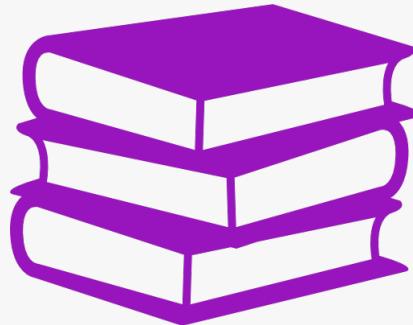




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# دوسية الزيود في : مختبر علم القياس

Metrology lab

إعداد : سيف الزيود

اللجنة الأكاديمية لقسم الهندسة الصناعية

2023



## ما هو الغرض

Q: what is the purpose of measuring devices?

- ch1 - Measurement systems are used for many detailed purposes in a wide variety of Application area.

ستقام انتظام العينات للعديد من الامراض الفيروسية في مجموعة متنوعة من مجالات التطبيق.

Q: define the measurement system?

- the term "measurement system" is meant to include all component in a chain of hardware and software that leads from the measured variable to processed data,  $\rightarrow$  number and unit

Q: How do measuring devices show the measured variable?

يعتمد بعض طلائع نظام العينات انه يشمل جميع المكونات وتحل محل المدخلات من الاجزاء طاربة الى البرامج التي توددي من اجل انجاز المعايس او لبيان عدد معايير

مثال: لتشغيل بسيط بسيط ((run a production line))  $\rightarrow$  لازم اعمل ((controlling)) انة بعين الاعتبار سوا الكيارة الى انابيب اعنيها معايير انتاج製品用 فلازم معايير  $\rightarrow$  التباين تكون معنوية compatible مع خط الانتاج.

Information: the modern automobile uses as many as 40 or 50 sensor (measuring devices)

Q: What should be the relationship between the designer and measuring instruments?

- The designer must be aware of the instruments available for a various measurements and how they operate and interface with other parts of the system.
- and aware in sensor that allow improvement.

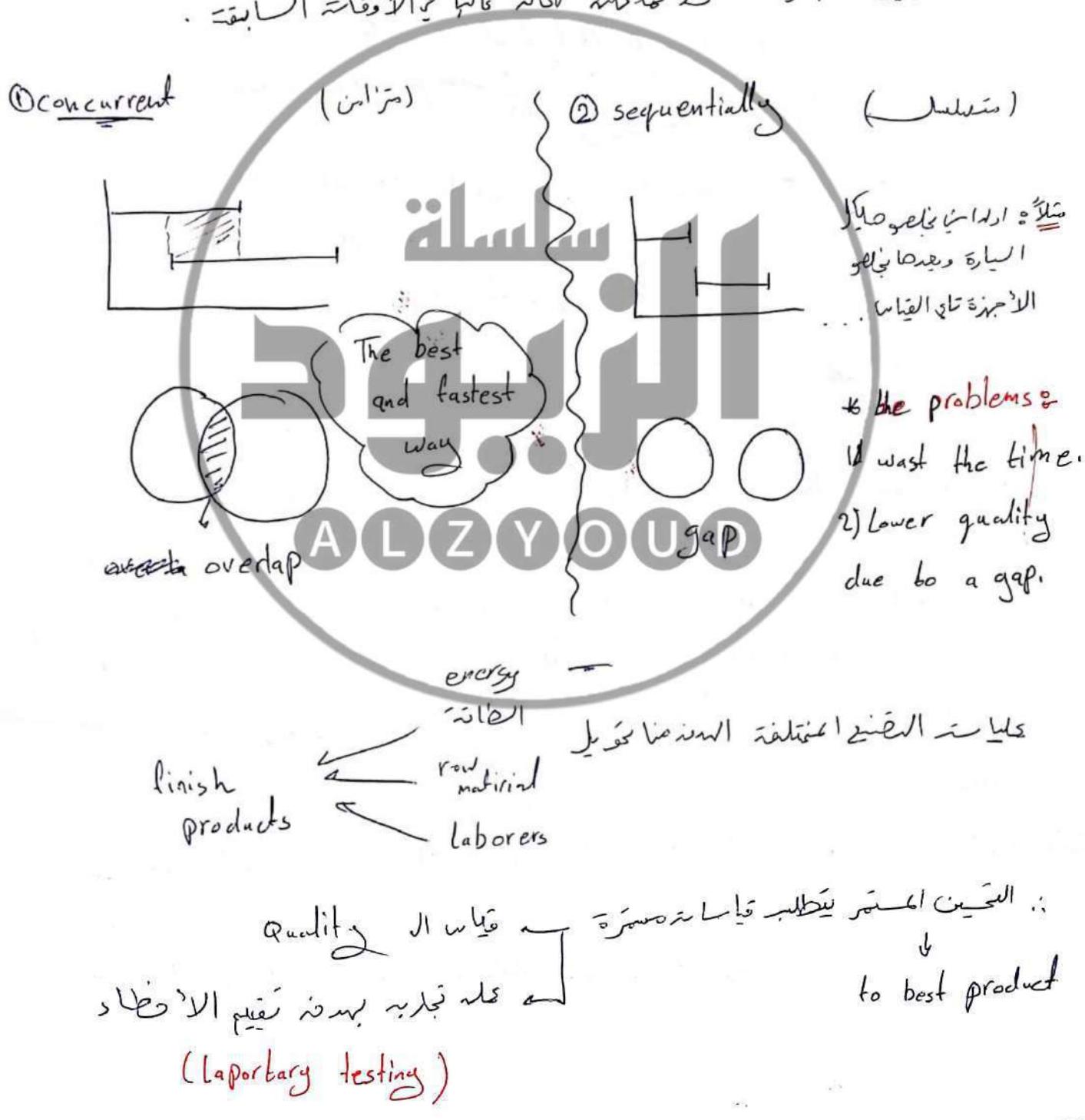
يجب على المصمم أن يكون على دراية بالادوات المتاحة لقياساته المختلفة ويفهم ما هي ومتى يمكنه الالتفاف حولها.

وغيرها من اجزاء النظام.

وغيرها من اجزاء النظام.

\* Many engineered products are nowadays designed using the methods of concurrent engineering where design and manufacturing are integrated, ~~rather~~ rather than being considered sequentially, as was often the case in the earlier time.

يتم في الوقت الراهن توضع العديد من المنتجات الجديدة باعتماد طرق الهندسة المترادفة بدلاً من النظر إلى بدل متسلسل حيث غالباً في الأوقات السابقة.



(2)

- Both functionality and manufacturability consideration often require the design process to include laboratory testing of one kind or another.
- When we ~~examine~~ actual production machinery and processes, we often find that these manufacturing tools are controlled by a so-called feedback mechanism. In such a scheme some ~~of~~ quality parameter of the part produced is measured with appropriate sensors.

لكل من الوظيفة وقابلية المصنع أن تجري <sup>(روبيان)</sup> تجربة في المختبرات <sup>لبيان</sup> قبل إنتاجها في المعمل. <sup>لذلك</sup> يتطلب إنجاز المصنع أن تجري تجربة في المختبرات قبل إنتاجها.

### <sup>٣</sup> classification of Types of Measurement Applications.

1. Monitoring of process and operations.
2. control of process or operations.
3. experimental engineering ~~and~~ analysis.

① Monitoring of process and operation, refers to the situation where the Measuring device is being used to keep ~~track~~ track of some quantity

- |                |              |                  |                 |
|----------------|--------------|------------------|-----------------|
| 1] thermometer | 2] barometer | 3] radars        | 4] anemometers  |
| متر حرارة      | متر ضغط      | متر اتجاه الرياح | متر سرعة الرياح |

- (indicate the condition of the environment but do not serve any control function in the ordinary sense )

- تشير إلى الظروف التي تقام بها أجزاء القياس يتبع المعايير.
- يشير إلى كل البيئة التي يمكن أن ينبع منها دatas من الممكن تأكيم.

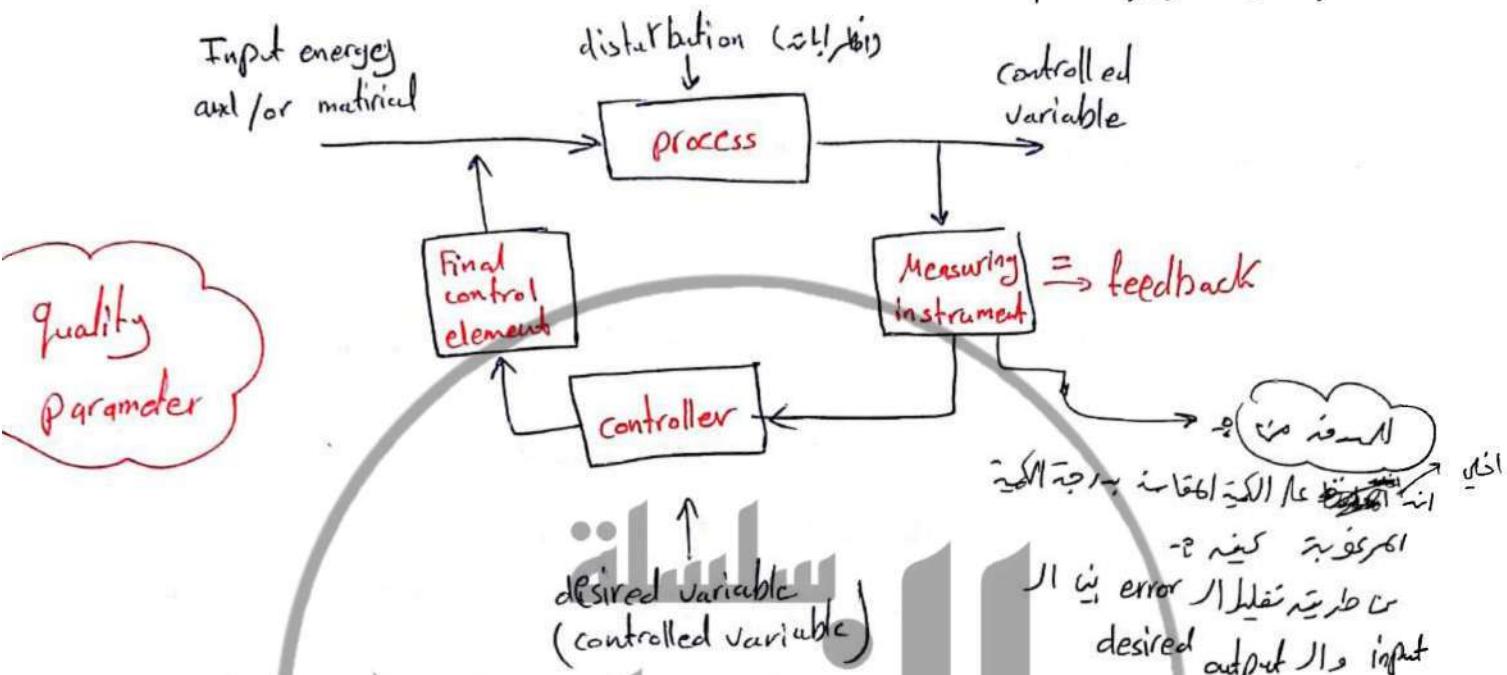
③

- 1) speed control system in car.  
 2) Antilock braking system  
 3) coolant temperature regulating system.  
 4) air conditioning system.  
 5) engine position controls

control of process and operation. This is usually referred to

to an automatic feedback control system.

نماية تعودوا / أحجزه الحكم الراجعة الأوتوماتيكية .



اهم امر في اجزء المقياس وحدة المقارنة بين ال input وال output

و بعد ما يعطيها measuring input بعد ما بتعمقها الى measuring instrument

هي الاخر جزء النتيجة للcontroller الذي يقرر اذا يوقفه العملية اخر جلبي ستر (يعطي امر) لل

وصول الى بجي وبنفسه وجدت اعواده تذكر لم ما امر للنتيجة اطرافه  
desired = controlled variable  
variable

(1) measuring instrument  
(2) controller

الفرقه بين ال monitoring



العواجم صورة تكونه sensor بباب controller خذ اذا دخل ام اي الماء الى بجي ايه ضغطى رفع بعده اسارة  
والصورة صوره العواجم الى رفع يام فال controller فالصورة صوره العواجم الى رفع يام فال controller

- صاصونه مع انه عندي جهاز قياس الى الماء / العواجم بباب كمانه عندي controller الى يعطي امر حسب نتائجه المقياس ملو اشي قليلة بالعكس انه رفع يفتح فال Valve ولو فال Valve الماء ما رفع يفتح فال Valve .

(4)

معزى كنه عندي دوامة رفع يفتح لوبينا الماء وصل بباب منشئ فال Valve

Monitoring

hot-wire anemometer } monitoring  
or controller → If we used the gas valve.

3) experimental engineering analysis: is the part of engineering design, development, and research that relies on laboratory testing of one kind or another to answer questions.

الذى يجرى على المدى القصير والجهاز الذى يجرى على المدى الطويل.

experimental engineering analysis is divided into types:

1) theoretical methods.

1) often give result that are general use rather than restrictive application.

التي تطبق على صناعات العالم بخلاف نتائج المعيار.

2) experimental methods.

1) often give result that apply only to the specific system being tested.  
in techniques such dimensional analysis may allow some generalization.

التي تطبق على نظام معين.

2) Invariably require the application of simplifying assumptions.

التي تجعل النتائج ملائمة.

mathematical model

actual system

The ~~theoretical~~ theoretically predicted behavior is always different from the real behavior.

فلا يتحقق التطابق.

3) In some cases, may lead to complicated mathematical problems.

التي تؤدي إلى صياغة معقدة.

3) Accurate measurement necessary to give true picture.

التي تؤدي إلى صياغة صحيحة.

3) variable - manipulation element & By "manipulation" we mean specifically a change in numerical value according to some definite rule but a preservation of the physical nature of the variable.

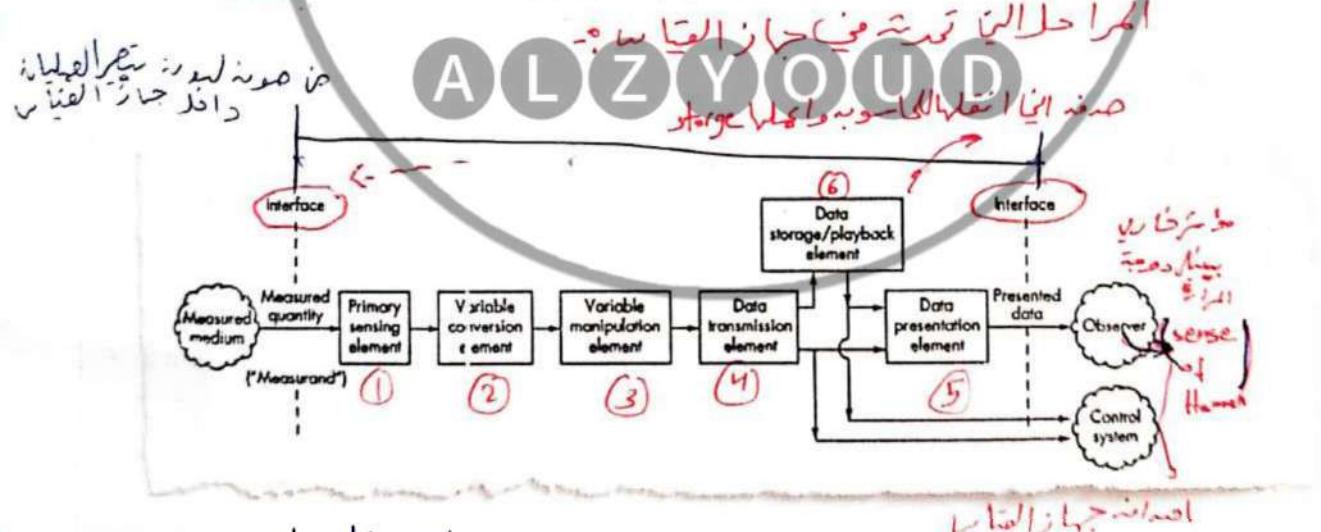
• Variable يغير قيمه العددية او يعدلها فيما يزيد عن مقدارها

4) data - transmission element & transmit the data from one to another (data ١ لـ data ٢)

5) data - presentation element & to be communicated to a human being for monitoring, control, or analysis purposes, it must be put into a form recognizable by one of a human sense. (translation)

• يمثل المترجم من الـ data ٣ الى ادوات لاصحاح حواس الانسان

(( Functional element of an instrument or a measurement system ))



block  
1. مدخل جهاز المعايرة  
2. الفيزيوم

اصوات جهاز المعايرة

1) Measured Medium

2) controller تكونه تكيي الحكم فيه لوبيه control

feedback

3) Data storage (play back element)

most majority of instruments communicate with people through

the visual sense:

جهاز يرسل رسائل في شكل صورة

ex: digital voltmeter or printer

6) data storage/play back :- we need function can easily recreate the stored data upon command 1) the magnetic tape recorder/reproducer

is classical example here

2) many recent instruments digitize the electric signals and store them in a computer like digital memory (RAM, hard drive, floppy disk, etc)

التي تسمى الـ مسح و موري وهي ممكنة امكانيات في الواقع - يمكنه

in this

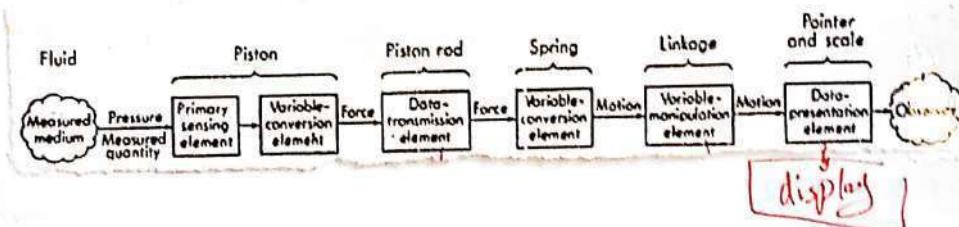
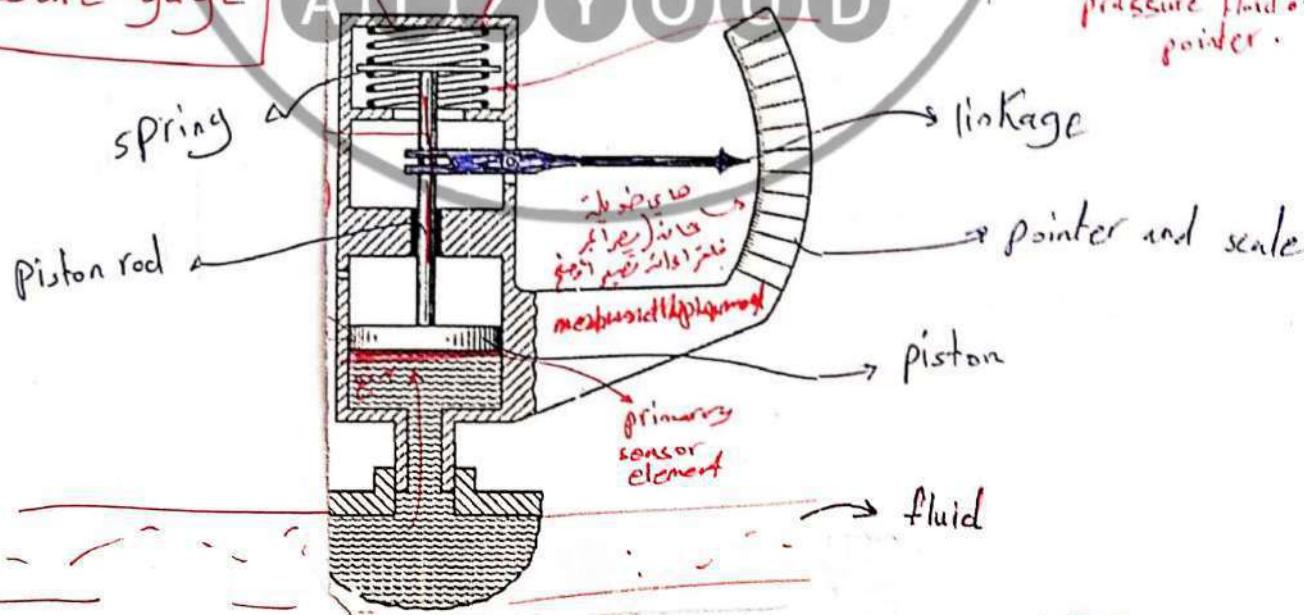
1) have a basic function in any number and combination

2) they not need appear in above order.

3) the physical component may serve several of the basic functions.

pressure gage

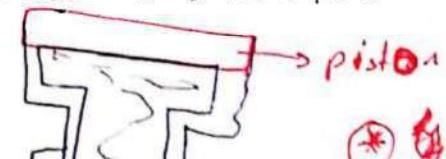
input: pressure fluid  
output: the scale of pressure fluid on pointer.



شكل بيبي حربة ببر اليمار و((الابياز مهندس)) ~ لأنها section من الوسط  
هي كأنها قسموا الكباريز من الماء  
يكونوا اجزاءه الداخلية  
Measured medium

(4) اليمار بين مقاييس المسالك اسائل ماء اليمار

ولم يلحد ظرسته عنه لأنها ابنة برا اليمار



صالة الفكرة صورة تكونه فيه فتحة باليمار مما

يعني للسائل بلورته والذات لم يعود

لل piston position وصدره صادر

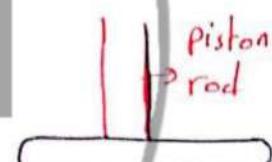
(piston) (فأمثلة على بعدها صيفاً اعاد مع

ذلك يغير primary sensing element فالظواهد على piston صادراته هي سيد بالضغط وصاف

Variable conversion element (displacement) (hydraulic energy)  
Mechanical variable ~ mechanical variable

piston rod

هي صافي الdisplacement وتنقل بواسطه

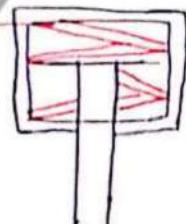


Data transmission element piston rod يغير ذلك الزناد لبيان

piston rod spring صدره يعني الـ spring في ناتج منه

بسـ الـ spring مـا يـ خـ لـ مـ بـ زـ مـ حـ مـ لـ اـ طـ اـ قـ مـ

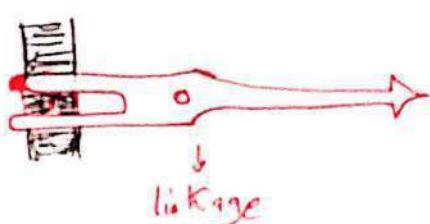
spring pressure → position energy لـ هـ اـ دـ سـ



يغير هذا variable conversion element

صـ صـادـ كلـ مـاـ رـاـدـ طـوـلـهـ  
روـ يـكـرـ اليـ هـبـهـ اليـ بـعـدـ

الـ فـرـادـةـ صـرـوـنـهـ صـرـدـ الـ عـلـقـةـ يـكـرـ العـادـهـ اوـ بـعـدـ

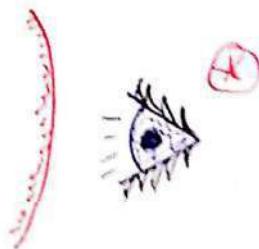


(10) Variable manipulation element

Amplifier المعايرة في معايرة انه دائمًا بالآخر بـ معايرة يكرر المعايرة يستخدموا

الجهاز الذي رفع تكوفه يعني دستيرهم لمعنى المعايرة فما نفهم كل ما زاد حجمه يغير المعايرة اسل.

data representation element لذلك يعبر pointer و صر and scale

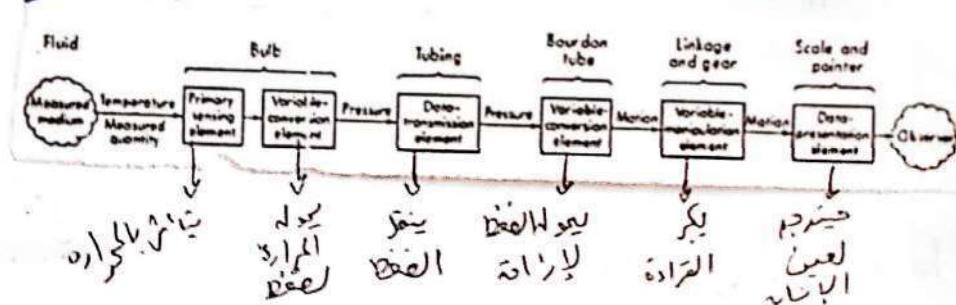
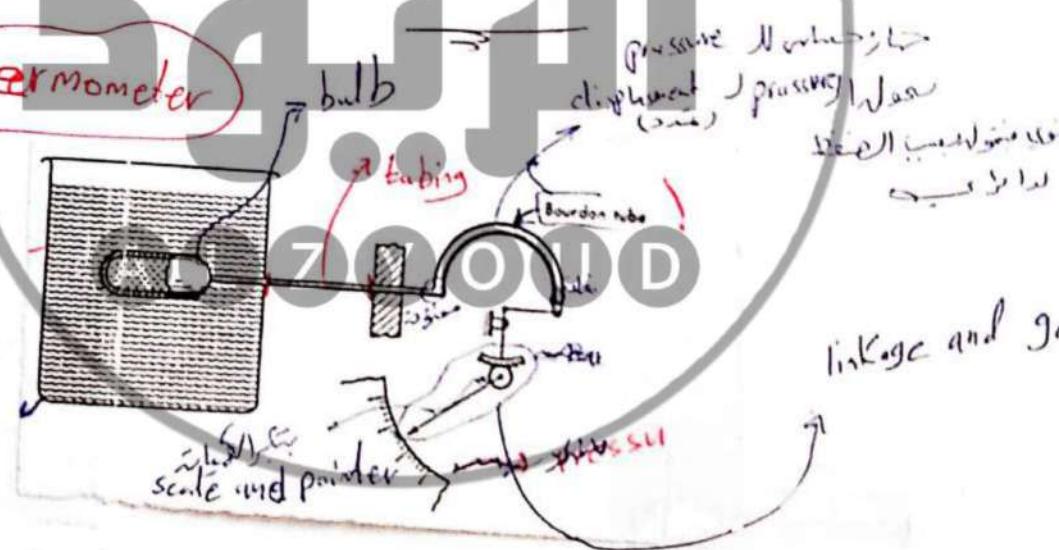


عند معايرة كمية تعبر observer دبر خلوله تتناسب بالرسالة حاطتها بقيمة  $\frac{1}{k}$  يقال انه موجز من الجهاز.

$$\text{potential energy} = \text{spring} = \text{displacement} = \text{hydrolic force}$$

ماد مثل سوكه انه اجزء القياس سوكه يغير العناصر كعدد او حتى اضافة  $\frac{1}{k}$  not data storage و data retrieval variable conversion element ...

pressure thermometer



ويعبر عن المعايرة فاما ان يكون طبع ماء في المعايرة من حرارة - بسبب التكثيف الحراري -

وينقل العزارة displacement / gear linkage بسبب التكثيف الحراري،

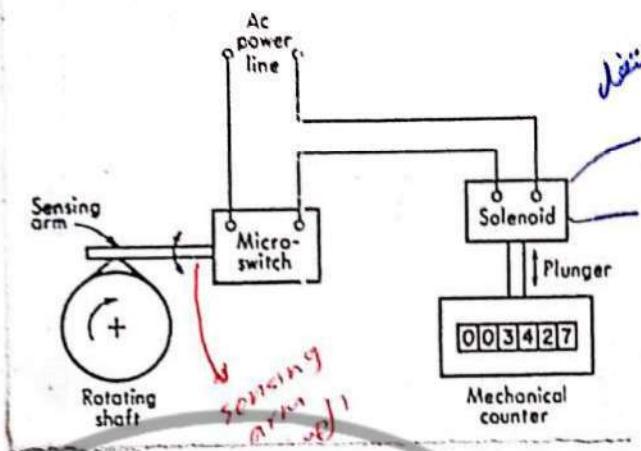
ويعبر عن المعايرة scale and pointer.

## Digital revolution counter

جهاز يقيس عدد الدورات التي يدور بها محور  
rotation shaft

Microswitch

صوت حارقة كهربائية بتسلقها  
يضرر امدادات بال  
دستع ملائكة يدور به دورة  
ما يضر بها

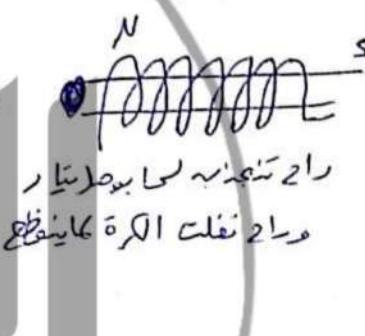


Microswitch  
solenoid لـ ٢٠٢٠

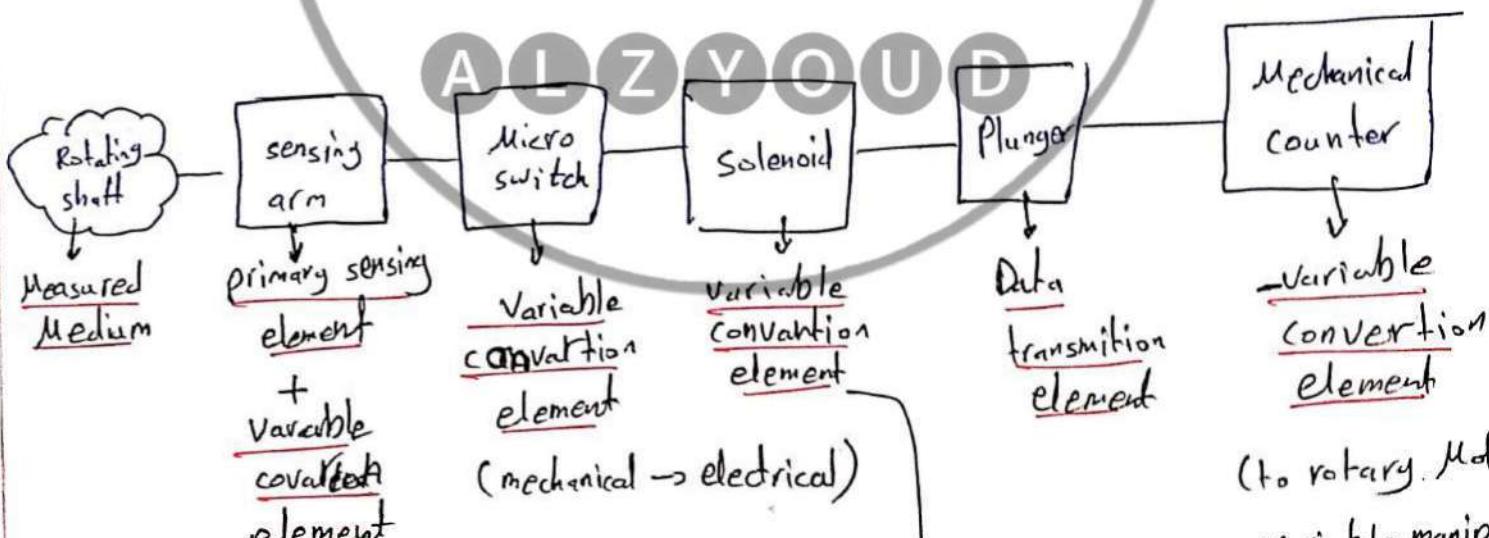
نفخ

داخل الـ (solenoid) فيه زี่ مركبة

(مagnets) (Magnets عن دائمة)  
Comparison



input → (Number of turns) Rotating shaft



(rotary displacement to linear displacement)

صادر عن اسلاك كهربائية بتسلقها  
Rotating shaft  
بـ (أ) بـ (B) sensing arm  
فيستعمل مفاتحة دارة وفتحها  
Micro-switch  
تـ (A) مفاتحة دارة فتحها فتحها  
تـ (B) مفاتحة دارة وفتحها  
Micro-solenoid  
تـ (A) مفاتحة دارة وفتحها  
Micro-counter  
تـ (B) مفاتحة دارة وفتحها

(electrical → mechanical)

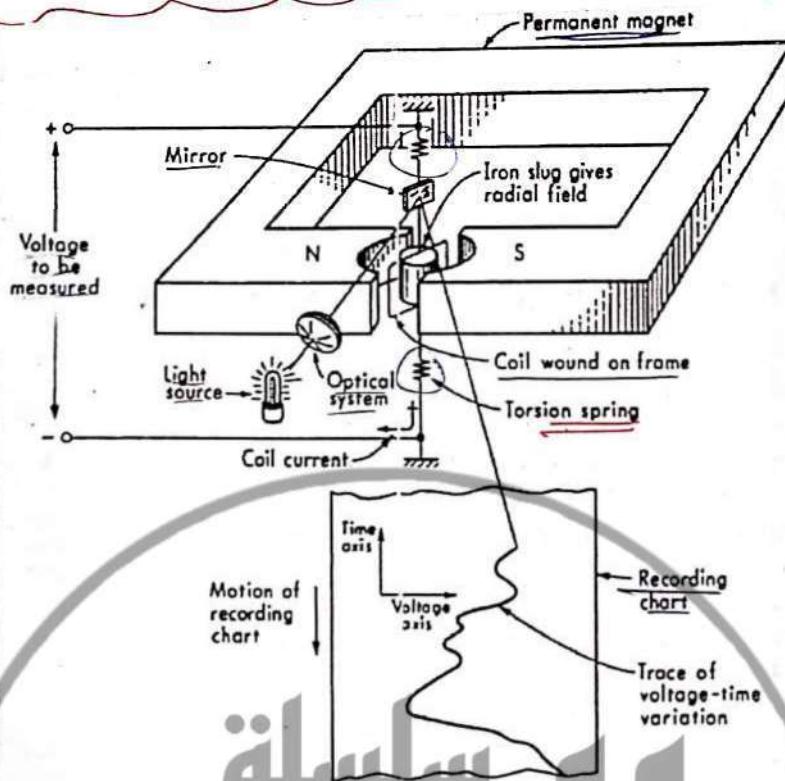
(rotary motion to decimalized  
rotary motion)

and

- data presentation

# D' Arsonval galvanometer

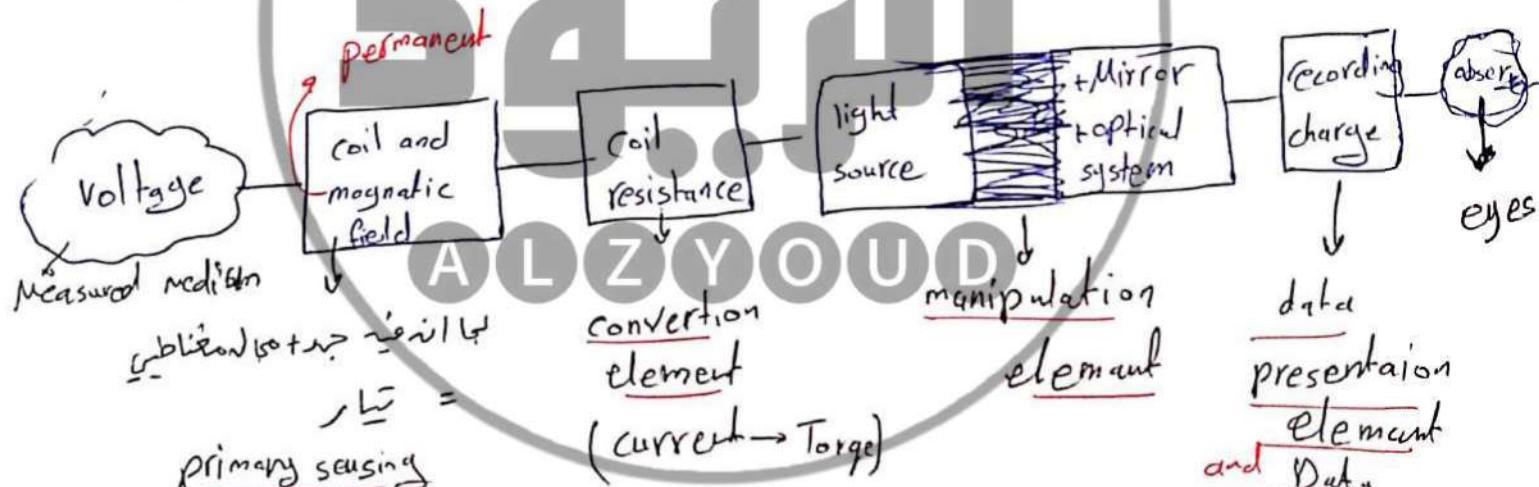
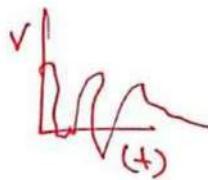
Input → voltage  
output → deflection  $\theta$



