\boxed{v_B = 1.667 \uparrow m/s}$$

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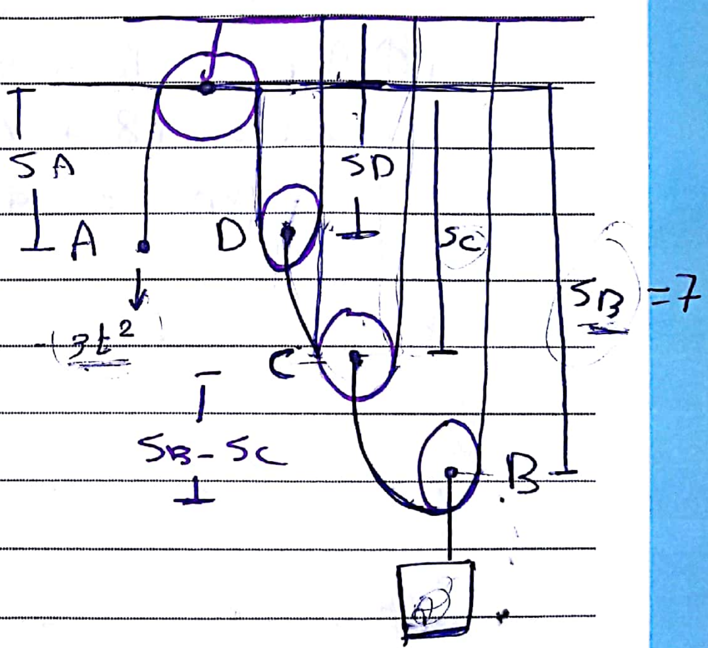
12 – 201 $v_A = 3t^2$

$t_o = 0$

• Def the time needed to lift the load 7m.

ارفع حمل



$S_B + S_B - S_C = L_1$

$S_C + S_C - S_D = L_2$

$S_A + 2S_D = L_3$

$2 v_B - v_C = 0$

$2 v_C - v_D = 0 \rightarrow v_C = v_D / 2$

$2 v_D + v_A = 0 \rightarrow v_D = -v_A / 2$

$\Rightarrow v_C = \dfrac{-v_A}{4}$

$\Rightarrow 2v_B + v_A/4 = 0 \Rightarrow v_A = -8 v_B \Rightarrow 3t^2 = -8 v_B$

$\int_0^{s_B} v_B = \int_0^{t} -3/8\, t^2 \Rightarrow S_B = -\frac{1}{8} t^3 \Rightarrow -7 = -\frac{1}{8} t^3$ $\boxed{t = 3.83}$

12-205

$v_{B/C} = ?!$

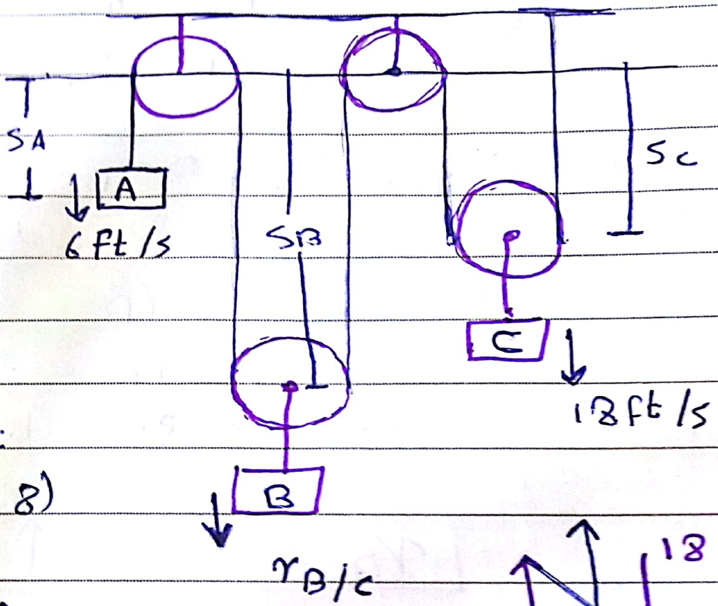
$l_{total} = S_A + 2S_B + 2S_C$

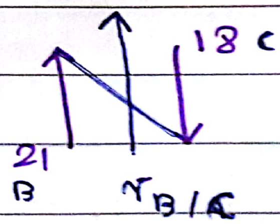
$0 = v_A + 2v_B + 2v_C$

$0 = 6 + 2v_B + 2(18)$

$v_B = -21$ ft/s

$= 21$ ft/s $\uparrow$





$+\uparrow \; v_B = v_C + v_{B/C}$

$21 = -18 + v_{BC}$

$v_{BC} = 39$ ft/s ans.

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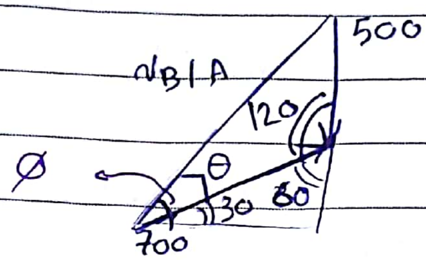
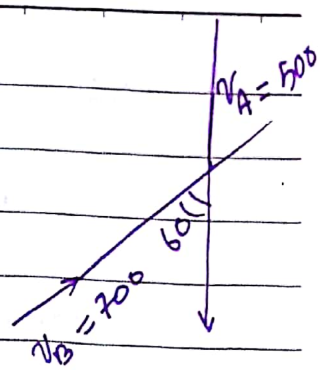
12-218/ find $v_{B/A} = ?!$

Sol:-





$$v_{B/A} = \sqrt{700^2 + 500^2 - 2(700)(500)\cos(120)}$$

$$= 1044.03 \text{ m/s}$$ ~ ans.

$$\frac{500}{\sin\theta} = \frac{1044.03}{\sin 120} \Rightarrow \theta = \sin^{-1}\left(\frac{500\sin(120)}{1044.03}\right)$$

$$\theta = 24.50$$

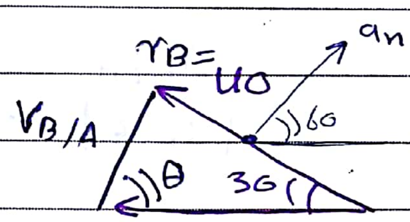
$$\phi = 24.50 + 30 = 54.50°$$ ~ ans.

12-219/ $\rho = .5$

$a_{B_t} = 1200$

$a_A = 0$

$v_{B/A} = ?!$ $a_{B/A} = ?!$



$v_A = 55$

Sol:- $v_{B/A} = \sqrt{(40)^2 + (55)^2 - 2(55)(40)\cos 30}$

$= 28.54$ m/s

$40/\sin\theta = 28.54/\sin 30 \Rightarrow \theta = 44.48°$ ~ ans

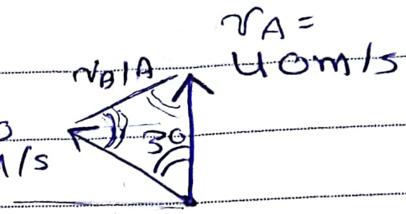
$a_{Bt} = -1200\cos 30\, \hat{i} + 1200 \sin 30 = -1039.23\,i + 600\,j$

$a_n = 40^2/.5 = 3200\cos 60 + 3200 \sin 60$

$= 1600\,i + 2771.28\,j$

$\vec{a} = 560.77\,i + 3371.28\,j \Rightarrow \theta = 80.6°$ ~ ans

12-228/ $\rho = 200$



$v_{B/A} = ?!$ $a_{B/A}?!$

$$v_{B/A} = \sqrt{30^2 + 40^2 - 2(30)(40)\cos(30)}$$
$$= 20.53$$

$$\frac{20.53}{\sin 30} = \frac{40}{\sin\theta} \Rightarrow \theta = 76.95$$
$$\phi = 16.95^\circ$$
$$\phi = 43.06$$

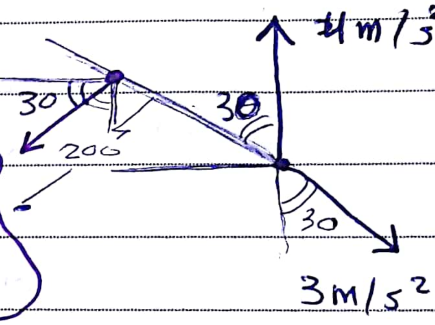
$a_A = 4\,j$

$a_{B_t} = 3\sin 30\, i - 3\cos 30\, j$

$a_{Bn} = -4.5\cos 30\, i - 4.5\sin 30\, j$

$a_n = \frac{30^2}{200}$

$a_n = 4.5$



$a_B = -2.398\, i - 4.848\, j$

$a_A = 0\, i + 4\, j$

$a_{B/A} = -2.398 - 8.898\, j$

$|a_{B/A}| = 9.2$ m/s²

$$\theta = \tan^{-1}\left(\frac{-8.898}{-2.398}\right) = 74.9^\circ$$

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Fun 12-28

$v_{ox} = v_A \cos 30$ $\quad x_o = 0$ $\quad x_f = 10$

$v_{oy} = v_A \sin 30$ $\quad y_o = 0$ $\quad y_f = 1.5$

$10 = 0 + v_A \cos 30 \; t \rightarrow t = 10 / v_A \cos 30$

$1.5 = 0 + \frac{\cancel{v_A} \sin 30}{\cancel{v_A} \cos 30} * 10 - .5 * 9.81 * \left(\frac{10}{\cos 30}\right)^2 * \frac{1}{v_A^2}$

$1.5 = 10 \tan 30 - 654 * 1/v_A^2$

$v_A = 12.37$ m/s

12-92

$x_o = 0$ $\quad x_f = 15$ $\quad v_{ox} = v_A \cos 30$

$y_o = 0$ $\quad y_f = -9$ $\quad v_{oy} = v_A \sin 30$

$15 = v_A \cos 30 \, t \rightarrow t = 15 / v_A \cos 30$

$-9 = \frac{\cancel{v_A} \sin 30 * 15}{\cancel{v_A} \cos 30} - .5(32.2) * \left(\frac{15}{\cos 30}\right)^2 * \frac{1}{v_A^2}$

$-9 = 15 \tan 30 - 4830 * \frac{1}{v_A^2}$

$v_A = 16.53$ ft/s

$\Rightarrow t = 1.047$

$v_{Bx} = 16.53 \cos 30 = 14.32$

$(v_{By}) = v_o - 32.2 \, t = -25.45$

$v_B = \sqrt{14.32^2 + 25.45^2} = 29.2$ ft/s.