معنى  
ذلك

- Torsion springs
- بعد ما ينطبق الزبروك برج  
لورجفه المطابق

السفن هنا  
الدافي علاقته  
المحبس مازمن



+  
conversion  
element  
(V+Magnetic  $\rightarrow$  current)

في البداية و بسب دخول Voltage في مخاطب من الماء  
سيوله تيار و بسبب مقاومة الماء سيوله اكي

صاعي التردد رجاع (رجاع)  
 $\theta$

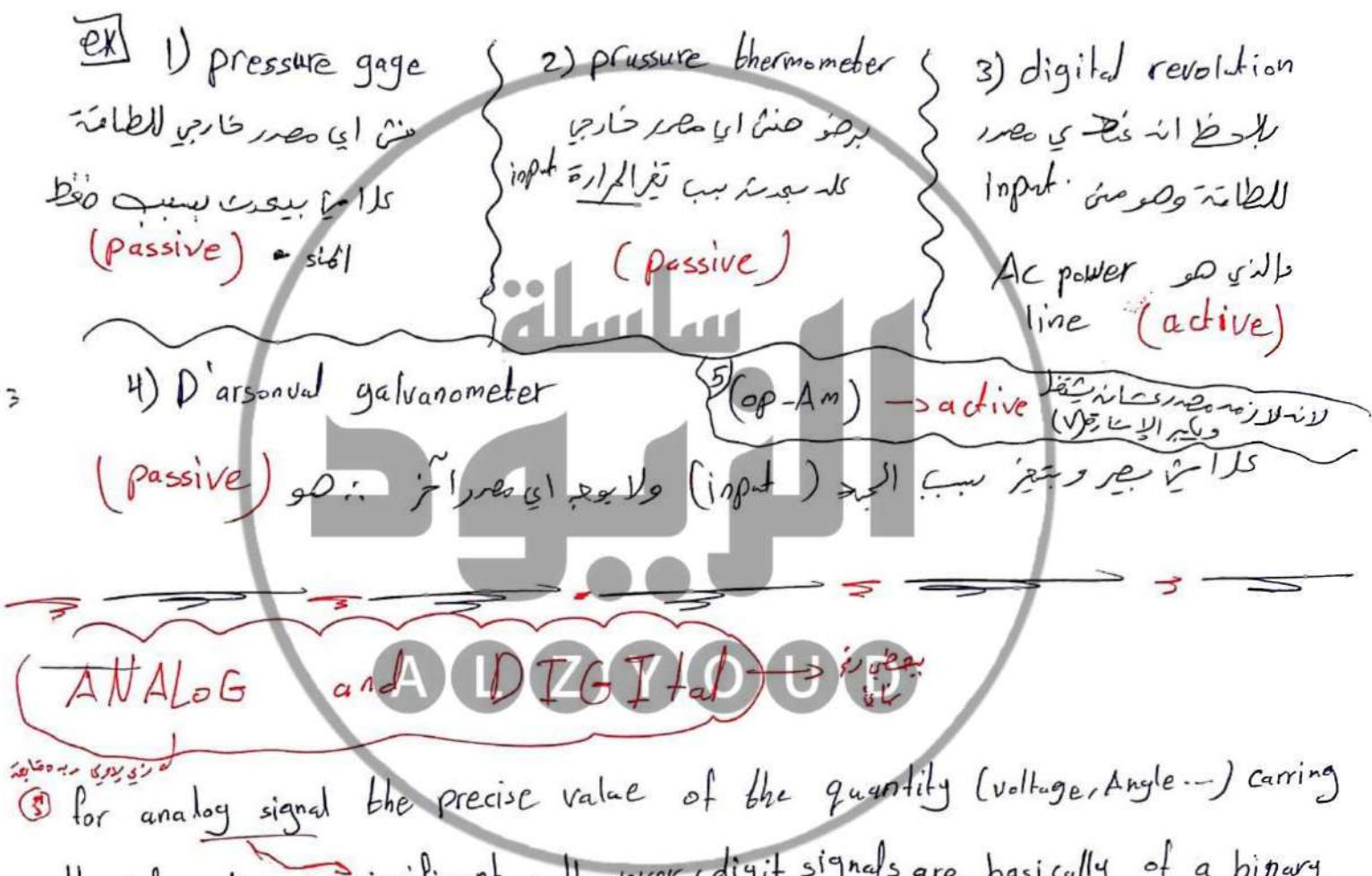
وصوئه يكونوا راضي الماء ب درجة نظر الماء  
عندما تكون قوية او  $\theta$  درجة نظر الماء

(B) Recording chart . (الراجع رد تطوره فشارها شبکوه الکمیوزر لـ Recording As chart

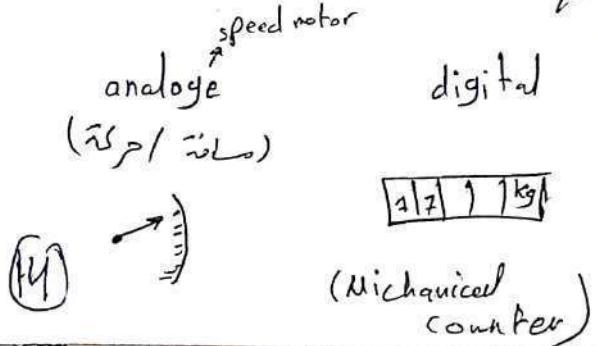
⇒ passive و active II

**passive transducer**: A component whose output energy supplied entirely or almost entirely by its input signal.

((بُنادِ الصَّافِي مِنْ إِنْدِ الْمُهَاجِرِ))  
**active transducer**: has an auxiliary source of power ~~energy~~  
((لِذِي مُكَوِّنٍ فِي مُهَاجِرِهِ خَارِجِيًّا مِنْ إِنْدِ الْمُهَاجِرِ))

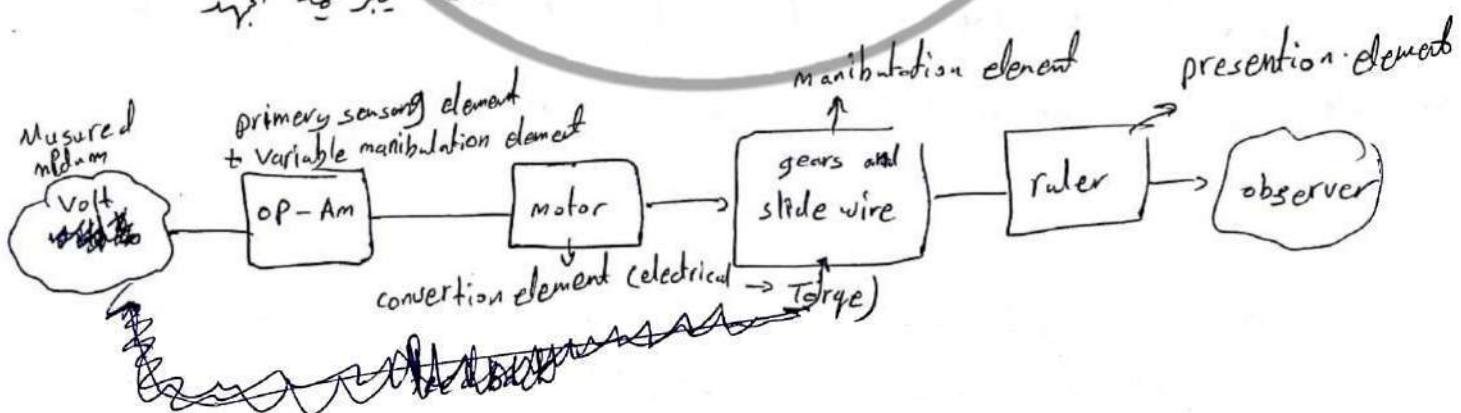
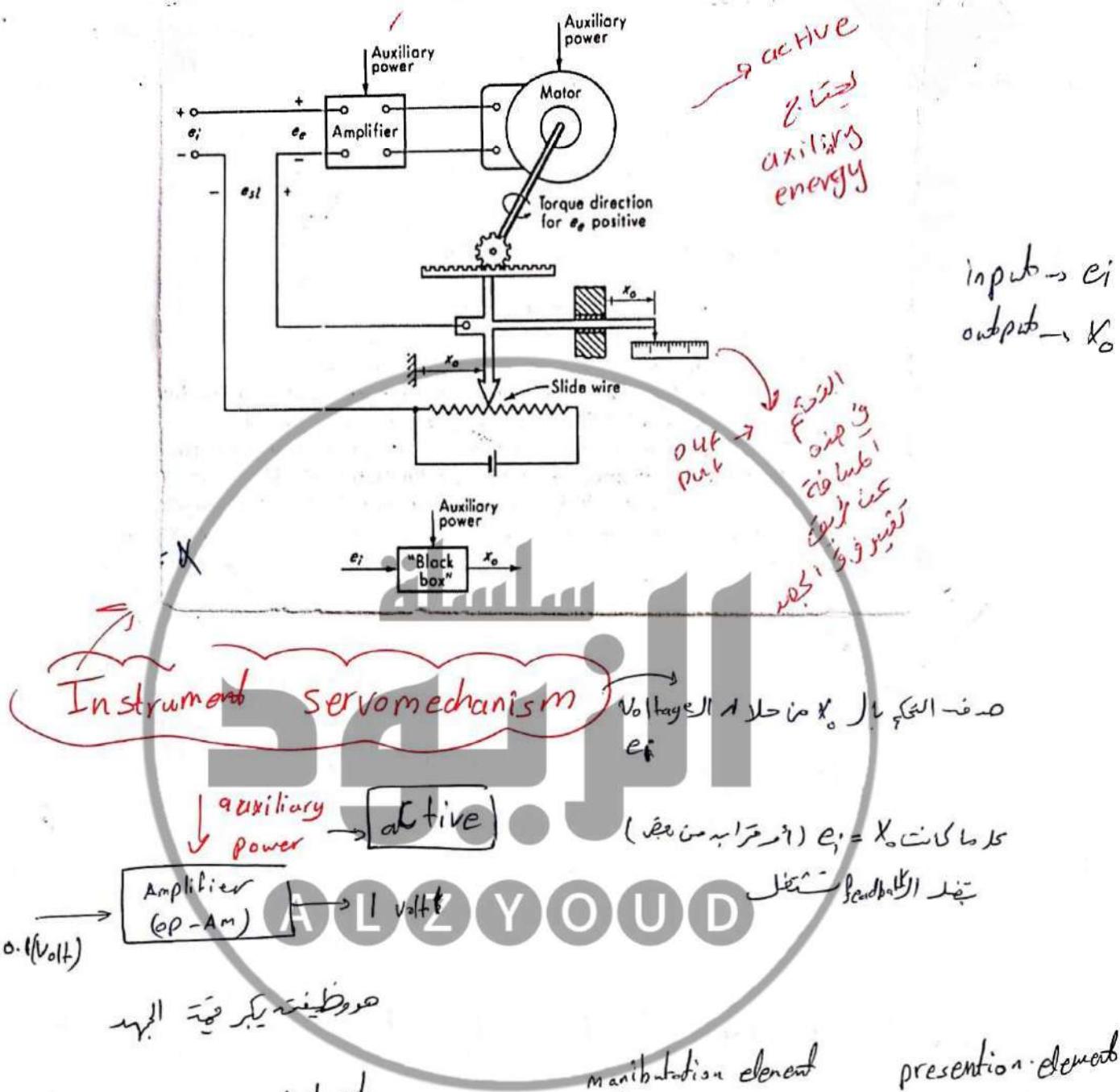


الـ analog معرفة وهي تغير موجات الموجات الموجية والalogarithmic حسناً، اساس طبيعة ثابتة (ستيل، ديناميك) والـ digital معرفة في العدة العددية ترتبط في اتجاه اطيفية (صوابها / خطأها) لغير المعايس.



it necessary to have both  
- analog to digital convert (at input to computer)  
- digital to analog convert (at output or " ")  
that enable the computer to communicate with the outside world.

# Amplifier (Op-Am)

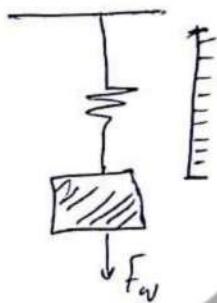


## Null and deflection Methods

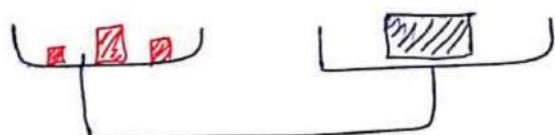
الـ null مصطلحه لا يقتصر عن طريقة التأثير والاعتراضة اما الـ deflection هو جهاز يعين ببساطة في الميزان .

deflection instrument

ex



null instrument



في الميزان خذ الكفتنا ما زلت افترضه لا من طرفيه تغير الميزان فهل اخفيه او رأته لم يتأثر وبعد ما وزنته الجسم = وزنة الموزان

يقدر ما يغير في deflection في الزنبرك بفرق اثادين وزنه الجسم

deflection Type device, the measurement quantity produces some physical effect that engender a similar but opposing effect in some part of the instrument.

(في اداة الميزان الكهربائية اتفاقية تلزم بعض التأثير العلوي (الايجابي) الذي يولده جسم ما بـ  $\rightarrow$  لكن سعى في جزء ما من اداة ،

a null-Type device attempts to maintain deflection at zero by suitable application of an effect opposing that generated by the measured quantity.

بيان الميزان من النوع الصفرى انه يحافظ على (الاينفارمة) ~~التأثير المعاكس~~ من خلاله التأثير اomba للتأثير المعاكس الناتج عن الكمية .

لو زر جع قبل درجة الأذى

1) pressure gage → deflection

2) pressure thermometer → deflection

3) Digital revolution counter → deflection

4) D'Arsonval galvanometer

→ deflection

بالخط انة كلما كانت سبب تغير جزء يابي او مادي (مترتب او زانه او اي انصرافه)  
للحاجز

عسانه يكون null ففي او زانه او مقادماته ضئلاً عسانه اصراً للجهاز ومن الاذرانه اد اعتماده

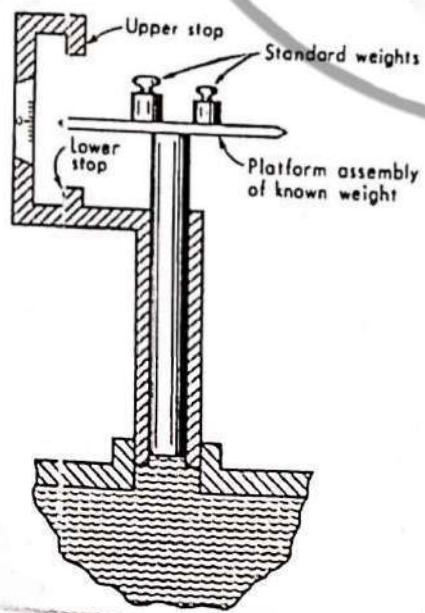
برقة العياب

اما الـ deflection فهو تغير داخل الجهاز (مثل انفصال او انفصال الزنبرك) الى بدل

الجهاز كال طبل على القبة تابع القباب

(مقادماته تعادل دفعه  
له صفره با مقادماته )  
بال null تكونه الجهاز  
ميكانيكي سه بضرره بالامزانه.

ALZYOUD



dead weight pressure gage

اصدرنه بظل اذانه او زانه حتى يتصارع

لما يتصارع تكونه طبقت F وضد اطبه كالقانون

صاعيل لوكال سجبي بتغير الصقط بجي حد بخل حوقه الجهاز

وكل سوي بيقدر يجيب لسلبيه الظرفية not practical

اما بالنسبة للدقة خار null سه ادقه more accurate

لبي ٩٩ لوزانات لاصغر الاذرانه نسبة اكتفاء يقاد

لكونه معروفة

اما لو اجي لل deflection فالقصة زنبرك وده كلاته كذا نطلع

ورفع سطر ستره سه نسبة انتظام ادن

للحاجز بحسب F سه الاذرانه

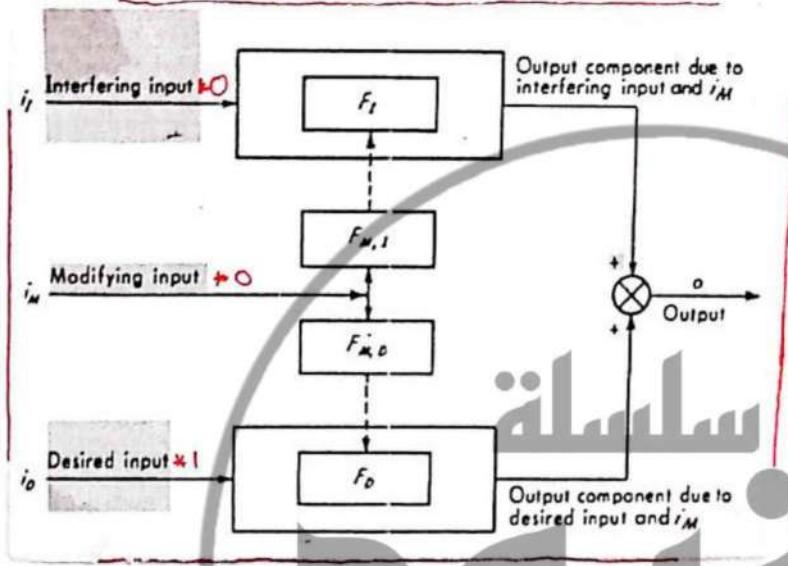
$$P = \frac{F}{A}$$

deflection null عارض معه اجهزة الـ

Input quantities are classified in to three categories :-

- 1) Interfiring input
- 2) Modifying input
- 3) Desired input

### Generalized (input - output) configuration



#### 1) desired output input

حالاتي أنا بـيـا إـلـاه طـلـعـيـةـتـيـةـ فـطـبـطـ كـلـ الـظـرـوـفـهـ دـيـارـهـ مـاـيـهـ اـنـيـ يـمـرـبـهـ الـتـسـبـيـهـ (عـكـاـنـهـ بـيـاـلـهـ بـزـبـبـ بـ1ـ)

#### 3) Modifying input

(desired input) اـنـيـ مـزـبـعـ (رـعـ يـتـصـرـبـ زـيـ كـذـنـ) صـاصـدـ كـنـيـلـ ثـيـرـ (output) كـالـ حـنـالـهـ ماـكـانـهـ فـيـ (desired input) لـيـكـتـ زـيـ كـانـهـ input جـديـهـ غـيرـ مـعـوـبـهـ فـيـ

وطـبـعـاـ بـجـاءـهـ أـفـلـهـ لـهـكـ بـرـسـهـ مـفـزـوـبـهـ بـ0ـ

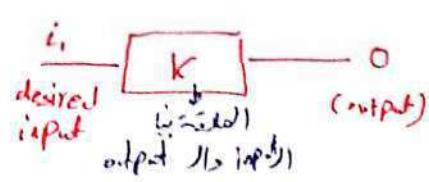
#### 2) Interfiring input

بعد تـقـيـادـكـ تـيـقـنـرـ تـصـرـهـ كـانـهـ (desired input)

صـاصـدـ دـوـرـيـ كـاـيـوـهـ عـنـكـ طـرـفـهـ ثـيـرـ مـاـنـهـ (output) لـيـكـ مـسـرـحـ طـلـعـيـهـ (output)

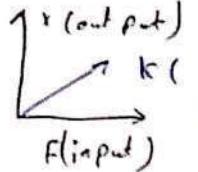
قطـبـعـاـ اـنـاـجـارـلـ اوـلـهـ لـهـكـ بـرـسـهـ بـدـرـ

0-

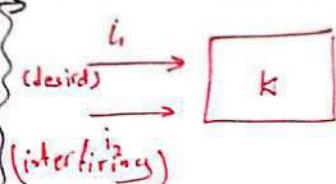


بنـالـهـ اـنـهـ بـيـ اـشـهـدـهـ اـسـطـالـهـ الزـبـرـكـ

بـيـبـ بـلـيـزـ عـقـدـهـ



(18)

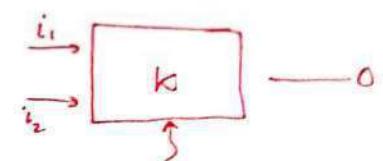


متـلـاـ لـوـ صـارـ عـلـ الزـبـرـكـ مـفـزـوـبـهـ رـعـ

تـادـلـ بـيـتـيـهـ Kـ درـاجـ تـيـزـ الرـسـهـ

صـونـهـ زـيـ مـخـلـعـهـ الصـراـ

modifying ~ interfiring ~ interfiring



(Modifying)

متـلـاـ لـوـ صـارـ عـلـ الزـبـرـكـ حـرـاءـ خـلـتـ

سـهـتـ (صـونـهـ الـحـرـاءـ مـنـاـ تـأـثـرـ بـخـلـتـ)ـ كـاـ

بـوـنـهـ الـFـ مـخـارـهـ زـيـ لـكـاـنـهـ دـعـهـ

modifying a man noo

## Spurious input for manometer

العلاقة بين  $\Delta P$  والـ  $X$



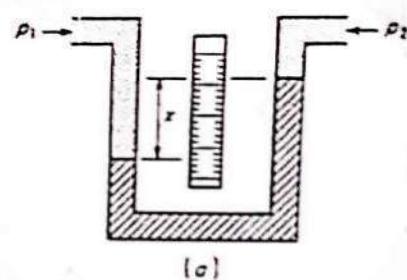
$$\Delta P \rightarrow \text{desired input}$$

العلاقة بين  $\Delta P$  وـ  $X$  بجهة الـ  $K$   
عندما يكون الماء في المانوميتر

$$\Delta P = \rho g \Delta H$$

النقطة الحبرية

$$P_1 - P_2$$



(a)

interfacing input

لعمد انتشار في الماء من 20 إلى 25 درجة مئوية  
عندما يكون الماء في المانوميتر

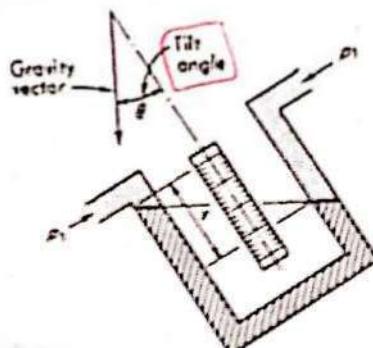
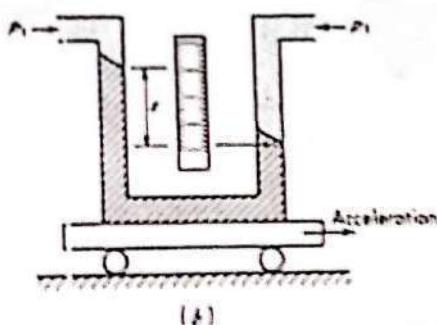
علاقة بين  $\Delta P$  وـ  $X$  (العلاقة) بين العمق والـ  $K$  (العلاقة)

interfacing input

عندما يكون الماء في المانوميتر

$$\text{interfacing input} = g \cdot x$$

modifying input



فرض لوحة مدار عادي ومحببة السائل

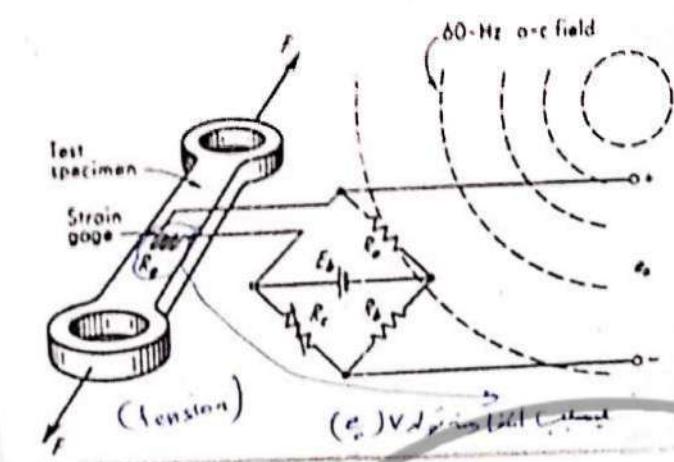
فهي تغير درجة حرارة X بدوره هنا الماء

عندما يكون الماء في المانوميتر

modifying input  $\rightarrow$  acceleration pressure

(c)

Introducing input for strain gage circuit



input → strain  
output → voltage

$$\epsilon = \text{strain} = \frac{\Delta L}{L}$$

٤) تغير درجة حرارة (desired input ) Force is a function of Temperature (الذى يدخل معه انتقال

Interfiring input ~ اخراج میان

\* لاملاكها في force ويزرت (الجامعة) بسبب المراجعة تغيرت الاتصالات وكانت الـ ٧ مومناً للمرأة

outPut gets input into next output in the sequence

((Temperature also acts as modulating input since the gage factor is sensitive to temperature))

new ( output ) will be produced → magnetic field

interfiring input

the gas's resistance changes according to the relation

$$\Delta R_e = (GF)R_e \epsilon \quad (2.1)$$

$$\Delta R \triangleq \text{change in gage resistance, } \Omega \quad (2.2)$$

$$GF \triangleq \text{gage factor, dimensionless} \quad (2.3)$$

$$R_s \triangleq \text{gage resistance when unstrained, } \Omega \quad (2.4)$$

$$\epsilon \triangleq \text{unit strain, cm/cm} \quad (2.5)$$

The voltage  $e$  is given by :-

$$e_e = -(GF)R_t \epsilon E_b \frac{R_a}{(R_t + R_a)^2}$$

## Methods of correction for interfering and modifiability input.

1] The method of inherent insensitivity proposes the obviously sound design philosophy that the element of the instrument should inherently be sensitive to only the desired input.

- تفعيل طريقة عدم الماء في التصميم بـ كل اطاعه مقاديره ائنه عاصمه

يجب ان تكون بطيئاً حساساً للمدخلات المترتبة فقط  
we might try to find some gage material that exhibits an

extremely low temperature coefficient of resistance while retaining its sensitivity to strain. If such a material can be

found, the problem of interfering temperature inputs is at least partially solved.  
حاجه العوامل على بعض المواد التي لها معامل حرارة مقلل لتنقية في اتفاق و معا

ترخى الحرارة يتم حلها جزئياً على الأقل.

Dinvar

ALZY OUD

2) Zerodur

3) Alloy Ni-span C

2] the method of high-gain feedback

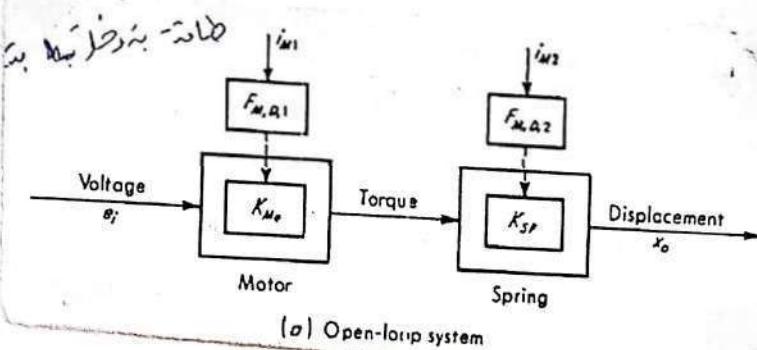
$$X_0 = (K_m, K_{sp}) e_i$$

open loop مصطلح

feedback هو مفهوم open loop

$e_i$  ~ input مدخل

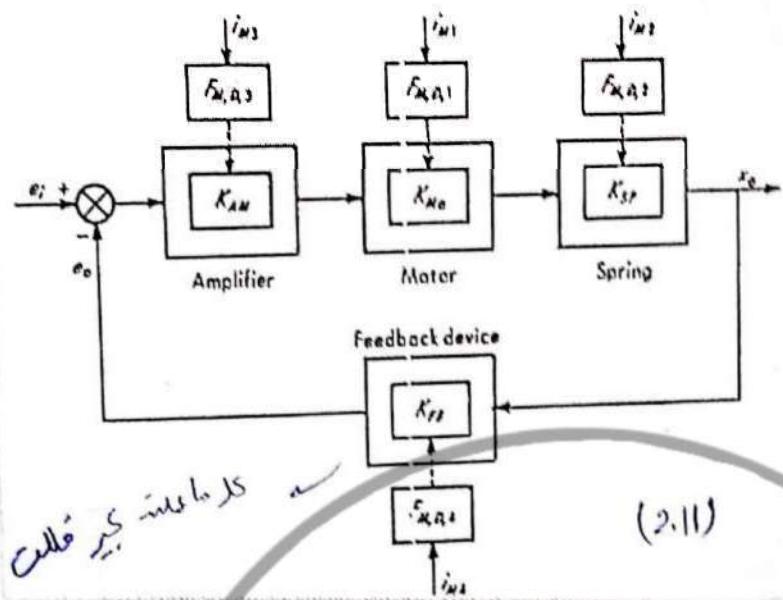
$i_{m1}, i_{m2}$  ~ دعنه بـ جرمو مدخل استمرار



صفدي انتر المدخل spring دخل خرج طبعي  $i_{m2} = i_{m1}$  اثرت على motor  $i_{m1}$   $i_{m2}$   $i_{m1}$

(ب) مركب بـ loop open (مخرج المدخلات) displacement  $x_0$

## Close loop system و دوائر مغلقة



interfering signals

modify the desired output to reduce the effect of spurious inputs.

use a feedback to reduce effect of spurious inputs.

$$(e_i - e_o)K_{AM}K_{MO}K_{SP} = (e_i - K_{FB}x_o)K_{AM}K_{MO}K_{SP} = x_o \quad (2.8)$$

$$e_i K_{AM}K_{MO}K_{SP} = (1 + K_{AM}K_{MO}K_{SP}K_{FB})x_o \quad (2.9)$$

$$x_o = \frac{K_{AM}K_{MO}K_{SP}}{1 + K_{AM}K_{MO}K_{SP}K_{FB}} e_i \quad (2.10)$$

Suppose, now, that we design  $K_{AM}$  to be very large (a "high-gain" system), so that  $K_{AM}K_{MO}K_{SP}K_{FB} \gg 1$ . Then

$$x_o \approx \frac{1}{K_{FB}} e_i \quad (2.11)$$

we now require only that  $K_{FB}$  stay constant (unaffected by  $i_m4$ )

in order to maintain constant (input-output) calibration as shown in (2.11)

نهاية في طلب منعاً لـ  $i_m4$  تابعة غير متغيرة بـ  $i_m4$  من أجل الكفاءة القدرة على إدخال إدخال خارجي (Feedback).

جذب ٢ - طرق الكاشف

→ method of correction for interfering and modifying input.

### ٣) the method of calculated output correction

٣) The method of calculated output corrections requires one to measure or estimate the magnitudes of the interfering and/or modifying inputs and to know quantitatively how they affect the output. With this information, it is possible to calculate corrections which may be added to or subtracted from the indicated output so as to leave (ideally) only that component associated with the desired input. Thus, in the manometer of Fig. 2.10, the effects of temperature on both the calibrated scale's length and the density of mercury may be quite accurately computed if the

temperature

تحوّل انتقال زم تحرّفه المترافق مع المدخلات المُنافِع (interfering input) والمُغيّرات (modifying input) على الناتج (output) (الناتج المُغيّر (output) (output المترافق (interfering output) (output المُغيّر (modifying output)).



### ٤) the method of signal filtering

The method of signal filtering is based on the possibility of introducing certain elements ("filters") into the instrument which in some fashion block the spurious signals, so that their effects on the output are removed or reduced. The filter may be

طريقـةـ الفلترة بـعـدـ الـ إـدخـالـ مـوـادـ مـعـيـةـ (بـتـهـ رـتـدـ خـلـ كـاـشـ مـعـيـةـ) وـعـجـبـهـ الـ فـلـتـرـةـ بـعـدـ إـزـالـةـ أـوـ تـقـلـيلـ تـأـثـيرـهـ عـلـىـ النـاتـجـ.

#### \* general principle of filtering

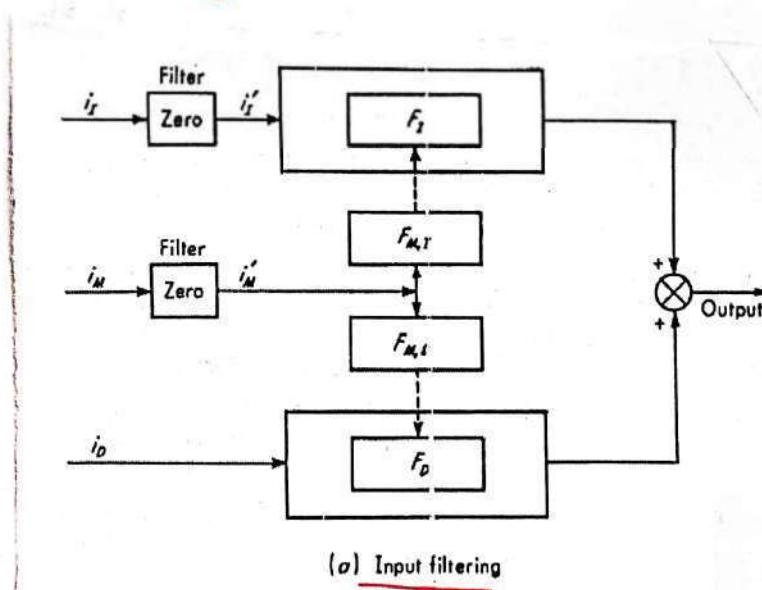
١) أي نجت

غير input

modifying (interfering)

بـ ٠

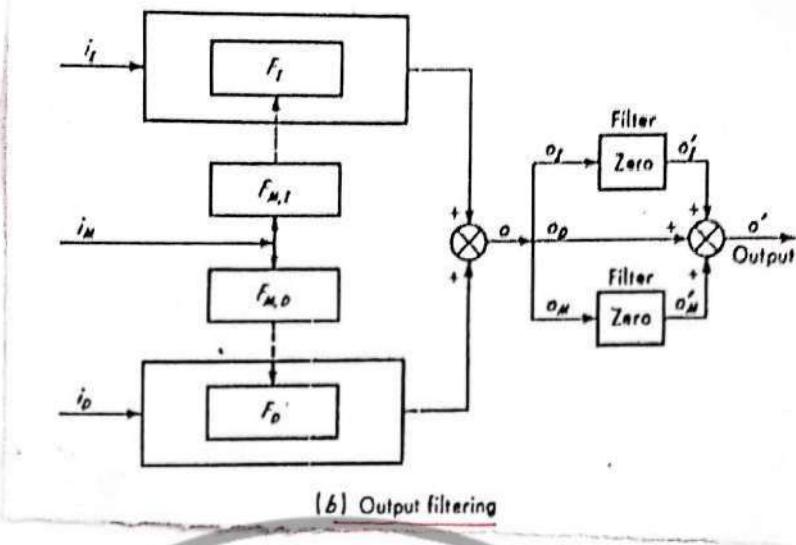
١) بـ dDesired



2) output ای نیکلر ای

الدستیابی بین ایام

الدستیابی بین ایام



- to block completely the passage of the signal: ideally  $\rightarrow$  filter  $\downarrow$   
: selective  $\rightarrow$  filter  $\downarrow$

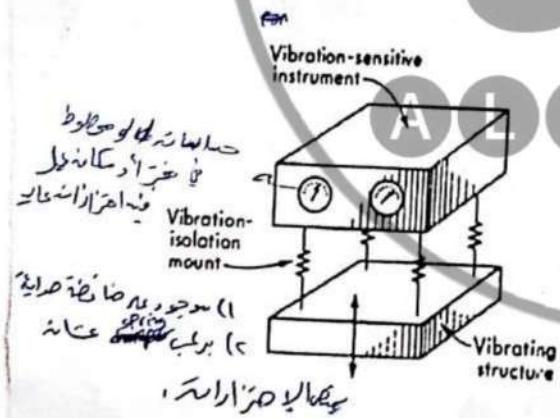
it must pass the desired component essentially  $\downarrow$  unaltered while effectively  
~~supp~~ suppressing all other.  $\downarrow$

example of filtering

interfiring input  $\rightarrow$  vibrating

هذا الجهاز يوجد في أماكن فيها اتصالات دائمة مثل طائرة وصاروخ  $\rightarrow$  aircraft and missile

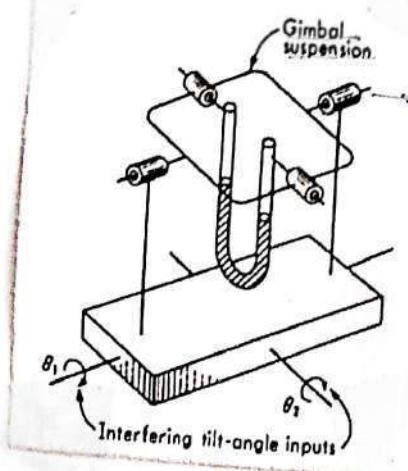
كينة بسيطة  $\rightarrow$  filtering mount



The mass-spring system is actually a mechanical filter which passes on the instrument only a negligible fraction of the motion of the vibration structure.

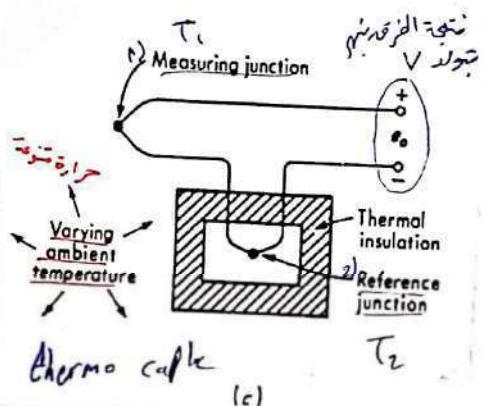
instrument  $\rightarrow$  manometer (بخار ماء)  
interfiring input  $\rightarrow$  tilted tilt-angle inputs

(by gimbals mounting)  $\rightarrow$  filtering scheme



IP The gimbal bearings are essentially frictionless  $\rightarrow$  the rotation  $\Theta_1$  and  $\Theta_2$  can not be communicated to the manometer. Thus it is always vertical.

صورة هيئي لوعين من المكثف بتردد عالي الدارة أو لوحدة



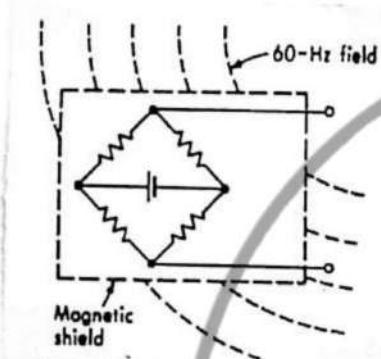
ـ ودي احافظ على  $\rightarrow$  Measuring junction

ـ بي اخلي او القيا بقطعا داخل مادة او  
ـ صدرقة عازلة للدارة (زي عزلها باعزم)  
ـ ما يطلع بالطبع بالطبع.

$\rightarrow$  Interfacing input  $\rightarrow$  Reference junction

ـ كينة تغافل  $\rightarrow$  by thermal insulation,

EX Ice box

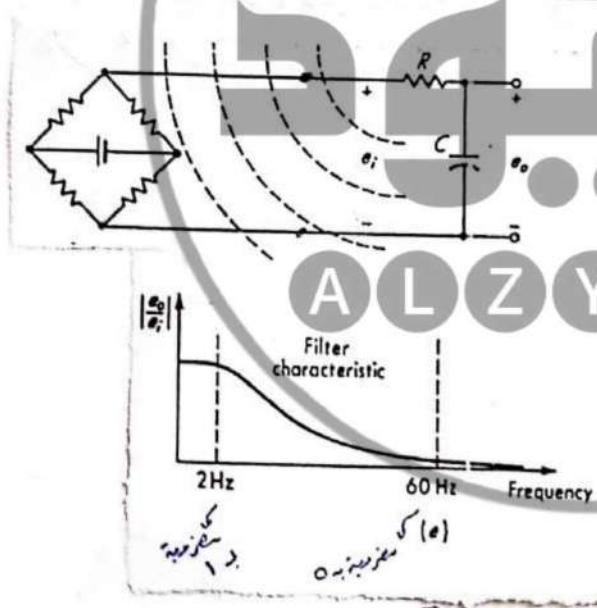


الدالة الالكترونية صورة سوية داخل مكانه من جبال هضابي

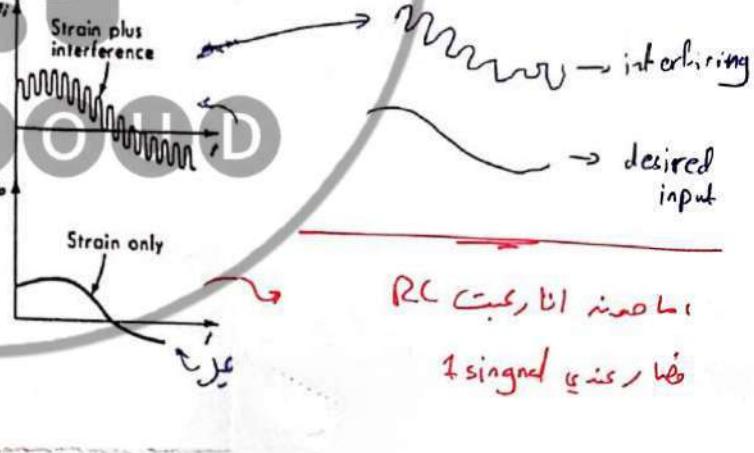
$\rightarrow$  Interfacing input  $\rightarrow$  60-Hz field

ـ كينة تغافل  $\rightarrow$  by enclosing it in a metal box.

instrument  $\rightarrow$  the strain-gage circuit



صورة اجمال اقطاطي  $\rightarrow$  sumation two signal



ـ حاصنة الـ RC

ـ فارع لـ RC

صورة اجمال اقطاطي  $\rightarrow$  العزاء

~~Interfacing input~~  $\rightarrow$  60-Hz

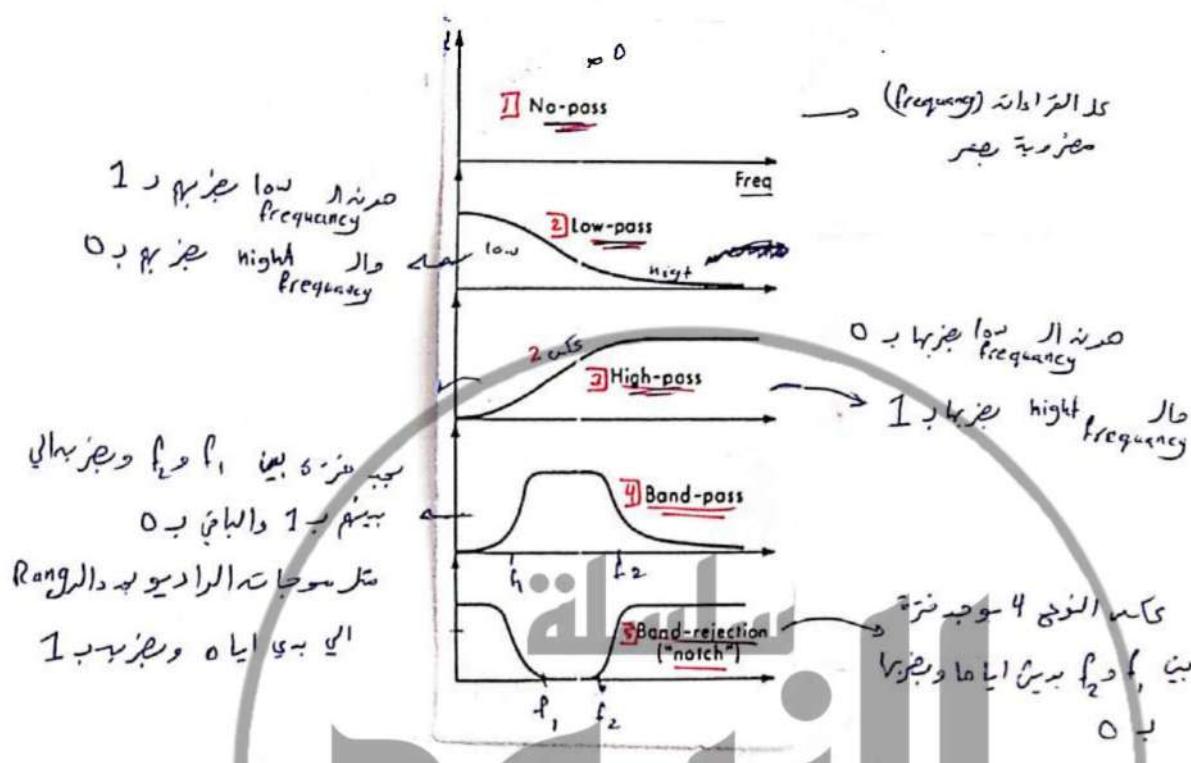
ـ كينة تغافل  $\rightarrow$  RC (Resistor & conductor circuit)

ـ ملحوظ

ـ ملحوظ

## the method of opposing input

### the Basic Types of filter :-



### 5 the method of opposing input

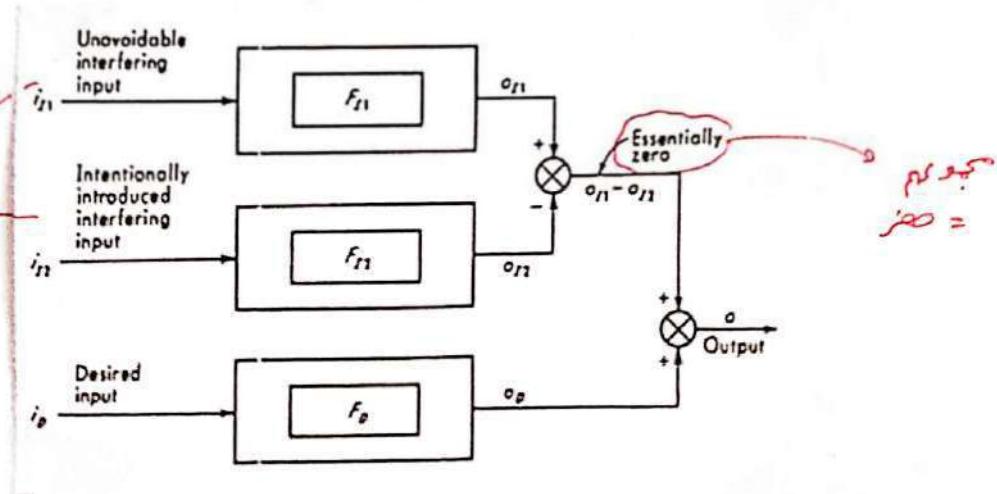
The method of opposing inputs consists of intentionally introducing into the instrument interfering and/or modifying inputs that tend to cancel the bad effects of the unavoidable spurious inputs. [Example](#)

مېلګاندا ایما و پیزېرا  
بى جى بى باند رېجىشن  
بى جى بى باند پاس  
بى جى بى هاي فرېزى را مىنځیزد  
بى جى بى لار فرېزى را مىنځیزد

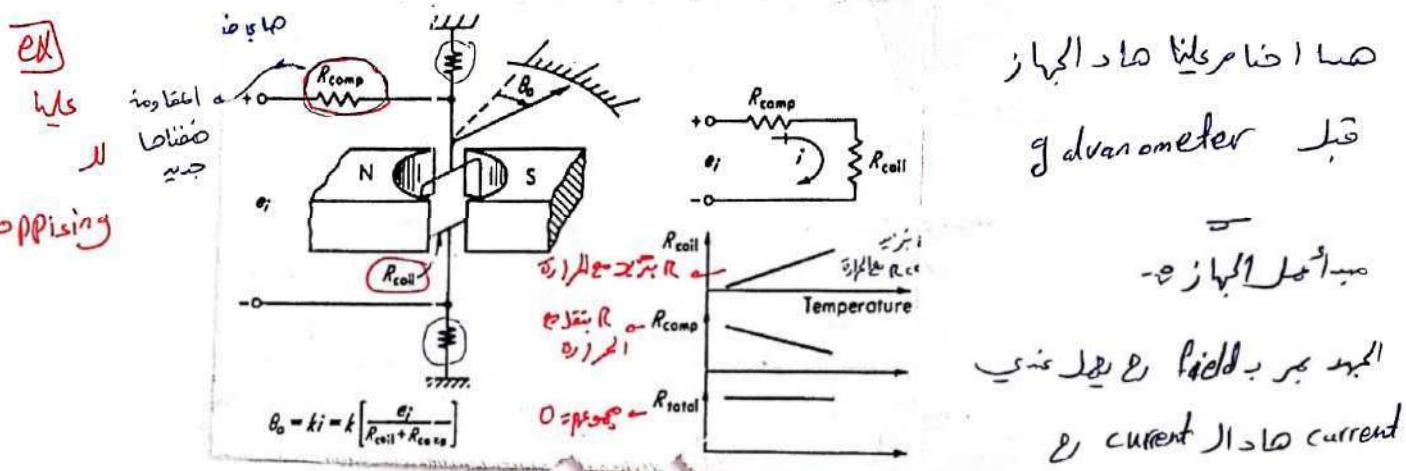
0 =

interfering input

all we do is



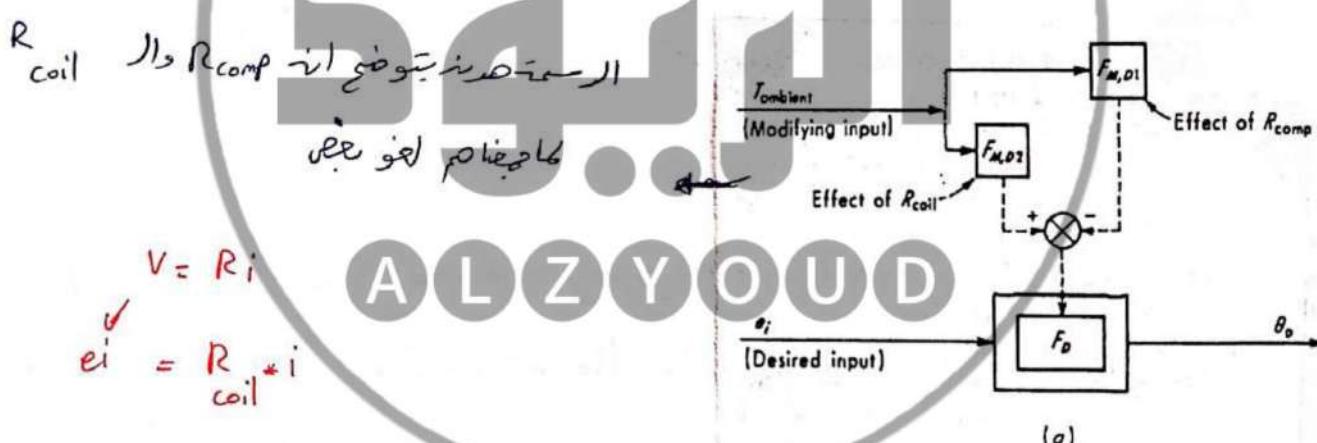
Vertical



+ صورة ضئلاً مقاومة ثانية بتغير الامثلية بناءً على تغير درجة الحرارة  $R_{coil}$  (ارتفاع درجة حرارة  $R_{coil}$ ) حيث أنا مقاومة ثانية من مادة ثانية

زيادة درجة الحرارة  $\rightarrow$  تغير في التردد  $\rightarrow$  تغير في الميل  $\rightarrow$  تغير في المعاين

الصلة  $\rightarrow R_{coil} + R_{comp} \uparrow$



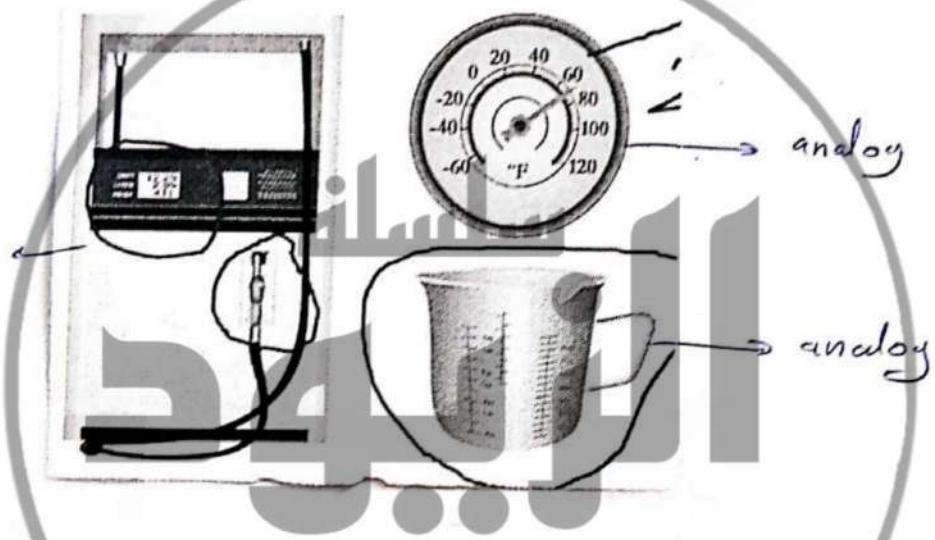
## Ch 1: Basic concept of Measurement Methods

$$15 \pm 0.5 \text{ kg}$$

نسبة المترية  
uncertain  
uncertainty error  
(خطاء غير مؤكد)

لما ذكرنا في قياسها بغير ذلك

- \* comment devices that involve measurements &  
(digital) or analog (الجهاز)



As a measurement is an act of assigning specific value to a physical variable, that physical variable is the measured variable. A measurement system is tool used for quantifying the measured variable.

صريطة لمعنى مفهوم مترية (متر) (Measurement) من نظام القياس صرورة تستخدم لتمرير

value = act.

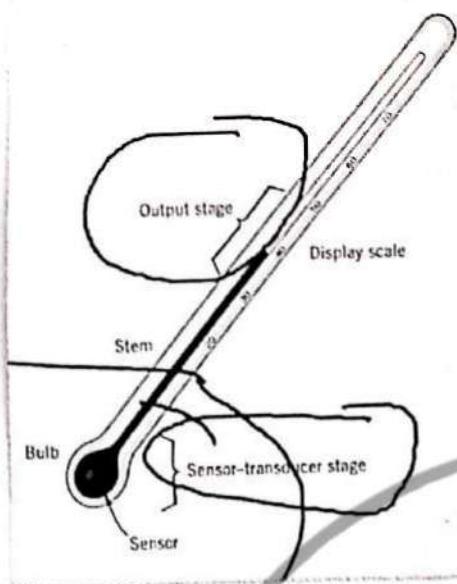
Sensor and transducer

متر  
(يقدم فحصاً بدلاً من متر)

وظيفة يغير ويدخل من تكل من اسعار الاطاقة لآخر  
converstion  
ذلك هو شبيه بـ

## bulb thermometer

component of bulb?



1) sensor (المراد) رجيم بارومتر امداد حساس

2) sensor-transducer stage bulb

which may contain a sensor, transducer and even some signal condition element bulb

3) signal condition

bulb will increase the temperature

(modifies) اوضاع

4) output stage

(bulb is part of the system) (bulb is part of the system)

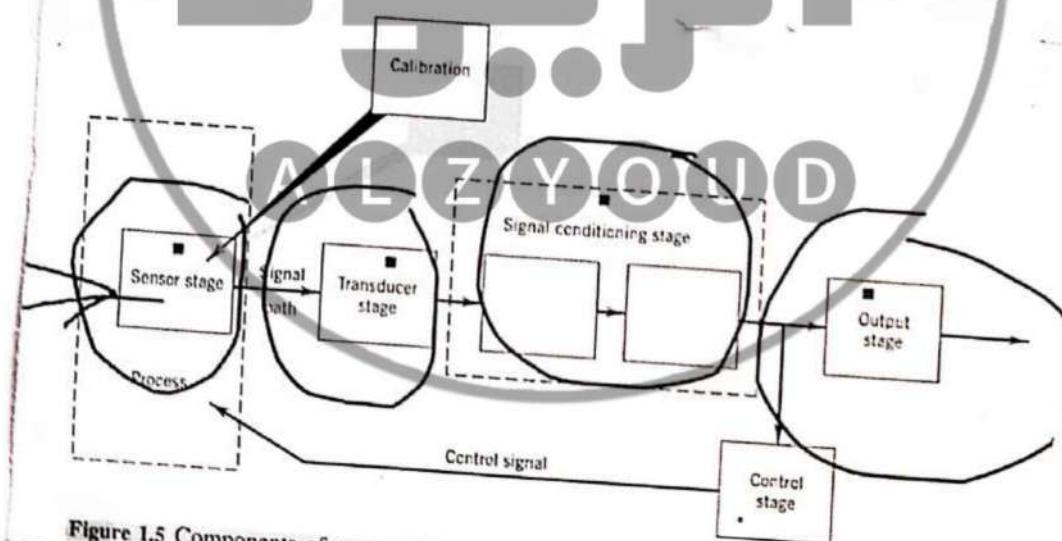


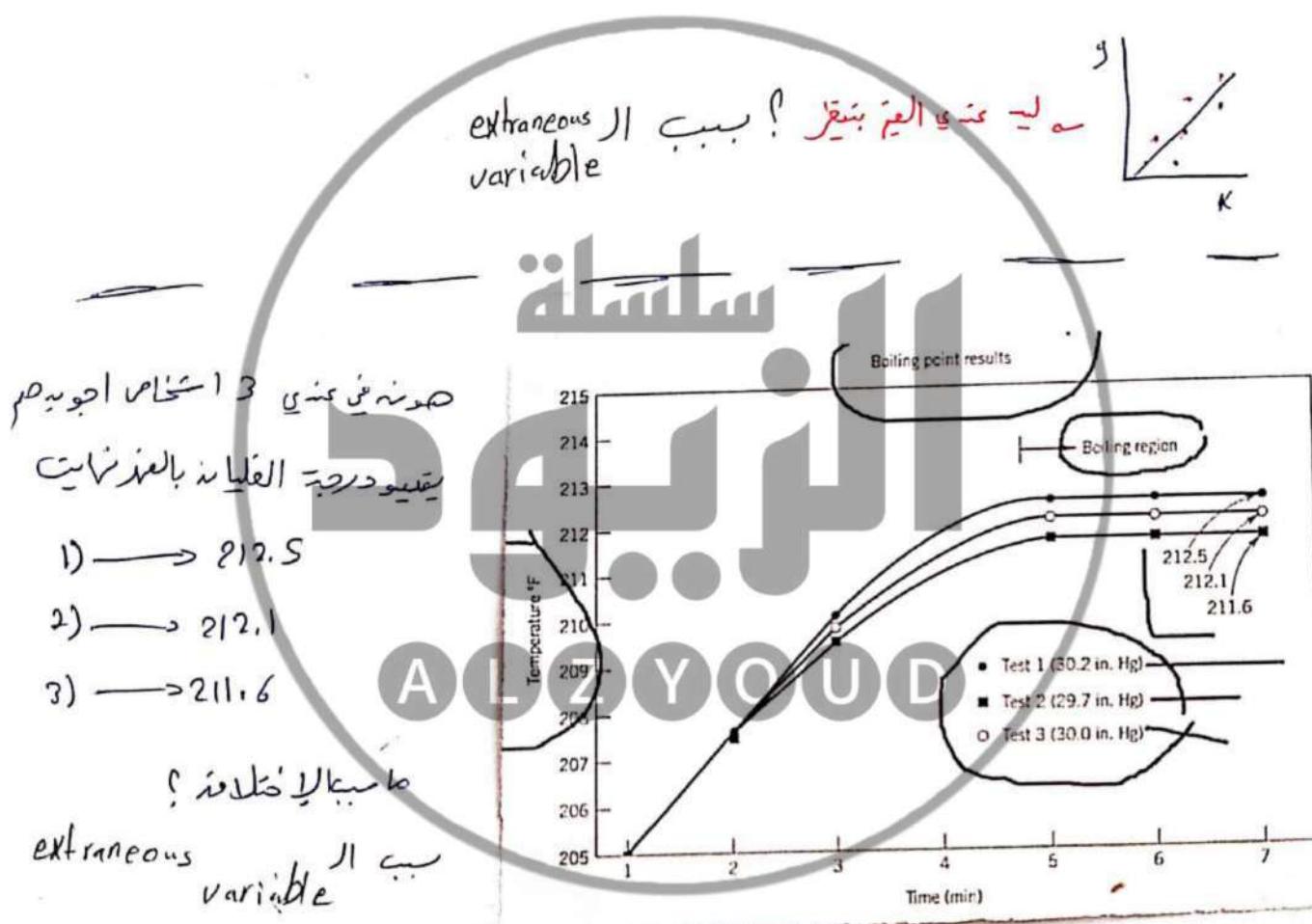
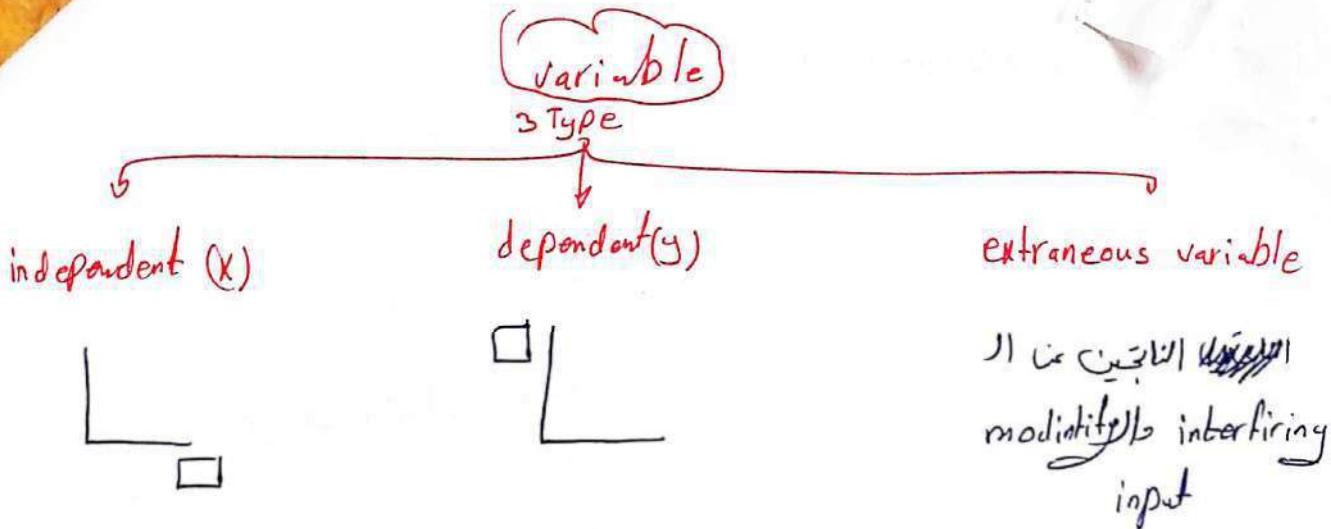
Figure 1.5 Components of a general measurement system.

~~experimental~~ experimental test plan

(خطه افثار تجربة)

the test should be design and executed to answer the question

↙ (experimental engineering analysis)



Parameter

define: a parameter as a functional grouping of variables.

\* parameter that has an effect on the behavior of the measured variable is called control parameter

صورة بسيطة لparameter هو مفهوم له معنى

Variable

مقدار المعرفة مني اعبر عن

$$C_1 = Q / nd^3$$

flow coefficient (parameter)

rotational speed  
diameter  
flow

كمال امروحة

احدى بنية الماء في 3D

### Noise and Interference

$$0.51 \pm 0.01$$

نسبة خطأ ممكنة

$$0.5 \pm 0.01$$

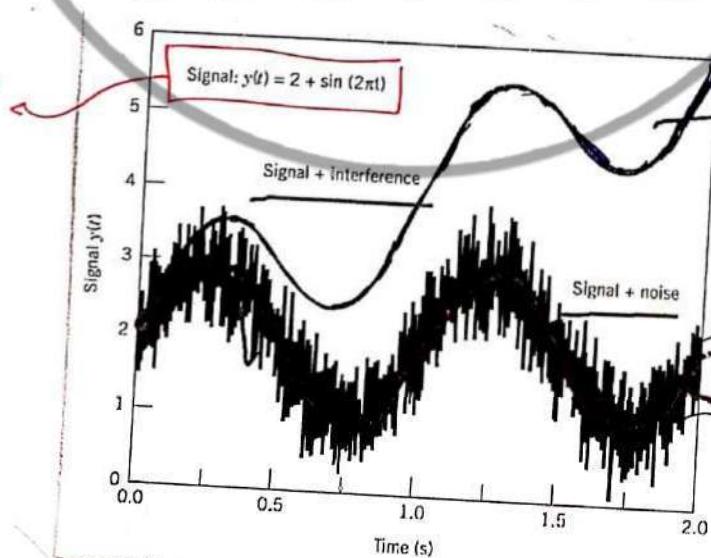
حيث 2 من المعرفة بينها ممكن يطلع

الصلة

اذا بطيء اهل مافعل من المعرفة الصغيرة  
اما Interference كل القيم في نسبة خطأ ثابتة هنا  
كل العناصر لها فوقة الـ 80 تكون  
في الغلاف سرير العناصر يعطي به ~ 85 cm  
راية نسبة ثابتة على حدا

ALZYOUD

تحدى طلب الامارة



صورة الدالة صو  
الى Interference و يكون  
كل عنصر الى  
Interference فيه  
ذاته

noise الـ  
بوي

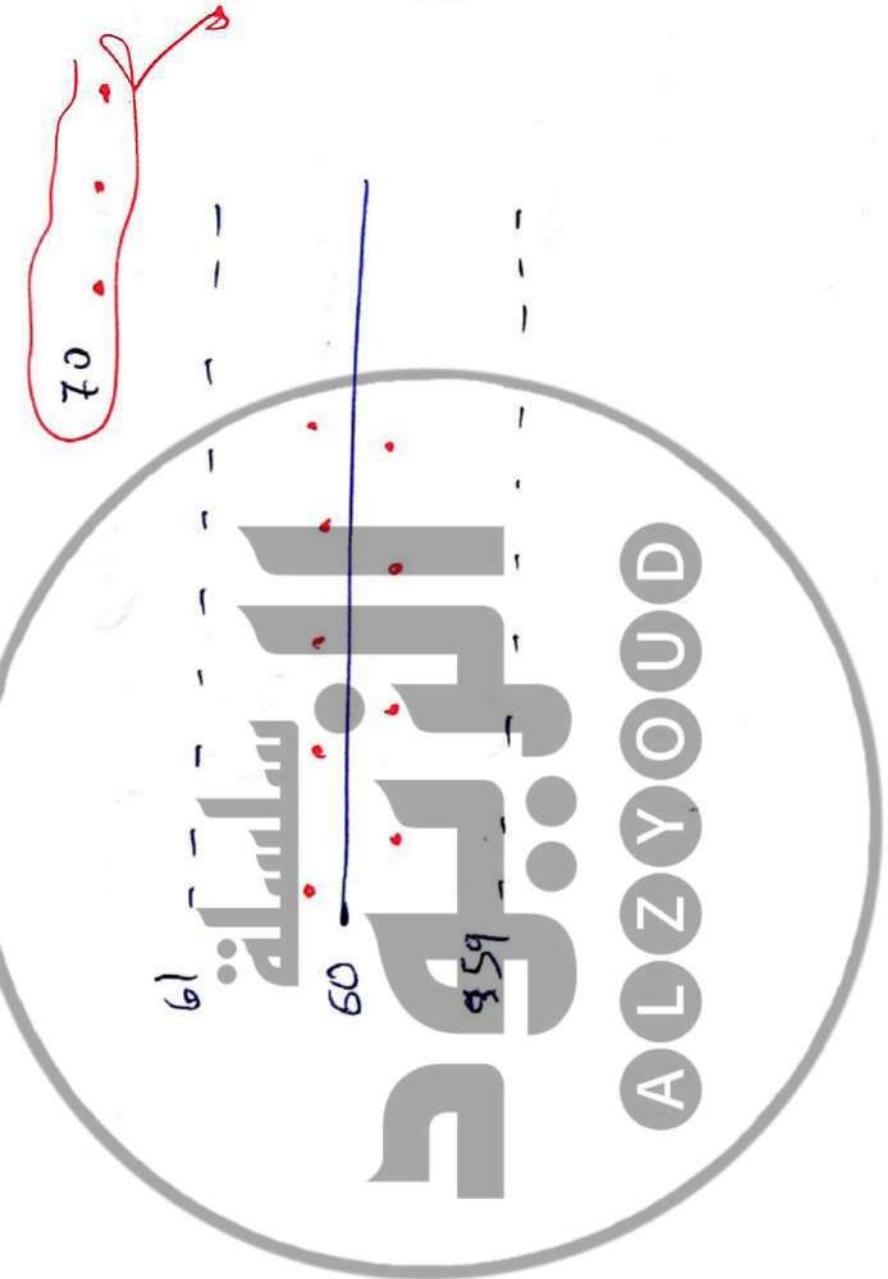
الـ  
باللغة

صواباً بما يجيء

noise الـ  
جوى

اند  
جوى

+ كل قراءة هي Al noise منها سبب واحد من سبب



جا

لـ

60

هـ

ALZYOOD

جا اند خناده ارسخت

وا باید سو بیکار ضمیر

interference

(systemic error)

## Ch2:Static and Dynamic Characteristics Signals إشارات الخصائص الثابتة والдинاميكية

measurement system takes an input quantity and transforms it into observed or recorded , such as the movement of a pointer on a dial or the magnitude of a digital display .