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CH.13:- Kinatics of particle:

- Force and acc.

※ Kinatics :- relationship between the motion ~~and~~ of bodies and it's causes namely forces and turques.

*— 13.1:- Newten's 2nd Low of motion.

$$\underline{\Sigma F = ma}$$

- Newten's Low of gravitational attraction.

قوة التجاذب بين جسمين . ← $F = G \frac{m_1 m_2}{r^2}$

G:- $66.73 * 10^{-12}$ m³/Kg.s
↳ universal constant.

F:- force attraction between two particles.

m_1 / m_2 : mass of each of the two particles.

r:- ~~dis~~ distance betw. the centers of the two Particles.

*— 13.2 :- The equation of motion.

$$\Sigma F = ma$$

[1] F.B.D

- apply all forces actiong on the particles
↳ external motion. ; weight
↳ μ_k : friction factor. ; normal force.

$\boxed{2}$ Kinetics : $\Sigma F = ma$

$r - \theta$ Rectangular component. $n - t$

$\boxed{3}$ Kinamatics :- Apply equation of motion

* $a = f(t) \Rightarrow$ use $a = dv/dt$
  $v = ds/dt$

* $a = f(s) \Rightarrow$ use $a\,ds = v\,dv$

* a ; constant $\Rightarrow$ $v = v_o + at$
  $v^2 = v_o^2 + 2a(s - s_o)$
  $s = s_o + v_o t + .5 a t^2$

* Frictional force $= F_R = \mu_K . N$

* Spring force :- $F_s = \underline{\underline{K}}\, S$
  ↳ spring constant

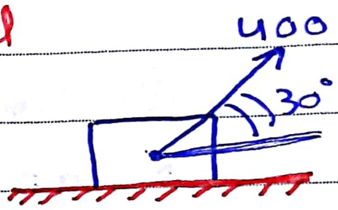
* $S = L - L_o$ → streched مشدود
  → compresed مضغوط

* 13.4 :- The eq of motion : Rectangular comp.

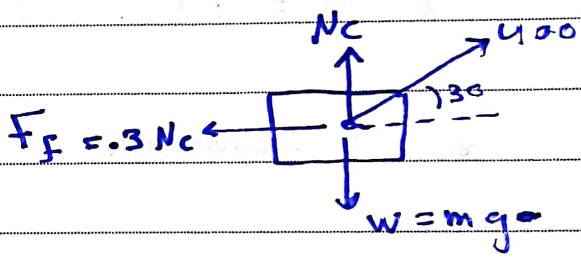
$$\Sigma F_x i + \Sigma F_y j + \Sigma F_z \hat{k} = m(a_x i + a_y j + a\hat{k})$$

Ex:- 13.1) The 50 kg crate rests on a horizantal surface withe coef of kinetic fraction $\mu_k = .3$.

If the crate is subjected to 400 N twoing force.



Det. the velocity of the crate in 3 sec. starting from rest ?!



[1] F.B.D

$\xrightarrow{+} \Sigma f_x = m a_x$

$400 \cos 30 - .3 \, \cancel{N_c}^{290.5} = 50 \, a_x$

$+\uparrow \Sigma f_y = m a_x$

$N_c + 400 \sin 30 - 50 \, \cancel{g}^{9.81} = 0$

$\Rightarrow N_c = 290.5 \ N$

$a_x = 5.2 \ m/s^2$

$a =$ constant.

$\Rightarrow$ Kinamatics:-

$v = v_\circ + at$

$v^2 = v_\circ^2 + 2a \Delta s$

$\Delta s = v_\circ t + .5 a t^2$

Ex :- 13.2 : A 10kg is fired vertically upword from the ground with an initial velocity of 50 m/s.

- Det. max hight to which it will travel ; If

(a) atmospharic resistance is rejected.

(b) " " " measured.

$F_D = (0.01\, v^2)\ N$

F.B.D

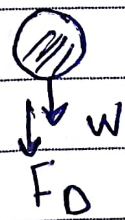
SOL:

(a) $v^2 = v_0^2 - 2gh$

$0 = 50^2 - 2(9.81)\,h$

$\boxed{h = 127\ m.}$

F.B.D



$a \neq g$

(b) [1] F.B.D

[2] $+\uparrow \Sigma F_y = ma$

$-W - F_D = ma$

$-98.1 - 0.01\, v^2 = 10\, a$

$a = -9.81 - 0.001\, v^2$

[3] $a\,ds = v\,dv \rightarrow$ E.L

$$a\,dy = v\,dv$$

$$\int_0^h dy = \int_{50}^{0} \frac{v}{a}\,dv$$

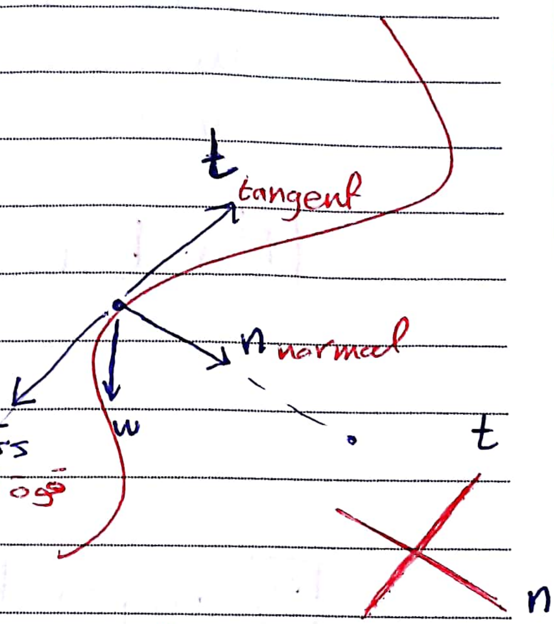
$$h = \int_{50}^{0} v/(-9.81 - 0.001v^2)\;.dv$$

$$\boxed{h = 114\text{ m}} \quad — *$$

Sec 13.5 / Equation of motion: Normal and tangent Component.

- * n عمودي ع T
- * $y = f(x)$
- * $\tan\theta = y'$
- * $\theta = \tan^{-1}(y')$
- * $S = r\theta$

  $ds = r\,d\theta$



- $ds = \rho\, d\theta$ In general. ※

$\Sigma F_t = m a_t$

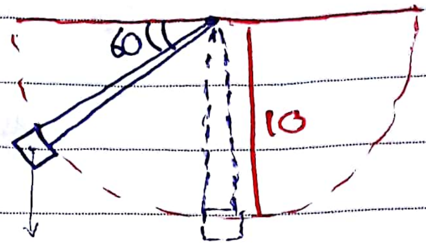
$\Sigma F_n = m a_n$

※ Normal force is $\perp$ to the tangent and passes the center.

※ Friction force is opposite to direction motion of motion along the tangent.

$y = f(x)$ $\rho = \left[1 + (dy/dx)^2\right]^{3/2} \Big/ d^2y/dx^2$

Ex:-



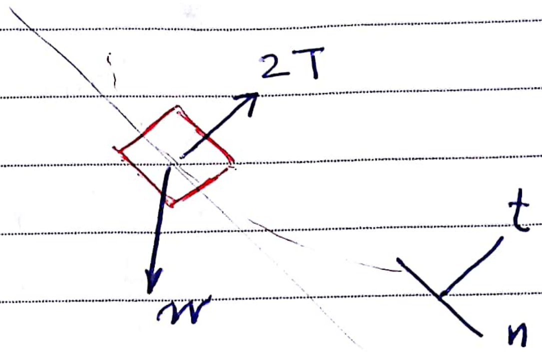
$\theta = 60$

Find at $\theta = 90$

$w = 60$ lb

$\rho = 10$

$\boxed{1}$ F.B.D



$\boxed{2}$ Apply Kinetics.

$$\sum F_n = m\, a_n$$

$$2T - w \sin\theta = m\, a_n \qquad a_n \to v^2/\rho$$

$$2T - 60 \sin\theta = \left(\frac{60}{32.2}\right) * \frac{v^2}{10} \quad \text{———} *$$

$$\sum F_t = m\, a_t$$

$$w \cos\theta = m a_t$$

$$\cancel{60} \cos\theta = \left(\frac{\cancel{60}}{32.2}\right) a_t \Rightarrow \underline{a_t = 32.2 \cos\theta}$$

$\boxed{3}$ Apply knematics:-

$$a_t * ds = v\, dv \qquad ع.ت \Rightarrow$$

$\Rightarrow$ $a_t \, ds = v \, dv$; $ds = r \, d\theta$

$$\int_{60}^{90} 32.2 \cos\theta * (\underset{10}{r} \, d\theta) = \int_{0}^{v} v \, dv$$