يأخذ نظام القياس كمية إدخال ويحولها إلى ملحوظة أو مسجلة ، مثل حركة المؤشر على قرص أو حجم العرض الرقمي

characteristics of both the input and the resulting output signals The shape and form of a signal are often referred to as its waveform . The waveform contains information about the magnitude and amplitude , which indicate the size of the input quantity and the frequency , which indicates the way the signal changes in time . An understanding of waveforms is required for the selection of measurement systems and the interpretation of measured signals .

خصائص كل من إشارات الإدخال والإخراج الناتجة غالباً ما يشار إلى شكل وشكل الإشارة على أنها شكل الموجة الخاص بها. يحتوي شكل الموجة على معلومات حول الحجم والسعة ، مما يشير إلى حجم كمية الإدخال والتردد ، مما يشير إلى الطريقة التي تتغير بها الإشارة بمرور الوقت. مطلوب فهم أشكال الموجة لاختيار أنظمة القياس وتفسير الإشارات المقاسة.

Two important tasks that engineers face in the measurement of physical variables are (1) selecting a measurement system and (2) interpreting the output from a measurement system .

مهمتان مهمتان يواجههما المهندسون في قياس المتغيرات الفيزيائية هما (1) اختيار نظام القياس و (2) تفسير الناتج من نظام القياس.

## Classification of Waveforms:

1) Analog describes a signal that is continuous in time . Because physical variables tend to be continuous , an analog signal provides a ready representation of their time - dependent behavior .

يصف التنازيرية إشارة مستمرة في الوقت المناسب . نظراً لأن المتغيرات المادية تميل إلى أن تكون مستمرة ، فإن الإشارة التنازيرية توفر تمثيلاً جاهزاً لسلوكها المعتمد على الوقت .



Figure 2.2 Analog signal concepts.

عبارة عن continuous waveform يعني بأي لحظة القراءة الي بشوفها عجهاز القياس بشوفها على ال waveform

Time → continuous..... Signal → continuous

2) A discrete time signal usually results from the sampling of a continuous variable at repeated finite time intervals .

تنتج إشارة الوقت المنفصلة عادةً عن أخذ عينات من متغير مستمر في فترات زمنية محددة متكررة .

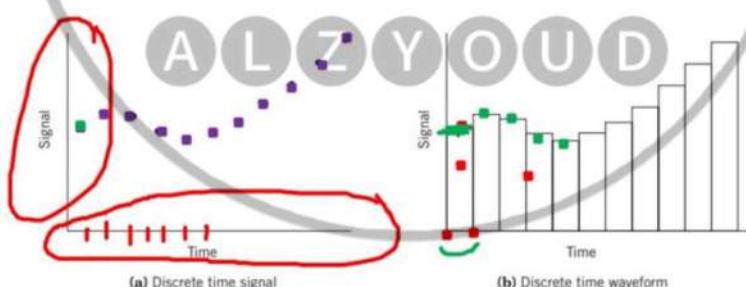


Figure 2.3 Discrete time signal concepts.

هون بتميز انه ال Time ما يكون على continuous تكون على discrete

Time → discrete..... Signal → continuous

3) Digital signals are particularly useful when data acquisition and processing are performed using a digital computer .

تكون الإشارات الرقمية مفيدة بشكل خاص عند الحصول على البيانات ومعالجتها باستخدام جهاز كمبيوتر رقمي .

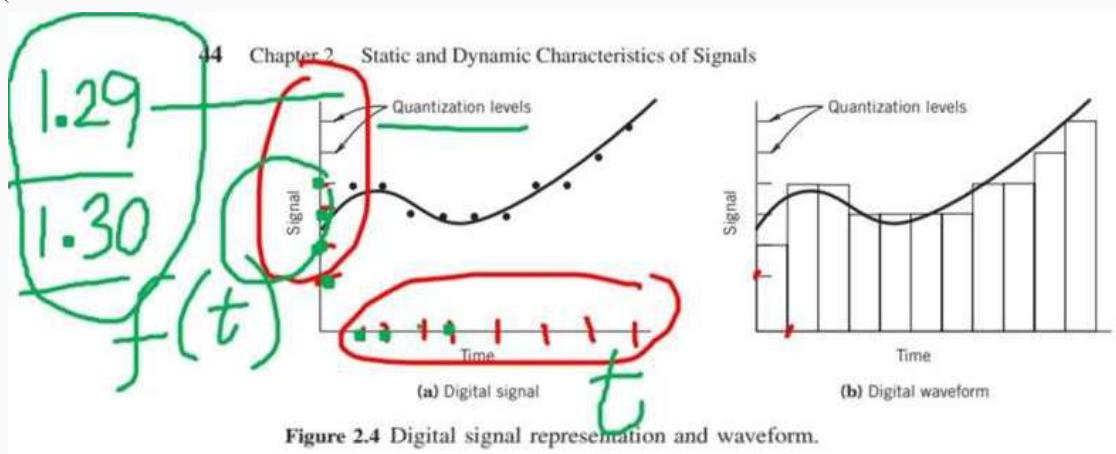


Figure 2.4 Digital signal representation and waveform.

لأهظ انه عبر عن ال  
y(t) وعبر عن  
t ب time

هون الثنين (على فترات)  $\leftarrow$  discrete  
 Time  $\rightarrow$  discrete..... Signal  $\rightarrow$  discrete  
 مثلاً بهاي الطريقة ممكن الاقي ال signal (1.29) والاقي signal (1.30) وبدي بينهم مثلاً (1.28)  
 فبقر لأنه هون ال signal discrete

مشكلة هاي الطريقة اني بعتبر signal فترات فما بيعطيني بدقة (Accuracy) ال

Sampling of an analog signal to produce a digital signal can be accomplished by using an analog - to - digital (A / D) converter , a solid - state device that converts an analog voltage signal to a binary number system representation

يمكن أخذ عينات من إشارة تنازلي لإنتاج إشارة رقمية باستخدام محول تنازلي إلى رقمي (A / D) ، وهو جهاز الحالة الصلبة الذي يحول إشارة الجهد التنازلي إلى تمثيل نظام الأرقام الثنائية



~~Table 2.1 Classification of Waveforms~~

I. Static

$$y(t) = A_0$$

II. Dynamic

Periodic waveforms

Simple ~~periodic waveform~~

$$y(t) = A_0 + C \sin(\omega t + \phi)$$

Complex periodic waveform

$$y(t) = A_0 + \sum_{n=1}^{\infty} C_n \sin(n\omega t + \phi_n)$$

Aperiodic waveforms

Step<sup>a</sup>

$$\begin{aligned} y(t) &= A_0 U(t) \\ &= A_0 \text{ for } t > 0 \end{aligned}$$

Ramp

$$y(t) = A_0 t \text{ for } 0 < t < t_f$$

Pulse<sup>b</sup>

$$y(t) = A_0 U(t) - A_0 U(t - t_1)$$

III. Nondeterministic waveform

$$y(t) \approx A_0 + \sum_{n=1}^{\infty} C_n \sin(n\omega t + \phi_n)$$

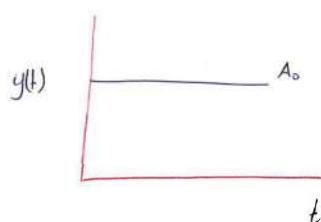
راحلاحظ وجود ثلث اشكال تين منهم dynamic الى هما ال deterministic وال static واخر نوع هو nondeterministic فايس معناهم راح نتفصل تحت فيهم

I. Static

$$y(t) = A_0$$

A static signal does not vary with time  
لا تتغير الإشارة الساكنة بمرور الوقت

$$y(t) = \text{ثابت}$$



مثال عليها لو انا بغرفة وكانت الحرارة 25 ووصلت 25 static signal ←

## II. Dynamic

### Periodic waveforms

#### Simple periodic waveform

$$y(t) = A_0 + C \sin(\omega t + \phi)$$

#### Complex periodic waveform

$$y(t) = A_0 + \sum_{n=1}^{\infty} C_n \sin(n\omega t + \phi_n)$$

### Aperiodic waveforms

#### Step<sup>a</sup>

$$\begin{aligned} y(t) &= A_0 U(t) \\ &= A_0 \text{ for } t > 0 \end{aligned}$$

#### Ramp

$$y(t) = A_0 t \text{ for } 0 < t < t_f$$

#### Pulse<sup>b</sup>

$$y(t) = A_0 U(t) - A_0 U(t - t_1)$$

A dynamic signal is defined as a time - dependent signal  
يتم تعريف الإشارة الديناميكية على أنها إشارة تعتمد على الوقت

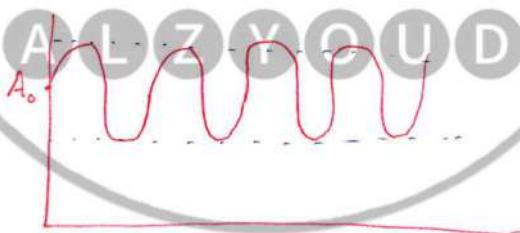
نلاحظ أنه ال dynamic و periodic نوعين

#### Simple periodic waveform

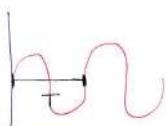
$$y(t) = A_0 + C \sin(\omega t + \phi)$$

periodic

$$y(t) = A_0 + C \sin(\omega t + \phi)$$



w ← لها علاقة بال frequency وكم بتتكرر (signal) القمة والقاع مع الوقت  
ϕ لها دخل بنقطة البداية عن الصفر تحت الصفر او فوقه وكم تبعد عنه



اما ال T فهو الزمن المستغرق لقمة وقاع كاملين

الـ simple لها periodic و complex والإختلاف بينهم

Signal ← Simple

Signal ← Complex

لوبتلاحظ ال periodic بتعمل عندي نمط متكرر اما ال Aperiodic ما بتعمل عندي نمط

### Aperiodic waveforms

Step<sup>a</sup>

$$y(t) = A_0 U(t)$$

$$= A_0 \text{ for } t > 0$$

Ramp

$$y(t) = A_0 t \text{ for } 0 < t < t_f$$

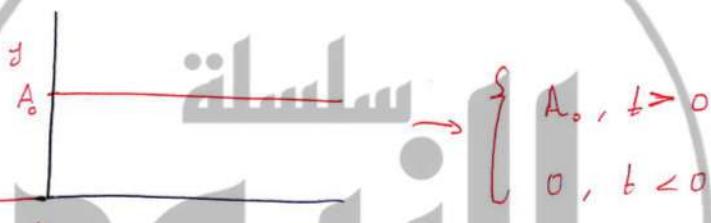
Pulse<sup>b</sup>

$$y(t) = A_0 U(t) - A_0 U(t - t_1)$$

Step<sup>a</sup>

$$y(t) = A_0 U(t)$$

step  $\rightarrow y(t) = A_0 * \underline{U(t)}$  unit step function



القيمة تساوي صفر قبل الصفر وبعد الصفر يساوي  $A_0$ .

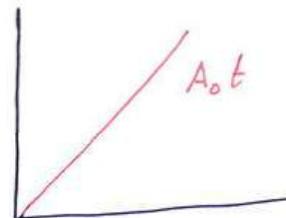
مثال عليه لو حطيت ميزان حرارة بكasaة ثلج وبعدها شلتها وحططيه بماء مغلي

Ramp

$$y(t) = A_0 t \text{ for } 0 < t < t_f$$

Ramp  $\rightarrow$  integration  
of step function

(Ramp is the integral of step function)



Pulse<sup>b</sup>

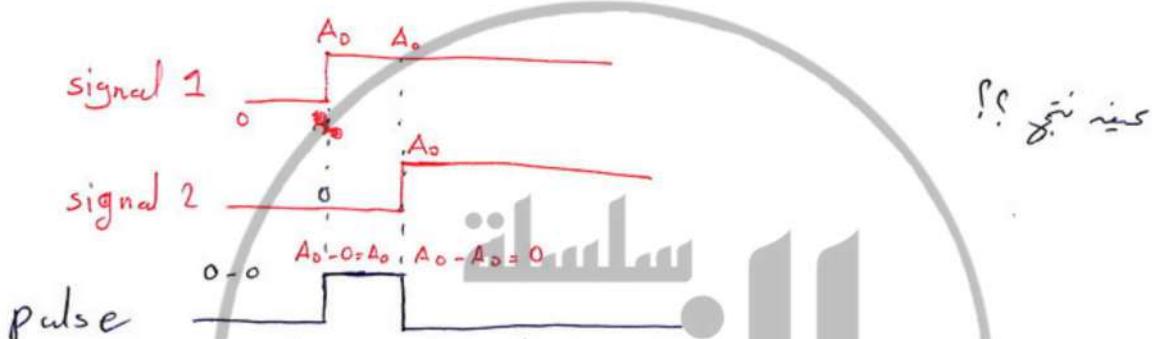
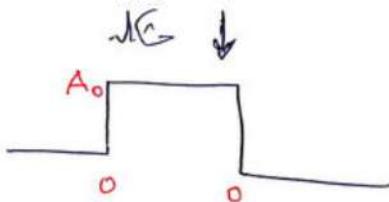
$$y(t) = A_0 U(t) - A_0 U(t - t_1)$$

pulse → step input من 2 signal (step signal)  
رذاذ خط از زی کن 2 signal من (step signal)

ومطروص من بعض فقط عنده ال pulse

لأن  $A_0$  يبعد  $t_1$  و  $t_0$   
كون صفر بعد  $t_1$  و  $t_0$

برجع صفر



A simple periodic waveform contains only one frequency. A complex periodic waveform contains multiple frequencies and is represented as a superposition of multiple simple periodic waveforms. Aperiodic is the term used to describe deterministic signals that do not repeat at regular intervals, such as a step function.

يحتوي الشكل الموجي الدوري البسيط على تردد واحد فقط. يحتوي الشكل الموجي الدوري المعقد على ترددات متعددة ويتم تمثيله على أنه تراكب لأسكال موجية دورية متعددة بسيطة. Aperiodic هو المصطلح المستخدم لوصف الإشارات القطعية التي لا تتكرر على فترات منتظمة، مثل دالة الخطوة.

كل اللي قبل كانوا deterministic بس ايش يعني؟؟؟

A deterministic signal varies in time in a predictable manner, such as a sine wave, a step function, or a ramp function, as shown in Figure 2.5.

تحتفل الإشارة القطعية في الوقت بطريقة يمكن التنبؤ بها، مثل موجة جيبية، أو وظيفة خطوة، أو وظيفة منحدر، كما هو موضح في الشكل 2.5.

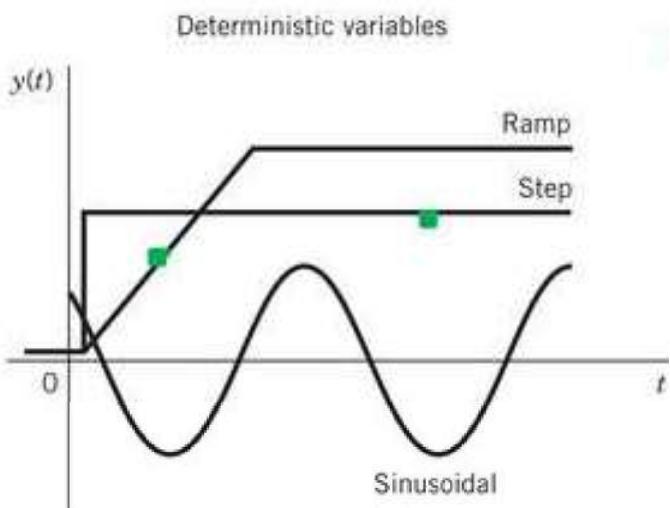


Figure 2.5 Examples of dynamic signals.

مثال عليها كل الى اخذناهم قبل  
نجي لآخر نوع والي هو ال nondeterministic

### III. Nondeterministic waveform

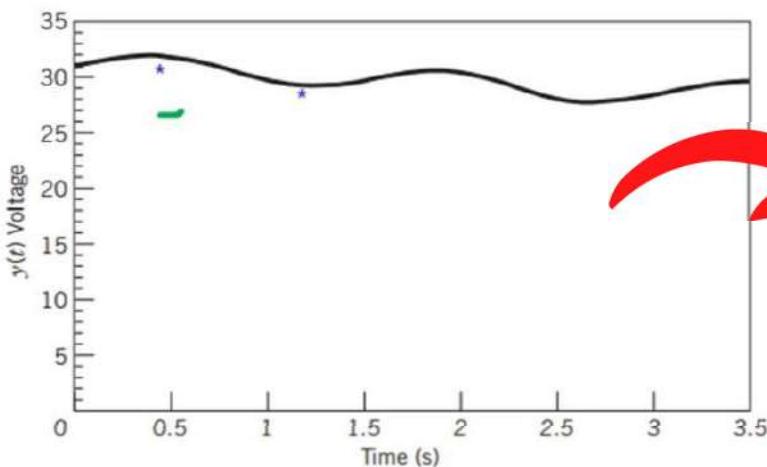
$$y(t) \approx A_0 + \sum_{n=1}^{\infty} C_n \sin(n\omega t + \phi_n)$$



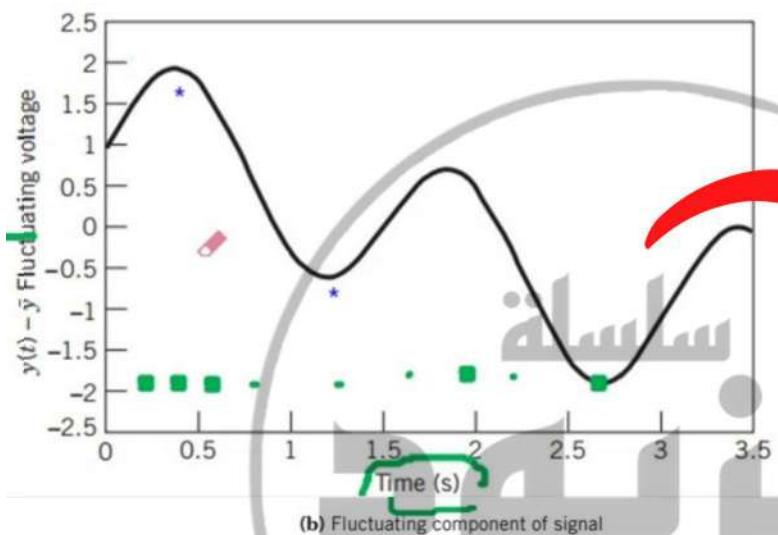
the sumation of  
the deterministic  
signal  
هي مجموع للإشارات

Also described in Figure 2.5 is a nondeterministic signal that has no discernible pattern of repetition . A nondeterministic signal cannot be prescribed before it occurs , although certain characteristics of the signal may be known in advance .

موصوفة أيضاً في الشكل 2.5 هي إشارة غير حتمية ليس لها نمط تكرار واضح. لا يمكن وصف إشارة غير حتمية قبل حدوثها ، على الرغم من أن بعض الخصائص المميزة للإشارة قد تكون معروفة مسبقاً.



(a) Signal prior to subtracting DC offset



لاحظ النجوم برسمتين  
الأفرج تاعها من (0-2) إذن 0±2

Figure 2.7 Effect of subtracting DC offset for a dynamic signal.

Simple periodic waveform

$$y(t) = A_0 + C \sin(\omega t + \phi)$$

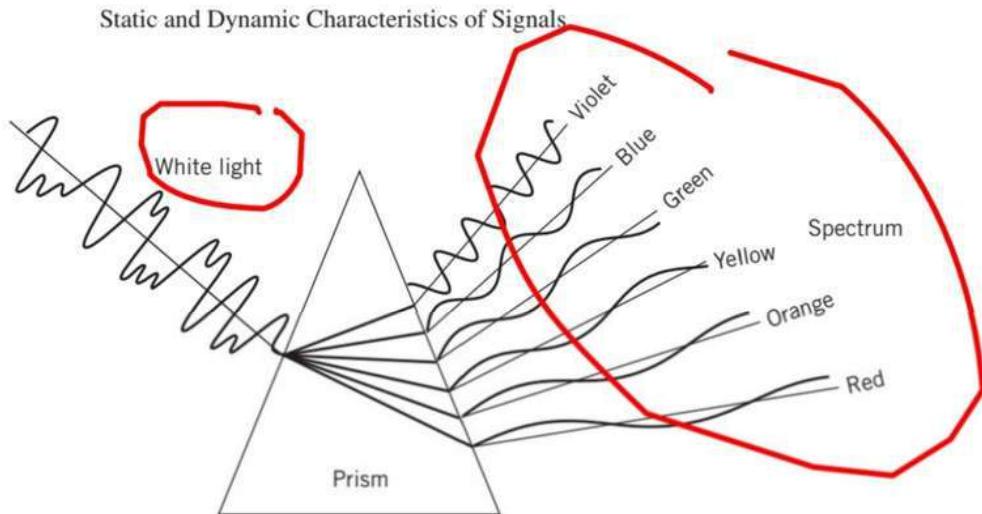
Dc  
Constant

Periodic  
variable او  
Ac او

فهون الي صار انهم حلووا الرسمة الأولى لرسمة تانية  
عن طريق انهم طرحوا 30  
وغيروا ال scale فالرسمة كبرت وصارت اوضح



هذا الهدف من هاد الشابتر انه اي signal اقدر اعرف مكوناتها ببسط زي مثال المنشور وضوء الشمس



زي ضوء الشمس لما امره داخل منشور رح يتحلل لعدة اطیاف بترددات وطول موجي مختلفين

Figure 2.8 Separation of white light into its color spectrum . Color corresponds to frequency or wavelength ; light intensity corresponds to varying amplitudes .

الشكل 2.8 فصل الضوء الأبيض إلى طيف ألوانه. يتافق اللون مع التردد أو الطول الموجي ؛ شدة الضوء تتافق مع السعات المتغيرة.

SIGNAL AMPLITUDE AND FREQUENCY : The method of expressing such a complex signal as a series of sines and cosines is called Fourier analysis .

اتساع وتواتر الإشارة: تسمى طريقة التعبير عن مثل هذه الإشارة المعقدة كسلسلة من الجيب وجيب التمام بتحليل فورييه.

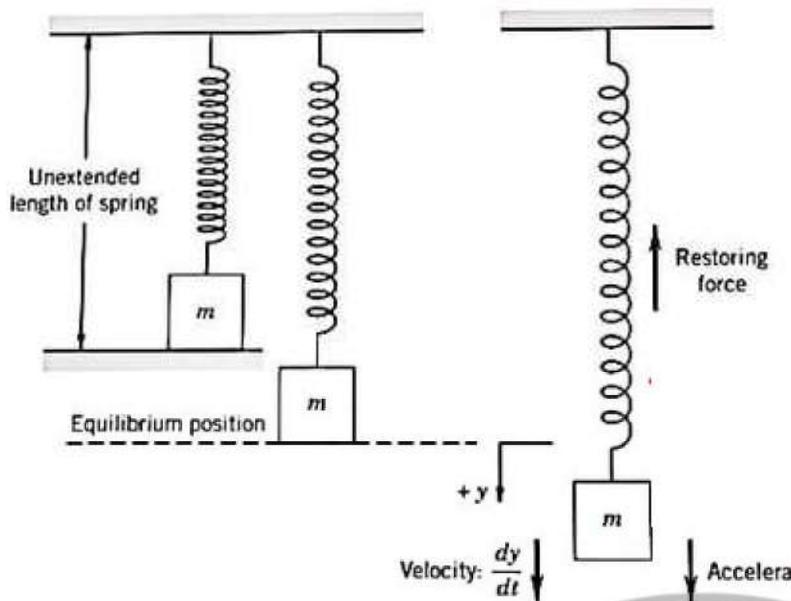
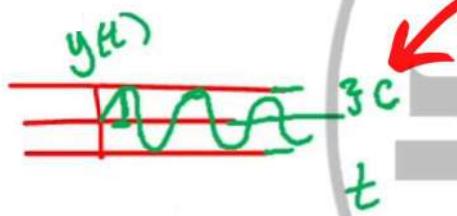


Figure 2.9 Spring-mass system.

هون انا عندي (mechanical system) لما اضغته واتركله مجال مشان يتحرك  
رنلاقني انه حيوصل لنقطة توازن وبعدها يصير يطلع وينزل الحركة تاعه هاي عبارك عن periodic motions



لوبدي اكتب معادلته التفاضلية:

$$m \frac{d^2y}{dt^2} + ky = 0$$

الهدف من هاي المعادلة اني اقدر احسب قيمة ال y

$$y = A \cos \omega t + B \sin \omega t$$

ال A وال B بما انهم نفس ال frequency بقدر اختصرهم ب c

$$y = C \cos (\omega t - \phi)$$

$$y = C \sin (\omega t + \phi')$$

The sine and cosine terms can be combined  
If a phase angle is introduced such that  
يمكن الجمع بين شرط الجيب  
وجيب التمام  
إذا تم إدخال زاوية المرحلة من  
هذا القبيل

$$c = \sqrt{a^2 + b^2} \leftarrow \text{AMPLITUDE}$$

وال w هي ال Anguler circuit frequency وال ووحدتها rad/s وتقرأ أوميغا

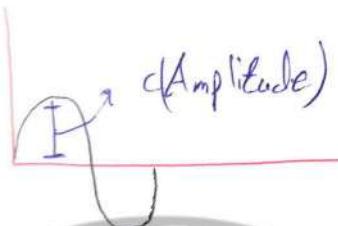
وال T عرفناه قبل بأنه الزمن تاع دورة كاملة(قمة وقاع) ووحدته ال الثانية s

$$F = \frac{1}{T} \quad \text{f هي مقلوب ال frequency}\quad \text{وال} \quad \text{Hz}\quad \text{ووحدتها}$$

ال w ال circuit frequency Anguler لها علاقك بال frequency كقانون

$$W=2\pi F$$

اما ال C والي هو ال amplitude الي هو السعة



ضل اخر اشي الي هو ال face angle  $\Phi$  ← اديه انا ببعد عن zero

The values of  $C$ ,  $\phi$ , and  $\phi^*$  are found from the following trigonometric identities:

$$A \cos \omega t + B \sin \omega t = \sqrt{A^2 + B^2} \cos(\omega t - \phi)$$

$$A \cos \omega t + B \sin \omega t = \sqrt{A^2 + B^2} \sin(\omega t + \phi^*)$$

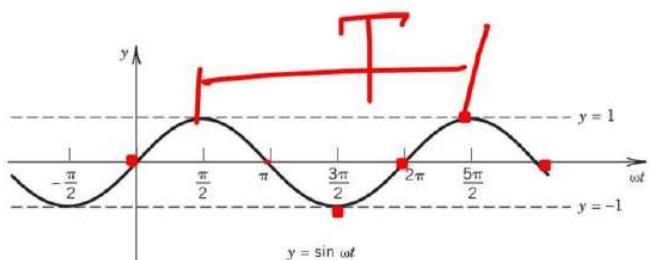
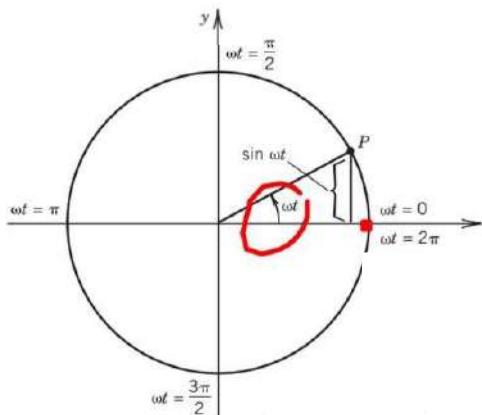
$$\phi = \tan^{-1} \frac{B}{A} \quad \phi^* = \tan^{-1} \frac{A}{B} \quad \phi^* = \frac{\pi}{2} - \phi$$

The fundamental concepts of frequency and amplitude can be understood through the observation and analysis of periodic motions.

يمكن فهم المفاهيم الأساسية للتعدد والاسعة من خلال مراقبة وتحليل الحركات الدورية.

هسا هون بحكيلى انه التعريفات هاي وصلتنا من الفيزياء

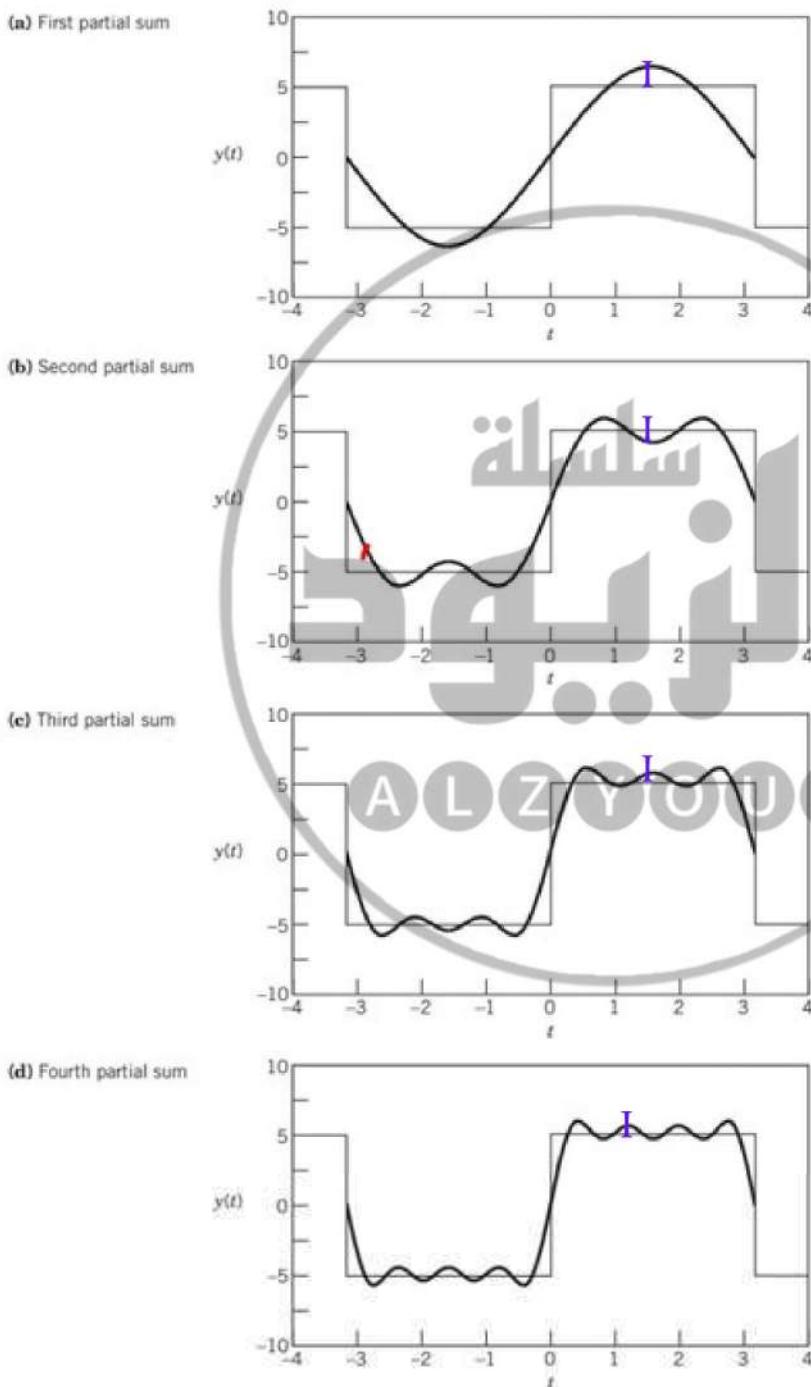
#### Static and Dynamic Characteristics of Signals



The size of the maximum and minimum displacements from the equilibrium position , or the value C , is the amplitude of the oscillation . The concepts of amplitude and frequency are essential for the description of time - dependent signals .

حجم الحد الأقصى والحد الأدنى من الإزاحة من موضع التوازن ، أو القيمة C ، هو سعة التذبذب. تعتبر مفاهيم الاتساع والتعدد ضرورية لوصف الإشارات المعتمدة على الوقت.

## 60 Chapter 2 Static and Dynamic Characteristics of Signals



الخط الملتوى هو عبارة عن  
analoge signal  
اما خط المربعات هو  
approximation فهون عندي  
ل واحدة Signal

هون صارت  
approximation  
ل Two signal

هون صارت  
approximation  
ل Three signal

هون صارت  
approximation  
ل signal 4

Figure 2.15 First four partial sums of the Fourier series  $(20/\pi)(\sin t + 1/3 \sin 3t + 1/5 \sin 5t + \dots)$  in comparison with the exact waveform.

كل ما زادت ال signal في ال approximation يقل error بصير ال

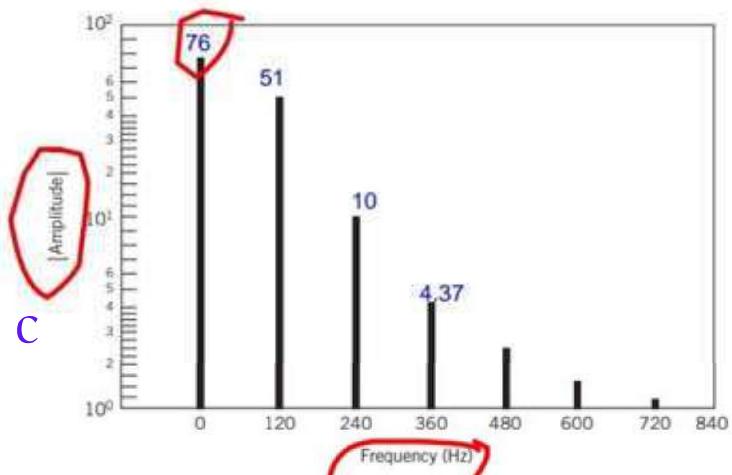


Figure 2.17 Frequency content of the function  $y(t) = |120 \sin 20\pi t|$  displayed as an amplitude-frequency spectrum.

w او f اما

هون الرسمة بتوضح العلاقة بين ال frequency وال amplitude على حسب المعادلة الي تحت

The Fourier series for the function  $|120 \sin 120\pi t|$  is

$$76.4 - 50.93 \cos 240\pi t - 10.10 \cos 480\pi t - 4.37 \cos 720\pi t \dots$$

↓      ↓      ↓      ↓

76      51      10      4

من المعادلة قدرت اطلع قيم ال amplitude على الترتيب  
frequency ← 0, 240, 480, 720  
و دلتني عقيم ال 76, 51, 10, 4

A L Z Y O U D

## Ch3:measurement system behavior

Each measurement system responds differently to different types of input signals and to the dynamic content within these signals . So a particular system may not be suitable for measuring certain signals or at least portions of some signals .