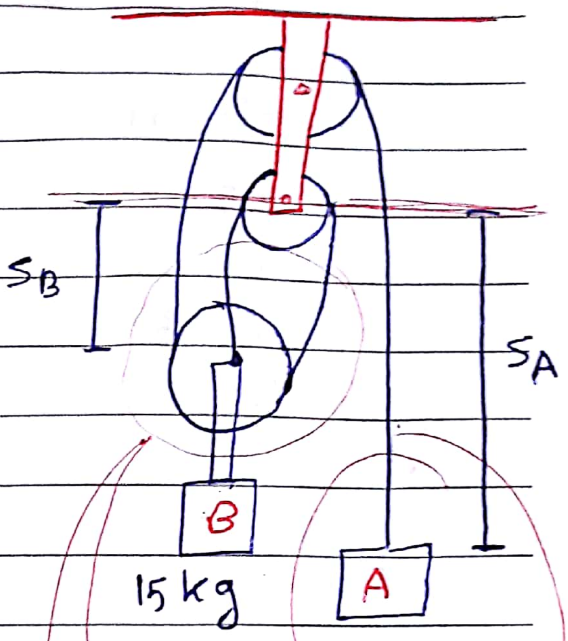
$\boxed{v = 2.28}$ ft/s

$$2T = 60 \sin\theta + \frac{60}{32.2} * \frac{v^2}{10}$$

$$2T = 60 \sin 90 + \frac{60}{32.2} * \frac{(2.28)^2}{10}$$

$\boxed{T = 38 \text{ lb}}$

Ex: P13-22 $t = 2s$



$3S_B + S_A = l_{tot}$

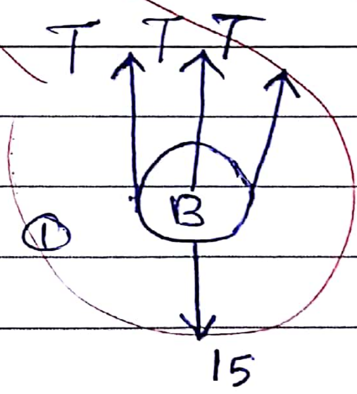
$3v_B + v_A = 0$

(1) — $3a_B + a_A = 0$

$a_A + 3a_B + 0T = 0$

$\Sigma F_{y_1} = m a_B$

$3T - W = m a_B$



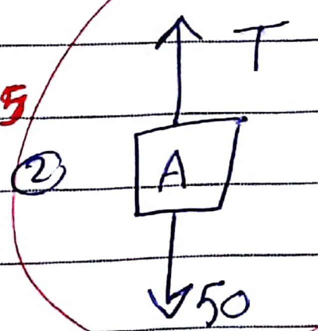
(2) — $3T - 15(9.81) = \left(\frac{15}{\cancel{\quad}}\right) a_B$

$0a_A + 15 a_B - 3T = -147.15$

$\Sigma F_{y_2} = m a_A$

(3) — $T - 50(9.81) = \left(\frac{50}{\cancel{\quad}}\right) a_A$

$50 a_A + 0 a_B - T = -490.5$



$a_A = 8.554$

$a_B = 2.848$

$T = 63.29$

$v_B = v_{A_0} + a_B(t)$

$v_B = 0 + 2.848(2) = 5.696 \text{ m/s} \uparrow +$

H.W and sug. prob.

13.11 13.56

13.23 13.59

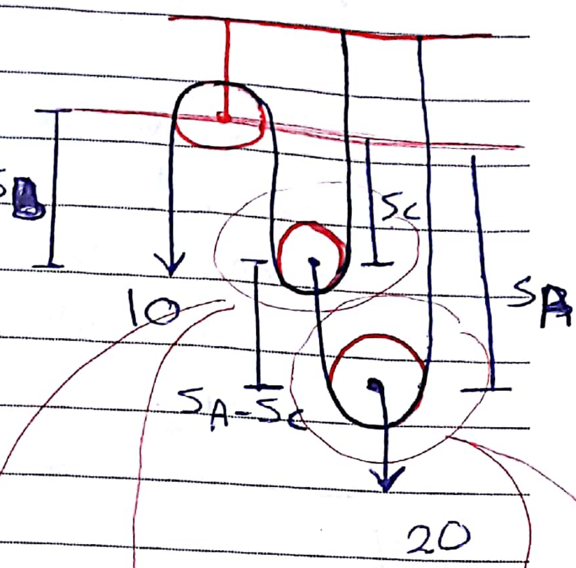
13.28 13.60

13.36

prob 13-11

find the time needed to cord at B down 4 ft.

and $s_o = 0$ $v_o = 0$



Sol:

2 robe:-

$s_B + 2s_C = l_1$

$v_B + 2v_C = 0$

$a_B = -2a_C$

$2s_A - s_C = 0$

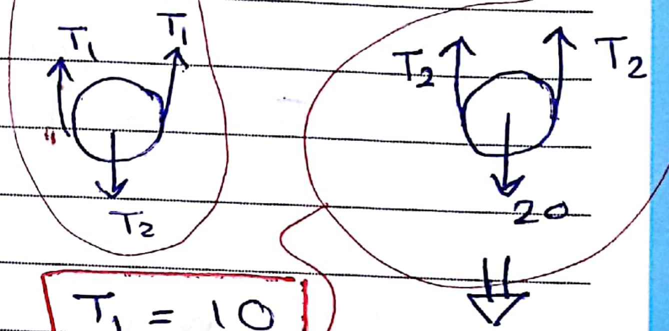
$2v_A - v_C = 0$

$v_C = 2v_A$

$a_C = 2a_A$

$\boxed{a_B = -4a_A}$

$\boxed{a_B = 128.8 \text{ ft/s}^2}$



$\boxed{T_1 = 10}$

$T_2 = 2T_1$

$\boxed{T_2 = 20}$

$+\uparrow \Sigma F_y = 0$

$2T_2 - 20 = ma_y$

$40 - 20 = \frac{20}{32.2} a_y$

$\boxed{a_{y_A} = 32.2 \text{ ft/s}^2}$

$S = S_o + v_o t - .5(128.8)t^2$

$4 = 0 + 0 - .5(128.8)t^2$

$t = 0.249 \text{ s}$ ans.

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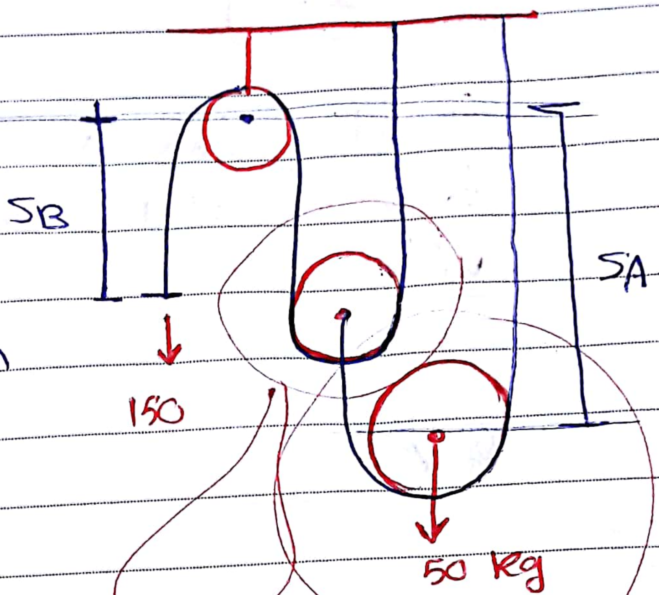
prob 13 - 23

find the velocity of A ! when it has risen 3m. start from rest. $v_0 = 0$, $S_0 = 0$

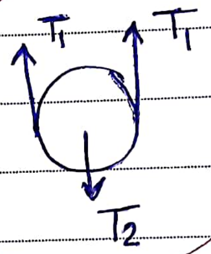


Sol:

$T_1 = 150$

$T_2 = 2(150)$

$\boxed{T_2 = 300}$

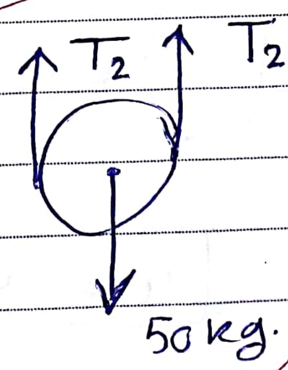


$\sum f_y = m a_A$ ①

$2T_2 - 50(9.81) = 50\,a_A$

$2(300) - 490.5 = 50\,a_A$ ②

$\boxed{a_A = 2.19}$



$v_f^2 = v_0^2 - 2\,a_A\,(S - S_0)$

$v_f^2 = 0 - 2(2.19)(3)$

$v_f = 3.625$

ans.

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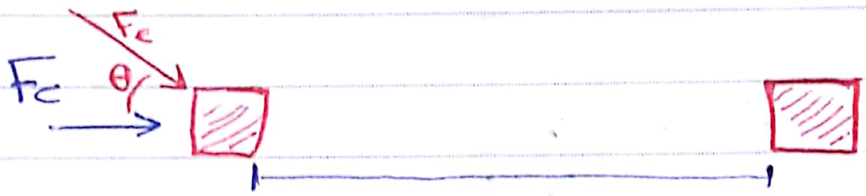
CH. 14 : Kinetics of a particle: work and energy.

※ A Force does work on a particle only when the particle undergoes a displacement in the direction of force.

※ Work of a constant force along a straight line.



$U_{1-2} = F_c \, \Delta s$ Unit → SI → J → N.m

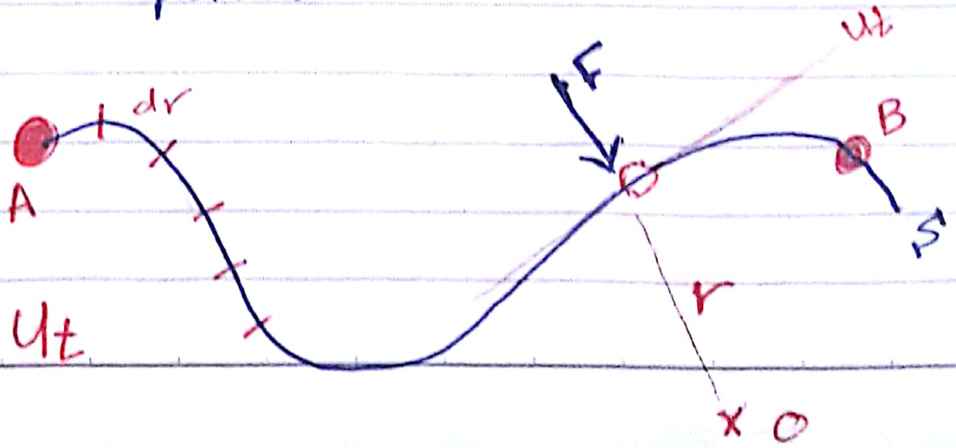
↳ work.   ↳ FPI → Ib.ft

$U_{1-2} = F_c \cos\theta \, \Delta s$ → +ve $0 < \theta < 90$

↳ 0 $\theta = 90$

↳ −ve $90 < \theta < 180$

※ work of a variable force along a curv path.



$\vec{dr} = ds \, u_t$

$$U_{1-2} = \int_{r_1}^{r_2} \vec{F} \cdot \vec{dr} = \int_{s_1}^{s_2} F \cdot ds \; u_t$$

$$U_{1-2} = \int_{s_1}^{s_2} F \cos\theta \; ds.$$

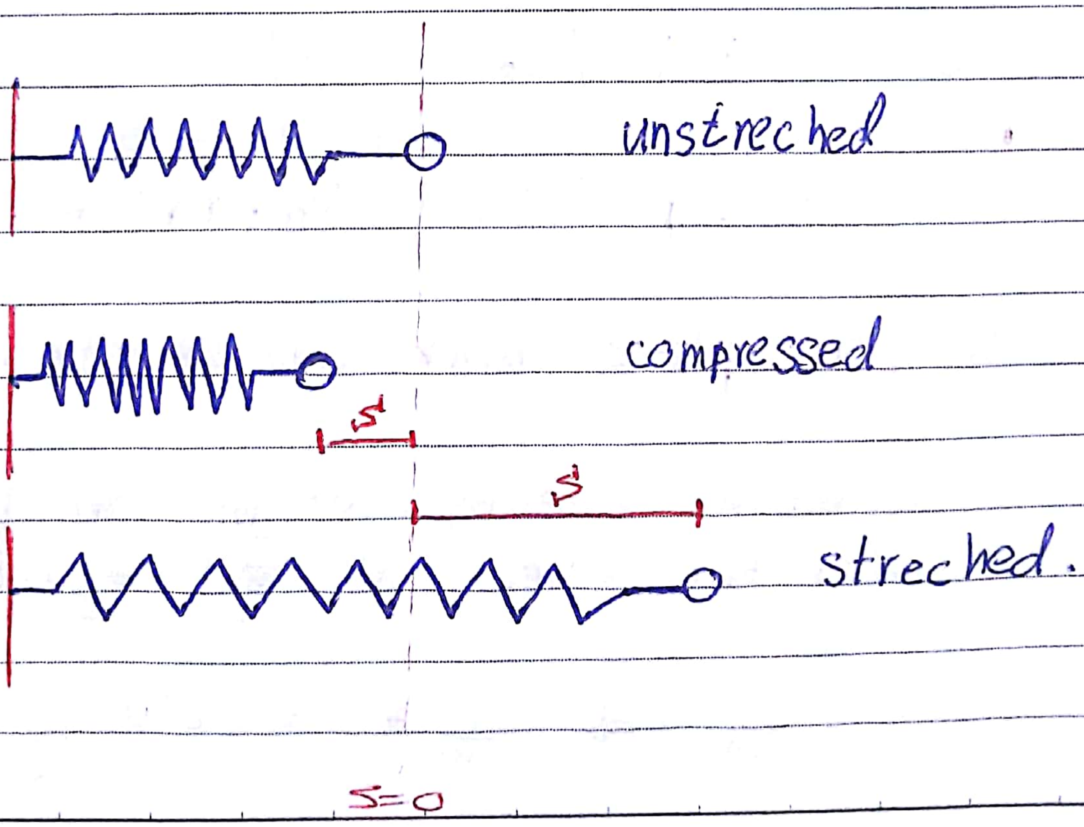
※ work of weight.

$\uparrow + y$ $$U_{1-2} = \int_{y_1}^{y_2} -W \; dy$$

$$= -W \, \Delta y$$

- IF Δy is upword $\Rightarrow U$ -ve
  Δy is downword $\Rightarrow U$ +ve

※ Work of spring.



※ Work done by spring Force.

$F_s = K \cdot s$; k: spring constant.
s: displacement.

$$U_{1-2} = \int_{s_1}^{s_2} F_s \, ds$$

$$= \int_{s_1}^{s_2} K \cdot S \, ds$$

$$\boxed{U_{1-2} = \frac{1}{2} k (s_2^2 - s_1^2)}$$

※ F_s exerted on the particle is aposite to that exerted on the spring.

$$\boxed{U_{1-2} = -1/2 \, K (s_2^2 - s_1^2)}$$

- Initial position is important.

※ principle of work and energy.

- Combine the eq. of motion $\boxed{F = ma}$ with the Kinetic eq $\boxed{a\,ds = v\,du}$

$a = \frac{F}{m} \Rightarrow \frac{F}{m} ds = v\,du$ تابع ⟹

$$\int_{s_1}^{s_2} F\,ds = \int_{v_1}^{v_2} mv\,dv$$

$$F\Delta S = 1/2\, m(v_2^2 - v_1^2)$$

$$\boxed{U_{1-2} = T_2 - T_1}$$; T: kinetic energy
$T = 1/2\, mv^2$

$$\boxed{T_2 = T_1 + \Sigma U_{1-2}}$$

sec: 14.4 : Power and efficiency.

P and ε

↳ time rate of work (watt)

$P = \frac{dv}{dt} = \vec{F} \cdot \vec{V}$; F: force V: velocity

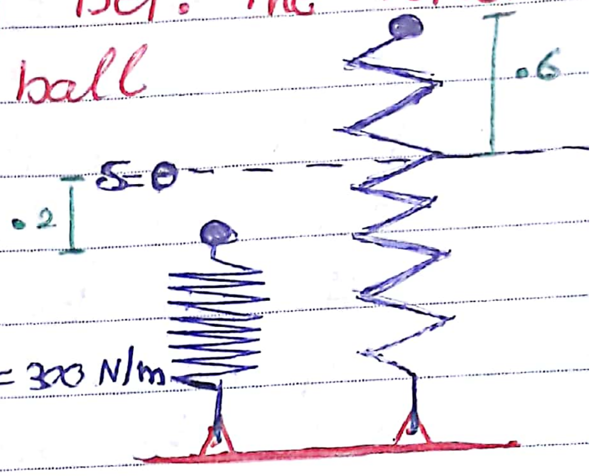
P: SI → $J/s = \frac{N.m}{s} = W$

FPS: $1\,hp = \frac{b.ft}{s}$; hp:- horse power
$hP = 746\ W$

ε : Efficiency : rate of useful out pot power ; It measers the losses in a system.

$\varepsilon = P_{out}/P_{in} = E_{out}/E_{in}$ $E < 1$

*ex:- 8 kg ball is connected with a spring as shown Intially the spring is compressed by (.2) m then under the applied force = 400 N the spring streched to (.6) m Det. the total work done be the ball



SOL:- $F = 400\ N$ $m = 8\ kg$

$k = 300\ N/m$

\# work done by weight:-

$$U_{weight\ 1-2} = -w(S_2 - S_1)$$
$$= -8(9.81)(.6 - -.2)$$
$$= -68.2\ J$$

\# Work done by spring:-

$$U_{spring\ 1-2} = -1/2\ k(S_2^2 - S_1^2)$$
$$= -1/2(300)((.6)^2 - (-.2)^2)$$
$$= -48\ J$$

& b ⟹

* Work done by the force:-

$U_{F\,1-2} = F(S_2 - S_1)$
$= 400(.6 - -.2)$
$= 320\ J$

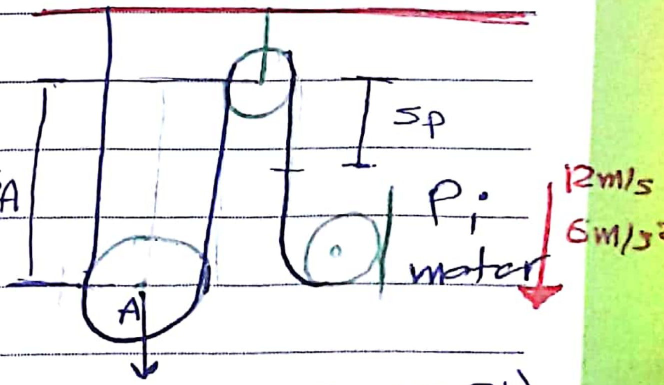
* the total work:- $\Sigma U = U_{S\,1-2} + U_{F\,1-2} + U_{W\,1-2}$

$320 - 68.2 - 48 = 209\ J$.