يستجيب كل نظام قياس بشكل مختلف لأنواع مختلفة من إشارات الإدخال والمحظى الديناميكي داخل هذه الإشارات. لذلك قد لا يكون نظام معين مناسباً لقياس إشارات معينة أو على الأقل أجزاء من بعض الإشارات.

As pointed out in Chapter 2 , all input and output signals can be broadly classified as being static , dynamic , or some combination of the two . For a static signal , only the signal magnitude is needed to reconstruct the input signal based on the indicated output signal

كما هو موضح في الفصل 2 ، يمكن تصنيف جميع إشارات الإدخال والإخراج على نطاق واسع على أنها ثابتة أو ديناميكية أو مزيج من الاثنين. بالنسبة للإشارة الثابتة ، لا يلزم سوي حجم الإشارة لإعادة بناء إشارة الإدخال بناءً على إشارة الخرج المشار إليها

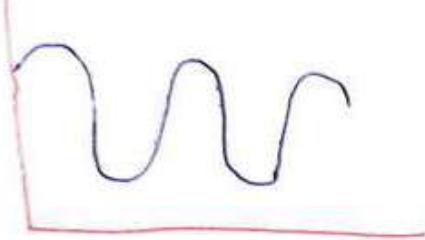
أخذنا في الشابتر الي قبل انواع ال signal وانواع ال input

Vibration signals vary in amplitude and time , and thus are a dynamic input signal to the measuring instrument .

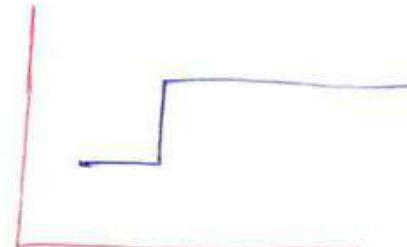
تختلف إشارات الاهتزاز في السعة والوقت ، وبالتالي فهي إشارة دخل ديناميكية لأداة القياس.

رح نركز بها الشابتر على نوعين من signal

Periodic  
or  
Sin input



Step  
input



نتيجة عملية القياس رح تعتمد على شغلتين :

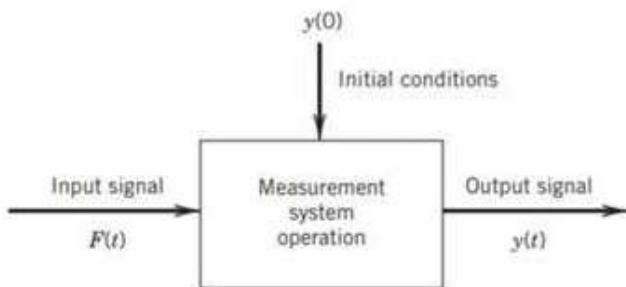


Figure 3.2 Measurement system operation on an input signal,  $F(t)$ , provides the output signal,  $y(t)$ .

الى هي ال input signal كمثال لو كانت  $\leftarrow G_1$  أو periodic  
measurement system operation وال  $\leftarrow G_2$  هي مكونات نظام القياس

$$G \text{ results} = G_1 * G_2$$

## Dynamic Measurements

Dynamic Measurements For dynamic signals , signal amplitude , frequency , and general waveform information is needed to reconstruct the input signal . Because dynamic signals vary with time , the measurement system must be able to respond fast enough to keep up with the input signal .

القياسات الديناميكية ضرورية للإشارات الديناميكية ، وسعة الإشارة ، والتردد ، ومعلومات شكل الموجة العامة لإعادة بناء إشارة الدخل . نظرًا لاختلاف الإشارات الديناميكية مع مرور الوقت ، يجب أن يكون نظام القياس قادرًا على الاستجابة بسرعة كافية لمواكبة إشارة الإدخال .

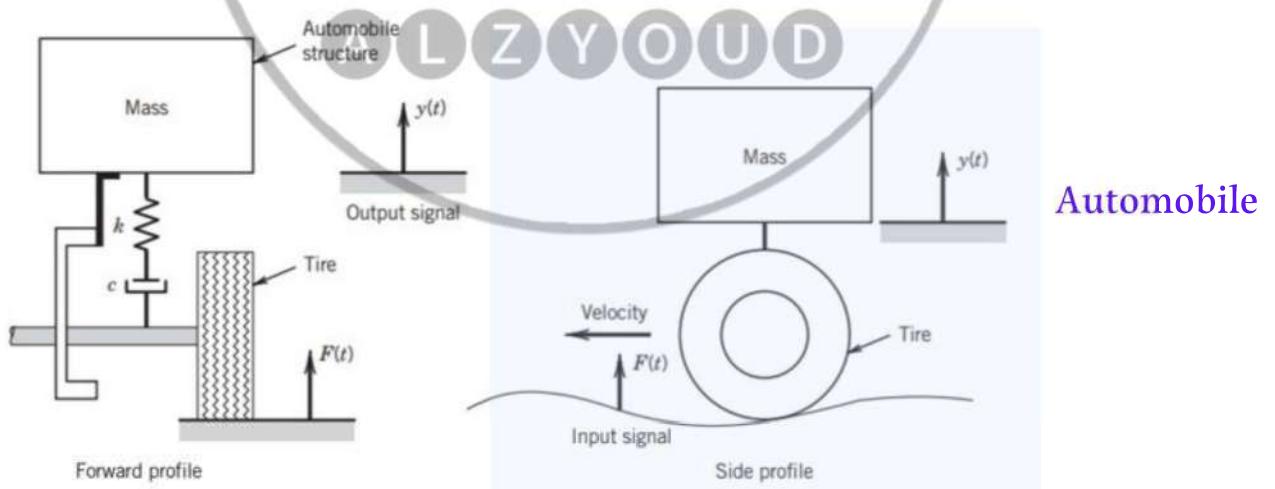


Figure 3.1 Lumped parameter model of an automobile suspension showing input and output signals.

لنفرض نظام داخل سيارة مممن تكون مرتاح او مش مرتاح حسب شغلتين  $y(t)$  وال displacement  $\uparrow$  وهما اشياء بعتمدو على ال signal (الطريق)

كل ما تكون  $y(t)$  ثابتة رح تكون قيمة  $y(t)$  ثابتة وتكون انت مرتاح اكتر  
هسا حكينا ال signal مهم لأنها هي الطريق وهي العامل الأول  
العامل الثاني جهاز القياس الي هو هون ال damper وال automobile تاعها

فهون ممكن تكون الطريق سين والسيارة كويستة فما تحس بالطريق  
وممكنا السيارة(جهاز) كويستة والطريق سين فبرضو تحسش بالطريق(signal)

وممكنا السيارة قديمة والطريق مليان تعرجات  
 فهو جهاز القياس(t)y( ) رح يعطيني انه عندي مشكلة بالG1 والG2

هاد الموضوع هو موازي للموضوع تاع الكنترول الي اسمه transfer function

$$\frac{\text{Laplace output}}{\text{Laplace input}}$$

فال input يعتمد على output

**هسا رح نحاول نبدأ نعمل حسب ال G1 وال G2 model measurement system**

Consider the following general model of a measurement system, which consists of an nth-order linear ordinary differential equation in terms of a general output signal, represented by variable  $y(t)$ , and subject to a general input signal,

represented by the forcing function,  $F()$ :

ضع في اعتبارك النموذج العام التالي لنظام القياس، والذي يتكون من معادلة تفاضلية خطية عادية من الدرجة التاسعة من حيث إشارة خرج عامة، ممثلة بالمتغير  $y(t)$ ، وتتعرض لإشارة إدخال عامة ممثلة بوظيفة التأثير  $F()$ :

هسا بنكتب المعادلة التفاضلية للنظام وبنفرض انها linear system

ومارح نتعلم كيفية حلها لأنه حلها بده كورسات كاملة

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \dots + a_1 \frac{dy}{dt} + a_0 y = F(t)$$

where

$$F(t) = b_m \frac{d^m x}{dt^m} + b_{m-1} \frac{d^{m-1} x}{dt^{m-1}} + \dots + b_1 \frac{dx}{dt} + b_0 x \quad m \leq n$$

هسا هون المعادلة التفاضلية تتكون من جهتين جهة ال input الي هي ال  $F(t)$  والجهة الثانية هي عبارة عن معاملات (a) مضروبين بال input (a) هو  $y$  ومشتقاته

$F(t)$  هي عبارة عن Sumation لكل ال Input

وكتبوا لها معادلة تحت ولكن بالبحث تاعنا في هاد الكورس وجدوا انه فش داعي للكتابة للحد الأخير

هون لاحظ انه المعادلات التفاضلية كلها بال  $t$  لو بدبي احوالها لل domain s لازم اخذلها لابلس

(Physical Parameters) → خصائص منظومة القياس

Comparing this to

the general form for a second-order equation ( $n=2$ ;  $m=1$ ) from Equation 3.1,

$$a^2 \frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = b_1 \frac{dx}{dt} + b_0 x$$

### 3.3 SPECIAL CASES OF THE GENERAL SYSTEM MODEL

#### Zero-Order Systems

لما اخذ بس ال اول حد من ال input بهاي الحالة تكون zero order system

The simplest model of a measurement systems and one used with static signals is the zero - order system model . This is represented by the zero - order differential equation :

أبسط نموذج لأنظمة القياس وآخر يستخدم مع الإشارات الثابتة هو نموذج النظام الصفرى. يتم تمثيل ذلك بالمعادلة التفاضلية ذات الترتيب الصفرى:

$$a_0 y = F(t) \rightarrow a_0 y = b_0 x \rightarrow y = \left( \frac{b_0}{a_0} \right) x \rightarrow y = K X$$

هاد الأشي بيذكرني بأشي مهم  
لما اقسم المعادلة على  $a_0$ .

**Static sensitivity**  $\rightarrow y(t) = KF(t)$

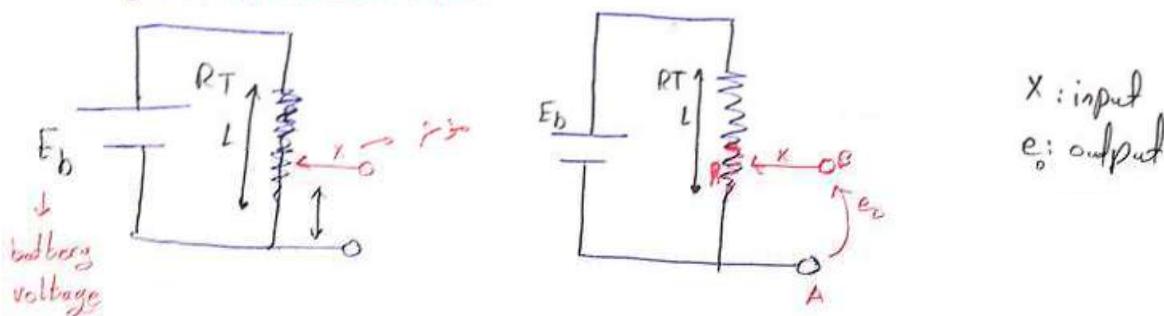
K is called the static sensitivity or steady gain of the system .

يسمى K بالحساسية الساكنة أو الكسب الثابت للنظام.

The static sensitivity is found from the static calibration of the measurement system . It is the slope of the calibration curve ,  $K = dy/dx$

تم العثور على الحساسية الساكنة من المعايرة الثابتة لنظام القياس. إنه منحدر منحنى المعايرة

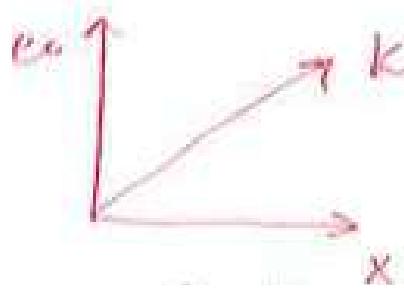
Example: Potentiometer



ال e\_o هو لو شبكت الدارة بفولتيميترا عشان اقيس فرق الجهد بين النقطة A والنقطة B وهو بلاقي علاقه بينهم

$$e_o = \frac{X}{L} \cdot E_b$$

لو فكرت ارسم العلاقة بين ال input وال output تكون العلاقة linear



وال output zero order system بظهرليسيطرة كاملة فأي تغير في ال input يتبعه تغير في ال output

لو سئلني عن قيمة ال K هون كيف يوجدها

Output      Input

$$e_o = \frac{K}{L} \cdot E_b$$

رسانني عن static sensitivity

$$e_o = \left( \frac{E_b}{L} \right) K$$

$$e_o = K \cdot X$$

$$\boxed{K = \frac{E_b}{L}}$$

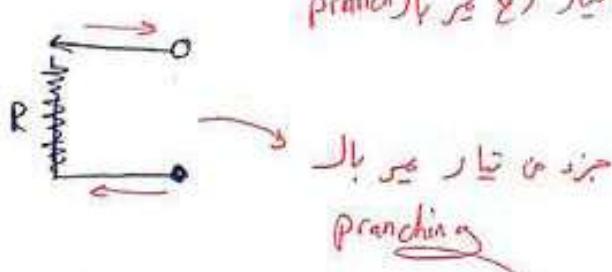
هسا في هاد المثال النظام ideal أو perfect لكن هل فعلاً في الحقيقة يوجد نظام ideal؟  
الجواب طبعاً هون افترضنا كثير افتراضات لطلع ideal منها

$$E_b = R_{total} = L \quad (1)$$

$$(R/R_T) \cdot E_b \quad \longleftarrow \quad R \text{ توزع جزء من } R_{total} \quad (2)$$

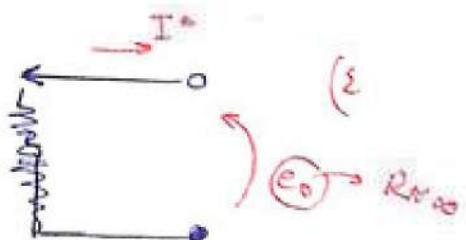
صبايا الحالات مرتنتا ال تيار الى مير في ال  $E_b$  منه بمح بال  $R$  ولكن هاد خنز

لأن فيه جزء من التيار ربع مير بال  $R_{branch}$

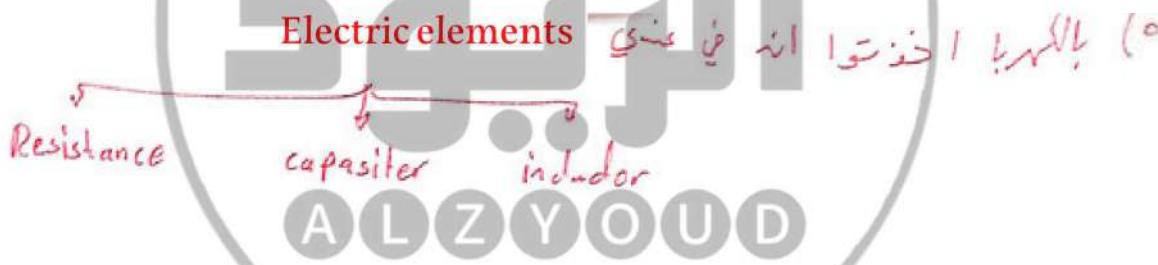


٣) ضماد يعني انه جزء من ال power رجع مفهوم ال Voltmeter ومفهوم ال ammeter  
 ويعني تكون العلامة linear ( $100/100$ ) مية باطية  
 فيه كلتي انا فرضت one loop لكن هي بالحقيقة  $2100\Omega$

الايسين اي مكان يقرب بالنظام من تكون العلامة  
 linear  
 ان تكون التيار غير مغير وانها تكون معاوته لمجاز  
 القاسم  $\infty$  (المادة جداً)



↓  
 دل مانطبقه صاد الشرط يكون له العلامة  
 (( يعني ملائمة خطيرة يعني ))



ضماد اعمالي أنا فرضت انه فيعني  
 صاد الصفر Resistor ( $100/100$ ) وصافرو لا  
 او لا مي موجودة (عجل العلامة zero)  
 resistance or inductor or capacitor

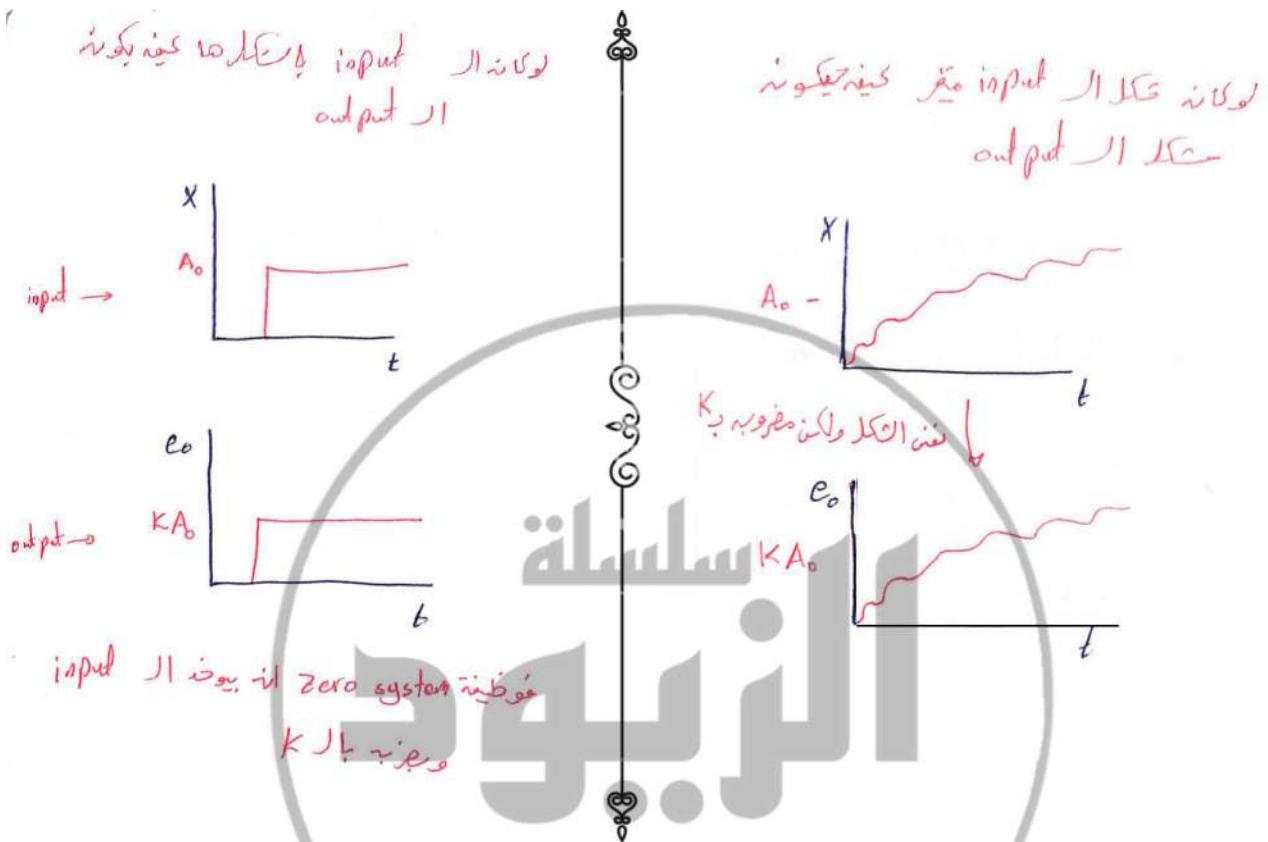
٤) الا مي موجودة (عجل العلامة zero)

مكان تغير ( first or second )  
 (من هو النظام )

٥) ما فرقنا اريفنا انه ابعد سر حجم صفر ما بهلا اكتواري ماصلنا الوزرة  
 ناس ما هملنا اسرية ذات مكان يغير من سرعة X

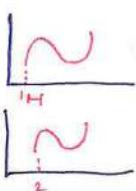
الهدف من كل هاد انه نفهم انه ما في ideal system مية بالمية فعشن هيك بحاول عقدر الإمكان اقل من كل الأخطاء الي ممكن تبعد النظام عن ال ideal وما يكون ال input يعطي output صحيح يعني نظام بدون اخطاء والمؤثر الوحيد على ال input هو ال output Ideal system

### لفهم فكرة السيطرة تابع ال zero-order



اسئلة على رسمات ال zero-order

• Time Lag / time phase  
كتير اكتر عن الـ phase



فنت الاشارة بين قبلي

كانت بتغير من زمن ١  
 وبعد بتغير من زمن ٢

مقدار تغيره

Perfect system  
ما يتفاوت في الحقيقة حل الـ

حيث انه تكونه موجود



yes ideal ✓ ideal instrument

perfect ✓ ? Dynamic response

هل فيه (output after direct input)

time lag / phase shift

X ? output after input

هل يوجد دivergence بين الـ input والـ output

هل هناك تغير في المقدار

Time lag

X distortion

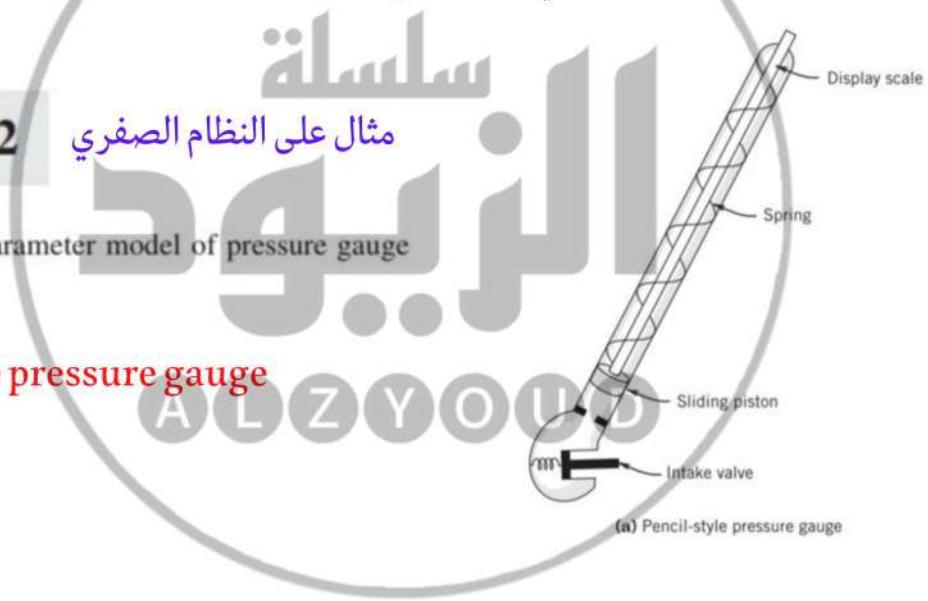
(هل تغير العجلة قبل بعد)

Fortunately , many measurement systems can be modeled by zero- , first- , or second - order linear . ordinary differential equations . More complex systems can usually be simplified to these lower orders . Our intention here is to attempt to understand how systems behave and how such response is closely related to the design features of a measurement system ; it is not to simulate the exact system behavior . The exact input - output relationship is found from calibration . But modeling guides us in choosing specific instruments and measuring methods by predicting system response to signals , and in determining the type , range , and specifics of calibration .

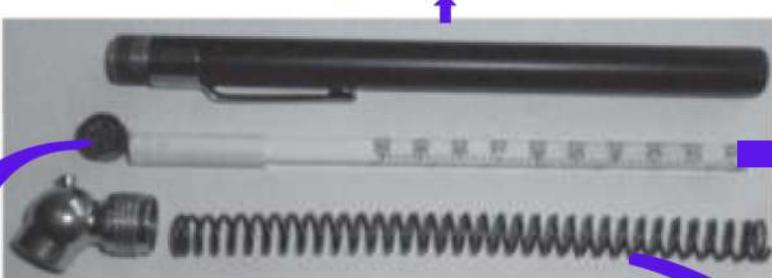
لحسن الحظ ، يمكن نمذجة العديد من أنظمة القياس باستخدام خطٍ من الدرجة الصفرية أو الأولى أو الثانية. المعادلات التفاضلية العادية . يمكن عادةً تبسيط الأنظمة الأكثر تعقيداً لهذه الطلبات الأقل. هدفنا هنا هو محاولة فهم كيفية تصرف الأنظمة وكيف ترتبط هذه الاستجابة ارتباطاً وثيقاً بميزات تصميم نظام القياس؛ ليس لمحاكاة سلوك النظام بالضبط. تم العثور على العلاقة الدقيقة بين الإدخال والإخراج من المعايرة. لكن النمذجة ترشدنا في اختيار أدوات وطرق قياس محددة من خلال التنبؤ باستجابة النظام للإشارات ، وفي تحديد نوع ونطاق وخصائص المعايرة.

### **Example 3.2** مثال على النظام الصفرى

**Figure 3.4** Lumped parameter model of pressure gauge



الأسود هو الغلاف الخارجي للجهاز

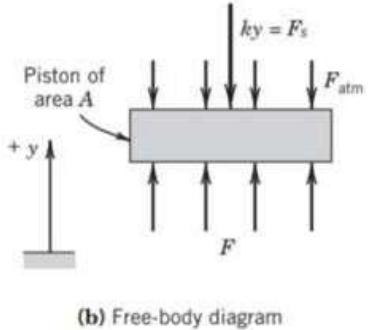


(c) Photograph of the components within a pencil-type pressure gauge

هون زى زى نرک بداخل الجهاز

Display scale هون عليه زي تدرج بيعطي قيمة ال pressure

لاحظ انه هاد ال peston عليه قوتين الأولى هو atmosphere pressure والثانية هو ال zernberk pressure



A pencil - type pressure gauge commonly used to measure tire pressure can be modeled at static equilibrium by considering the force balance on the gauge sensor , a piston that slides up and down a cylinder piston

يمكن نمذجة مقياس ضغط من نوع القلم الرصاص يستخدم عادة لقياس ضغط الإطارات عند توازن ثابت من خلال مراعاة توازن القوة على مستشعر القياس ، وهو مكبس ينزلق لأعلى ولأسفل مكبس أسطوانة

$$\sum F = 0, \text{ gives}$$

$$k_y = F - F_{atm}$$

$$K_y = p^* A$$

الى هي الضغط المؤثر على المساحة A في p هي القوة تابع الزنبرك ومساوية لل

Pressure is simply the force acting inward over the piston surface area , Dividing through by area provides the zero - order response equation between output displacement and input pressure and gives

الضغط هو ببساطة القوة المؤثرة للداخل على مساحة سطح المكبس ، وتقسيمها حسب المنطقة يوفر معادلة الاستجابة ذات الترتيب الصفرى بين وضع الإخراج وضغط الإدخال ويعطي

$$y = (A/k)(p - p_{atm})$$

The exact static input - output relationship is found through calibration of the gauge . Because elements such as piston inertia and frictional dissipation were not considered , this model would not be appropriate for studying the dynamic response of the gauge .

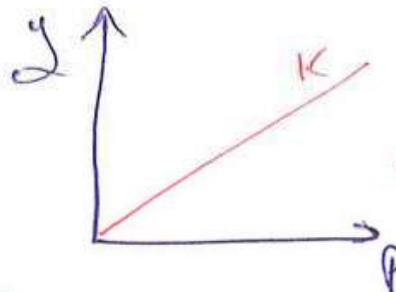
تم العثور على علاقة الإدخال والإخراج الثابتة الدقيقة من خلال معايرة المقياس. نظرًا للعدم مراعاة عناصر مثل القصور الذاتي للمكبس والتبييد الاحتكمائي ، فلن يكون هذا النموذج مناسًّا لدراسة الاستجابة الديناميكية للمقياس.

لوبدي ارسم العلاقة بينهم

$$Ky = p^* A$$

$$K = \frac{k}{A}$$

$y$ : output  
 $p$ : input



static sensitivity

static calibration curve

وفرضوا ي تغير بقيمة ال  $x$  بتغير بقيمة ال  $t$



لو بتلاحظ نفس ال  
wave form

## FirsOrder Systems

Measurement systems that contain storage elements do not respond instantaneously to changes in input.

أنظمة القياس التي تحتوي على عناصر تخزين لا تستجيب بشكل فوري للتغيرات في المدخلات.

first order system

$$a_1 y + a_0 y = F(t)$$

هاد الحد زائد عن ال zero-order



هسا اي معادلة احنا بنحاول نبسطها بحيث تبقا ال  $y$  لحالها  
زي ممثل ال zero-order قسمناها على  $a_0$ . وهيك بعمل هون

$$a_1 y + a_0 y = F(t)$$

$$x y + y = K f(t)$$

$$\downarrow \\ \text{Time constant} \\ = a_1 / a_0$$

$$\downarrow \\ \text{static sensitivity} \\ = K$$

may be modeled using a first-order differential equation of the form

$$a_1 \dot{y} + a_0 y = F(t)$$

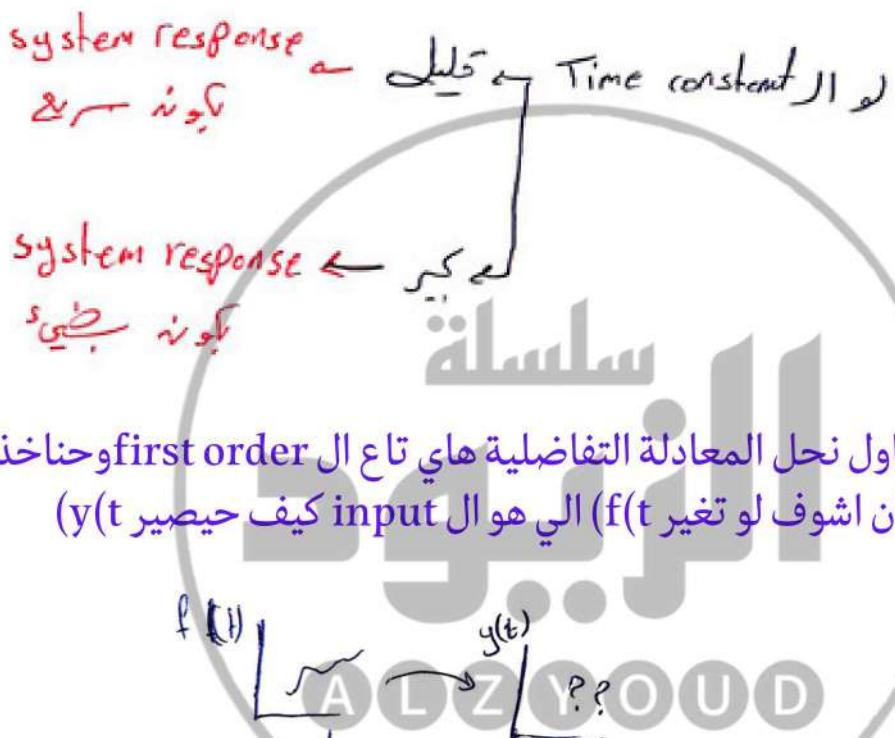
with  $\dot{y} = dy/dt$ . Dividing through by  $a_0$  gives

$$\tau y + y = KF(t)$$

The parameter  $T$  is called the time constant of the system.

تسمى المعلمة  $T$  ثابت الوقت للنظام.

تمام بس هسا شو الفائدة من ال time constant ال time constant بتعبيرلي عن ال Speed of system responds



### Step Function Input

The step function,  $AU(t)$ , is defined as

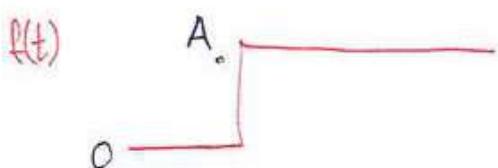
where  $A$  is the amplitude  
of the step function  
حيث  $A$  هي سعة دالة الخطوة

$$AU(t) = 0 \quad t \leq 0^-$$

$$AU(t) = A \quad t \geq 0^+$$

a sudden change in the input signal from a constant value of one magnitude to a constant value of some other magnitude

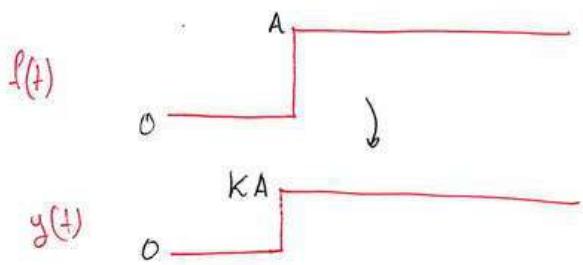
تغير مفاجئ في إشارة الدخل من قيمة ثابتة مقدارها إلى قيمة ثابتة لبعض المقدار الآخر



كمثال عليه لما انقل الميزان تابع الحرارة من كوب مثلج لکوب بغلی هاد شکله العام قبل ما اشوف استجابته بال FirstOrder

$$\begin{cases} t = A & , t > 0 \\ t = 0 & , t < 0 \end{cases}$$

تمام هسا قبل ما اسوف استجابته بال first order بتندر كر كيف استجابته بال

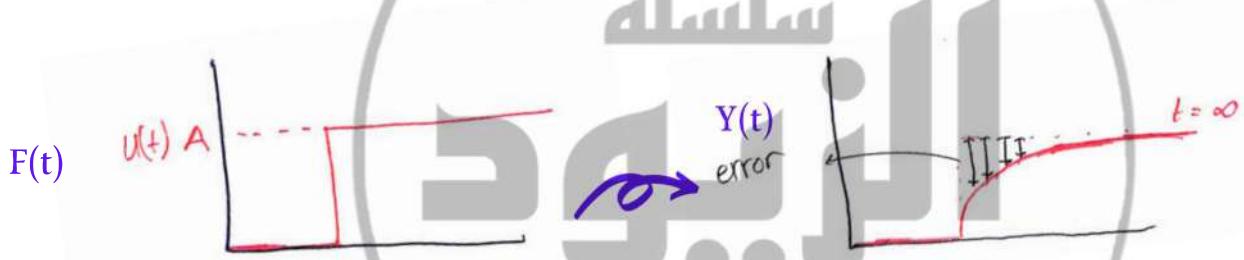


رح يصل نفس الشكل وبس ينضرب ب  $k$

طبعاً بالواقع ما عندي zero-order system بيرفكت مية بالمية يعطيوني إلا ما يكون عندي time constant لحد ما يستجيب ال



تمام هسا كيف يستجيب ال first order بال step function  
لاحظوا علماء الرياضيات انه بال step function الكيف رح يختلف عندي  
بال input عن ال output



ولاحظوا كمان انه صرح رح يتغير بس رح  
يثبت عند الأنفنتي او عند رقم كبير  
ويصير بشبه ال zero-order

تمام هسا لاقوا شكل الاقتران وبعدها حاولوا يحلوا المعادلة عشان يقدرو يعرفو  $y(t)$



$$y(t) = \underbrace{KA}_{\text{steady response}} + \underbrace{(y_0 - KA)}_{\text{initial condition}} e^{-\frac{t}{\tau}}$$

$$F(y) = A U(t)$$

$$(1) \quad \tau \dot{y} + y = KA U(t) = k F(t)$$

steady  
response

Transient  
response

بعد زمن طويل ال  
System  
رح يتصرف زي ال  
Zero-order

بعد فترة عشان ال  
System  
يتصرف زي ال  
Zero-order

وال input تتبع ال output

$$\tau \dot{y} + y = KAU(t) = KF(t)$$

with an arbitrary initial condition denoted by,  $y(0) = y_0$ . Solving for  $t \geq 0^+$  yields

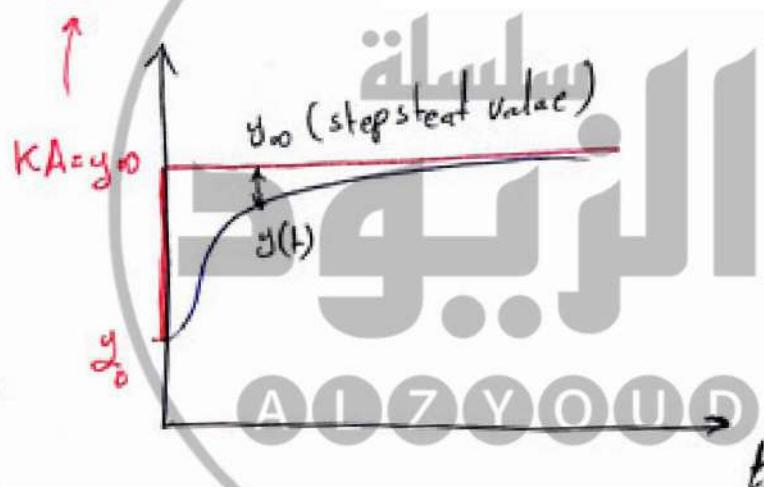
$$y(t) = \underbrace{KA}_{\text{Time response}} + \underbrace{(y_0 - KA)e^{-t/\tau}}_{\text{Transient response}}$$

The solution of the differential equation,  $T$ , is the time response (or simply the response) of the system.