---

qaz'il

ex: at the instant shown, point p on the cable has a velocity $v_p = 12\ m/s$
$a_p = 6\ m/s^2$
det. the power of the motor P_i of the motor?! $(\Sigma = .75)$ $(m_a = 50\ kg)$



$2S_a + S_p = L$
$2V_a + V_p = 0$
$2a_a + a_p = 0$

$V_a = \cancel{24}\ 6\ m/s$
$a_a = \cancel{12}\ 3\ m/s \uparrow$

$mg = 50(9.81)$
$= 490.5$

$\Rightarrow \Sigma f_y = ma$
$T_A - 490.5 = 50(3)$

$T_A = 640.5$ $\Rightarrow$ & L

$$P_{out} = T \cdot V$$
$$= (640.5)(12) = 3843 \text{ W}$$

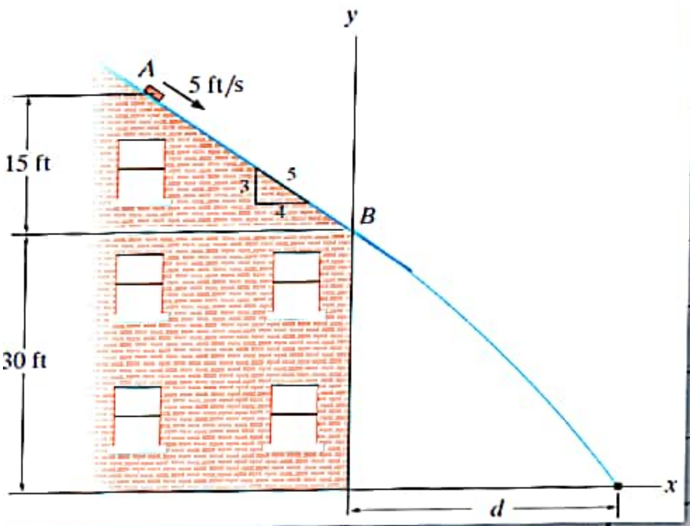
$$P_{in} = \frac{P_{out}}{\varepsilon} = 4803.75 \text{ W}$$

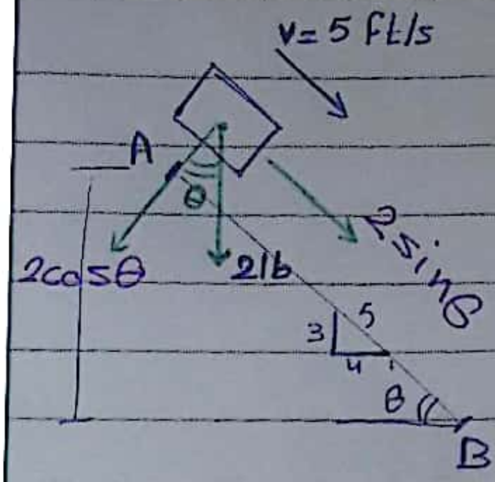
H.W 14.13
.17
.24
.31
.47
.68
.80
.84

end of 14

H.W 8-

14–13. The 2-lb brick slides down a smooth roof, such that when it is at A it has a velocity of 5 ft/s. Determine the speed of the brick just before it leaves the surface at B, the distance d from the wall to where it strikes the ground, and the speed at which it hits the ground.





$\cos\theta = (4/5)$

$\sin\theta = (3/5)$

$AB = \frac{15}{\sin\theta} \Rightarrow AB = \frac{15}{(3/5)}$

$AB = 25$ ft.

$T_A + U_{A\to B} = T_B$

$\Rightarrow (\frac{1}{2})(\frac{2}{32.2})(5)^2 + 2(3/5)(25) = (1/2)(2/32)(v_B)^2$

$\Rightarrow v_B = 31.48$ ft/s ans

$\Rightarrow y_0 = 0 \quad y_f = -30 \quad (v_B)_y = -31.48 \sin\theta \; (3/5)$

$x_0 = 0 \quad x_f = d \quad (v_B)_x = 31 \cos\theta \; (4/5)$

$\Rightarrow y_f = y_0 + (v_B)_y t - .5(32.2)t^2$ ($y_f = -30$, $(v_B)_y = -31.48(3/5)$)

$\Rightarrow t = 0.899$ s

$\Rightarrow x_f = x_0 + (v_B)_x t$ ($x_0 = 0$, $(v_B)_x = 31.48(4/5)$, $t = 0.899$)

$d = 22.64$ ft. ans

$\Rightarrow T_A + U_{A\to C} = T_c \Rightarrow (1/2)(\frac{2}{32.2})(5)^2 + 2(45) = \frac{1}{2}(\frac{2}{32.2})v_c^2$

$\Rightarrow v_c = 54.1$ ft/s

ans

CH. 15/ Kinetics of a particle.

* Impulse and Momentum.

الزخم.

Impulse:- (I): Is the integral of the force, (F) over the time interval, (t) for which it acts.

$$I = \int_{t_1}^{t_2} F \cdot dt \rightarrow \text{vector}$$

$$\Sigma F = m\underbrace{a}_{\frac{dv}{dt}} \rightarrow \Sigma \int_{t_1}^{t_2} F dt = \int_{v_1}^{v_2} m\, dv$$

momentum.

$$\Rightarrow I = \boxed{m\vec{v}_2} - mv_1 \Rightarrow \boxed{I = m(v_2 - v_1)}$$

$$I = \Delta L \Rightarrow L = mv$$

علاقة طردية
بين ال L وال m
وال V ← كلما زاد
ال m يزداد ال L

* Unit of momentum:-

SI → kg.m/s

FPS → slug.Ft/s

Momentum:-(L) A masure of How dfficult an object will be to stop.

↳ proportional relationship $m, v \rightarrow L$

- constant Force, $F_c \rightarrow I = F_c(t_2 - t_1)$
- Variable Force, $F \rightarrow I = \int_{t_1}^{t_2} F\,dt$.

→ In x - y - z componants.

$$mv_{1,x} + \sum \int_{t_1}^{t_2} F_x\,dt = mv_{2x}$$

$$mv_{1y} + \sum \int_{t_1}^{t_2} F_y\,dt = mv_{2y}$$

$$mv_{1z} + \sum \int_{t_1}^{t_2} F_z\,dt = mv_{2z}$$

ex:- A hockey puck is traveling to the left with a velocity of $V_1 = 10\,m/s$ when it is struck by hockey stick and given a velocity of $V_2 = 20\,m/s$ as shown.

Determine the magnitude of the net impulse exerted by the hokey stick on the puck.

hint:- the puck has a mass of 0.2 kg.

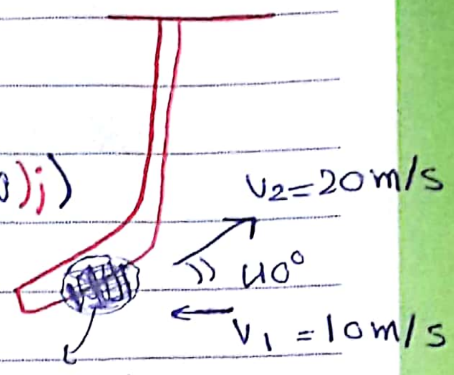
.dkl ⇒

Sol:

$$I = m(V_2 - V_1)$$

$$I = \overset{(.2)}{m}((20\cos 40 - 10)\,i + (20\sin 40)\,j)$$

$$I = 5.06\,i + 2.57\,j$$



$(V_1 = -10\,m/s\ i)$

$(V_2 = 20\cos 40\ i + 20\sin 40\ j)$

$$\Rightarrow I = \sqrt{(5.06)^2 + (2.57)^2}$$

as mag.

$$= 5.68\ N.S.$$

Exe:- 15.14 A tank car has a mass of 20 Mg $V_1 = .75\ m/s$; find (I) @ $k = \infty$, $k = 15\,kN/m$

sol:- $I = mV_1 = 20{,}000 * .75$
$= 15\ kN.s$

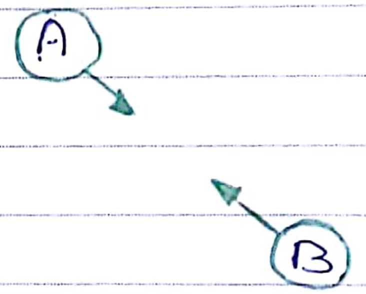
* The impulse of the same for both cases $k = 15\,kN/m$ the impulse is applied over a large period of time than $k = \infty$.

sec 15.4 | Impact.

※ Special case of impulse - momentum where two particles collide and interact during a very short period of time.

※ conservation of momentum.

$$m_A (V_A)_1 + m_B (V_B)_1 = m_A (V_A)_2 + m_B (V_B)_2$$



※ During the time of interaction, the $\int F \cdot dt$ acting on each particle is equal and opposite ⇒ They cancel out of the system momentum.

※ The exiting velocities are det. by the coeff. of restitution. (e)

الترتيب بالقانون مهم ←

$$e = \frac{(V_B)_2 - (V_A)_2}{(V_A)_1 - (V_B)_1} = \frac{\text{exiting velocite}}{\text{opproaching}}$$

$0 < e < 1$

if $e = 0$ $\Rightarrow$ plastic impuls. * مهم بيسى فرع دائرة.

the two particles will stuck together.

if $e = 1$ $\Rightarrow$ perfect elastic impact.

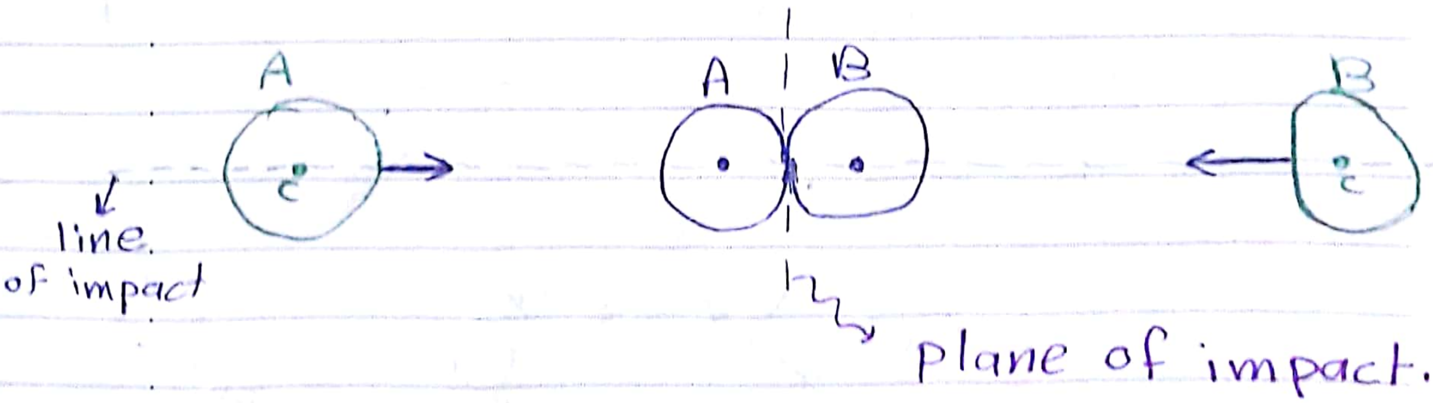
① * central impact.

occurs when the motion of the two mass centers is on aline connecting the two mass centers.

$\Downarrow$

this Line is called of impact.

* plane of impact. $\perp$ to the line impact
عمودي



1D : Dimension

البعد الأول.

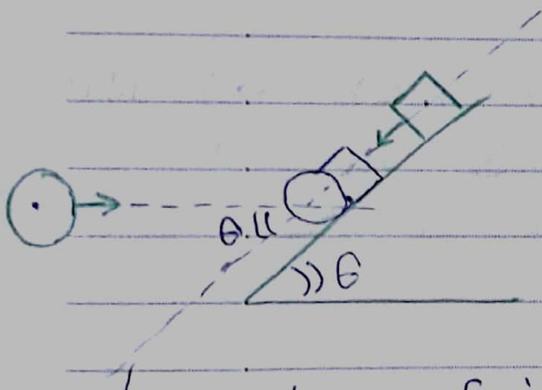
مهم بيجي فرع دائرة.

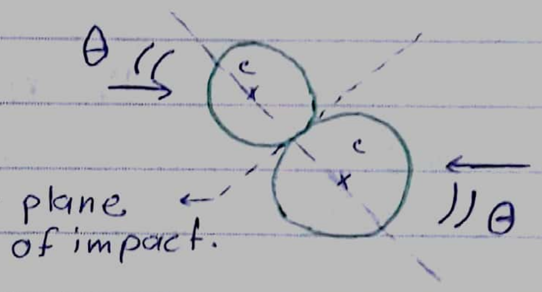
• centeral impact is one D impact in which the motion of the mass centers before & after impact are all on the line of impact.

مائل.

مهم فرع دائرة.

② **Oblique impact**: The line of impact occurs at an angle to the line of motion of one or both of the particles.





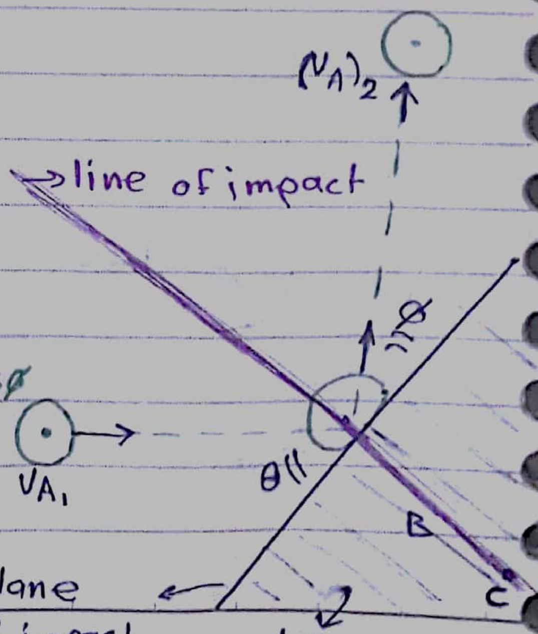
line of impact.

line of impact.

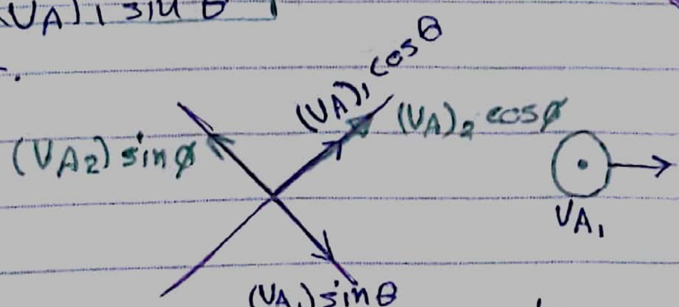
ex:-

$$e = \frac{0 - (V_A)_2 \sin\phi}{-(V_A)_1 \sin\theta - 0}$$

$$\boxed{e_1 = \frac{(V_A)_2 \sin\phi}{(V_A)_1 \sin\theta}}$$



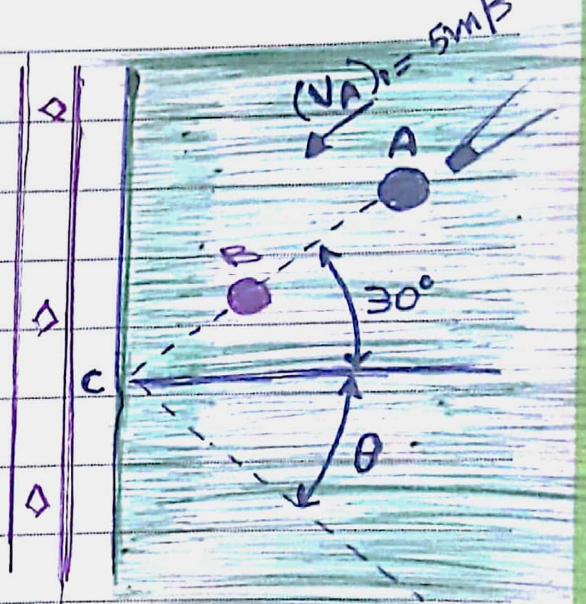
line of impact.



* $(V_A)_1 \cos\theta = (V_A)_2 \cos\phi$

plane of impact.

ex:- P.15.77:- The cue ball A is given an initial velocity $(V_A)_1 = 5\, m/s$ If It makes a direct collision with ball B $(e = .8)$, determine the velo. of B and the angle θ just after it rebounds from the cushion at C $(e' = .6)$. Each ball has a mass of $(.4\, kg)$ Neglect the size.



Sol: $\Sigma (m v_*)_1 = \Sigma (m v)_2$

$$\cancel{m}_A (V_A)_1 + \cancel{m}_B (V_B)_1 = \cancel{m}_A (V_A)_2 + \cancel{m}_B (V_B)_2$$

① — $5 + 0 = (V_A)_2 + (V_B)_2$

$$e = \frac{(V_B)_2 - (V_A)_2}{(V_A)_1 - (V_B)_1} \Rightarrow (.8) = \frac{V_{B2} - V_{A2}}{5 - 0}$$

② — $\Rightarrow 4 = (V_B)_2 - (V_A)_2$

$\Rightarrow (V_B)_2 = 4.5\, m/s$; $(V_A)_2 = .5\, m/s$

$$(\cancel{m}_B)(V_{By})_2 = \cancel{m}_B (V_{By})_3$$

$$4.5 \sin 30 = (V_B)_3 \sin\theta \Rightarrow \boxed{V_{B3} \sin\theta = 2.5}$$

$$e' = \frac{(V_c)_2 - (V_{Bx})_3}{(V_{Bx})_2 - (V_c)_1} \Rightarrow (.6) = \frac{-(-(V_B)_3 \cos\theta)}{4.5 \cos 30} \Rightarrow$$

$(V_B)_3 \sin\theta = 2.25$ —— (3)

$(V_B)_3 \cos\theta = 2.338$ —— (4)

$\Rightarrow \theta = \tan^{-1}\left(\frac{2.25}{2.338}\right) = \underline{\underline{43.9^\circ}}$ ans.

$\Rightarrow (V_B)_3 = \underline{\underline{3.24\ m/s}}$ ans.

end of CH.5.

CH.16 : Kinetics of arigid body.

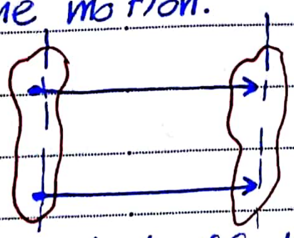
16.1 * **Rigid body:-** Solid body in which deformation is zero or so small that it can be neglected.

- or the distance between any two points remains constant intime regardless of external forces exacted on it.

* Rigid body planar motion:-

Translation

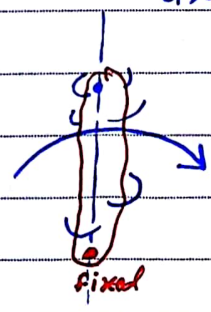
- aline in the body remains parallel to it's original orientetion throwout the motion.



All point off the body have asame velocity and acceleration.

Rotation

- When arigid body rotates about fixed axis.

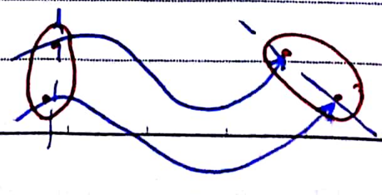


General.

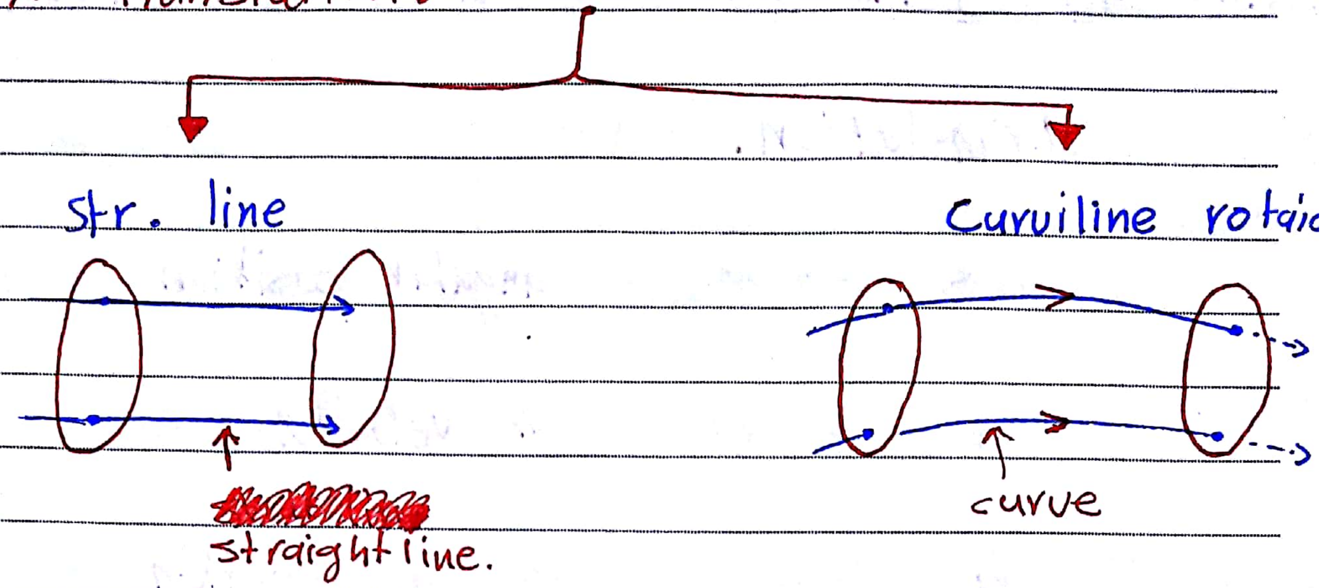
P. Motion.