حل المعادلة التفاضلية ،  $T$  ، هو استجابة للوقت (أو ببساطة استجابة) للنظام.

كتوضيح لفكرة ال error

لأنها  $\infty$  بعدين من  
ذاته  
كار 2nd order



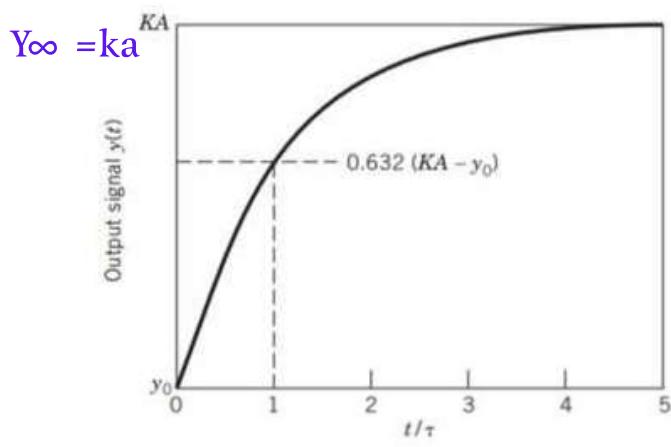
لاحظ بأنه كل ما يمر وقت  
رح يقل ال error

Suppose we rewrite the response Equation 3.5 in the form

$$\Gamma(t) = \frac{y(t) - y_\infty}{y_0 - y_\infty} = e^{-t/\tau}$$

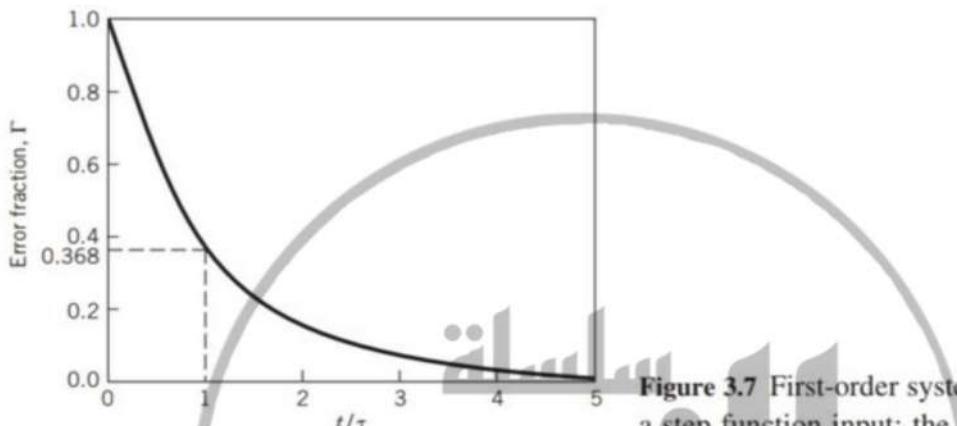
The term  $\Gamma(t)$  is called the *error fraction* of the output signal. Equation 3.6 is:

## توضيح لفكرة ال time constant



لاحظ انه بال x axis عندي  $t$  على تاو  
فمثلا عند الزمن 1 بتكون  $t=t\tau$

**Figure 3.6** First-order system time response to a step function input: the time response,  $y(t)$ .



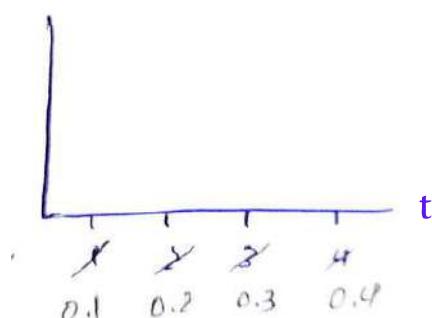
**Figure 3.7** First-order system time response to a step function input: the error fraction,  $\Gamma$ .

بعد مرور واحد second من time response في الرسمة الأولى رح يكون 0.632 إذن ال error كيف بحسبه

$$\text{المكمالة ببحكي } 1 - 0.632 = 0.368$$

الظاهو في الرسمة الثانية

حيينا قبل انه التاو بتعتمد عجهاز القياس طي لو الرسمة ال x axis بال  $t$   
مش  $t$  على تاو وحكالي انه التاو تاع الجهاز = 0.1



لاحظ انه ال a axis هو  $t$  بشطبة القيم وبرجع بعيهم حسب تاو  
لهي الكتاب مختصر القصة وفوراً حاطتها  $t$  على تاو

Table 3.1 First-Order System Response and Error Fraction

$t/\tau$	% Response	$\Gamma$	% Error
0	0.0	1.0	100.0
1	63.2	0.368	36.8
2	86.5	0.135	13.5
2.3	90.0	0.100	10.0
3	95.0	0.050	5.0
5	99.3	0.007	0.7
$\infty$	100.0	0.0	0.0

بـ ٥ وقتية كانه الـ Response

$$= (1 - 0) \rightarrow 1 \text{ error دار}$$

بـ ٦٣.٢ وقتية كانه الـ Response

$$0.368 = (1 - 63.2) \text{ error دار}$$

١٠٥ مكتبة

plot is equivalent to the transformation

$$\ln \Gamma = 2.3 \log \Gamma = -(1/\tau)t \quad (3.7)$$

which is of the linear form,  $Y = mX + B$  (where  $Y = \ln \Gamma$ ,  $m = -(1/\tau)$ ,  $X = t$ , and  $B = 0$  here). A linear curve fit through the data will provide a good estimate of the slope,  $m$ , of the resulting plot. From Equation 3.7, we see that  $m = -1/\tau$ , which yields the estimate for  $\tau$ .

الـ  $\tau$  اتبعت عجہاز وكلما كانت قلیلة الـ response يكون اسرع

الزمن 2.3 هو مهم النـ لأنـ تكون درجة الاستجابة عندـ 90 بالمـية والـ error فقط 10 بالمـية وهو حفـظ

The time required for a system to respond to a value that is 90% of the step input,  $\tau_{90\%}$ , is important and is called the rise time of the system.

يعد الوقت اللازم لاستعادة النظام لقيمة 90% من الخطوة الداخلية،  $\tau_{90\%}$ ، أمرًا مهمًا ويسمى وقت صعود النظام.

ممكن يسئلني بها الطريقة

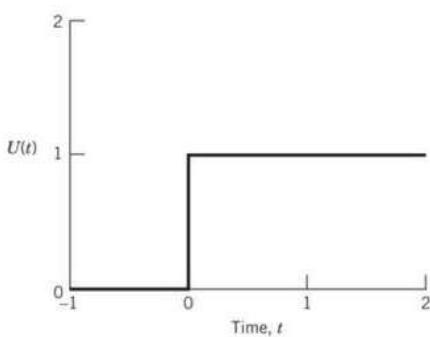
ALZOONUD

$$\tau_{90\%} = 2.3\tau$$

$$\tau_{95\%} = 3\tau$$

$$\tau_{Response} = \tau \rightarrow \tau$$

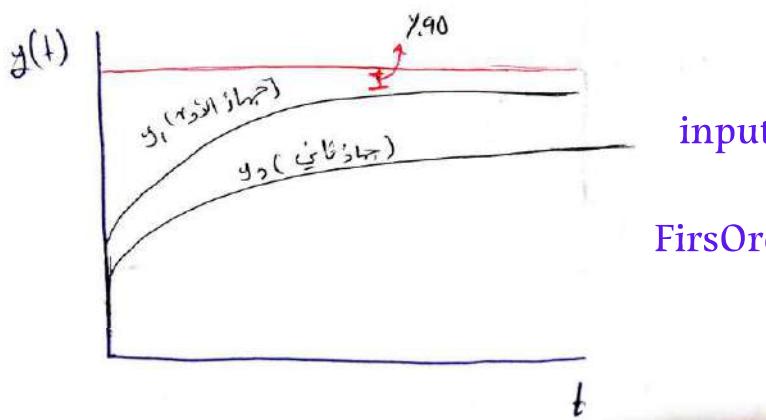
Chapter 3 Measurement System Behavior



معلومة للمراجعة: رسمة الـ step function

Figure 3.5 The unit step function,  $U(t)$ .

example :



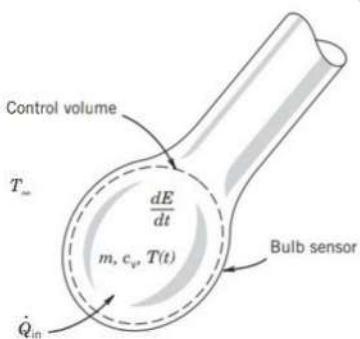
افرض عندي جهازين وهي رسماتهم  
فبدي اقارن بينهم

او الاشي بحدد نوع ال measurement وال  
من رسمة ال output

FirsOrder measurement انها عاوف  
step input هو input وال

السؤال من هنا صول المجهازين  
first order instrument كحاله على اسرع؟  
له ميزان الحرارة لادنه راح يوصل وقتها ليمشي  
السؤال الثاني هي المجهاز دلالة 2  
الحرارة ميوجده Time constant = (ادنه غالباً)  
First order دلالة 2 دلالة 2  
السؤال الثالث هي المجهاز دلالة 2  
لامنه اسرع ادنه التأثير على  
(thermometer)

## Examples



عملية انتقال الحرارة من ال  
للسائل الداخلي بتحتاج وقت معين (time constant)  
 فهي اكيد مهي zero-order  
FirsOrder وغالبا هي  
لما تقيس درجة حرارتكم الميزان بده وقت ليعطي انه ٣٧

Figure 3.9 Lumped parameter model of thermometer and its energy balance (Ex. 3.3).

Suppose a bulb thermometer originally indicating 20 C is suddenly exposed to a fluid temperature of 37 C. Develop a model that simulates the thermometer output response.

لنفترض أن ترمومتر لمبة يشير في الأصل إلى أن 20 درجة مئوية قد تعرضت فجأة لدرجة حرارة ماء تبلغ 37 درجة مئوية. طور نموذجاً يحاكي استجابة خرج مقياس الحرارة.

where

$m$  = mass of liquid within thermometer

$c_v$  = specific heat of liquid within thermometer

$h$  = convection heat transfer coefficient between bulb and environment

$A_s$  = thermometer surface area

The term  $hA_s$  controls the rate at which energy can be transferred between a fluid and a body; it is analogous to electrical conductance. By comparison with Equation 3.3,  $a_0 = hA_s$ ,  $a_1 = mc_v$ , and  $b_0 = hA_s$ . Rewriting for  $t \geq 0^+$  and simplifying yields

$$\frac{mc_v}{hA_s} \frac{dT(t)}{dt} + T(t) = T_\infty$$

From Equation 3.4, this implies that the time constant and static sensitivity are

$$\tau = \frac{mc_v}{hA_s} \quad K = \frac{hA_s}{hA_s} = 1$$

Direct comparison with Equation 3.5 yields this thermometer response:

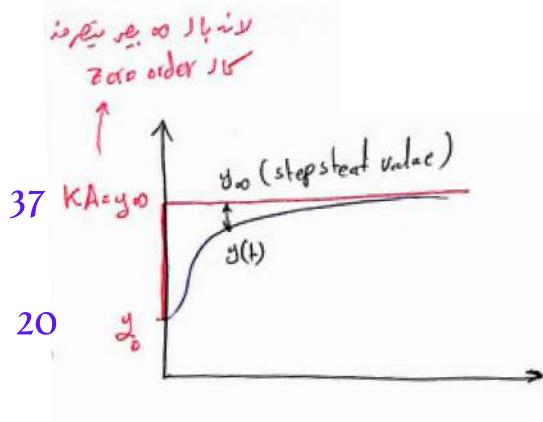
$$T(t) = T_\infty + [T(0) - T_\infty] e^{-t/\tau}$$

$$= 37 - 17e^{-t/\tau} \text{ [ } ^\circ \text{C}]$$



Also, it is significant that we found that the response of the temperature measurement system in this case depends on the environmental conditions of the measurement that control  $h$ , because the magnitude of  $h$  affects the magnitude of  $T$ .  
 أيًضاً، من المهم أننا وجدنا أن استجابة نظام قياس درجة الحرارة في هذه الحالة تعتمد على الظروف البيئية للقياس الذي يتحكم في  $h$ ، لأن حجم  $h$  يؤثر على حجم تأثير  $T$ .

تكرار لـ "فوق بأنه النظام يعتمد عوامل تأثير ومنها  $h$ "



هون انتقلت من 20 ل 37  
المعادلة تاع الرسمة هي حل المعادلة الي فوق

$$T(t) = T_\infty + [T(0) - T_\infty]e^{-t/\tau}$$

$$= 37 - 17e^{-t/\tau} \text{ [ } ^\circ \text{C}]$$

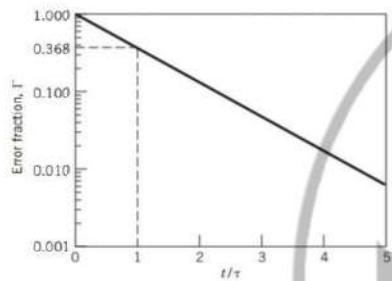


Figure 3.8 The error fraction played on semilog coordinates.

الشكل 3.8 جزء الخطأ الذي تم تشغيله على الإحداثيات شبه اللوغاريتمية.

خلصنا من الحالة الأولى لل FirstOrder وهي لما يكون ال

may be modeled using a first-order differential equation of the form

$$a_1 \dot{y} + a_0 y = F(t)$$

with  $\dot{y} = dy/dt$ . Dividing through by  $a_0$  gives

$$\tau y + y = KF(t)$$

بمعنى كنا نعوض بدل ال simple periodic function input  $f(t)$  ببعوض  $(f(t))$  هسا بدننا ننتقل للحالة الثانية الي بدل ال

وبشوف لو كانت ال  $y(t)$  شكلها  $\sin$  او  $\cos$  كيف تكون شكل ال

## امثلة مخططين بس الدكتور ما شرح عنهم

### Example 3.4

For the thermometer in Example 3.3 subjected to a step change in input, calculate the 90% rise time in terms of  $t/\tau$ .

**KNOWN** Same as Example 3.3

**ASSUMPTIONS** Same as Example 3.3

**FIND** 90% response time in terms of  $t/\tau$

**SOLUTION** The percent response of the system is given by  $(1 - \Gamma) \times 100$  with the error fraction,  $\Gamma$ , defined by Equation 3.6. From Equation 3.5, we note that at  $t = \tau$ , the thermometer will indicate  $T(t) = 30.75^\circ\text{C}$ , which represents only 63.2% of the step change from  $20^\circ$  to  $37^\circ\text{C}$ . The 90% rise time represents the time required for  $\Gamma$  to drop to a value of 0.10. Then

$$\Gamma = 0.10 = e^{-t/\tau}$$

or  $t/\tau = 2.3$ .

**COMMENT** In general, a time equivalent to  $2.3\tau$  is required to achieve 90% of the applied step input for a first-order system.

### Example 3.5

A particular thermometer is subjected to a step change, such as in Example 3.3, in an experimental exercise to determine its time constant. The temperature data are recorded with time and presented in Figure 3.10. Determine the time constant for this thermometer. In the experiment, the heat transfer coefficient,  $h$ , is estimated to be  $6 \text{ W/m}^2 \cdot ^\circ\text{C}$  from engineering handbook correlations.

**KNOWN** Data of Figure 3.10

$$h = 6 \text{ W/m}^2 \cdot ^\circ\text{C}$$

**ASSUMPTIONS** First-order behavior using the model of Example 3.3, constant properties

**FIND**  $\tau$

**SOLUTION** According to Equation 3.7, the time constant should be the negative reciprocal of the slope of a line drawn through the data of Figure 3.10. Aside from the first few data points, the data appear to follow a linear trend, indicating a nearly first-order behavior and validating our model assumption. The data is fit to the first-order equation<sup>2</sup>

$$2.3 \log \Gamma = (-0.194)t + 0.00064$$

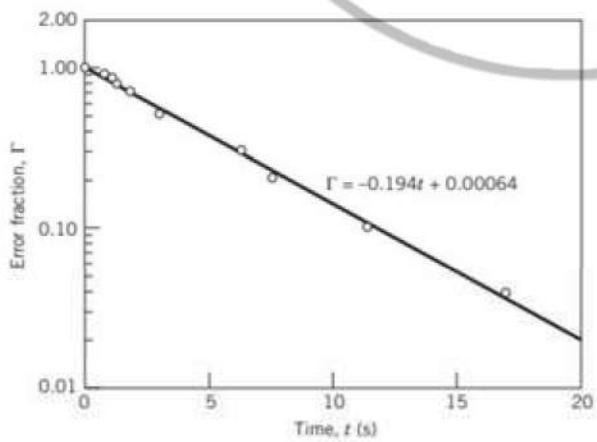


Figure 3.10 Temperature-time history of Example 3.5.

With  $m = -0.194 = -1/\tau$ , the time constant is calculated as  $\tau = 5.15$  seconds.

**COMMENT** If the experimental data were to deviate significantly from first-order behavior, this would be a clue that either our assumptions do not fit the real-problem physics or that the test conduct has control or execution problems.

may be modeled using a first-order differential equation of the form

$$a_1 \dot{y} + a_0 y = F(t)$$

with  $\dot{y} = dy/dt$ . Dividing through by  $a_0$  gives

$$\tau y + y = KF(t)$$

بمعنى كنا نعوض بدل ال  $f(t) \leftarrow$  step input

هذا بدلنا ننتقل للحالة الثانية التي بدل ال  $(f(t)) \leftarrow$  بعض simple periodic function input

وبشوف لو كانت ال  $f(t)$  شكلها sin او cos كيف تكون شكل ال  $y(t)$

measuring system to which an input of the form of a simple periodic function,  $F(t) = A \sin \omega t$ , is applied for  $t \geq 0^+$ :

هي مدخلات ال  $f(t)$ . هون ال

$$F(t) = A \sin \omega t$$

بعوضها في معادلة ال

$$\tau \dot{y} + y = KA \sin \omega t$$

حل المعادلة التفاضلية التي فوق

$$F(t) = A \sin \omega t$$

Amplitude

Frequency

angular frequency  
circular frequency  
(rad/s)

this differential equation yields the measurement system output signal, that is, the time response to the applied input,  $y(t)$ :

$$y(t) = Ce^{-t/\tau} + \frac{KA}{\sqrt{1 + (\omega\tau)^2}} \sin(\omega t - \tan^{-1} \omega\tau) \quad (3.8)$$

where the value for  $C$  depends on the initial conditions.

$\sin$  ← periodic

$$\tau \dot{y} + y = KA \sin \omega t$$

$$y(t) = C e^{-t/\tau} + \frac{KA}{\sqrt{1 + (\omega\tau)^2}} \sin(\omega t - \tan^{-1} \omega\tau)$$

Transient response

يعبر عن ما في الزمن

و يتغير مع الزمن

0.98

هذا فوق حكينا انه  $A \sin(\omega t)$  هو ال input فمن وين اجا كل الباقي

$$\frac{K}{\sqrt{1 + \omega^2 \tau^2}}$$

أحجامنا العاكس ( factor )

كمان ضغطنا عاكس

الرقم الي راح يطلع  
scaling of Amplitude (A)

إلي يتم فيه وهو  
steady state response

عرفنا ال periodic واخذنا مدخلاته وعوضناه بال FirsOrder هسا اكيد رح يخطرلك طب كيف حتصير FirsOrder output لـ



لقد الرسمة ولكن زيجه كنه ضربتها خـ k  
ولكن دع يكونه في طبعـ بين الـ input  
والـ output

هـاي هي رسمـة الـ periodic

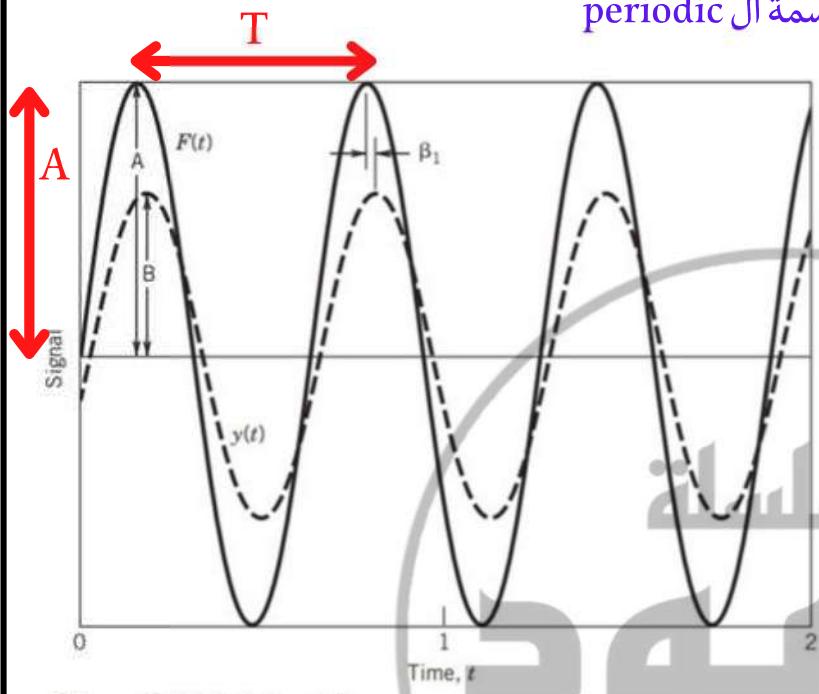


Figure 3.11 Relationship between a sinusoidal input and output: amplitude, frequency, and time delay.

هـسا نتفق عـاشـي الخط المتصل هو الـ inputs

$$F(t) = A \sin \omega t$$

وـالـoutput هو الـ

$$y(t) = Ce^{-t/\tau} + \frac{KA}{\sqrt{1 + (\omega\tau)^2}} \sin(\omega t - \tan^{-1}\omega\tau)$$

لو حبيت ابسطـها بـأنـه اشـيلـ الـ factors

$$y(t) = B \sin(\omega t + \phi)$$

$$\text{out~put~Amplitude} \quad B = \frac{\omega}{\sqrt{1 + (\omega\tau)^2}}$$

لـو دـحلـ الرـسمـة وـشـو عـملـ الـ periodic بالـ

ALZYOUD

(1) اـطـابـقـه فـنـ كلـ الـ input بــنـ يـهـ (2) صـارـبـتـ الـ output (B) زـيـ gabـهـ مـضـارـهـ السـكـلـ مـقـطـعـهـ (B) سـهـ تـسـاـبـقـهـ الزـنـهـ Response scaling

$$A > B \rightarrow \text{input}$$

(3) لا حـظـانـه المـبـدـأـهـ فيـ الـ input  
منـ فـنـ بـنـيـةـ الرـسـمـةـ فيـ الـ output

$$\tau = \frac{1}{f} \text{ (sec)}$$

$$(\text{Circular frequency}) \quad \omega = 2\pi f \text{ (rad/s)}$$

$$f = \frac{\omega}{2\pi} \text{ (Hz)}$$

applied is known as the frequency response of the system. The frequency affects the magnitude of amplitude  $B$  and also can bring about a time delay. This time delay,  $\beta_1$ , is seen in the phase shift,  $\Phi(\omega)$ , of the steady response. For a phase shift given in radians, the time delay in units of time is

$$\beta_1 = \frac{\Phi}{\omega}$$

that is, we can write

$$\sin(\omega t + \Phi) = \sin\left[\omega\left(t + \frac{\Phi}{\omega}\right)\right] = \sin[\omega(t + \beta_1)]$$

## First order and Zero order

---

- Signal types and what's difference between them ? ( Page 1+4+5+6)
- Types of differential equations and measurement operations ( page 2 )
- output depends on (page 3 + 5)
- Two types of input (page 4 )
- Each measurement system responds differently to ( page 4 )
- Definition of zero order system ( page 8 )
- Potensiometer ( 8+9+11 )
- Reasons for not having an ideal system ( 9+10 )
- Properties and questions of Zero order system ( 12 )
- Advantages and disadvantages of Zero order system ( 14+15 )
- Bandwidth of Zero order ( 16 )
- Modeling guide us / the exact relationship ( 17 )
- Pencil style pressure gage ( 17+18+19)

## First order system

---

- First order ( 20 and so on )
- Time constant ( 21 ) / first order system ( 20 – and so on )
- Step input ( 24 )
- Properties of Time constant 24-30
- Friction error and time / response and t 26-30
- Thermometer 31-32+34
- Properties of first order ( 33 )

- Periodic ( 38 and so on )
- Phase shift and magnitude ( 43+45+46 )
- Different between sin and step (44)
- Examples of 1<sup>st</sup> order and properties (47 )
- Problems 49 (and so on )

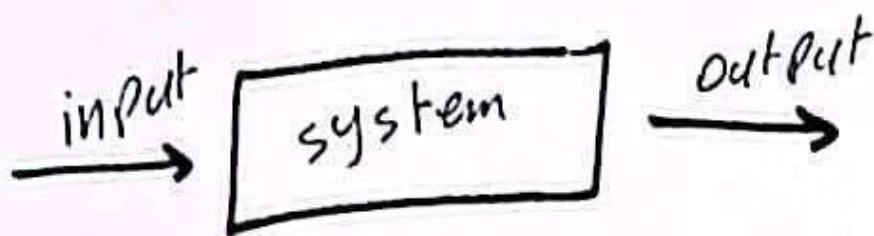
## Second order

---

- Examples of 2<sup>nd</sup> order (62+63)
- الصورة العامة لمعادلة السكند والتي يتم من خلالها حساب قيمة natural frequency ( 63 ) صفة رقم and damping
- Second order ( Step input ) \_ 64
- Three cases in step input in 2<sup>nd</sup> order ( 64-67 )
- Definition of ringing frequency ( 64 )
- Properties of under damped graph (64)
- Definition of damping (68 )
- Select the step input in second order \_ 71
- Problems of under and critical (71 )
- Rise and settling (71)
- Ringing frequency and period of ringing 74
- اخراج القيم من الصيغة العامة لل input 77+76+75 صفة رقم
- تأثير damping Ratio and ringing frequency 80 صفة
- كعلاقة عامة 81 Time response
- كيف تقوم بحساب rise and settling time 85-82
- Simple periodic ( 86 and so on )
- Graph of magnitude and Bd 88-93
- Phase shift 94
- Multiple and coupled 95
- Problems 96 and so on

• قانون M و Ø صفة 86

# Ch3: Measurement system behavior



\*Two Types for signal:

- ① static: not vary with time
- ② dynamic: vary with time

<u>dynamic</u>	<u>static</u>
· amplitude	· magnitude
· frequency	
· general waveform information	

## \* General model:

$n \rightarrow \infty$  order

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \dots + a_1 \frac{dy}{dt} + \underline{a_0 y} = b_m \frac{d^m x}{dt^m} + \dots + b_1 \frac{dx}{dt} + \underline{b_0 x}$$

constant (physical parameters)

$y$ : output

$x$ : input

$m \leq n$

general form second order

$$n=2, m=1$$

$$a_2 \frac{d^2 y}{dt^2} + a_1 \frac{dy}{dt} + \underline{a_0 y} = b_1 \frac{dx}{dt} + \underline{b_0 x}$$

## \* zero order:

$$\underline{a_0 y} = \underline{b_0 x}$$

output      input

## \* first order:

$$a_1 \frac{dy}{dt} + \underline{a_0 y} = \underline{b_0 x}$$

## \* second order:

$$a_2 \frac{d^2 y}{dt^2} + a_1 \frac{dy}{dt} + \underline{a_0 y} = b_1 \frac{dx}{dt} + \underline{b_0 x}$$

\* output of a measurement system:

dependent:  $\perp$  Type of system  
 $\approx$  system respons.

\* General model:

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \dots + a_1 \frac{dy}{dt} + a_0 y = b_m \frac{d^m x}{dt^m} + \dots + b_1 \frac{dx}{dt} + b_0 x$$

n: order of system

b<sub>m</sub>

y: output

x: input

m ≤ n

constant (physical parameters)

general form second order

$$n=2, m=1$$

$$a_2 \frac{d^2 y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = b_1 \frac{dx}{dt} + b_0 x$$

# Ch3:measurement system behavior

mention

explain

Each measurement system responds differently to different types of input signals and to the dynamic content within these signals. So a particular system may not be suitable for measuring certain signals or at least portions of some signals.

يستجيب كل نظام قياس بشكل مختلف لأنواع مختلفة من إشارات الإدخال والمحتمى الديناميكى داخل هذه الإشارات. لذلك قد لا يكون نظام معين مناسباً لقياس إشارات معينة أو على الأقل أجزاء من بعض الإشارات.

mention

As pointed out in Chapter 2, all input and output signals can be broadly classified as being static , dynamic , or some combination of the two . For a static signal , only the signal magnitude is needed to reconstruct the input signal based on the indicated output signal

كما هو موضح في الفصل 2 ، يمكن تصنيف جميع إشارات الإدخال والإخراج على نطاق واسع على أنها ثابتة أو ديناميكية أو مزيج من الاثنين. بالنسبة للإشارة الثابتة ، لا يلزم سوى حجم الإشارة لإعادة بناء إشارة الإدخال بناءً على إشارة الخرج المشار إليها

أخذنا في الشابتر الي قبل انواع ال signal وانواع ال input

\* Vibration signals vary in amplitude and time , and thus are a dynamic input signal to the measuring instrument .

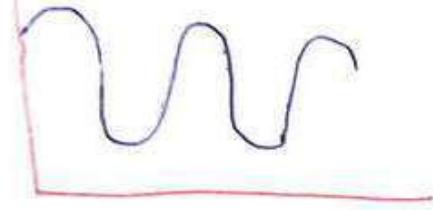
تحتفل إشارات الاهتزاز في السعة والوقت ، وبالتالي فهي إشارة دخل ديناميكية لأداة القياس.

## 2 types of input

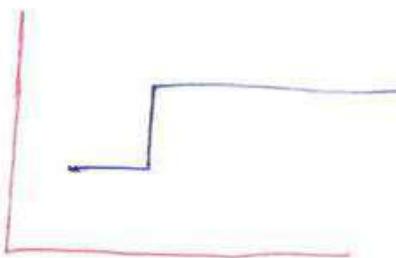
نركز بها الشابتر على نوعين من signal

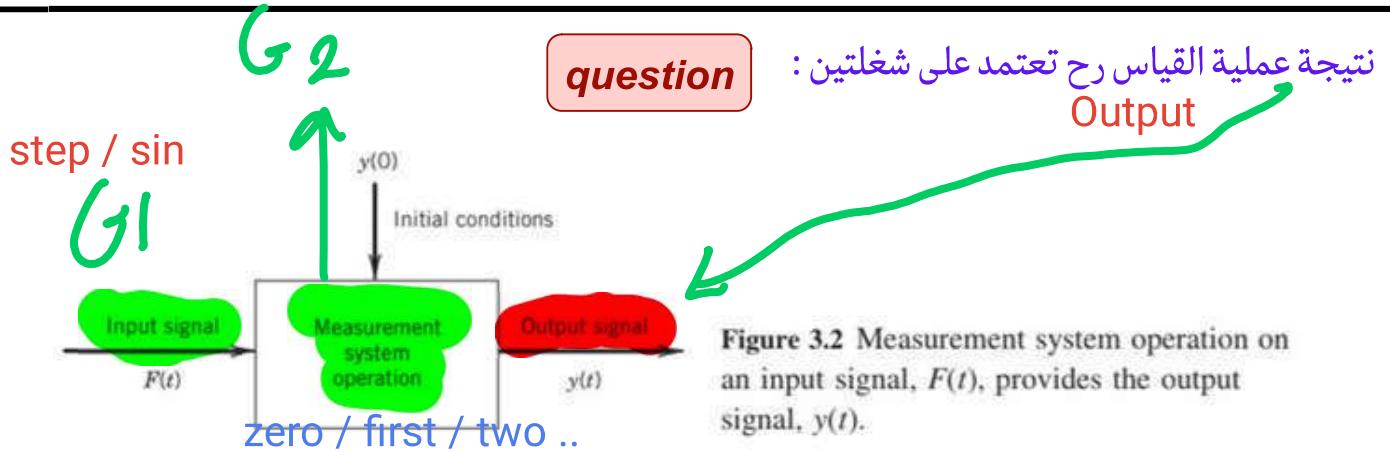


Periodic  
or  
Sin input



Step  
input





الى هي ال input signal كمثال لو كانت step periodic

measurement system operation هي مكونات نظام القياس وال

$$G \text{ results} = G_1 * G_2$$

mention

explain

## Dynamic Measurements

Dynamic Measurements For dynamic signals , signal amplitude , frequency and general waveform information is needed to reconstruct the input signal . Because dynamic signals vary with time , the measurement system must be able to respond fast enough to keep up with the input signal .