** General planar Motion

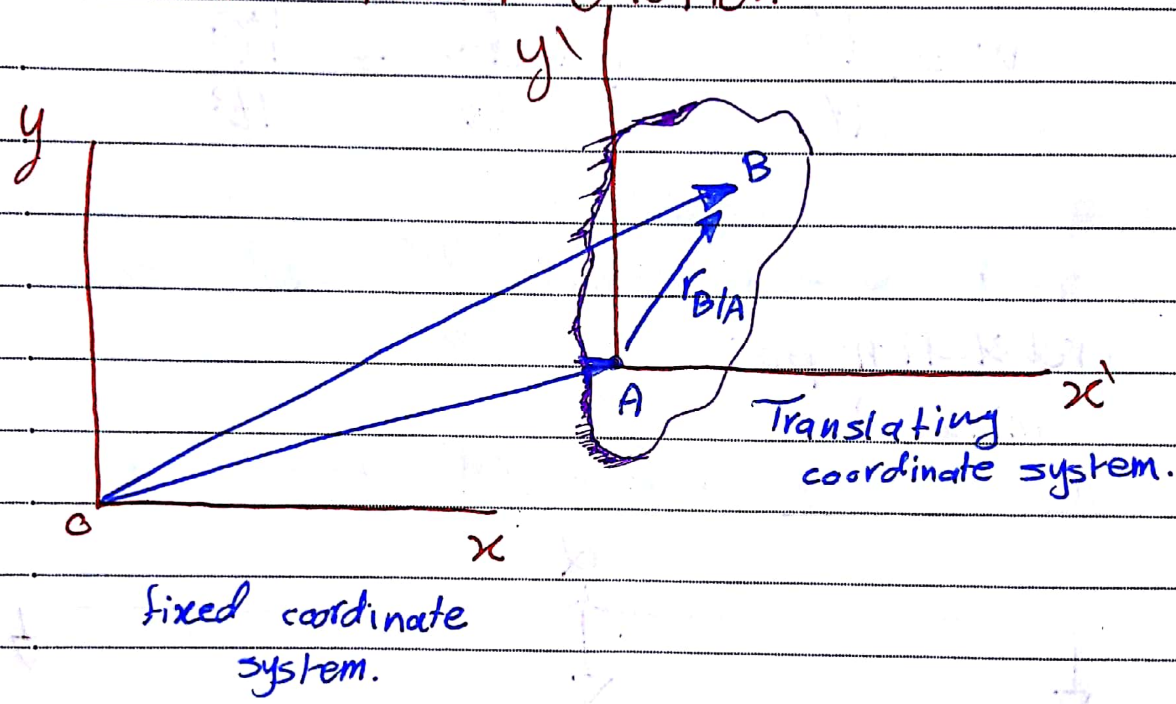
- combination between translaition and rotation



16.2 * Translation.

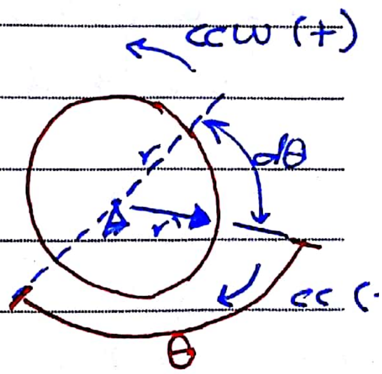


* General Planar Motion



* Rotation about fixed axis.

s	m	$\rightarrow$	θ	rad	
v	m/s		ω	rad/sec	
a	m/s^2		α	rad/sec^2	
t	sec		t	sec.	



section $\boxed{16.3}$ Rotation about a fixed axis.

translation.

$s \rightarrow m$	angular position θ rad.
$v \rightarrow m/s$	" velocity ω rad/sec.
$a \rightarrow m/s^2$	" acceleration α rad/sec^2.

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

rpm (revelution per minute) $\rightsquigarrow$ rpm $= \frac{2\pi}{60}$ rad/sec.

rev $\rightsquigarrow$ rev $= 2\pi$ rad/sec.

α

α: constant

$$w = w_o + \alpha_c t$$

$$\theta = \theta_o + w_o t + \frac{1}{2}\alpha_c t^2$$

$$w^2 = w_o^2 + 2\alpha_c(\theta - \theta_o)$$

$\alpha = f(t)$

θ

w

α

diff

اشتق

int.

تكامل

$\alpha = f(\theta)$

$$\alpha\, d\theta = w\, dw$$

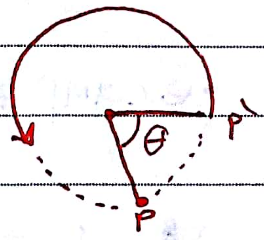
$w = d\theta / dt$

$\alpha = dw/dt$

※ Motion of apoint P.

When abody rotate about afixed axis any point (P) located on the body travels along acircular path.

اذا طلب مني كم الجسم قطع مسافة بنستخدم هذا القانون ← $\boxed{S_p = r\theta}$



$$\frac{dS_p}{dt} = \frac{r\,d\theta}{dt}$$

[*] $\boxed{v_p = r\omega}$

the velocity acts always tangent to the path of motion.

acceleration has two components.

$$a_p \longrightarrow a_{p,t} = \alpha r$$

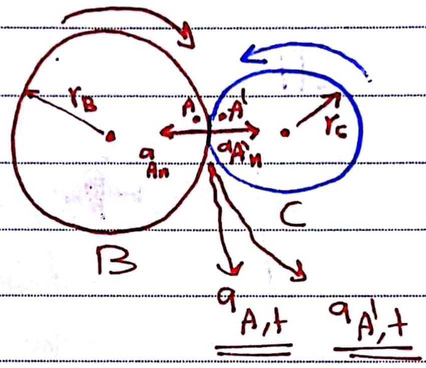
$$\qquad \longrightarrow a_{p,n} = \frac{v^2}{r} = \omega^2 r$$

$$a = \sqrt{(a_{p,t})^2 + (a_{p,n})^2}$$

※ If two rotating loodies contact one another.

⇒ the velocity and the tangentiel component of acceleration of the point's will be the same.

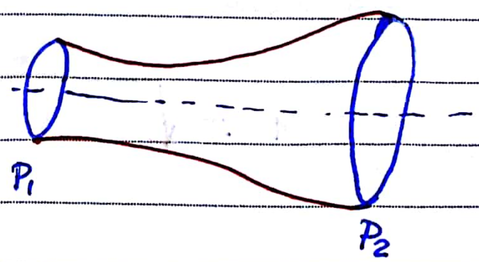
• example ① ※ Meshed gears.



⇒ دائماً تلفو عكس بعض

- $a_{An} \neq a_{A'n}$
- $V_A = V_{A'}$
- $a_{A,t} = a_{A't}$
- $\theta_B r_B = \theta_c r_c$
- $w_B r_B = w_c r_c$
- $\alpha_c r_B = \alpha_c r_c$

• example ② ※ Chain and sprocket

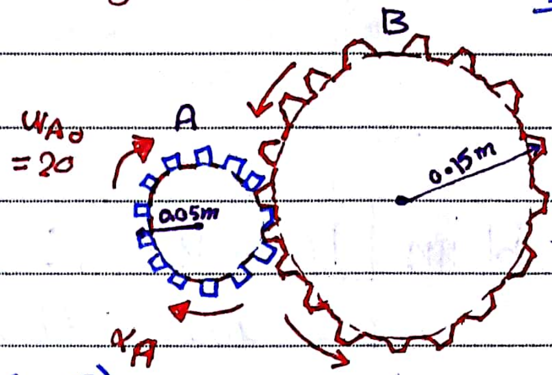


- $\theta_1 r_1 = \theta_2 r_2$
- $w_1 r_1 = w_2 r_2$
- $\alpha_1 r_1 = \alpha_2 r_2$

Ex:- Problem 16-20:

A motor given gear A an angular acceleration $\alpha = 4t^3$ rad/sec². If this gear is initially turning @ $(\omega_A)_0 = 20$ rad/sec. determine the angular velocity of gear B when $t = 2$ sec.

$\omega_B = ?!$



sol: $\omega_A r_A = \omega_B r_B$

$$\omega_B = \frac{\overset{?!}{\omega_A} r_A}{r_B} = \frac{(36)(.05)}{(.15)}$$

$$= (12) \text{ rad/sec}$$

to find ω_A ⇒ $\alpha = 4t^3$; $\alpha = \frac{d\omega}{dt}$; $d\omega = \alpha\, dt$

$$\int_{20}^{\omega} d\omega = \int_0^2 4t^3\, dt$$

عوضها فوق

$$\omega - 20 = t^4 \Big|_0^2 \Rightarrow \omega_A = (2)^4 + 20 = \boxed{36 \text{ rad/sec}}$$

H.W and suggest problem.