القياسات الديناميكية ضرورية للإشارات الديناميكية ، وسعة الإشارة ، والتردد ، ومعلومات شكل الموجة العامة لإعادة بناء إشارة الدخل. نظرًا لاختلاف الإشارات الديناميكية مع مرور الوقت ، يجب أن يكون نظام القياس قادرًا على الاستجابة بسرعة كافية لمواكبة إشارة الإدخال.

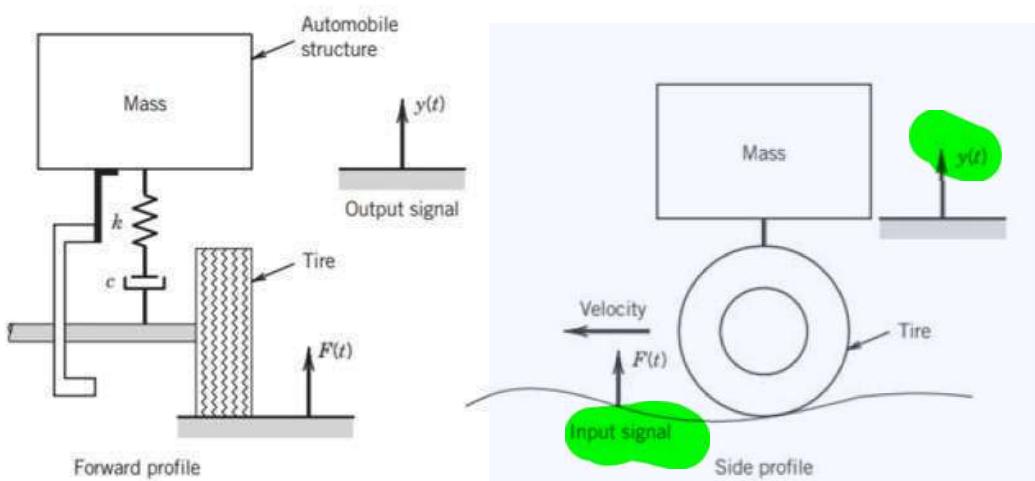


Figure 3.1 Lumped parameter model of an automobile suspension showing input and output signals.

لنفرض نظام داخل سيارة مممن تكون مرتاح او مش مرتاح حسب شغلتين  $y(t)$  وال displacement وهم اشياء بعتمدوا على ال signal (الطريق)

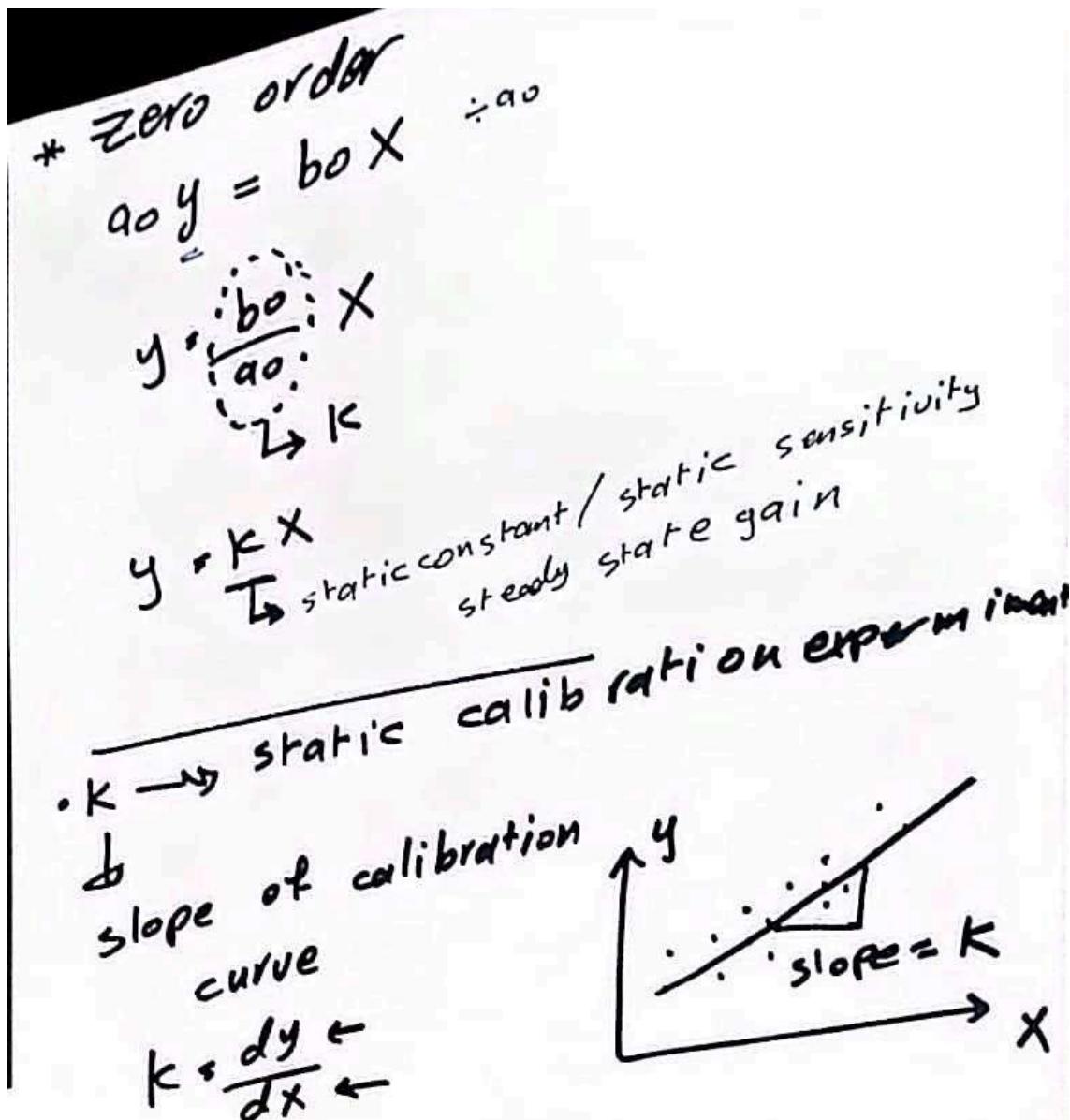
كل ما تكون  $y(t)$  ثابتة رح تكون قيمة ال  $y(t)$  ثابتة وتكون انت مرتاح اكتر

هسا حكينا ال signal مهمه لأنها هي الطريق وهي العامل الأول

العامل الثاني جهاز القياس الي هو هون ال automobile وال damper تاعها

For a static signal , only the signal magnitude is needed to reconstruct the input signal based on the indicated output sig

Dynamic Measurements For dynamic signals , signal amplitude , frequency , and general waveform information is needed to recon the input signal . Because dynamic signals vary with time , the measurement system must respond fast enough to keep up with the input signal .



فهون ممكنا تكون الطريق سين والسيارة كويسة فما تحس بالطريق  
وممكنا السيارة(جهاز) كويسة والطريق سين فبرضو تحسش بالطريق(signal)

وممكنا السيارة قديمة والطريق مليان تعرجات  
 فهو جهاز القياس  $y(t)$  رح يعطيني انه عندي مشكلة بال  $G_1$  وال  $G_2$

هاد الموضوع هو موازي للموضوع تاع الكنترول الي اسمه transfer function

$$\frac{\text{Laplace output}}{\text{Laplace input}}$$

فال output بيعتمد على input

انواع ال measurement systems

هس ارح نحاول نبدا نعمل حسب ال  $G_1$  وال  $G_2$  model measurement system

Consider the following general model of a measurement system, which consists of an nth-order linear ordinary differential equation in terms of a general output signal, represented by variable  $y(t)$ , and subject to a general input signal,

represented by the forcing function,  $F()$ :

ضع في اعتبارك النموذج العام التالي لنظام القياس ، والذي يتكون من معادلة تفاضلية خطية عادية من الدرجة التاسعة من حيث إشارة خرج عامة ، ممثلة بالمتغير  $t$  ( $y$ ) ، وتتعرض لإشارة إدخال عامة ممثلة بوظيفة التأثير  $F()$ :

هسابنكتب المعادلة التفاضلية للنظام وبنفرض انها linear system  
ومارح نتعلم كيفية حلها لأنه حلها بده كورسات كاملة

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \dots + a_1 \frac{dy}{dt} + a_0 y = F(t)$$

where

$f(t) = b_m x$

$$F(t) = b_m \frac{d^m x}{dt^m} + b_{m-1} \frac{d^{m-1} x}{dt^{m-1}} + \dots + b_1 \frac{dx}{dt} + b_0 x \quad m \leq n$$

هساهون المعادلة التفاضلية تتكون من جهتين جهة ال input الي هي ال  $F(t)$  والجهة الثانية هي عبارة عن معاملات ( $a$ ) مضروبين بال input الي هو  $y$  ومشتقاته

$F(t)$  هي عبارة عن Sumation لكل ال Input

وكبوالها معادلة تحت ولكن بالبحث تاعنا في هاد الكورس وجدوا انه فش داعي لكل هدول الا للحد الأخير

هون لاحظ انه المعادلات التفاضلية كلها بال  $t$  domain لو بدي احوالها لل  $s$  domain لازم اخذلها لابلس

(Physical Parameters) خصائص منظومة القياس  $a, a_1, a_n, \dots, b$

Comparing this to

the general form for a second-order equation ( $n=2$ ;  $m=1$ ) from Equation 3.1,

$$a^2 \frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = b_1 \frac{dx}{dt} + b_0 x$$

### 3.3 SPECIAL CASES OF THE GENERAL SYSTEM MODEL

#### Zero-Order Systems

definition

لما اخذ بس ال اول حد من ال input بهاي الحالة تكون zero order system

The simplest model of a measurement systems and one used with static signals is the zero - order system model . This is represented by the zero - order differential equation :

أبسط نموذج لأنظمة القياس وآخر يستخدم مع الإشارات الثابتة هو نموذج النظام الصفرى. يتم تمثيل ذلك بالمعادلة التفاضلية ذات الترتيب الصفرى:

important

$$a_0 y = F(t) \rightarrow a_0 y = b_0 x \rightarrow y = \left( \frac{b_0}{a_0} \right) x \rightarrow y = K x$$

هاد الأشي بيذكرني بأشيء مهم  
لما اقسم المعادلة على  $a_0$ .

Static sensitivity  $\rightarrow y(t) = KF(t)$

K is called the static sensitivity or steady gain of the system .

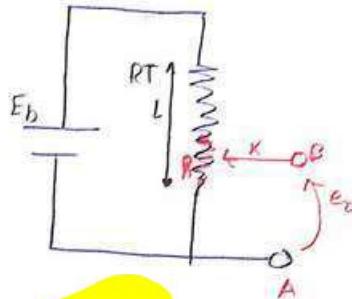
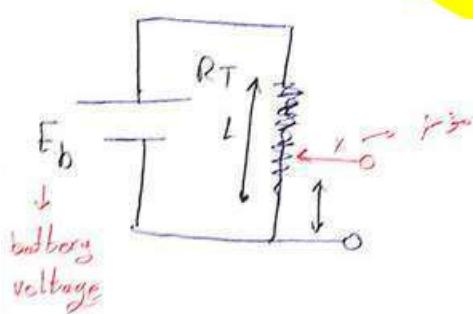
يسمى K بالحساسية الساكنة أو الكسب الثابت للنظام.

The static sensitivity is found from the static calibration of the measurement system . It is the slope of the calibration curve ,  $K = dy/dx$

تم العثور على الحساسية الساكنة من المعايرة الثابتة لنظام القياس. إنه منحدر منحنى المعايرة

example :

Example: Potentiometer

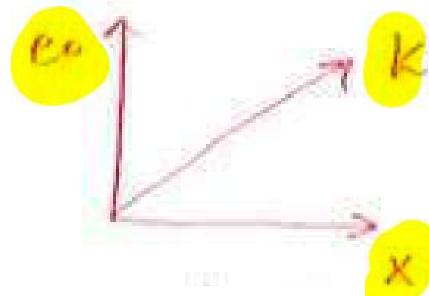


X : input  
e\_o : output

الـ  $e_o$  هو لو شبكت الدارة بفولتيميترا عشان اقيس فرق الجهد بين النقطة A والنقطة B وهون بلاقي علاقه بينهم

$$e_o = \frac{X}{L} \cdot E_b$$

لو فكرت ارسم العلاقة بين ال input وال output رح تكون العلاقة linear



وال zero order system بظوري سيطرة كاملة فأي تغير في ال input يتبعه تغير في ال output

لو سئلن عن قيمة ال K هون كيف بوجدها

$$\text{Output} = \text{Input}$$

static sensitivity میں چلے

## *important*

$$c_o = \frac{F_d}{k}$$

$$e_0 = k_s x$$

$$K = \frac{E_B}{L}$$

zero order  
( pointsmeter )  
is perfect or  
ideal

### **question**

هسا في هاد المثال النظام **ideal** او **perfect** لكن هل فعلاً في الحقيقة يوجد نظام **ideal**؟  
الجواب طبعاً لا هون افترضنا كثير افتراضات لطبع **ideal** منها

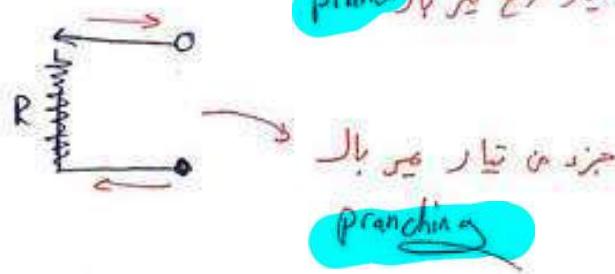
## **explain**

$$E_b = R_{\text{Total}} = L \quad \leftarrow \quad \text{اولہ اسٹر انٹر فٹ} \quad (1)$$

$$(R/R_T) \cdot E_h \leftarrow R \text{ لـ توحيد جزء من الـ المكـبة} \quad (1)$$

صهایی الالت فرماتا ال تیار الی هر فیال  $E_D$  مفت بمر بار R ولن هاد فرم

لأنه في جزء من التيار (ع) يمر بالـ



٣) ضياء محناء انه جزء من ال **Voltmeter** و محدد **الاخير** **power** رجع سهيل ال

رجع سهيل كونه العلاقة **linear** متناسبة  $(100/100)$

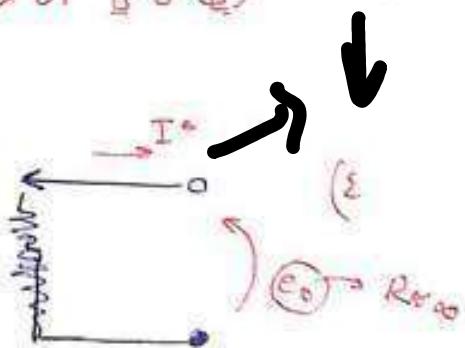
زيه كافى ان افترضنا لكن **is بالحقيقة**  $2100\Omega$

**perfect and ideal**

الاخير اي يمكن تقرير النظام **perfect** وكذا **is العلاقة**

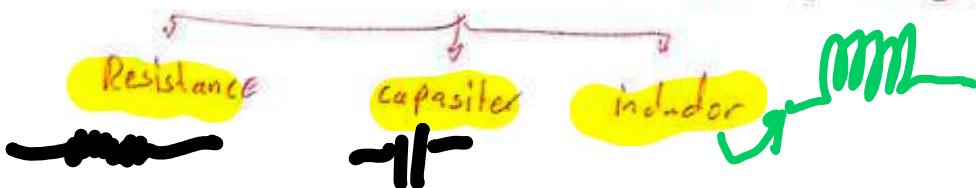
**linear**

انه ياخذ التيار **غير مفروض** و انتها تكونه مقاومة المعاو  
القياس  $\approx 20 \Omega$  (المية جداً)



**ideal** دل مانطبقة صاد المزط يكاد **is العلاقة**  
( يعني ملائمة مفروض )

٤) بالطبع اخذتوا انه في مني **Electric elements**



صياغ امثال افترضته انه **Resistor**  $(100/100)$  دعافته لا



او كانت برصو **inductor** او **capacitor**



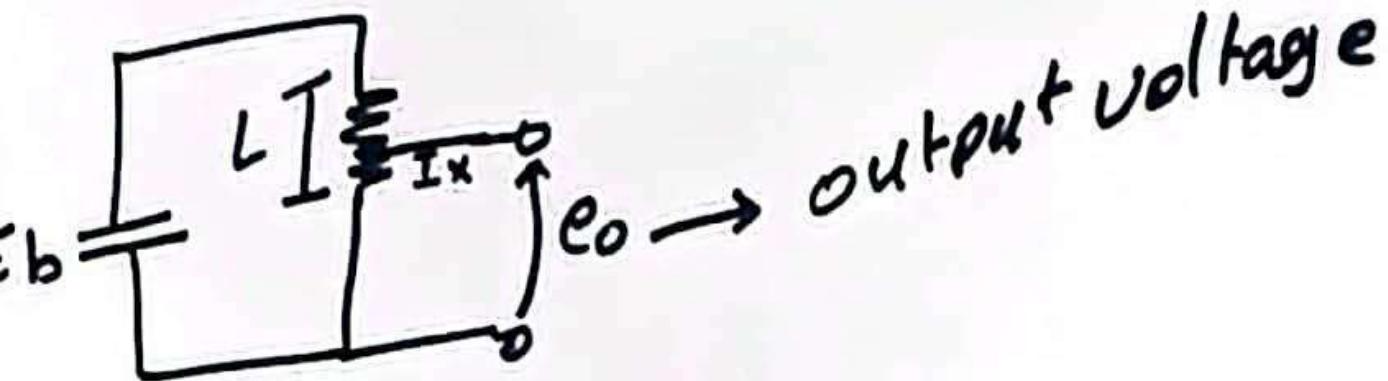
٥) الا **لا** مسوجدة **is** **zero** العلامة

مكان تغير **first** او **second**  
(نحو النظام)

٦) دافرضا اريفا انه اهدى سر **حجمه صغير** مابعد **اكله** **ما اكلنا** **الوزنة**

ناره ما اكلنا **اسرة** **واسع** **غير** **سرقة**

\* Example : Potentiometer  
displacement measuring



$$\frac{e_o}{x_i} = \frac{E_b}{L}$$

$$e_o = \frac{E_b}{L} \cdot x_i$$

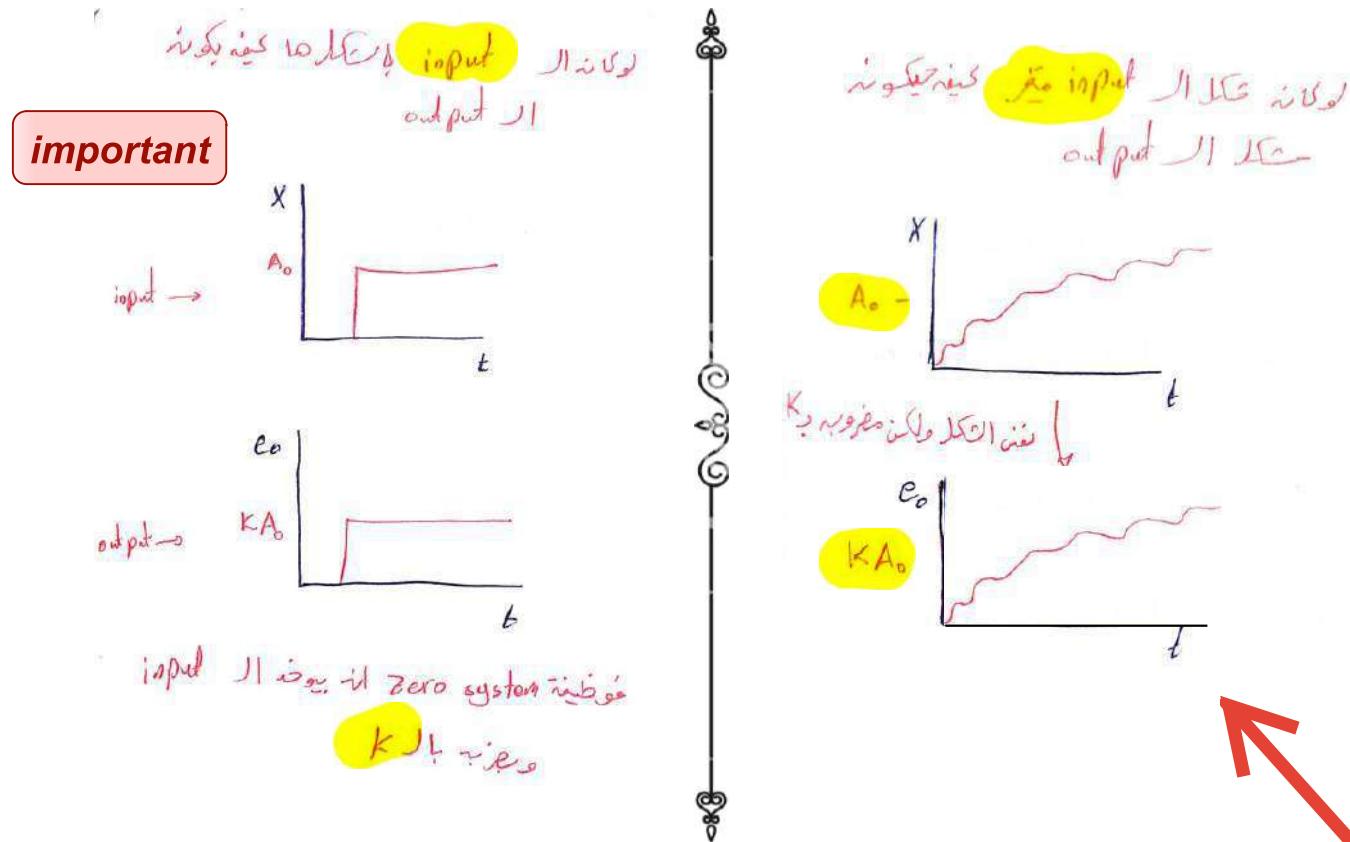
$$e_o = K \frac{x}{l} \text{ input}$$

static sensitivity

Potensiometer

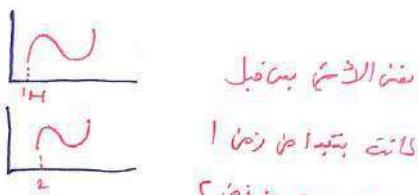
الهدف من كل هاد انه نفهم انه ما في ideal system مية بالمية فعشان هيكل بحاول عقدر الإمكان اقل ممن كل الأخطاء الي ممكن تبعد النظم عن ال ideal وما يكون ال input يعطي output صحيح يعني نظام بدون اخطاء والمؤثر الوحيد على ال input هو ال output Ideal system

## لفهم فكرة السيطرة تابع الـ zero-order



## اسئلة على رسمات ال zero-order

- $\therefore$  Time lag / time phase is 2512 ms



perfect system معايير دكتور في المكتبة حلول  
عالية لـ تكنولوجيا موجودة

نظام بدون اخطاء idea

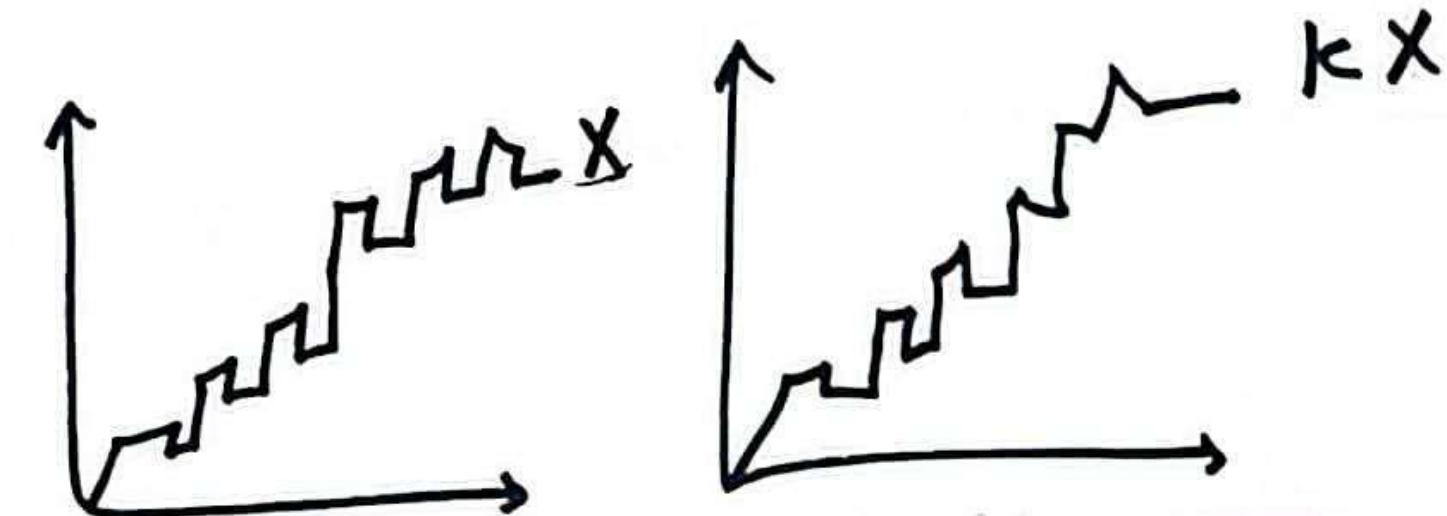
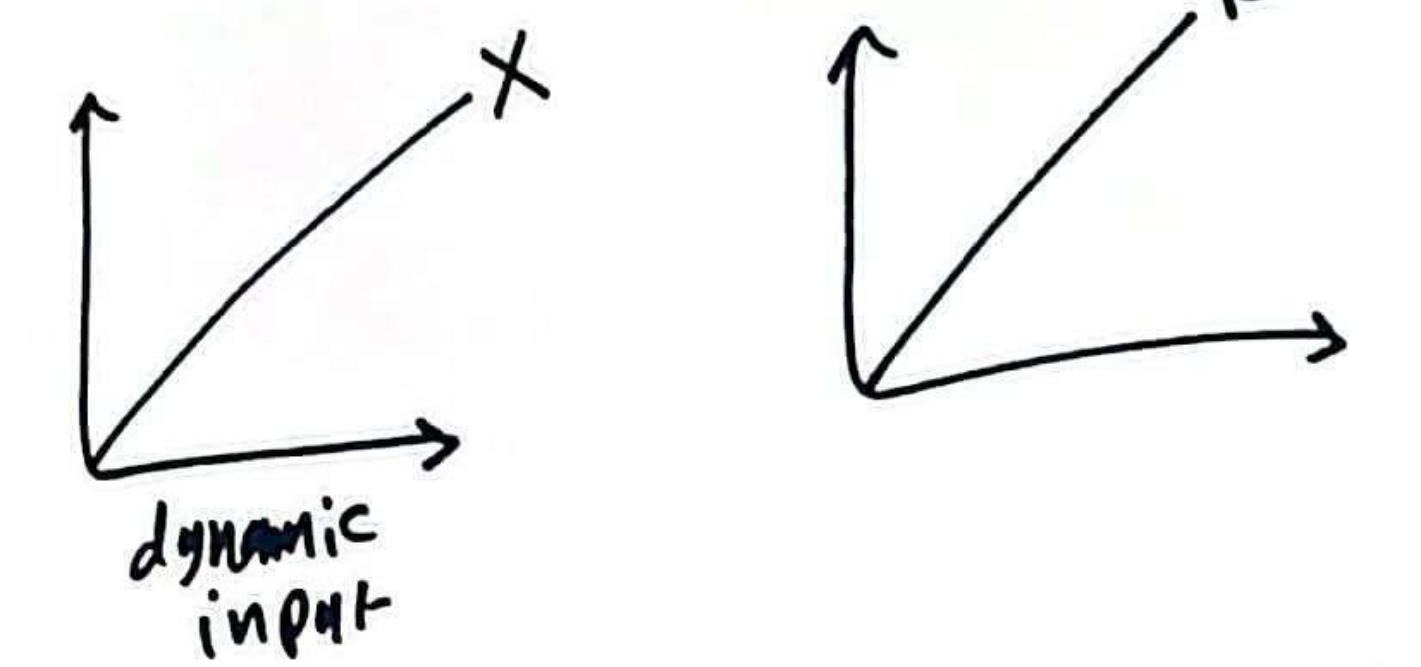
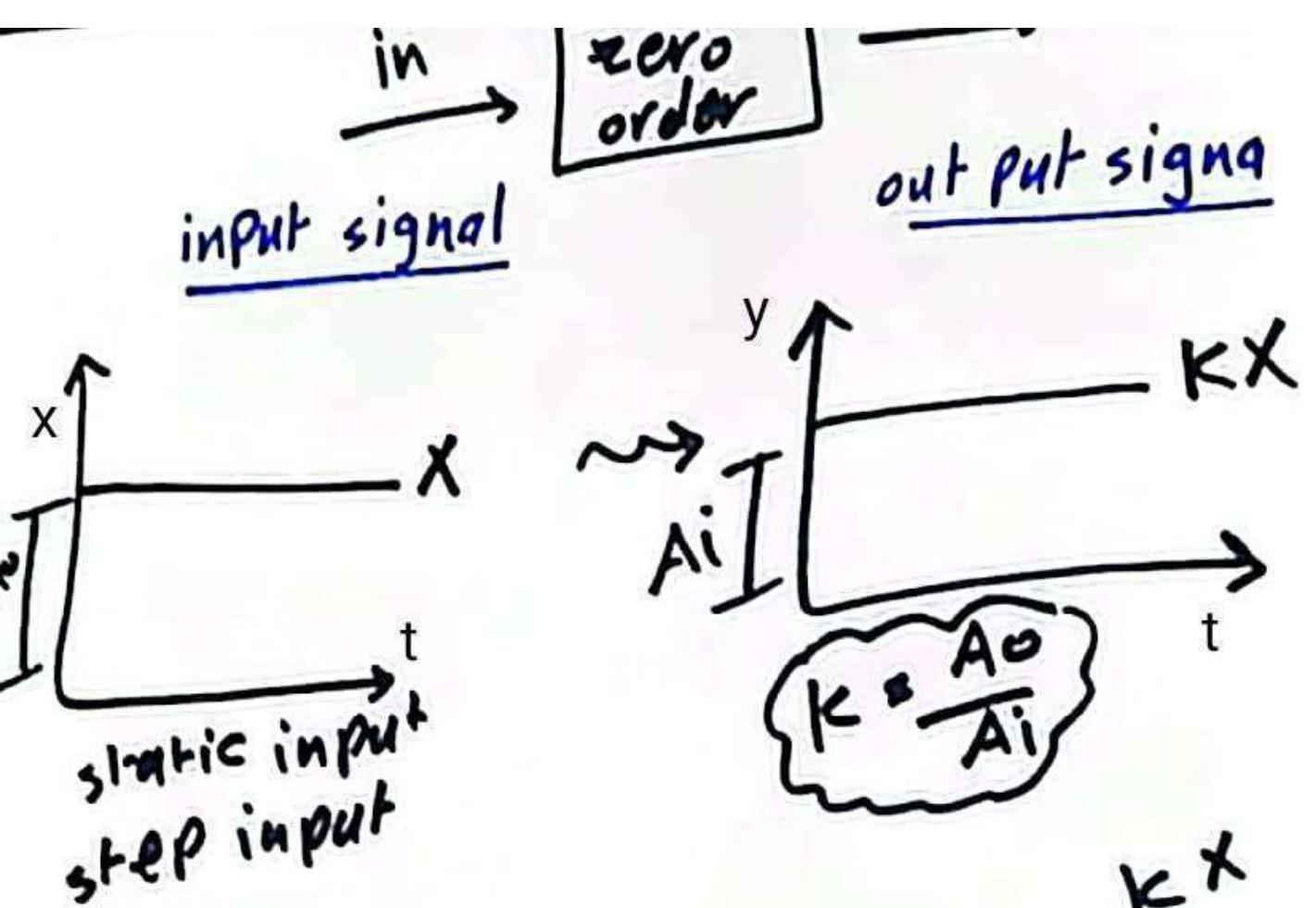
نظام بتأثر فيه الـ *perfec*  
ـ *input* بالـ *Out*

- ✓ yes ideal ✓? ideal instrument  $\rightarrow$   $\rightarrow$
  - ✓ perfect ✓ ? Dynamic response  $\rightarrow$   $\rightarrow$   
(Output  $\downarrow$   $\leftarrow$   $\leftarrow$   $\leftarrow$ , input  $\uparrow$   $\uparrow$   $\uparrow$ )

time lag / phase shift  $\rightarrow$  هل يوجد  
 $\times$   $\times$  output to input بين الـ  
output هنا يتأثر بما  
يحدث في input والـ

**بعض الحال لترسيم الفكرة الرايكور كما يلي (أحياناً بغير ترتيب Time lag بعد مثرة منه)**

$\times$   $\times$  distortion ملخص (عمل غير العدل قبل وبعد)

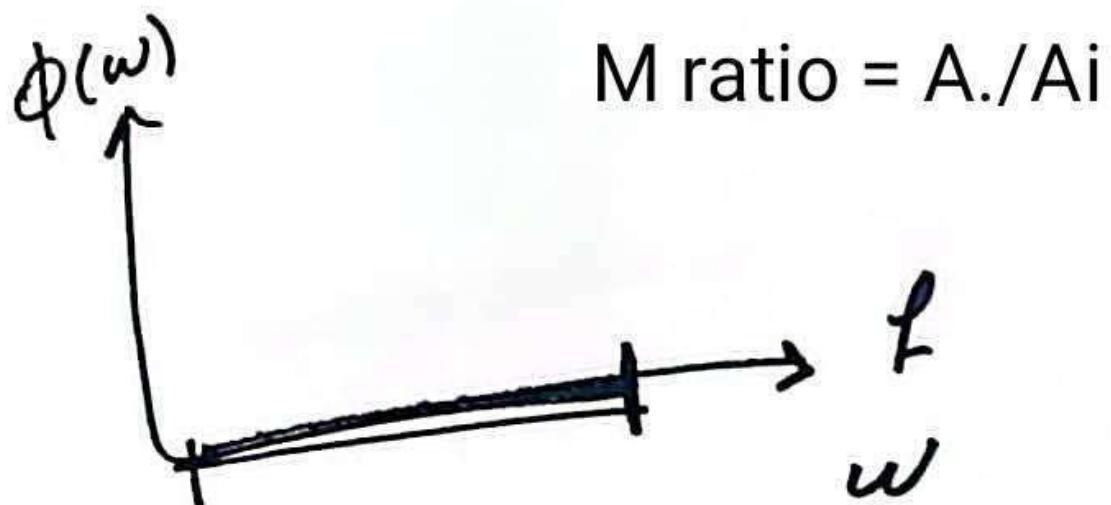
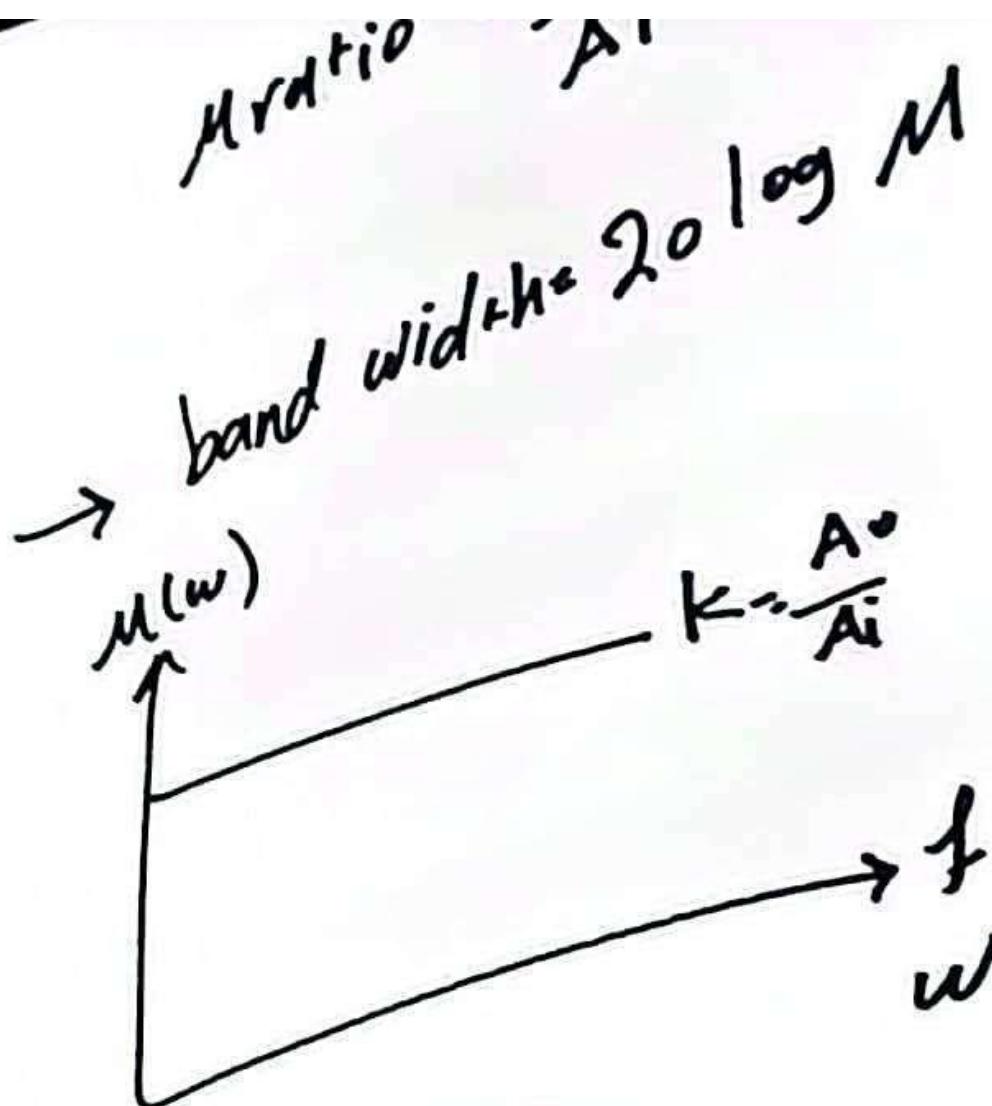


## advantages Z-0

1. ideal system
2. dynamic response perfect
3. there is no time lag in signal
4. there's no phase shift
5. output like input  $B = kA$
6. تكبير او تصغير الاشارة ما يأثر
7. infinite bandwidth and constant M Ratio

disadvantages for this system :

1. resistance isn't pure
2. friction and internal energy or force is almost zero
3. E. need a high energy to exit the system



*modeling guide us :*

Fortunately , many measurement systems can be modeled by zero- , first- , or second - order linear . ordinary differential equations . More complex systems can usually be simplified to these lower orders . Our intention here is to attempt to understand how systems behave and how such response is closely related to the design features of a measurement system ; it is not to simulate the exact system behavior . The exact input - output relationship is found from calibration . But modeling guides us in choosing specific instruments and measuring methods by predicting system response to signals , and in determining the type , range , and specifics of calibration .