F 16-6	P 16-43	P 16-70
P 16-17	P 16-59	P 16-103
P 16-22	P 16-111	
P 16-31	P 16-116	

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sec $\boxed{16.4}$ Absolute Motion Analysis.

$$s \rightarrow \theta$$

$$v = \frac{ds}{dt} \rightarrow \omega = \frac{d\theta}{dt} = \dot{\theta}$$

$$a = \frac{dv}{dt} \rightarrow \alpha = \frac{d\omega}{dt} = \ddot{\theta} = \frac{d^2\theta}{dt}$$

problem $\boxed{16-48}$: Determine the velocity and acceleration of the peg A, which is confined between the vertically guide and rotating slotted rod.

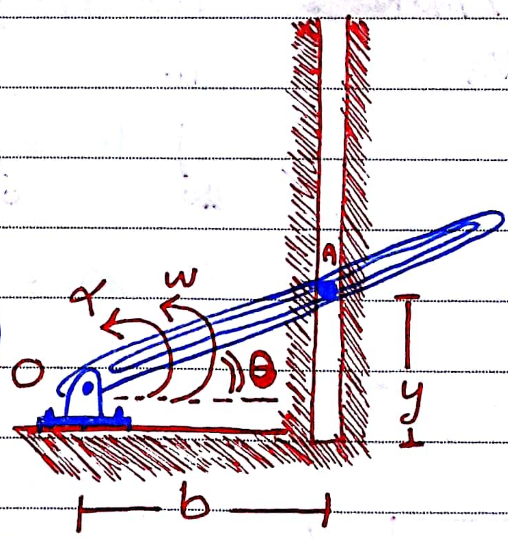
sol:

$$\tan\theta = \frac{y}{b}$$

$$y = b\tan\theta$$

$$\dot{y} = b\sec^2\theta\,\dot{\theta}$$

$$\ddot{y} = b(\sec^2\theta.\ddot{\theta} + 2\sec^2\theta\tan\theta.\dot{\theta}^2)$$



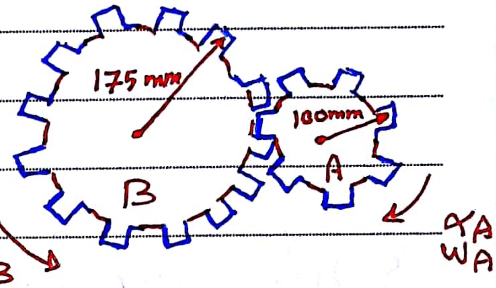
$$V = b\sec^2\theta\,\omega$$

$$a = b\sec^2\theta\,[\alpha + 2\tan\theta\,\omega^2]$$

suggest prob.

problem 16-17:- A motor gives gear A an angular acc. of $\alpha = (2 + .006\,\theta^2)\ rad/s^2$ where θ is in radians. IF this gears is initially turning at $w_A = 15\ rad/s$

\# determine the angular velocity of gear B after A undergoes an angular displacement of 10 rev.



$w_B = ?!$

$$\boxed{\theta_A = 10\ rev * 2\pi = 20\pi\ rad}$$

Sol:

$$w_A r_A = w_B r_B$$

$$w_B = \frac{\overset{?!}{w_A}\, r_A}{r_B} = \frac{(38.32)(.1)}{(.175)} = (21.89)\ rad/s$$

to find w_A $\Rightarrow$

$$\alpha_A\, d\theta = w_A\, dw$$

$$\int_0^{20\pi} (2 + .006\,\theta^2)\, d\theta = \int_{15}^{w_A} w_A\, dw$$

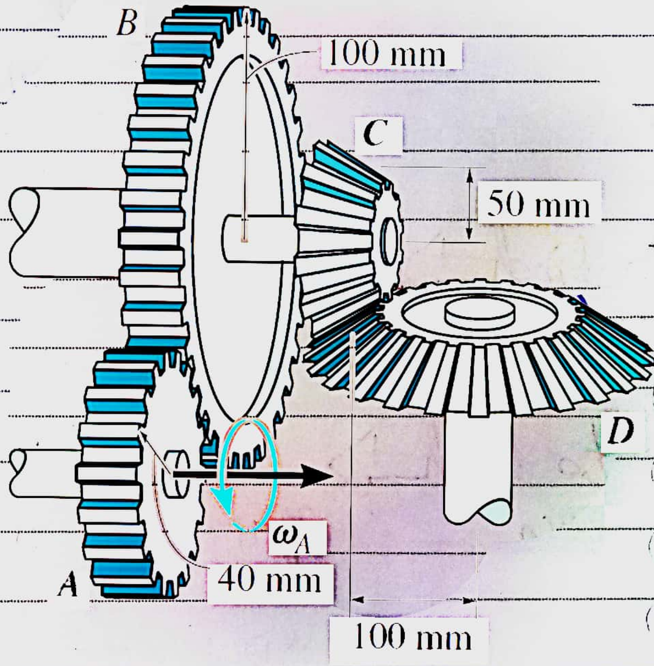
$$2\theta + \frac{\cancel{.006}^{\,2}}{\cancel{3}}\theta^3 \Big|_0^{20\pi} = \frac{w_A^2}{2}\Big|_{15}^{w_A}$$

$$*2 \left(\frac{w_A^2}{\cancel{2}} = \cancel{2}^{\,4}(20\pi) + \cancel{.002}^{\,.004}(20\pi)^3 + \frac{(15)^2}{\cancel{2}} \right)$$

$$w_A = 38.32138\ rad/sec.$$

عوضها
فوق

problem: 16-22: If the motor turns gear (A) with angular acceleration $\boxed{\alpha_A = 2\,rad/s^2}$ when the angular velocity is $\boxed{\omega_A = 20\,rad/s}$ determine the angular acceleration and angular velocity of gear (D).



Sol:

$\omega_A r_A = \omega_B r_B$

$$\omega_B = \frac{(20)(.04)}{(.1)}$$

$= 8\ rad/sec$

$\alpha_A r_A = \alpha_B r_B$

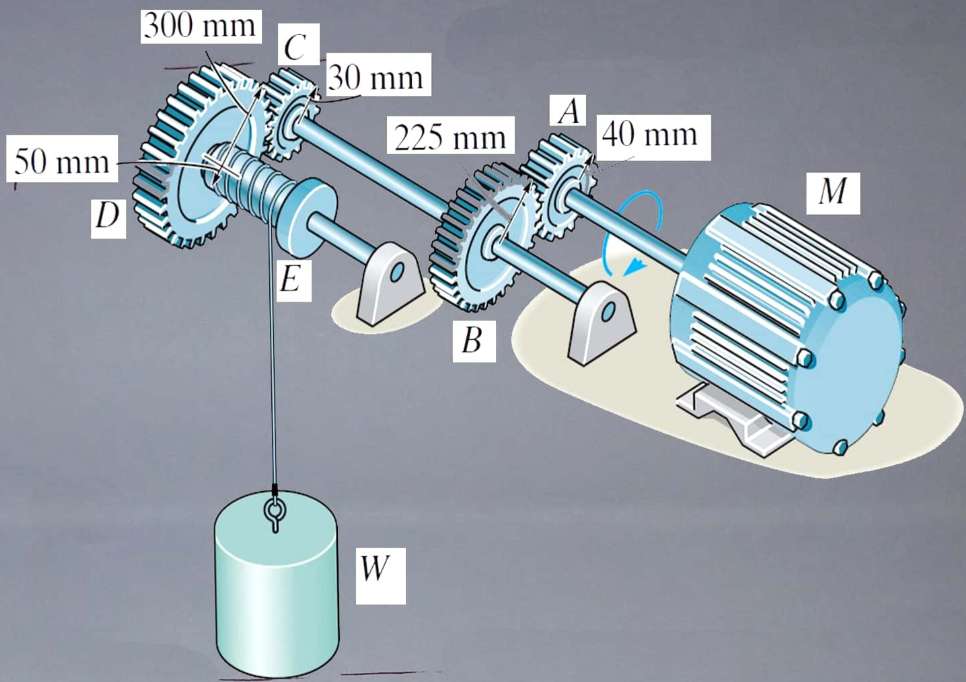
$$\alpha_B = \frac{(2)(.04)}{(.1)} = .8\ rad/sec^2$$

$\omega_B = \omega_C$; $\alpha_B = \alpha_C$

$$\omega_D = \frac{\omega_C r_C}{r_D} = \frac{8(.05)}{(.1)} = 4\ rad/sec$$

$$\alpha_D = \frac{\alpha_C r_C}{r_D} = \frac{0.8(.05)}{(.1)} = .4\ rad/sec^2$$

* Problem $\boxed{16-31}$ Determine the distance the load $\underline{W}$ is lifted in $\boxed{t = 5\,s}$ ~~hoist~~ Using the hoist. The shaft of the motor $\underline{M}$ turnes with an angular velocity $\underline{\omega = 100(4+t)\ rad/s}$ where t is in secounds.



Sol:

$$\omega = \frac{d\theta}{dt} \Rightarrow \int_0^{\theta_A} d\theta = \int_0^5 \omega\, dt$$

$$\Rightarrow \theta_A = 100\int_0^5 (4+t)\, dt = 100\left[4t + \frac{t^2}{2}\Big|_0^5\right]$$

$$\theta_A = 100\left[4(5) + \frac{(5)^2}{2}\right] = 3250 \text{ rad.}$$

$$\Rightarrow \theta_A r_A = \theta_B r_B \Rightarrow \theta_B = \frac{r_A}{r_B}\theta_A = \frac{40}{225}(3250) = \underline{577.78 \text{ rad}}$$

$$\Rightarrow \theta_B = \theta_C \Rightarrow \theta_C r_C = \theta_D r_D \Rightarrow \theta_D = \frac{30\,(577.78)}{300}$$

$$\boxed{\theta_D = 57.78}$$

$$\Rightarrow \theta_D = \theta_E \Rightarrow S_W = r_E\theta_E = (.05)(57.78)$$

$$\boxed{S_W = 2.89 \text{ m.}}$$ ans.