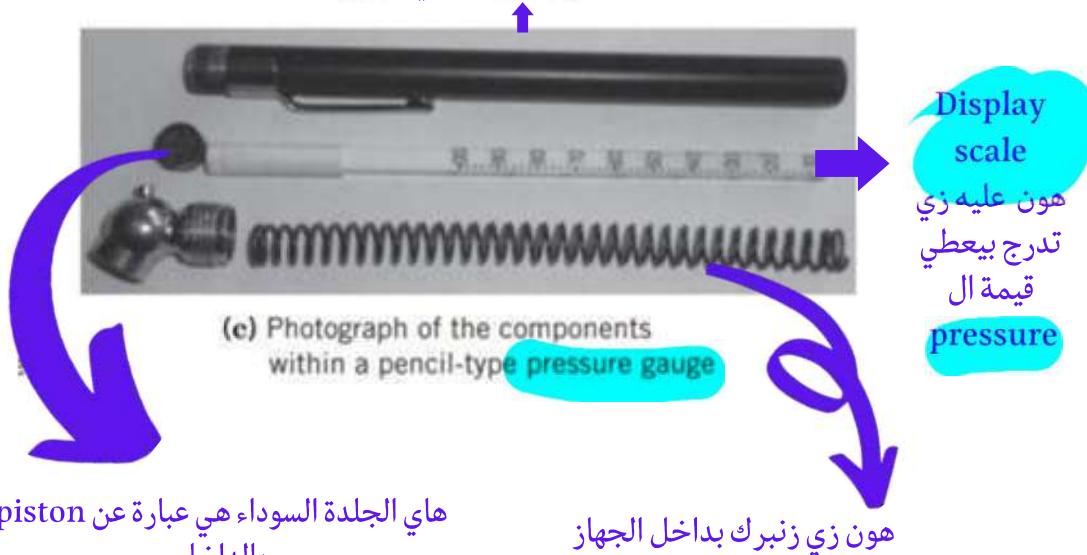
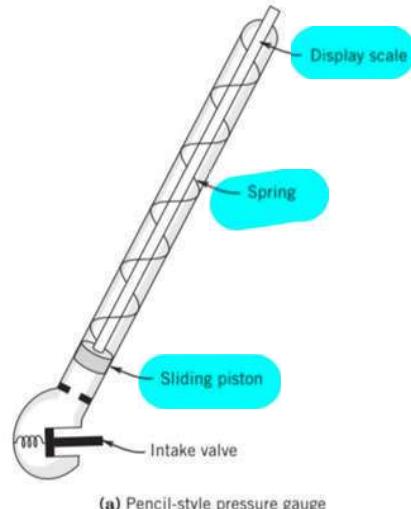
لحسن الحظ ، يمكن نمذجة العديد من أنظمة القياس باستخدام خطى من الدرجة الصفرية أو الأولى أو الثانية. المعادلات التفاضلية العادية . يمكن عادةً تبسيط الأنظمة الأكثر تعقيداً لهذه الطلبات الأقل. هدفنا هنا هو محاولة فهم كيفية تصرف الأنظمة وكيف ترتبط هذه الاستجابة ارتباطاً وثيقاً بميزات تصميم نظام القياس؛ ليس لمحاكاة سلوك النظام بالضبط. تم العثور على العلاقة الدقيقة بين الإدخال والإخراج من المعايرة. لكن النمذجة ترشدنا في اختيار أدوات وطرق قياس محددة من خلال التنبؤ باستجابة النظام للإشارات ، وفي تحديد نوع ونطاق وخصائص المعايرة.

### Example 3.2 مثال على النظام الصفيري

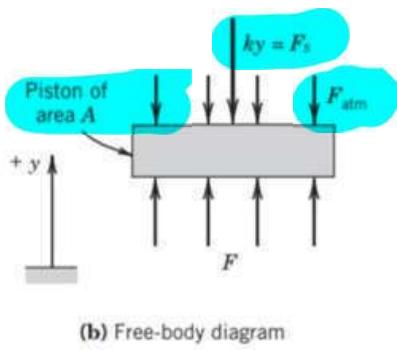
Figure 3.4 Lumped parameter model of pressure gauge

*example :*

#### Pencil style pressure gauge



لاحظ انه هاد ال peston عليه قوتين الأولى هو atmosphere pressure تاع ال



والثاني هو ال f تاع الزنبرك

والقوتين متساوين لأنه النظام متوازن

use in

A pencil - type pressure gauge commonly used to measure tire pressure can be modeled at static equilibrium by considering the force balance on the gauge sensor , a piston that slides up and down a cylinder piston

يمكن نمذجة مقاييس ضغط من نوع القلم الرصاص يستخدم عادة لقياس ضغط الإطارات عند توازن ثابت من خلال مراعاة توازن القوة على مستشعر القياس ، وهو مكبس ينزلق لأعلى ولأسفل مكبس أسطوانة

**important**  $\Sigma F = 0$ , gives

$$ky = F - F_{atm}$$

$$Ky = p^* A$$

الى هي الضغط المؤثر عالمساحة A في p هي القوة تاع الزنبرك ومساوية لل

Pressure is simply the force acting inward over the piston surface area , Dividing through by area provides the zero - order response equation between output displacement and input pressure and gives

الضغط هو ببساطة القوة المؤثرة للداخل على مساحة سطح المكبس ، وتقسيمها حسب المنطقة يوفر معادلة الاستجابة ذات الترتيب الصفرى بين وضع الإخراج وضغط الإدخال ويعطي

$$y = (A/k)(p - p_{atm})$$

لماذا هذا المودل مش مناسب للديناميك ؟

The exact static input - output relationship is found through calibration of the gauge . Because elements such as piston inertia and frictional dissipation were not considered , this model would not be appropriate for studying the dynamic response of the gauge .

تم العثور على علاقة الإدخال والإخراج الثابتة الدقيقة من خلال معايرة المقياس. نظرًا للعدم مراعاة عناصر مثل القصور الذاتي للمكبس والتبييد الاحتكمائي ، فمن يكون هذا النموذج مناسباً لدراسة الاستجابة الديناميكية للمقياس.

explain

Example 3.2: Pencil type Pressure.

$$\sum F = 0$$

$$K_y = F - F_{atm}$$

$$P = \frac{F}{A} \rightarrow \text{piston}$$

$$y = (A/K) (P - P_{atm})$$

↑ from calibration  
just for gague

static response of the gague

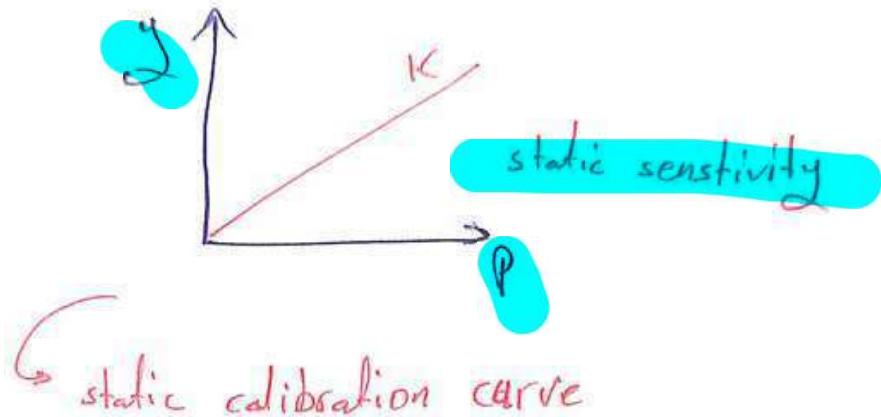
not suitable for dynamic

لو بدبي ارسم العلاقة بينهم

$$Ky = p^* A$$

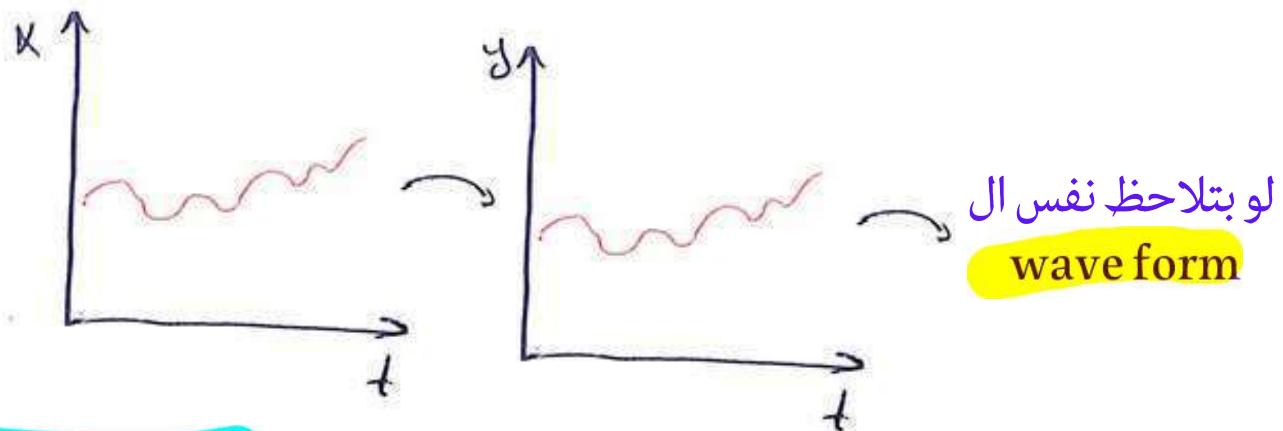
$$K = \frac{k}{A}$$

$y$ : output  
 $p$ : input



وبالرضا اي تغير بقيمة ال  $x$  بتغير بقيمة ال  $t$

**important**



## FirsOrder Systems

Measurement systems that contain storage elements do not respond instantaneously to changes in input.

أنظمة القياس التي تحتوي على عناصر تخزين لا تستجيب بشكل فوري للتغيرات في المدخلات.

first order system

$$a_1 \dot{y} + a_0 y = f(t)$$

هاد الحد زائد عن ال zero-order



هسا اي معادلة احنا بنحاول نبسطها بحيث تبقا ال  $y$  لحالها  
زي ممثل ال zero-order قسمناها على  $a_0$ . وهيك بعمل هون

$$a_1 \dot{y} + a_0 y = f(t)$$

$$\cancel{a_1} \dot{y} + y = K f(t)$$

Time constant  
 $= a_1 / a_0$

static sensitivity

may be modeled using a first-order differential equation of the form

$$a_1 \dot{y} + a_0 y = F(t)$$

with  $\dot{y} = dy/dt$ . Dividing through by  $a_0$  gives

important

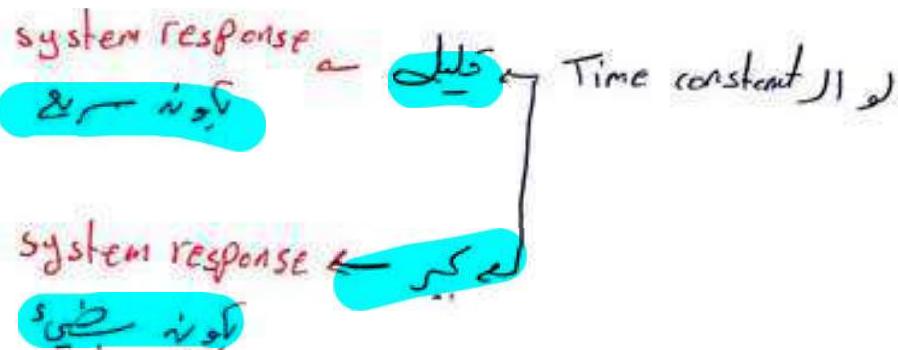
$$\tau \dot{y} + y = KF(t)$$

The parameter  $T$  is called the time constant of the system.

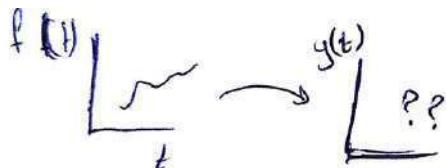
تسمى المعلمة  $T$  ثابت الوقت للنظام.

تمام بس هسا شو الفائدة من ال time constant من ال time constant بتعبرلي عن ال

Speed of system responds



هسا زي بدننا حاول نحل المعادلة التفاضلية هاي تاع ال first order وحناخذ حالتين عشان اشوف لو تغير  $f(t)$  الي هو ال input كيف حيصير  $y(t)$



### Step Function Input

The step function,  $AU(t)$ , is defined as

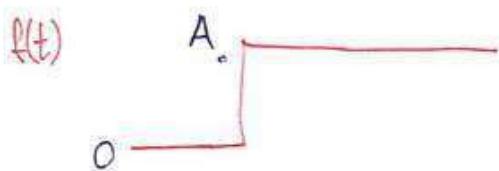
$$AU(t) = 0 \quad t \leq 0^-$$

$$AU(t) = A \quad t \geq 0^+$$

where  $A$  is the amplitude  
of the step function  
حيث  $A$  هي سعة دالة الخطوة

a sudden change in the input signal from a constant value of one magnitude to a constant value of some other magnitude

تغير مفاجئ في إشارة الدخل من قيمة ثابتة مقدارها إلى قيمة ثابتة لبعض المقدار الآخر

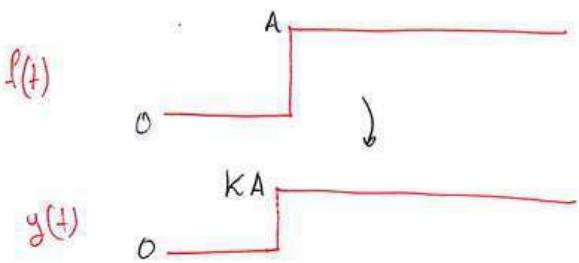


كمثال عليه لما انقل الميزان تابع  
الحرارة من كوب مثلج للكوب بغلبي  
هاد شكله العام قبل ما اشوف  
استجابته بال FirstOrder

important

$$\begin{cases} t = A & , t > 0 \\ t = 0 & , t < 0 \end{cases}$$

تمام هسا قبل ما اسوف استجابته بال first order بتندركر كيف استجابته بال



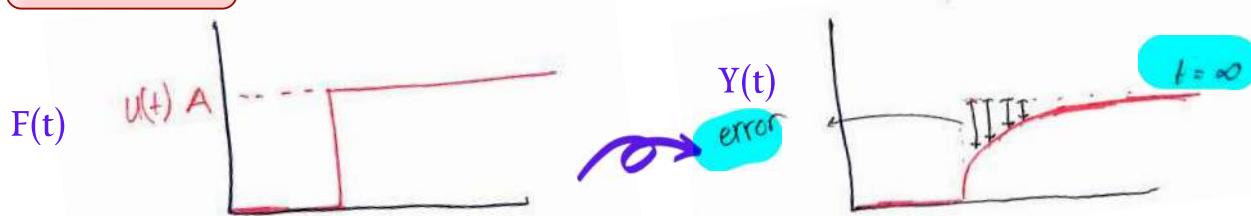
رح يصل نفس الشكل وبس ينضرب ب k

طبعاً بالواقع ما عندي system بيرفكت مية بالمية يعطيوني  
إلا ما يكون عندي time constant لحد ما يستجيب ال



تمام هسا كيف يستجيب ال first order بال  
لاحظوا علماء الرياضيات انه بال step function الكيف رح مختلف عندي  
بال input عن ال output

### important



ولاحظوا كمان انه صرح يتغير بس رح  
يثبت عند الأنفنتي او عند رقم كبير  
zero-order ويصير بشبه ال

تمام هسا لاقوا شكل الاقتران وبعدها حاولوا يحلوا المعادلة عشان يقدرو يعرفو y(t)



$$y(t) = KA + (y_0 - KA) e^{-\frac{t}{\tau}}$$

$$F(y) = A \cdot U(t)$$

$$\frac{dy}{dt} + y = KA \cdot U(t) = kF(t)$$

steady  
response

Transient

response

بعد زمن طويل ال  
System  
رح يتصرف زي ال  
Zero-order

بعد فترة عشان ال  
System  
يتصرف زي ال  
Zero-order

وال input output تتبع ال

الأساسية

$$\tau \dot{y} + y = KAU(t) = KF(t)$$

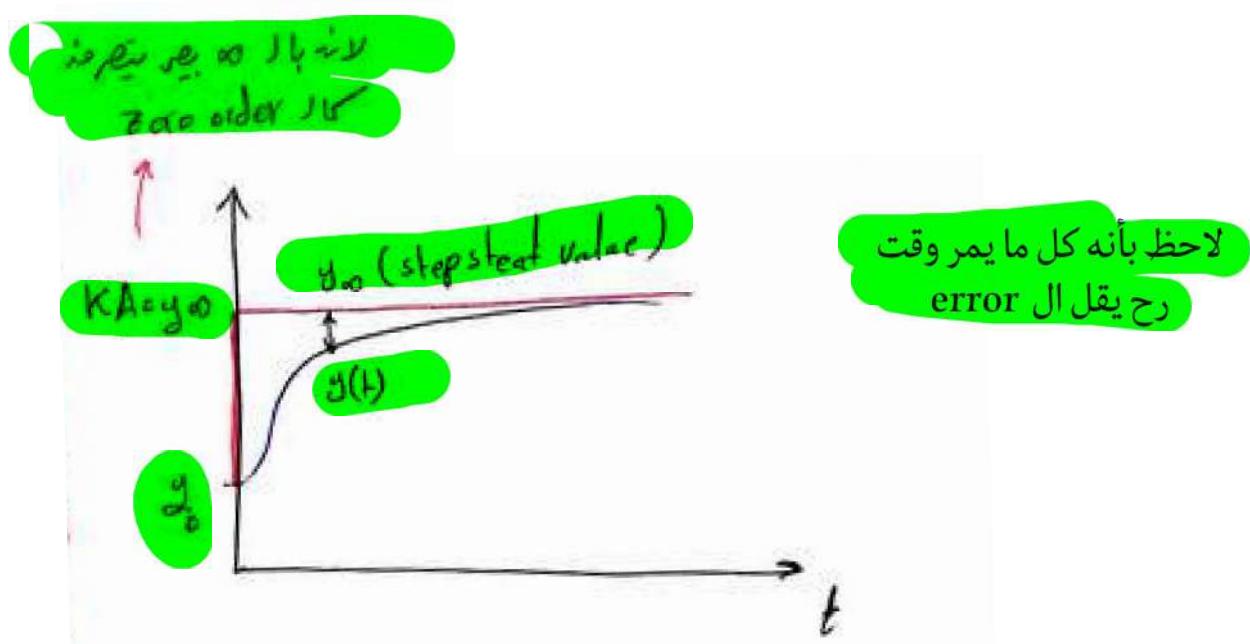
with an arbitrary initial condition denoted by,  $y(0) = y_0$ . Solving for  $t \geq 0^+$  yields

$$y(t) = \underbrace{KA}_{\text{Time response}} + \underbrace{(y_0 - KA)e^{-t/\tau}}_{\text{Transient response}}$$

The solution of the differential equation, T, is the time response (or simply the response) of the system.

حل المعادلة التفاضلية ، T ، هو استجابة للوقت (أو ببساطة استجابة) للنظام.

كتوضيح لفكرة ال error



Suppose we rewrite the response Equation 3.5 in the form

$$\Gamma(t) = \frac{y(t) - y_\infty}{y_0 - y_\infty} = e^{-t/\tau}$$

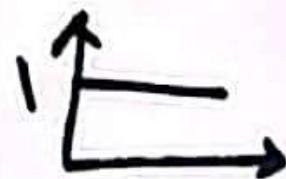
The term  $\Gamma(t)$  is called the *error fraction* of the output signal. Equation 3.6 is

$$\rightarrow x(t) = Au(t)$$

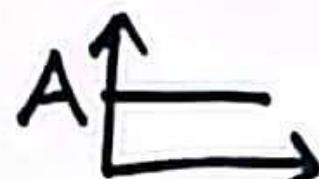
$$\rightarrow \tau \dot{y} + y = kx$$

$$\tau \dot{y} + y = kAu(t)$$

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$



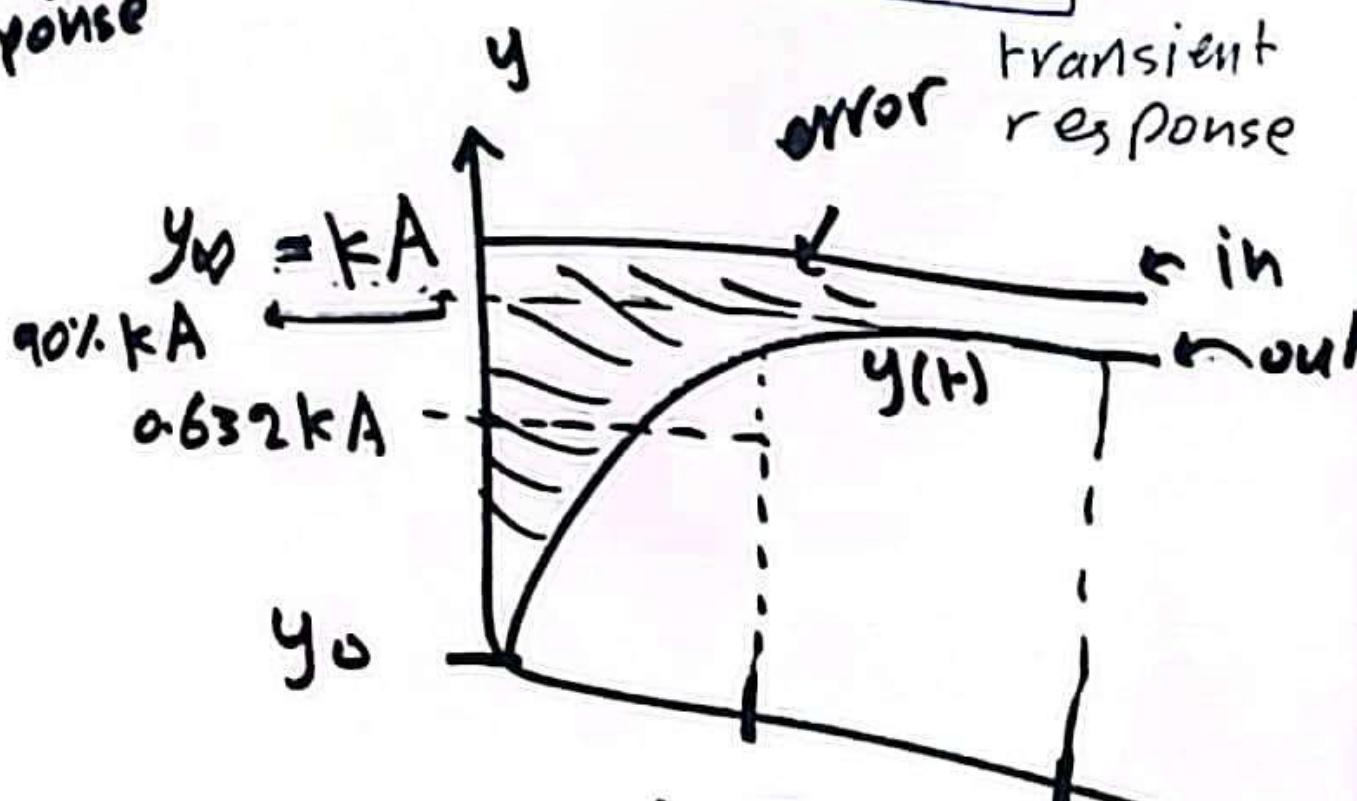
$$Au(t) = \begin{cases} 0, & t < 0 \\ A, & t > 0 \end{cases}$$



output / time response

$$\rightarrow y(t) = \underbrace{KA}_{\text{steady response}} + \underbrace{(y_0 - KA)}_{\text{error}} e^{-t/\tau}$$

steady response

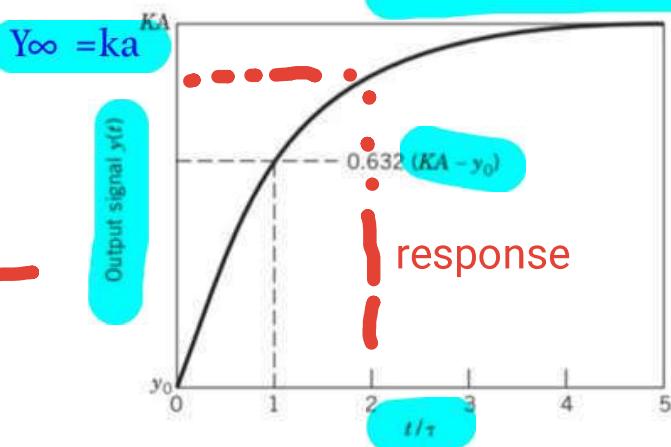


$$T_S \rightarrow 95\% KA$$

$$t = T$$

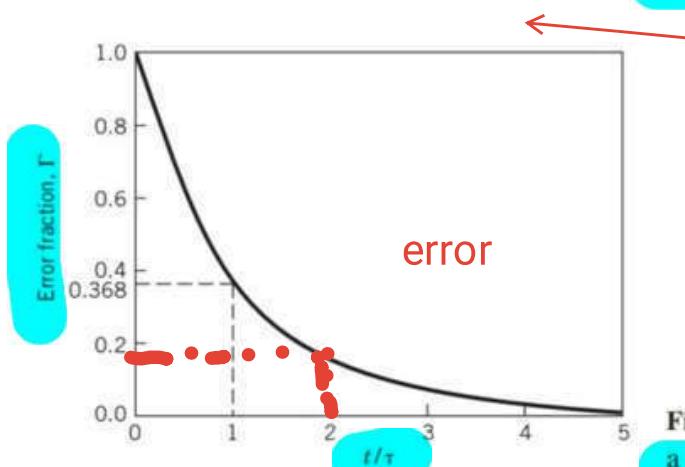
$$Tr \quad t/\tau$$

## توضيح لفكرة ال time constant



لاحظ انه بال x axis عندي  $t$  على تاو  
فمثلا عند الزمن  $t$  تكون  $t=t_\tau$

Figure 3.6 First-order system time response to a step function input: the time response,  $y(t)$ .



1- graph of  $(y(t) - t/t_\tau)$  =  
error friction

Figure 3.7 First-order system time response to a step function input: the error fraction,  $\Gamma$ .

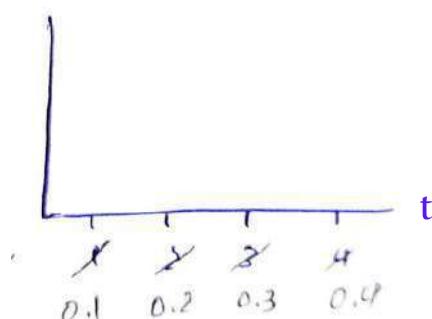
Note :

بعد مرور واحد second من time response في الرسمة الأولى رح يكون 0.632 إذن ال  
كيف بحسبه error

$$\text{المكملاة فبحكي } 1 - 0.632 = 0.368 \text{ هـ}$$

الظاهو في الرسمة الثانية

حيينا قبل انه التاو بتعتمد عجهاز القياس طي لو الرسمة ال x axis بال  $t$   
مش  $t$  على تاو وحاليا انه التاو تاع الجهاز = 0.1



لاحظ انه ال a axis هو  $t$  بشطب القيم وبرجع بعيهم حسب تاو  
لهي الكتاب مختصر القصة وفوراً حاطتها  $t$  على تاو

Table 3.1 First-Order System Response and Error Fraction

$t/\tau$	% Response	$\Gamma$	% Error
0	0.0	1.0	100.0
1	63.2	0.368	36.8
2	86.5	0.135	13.5
2.3	90.0	0.100	10.0
3	95.0	0.050	5.0
5	99.3	0.007	0.7
$\infty$	100.0	0.0	0.0

بـ 0 دقـنة كـانه الـ Response

$$1 = (1 - 0) \rightarrow 1 \text{ error}$$

بـ 63.2 دـقـنة كـانه الـ Response

$$0.368 = (1 - 63.2) \text{ error}$$

مـكـنـا

plot is equivalent to the transformation

$$\ln \Gamma = 2.3 \log \Gamma = -(1/\tau)t \quad (3.7)$$

which is of the linear form,  $Y = mX + B$  (where  $Y = \ln \Gamma$ ,  $m = -(1/\tau)$ ,  $X = t$ , and  $B = 0$  here). A linear curve fit through the data will provide a good estimate of the slope,  $m$ , of the resulting plot. From Equation 3.7, we see that  $m = -1/\tau$ , which yields the estimate for  $\tau$ .

الـ تـاوـ بـتـعـتمـدـ عـجـهـاـزـ وـكـلـماـ كـانـتـ قـلـيلـةـ الـ responseـ بـكـوـنـ اـسـرـ

الـ زـمـنـ 2.3ـ هـوـ مـهـمـ النـاـ لـأـنـهـ بـتـكـونـ درـجـةـ الـ استـجـابـهـ عـنـدـهـ 90ـ بـالـمـيـةـ وـالـ errorـ فـقـطـ 10ـ بـالـمـيـةـ وـهـوـ حـفـظـ

The time required for a system to respond to a value that is 90% of the step input, Yo-Yo, is important and is called the rise time of the system.

يـعـدـ الـوقـتـ الـلـازـمـ لـاستـعـادـةـ النـظـامـ لـقيـمةـ 90%ـ مـنـ الـخـطـوـةـ الدـاخـلـيـةـ، Yo-Yoـ، أـمـرـاـ مـهـمـاـ وـيـسـمـىـ وقتـ صـعـودـ النـظـامـ.

ممـكـنـ يـسـئـلـنـيـ بـهـاـ الطـرـيقـةـ

$$\tau_{90\%} = 2.32 \text{ مـلـفـلـلـ}$$

$$\tau_{\text{asy}} = 3\tau$$

$$\tau_{\text{Response}} = T \rightarrow \text{سـاحـرـ}$$

Chapter 3 Measurement System Behavior

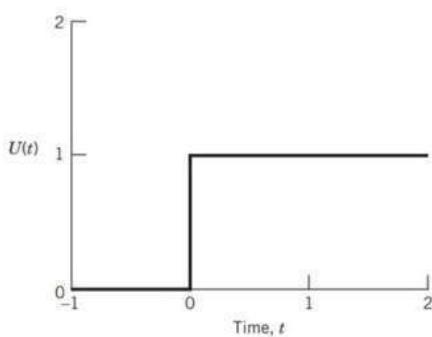


Figure 3.5 The unit step function,  $U(t)$ .

مـلـوـمـةـ لـلـمـراـجـعـةـ: رـسـمـةـ الـ step~function

$$\rightsquigarrow \text{fraction out} = \frac{y(t) - y_0}{y_\infty - y_0} = e^{-\frac{t}{T}}$$

$T_r \rightarrow$  rise time  $[90\%] kA$

$T_s \rightarrow$  settling time  $[95\%] kA$

$75\%, 25\%, 60\% \rightarrow$  response time

$90\%.kA \rightarrow$  error = 10%

$$\frac{10}{100} = e^{-t/T} \rightarrow t/T = 2.3$$

$$t = 2.3 T$$

$95\%.kA \rightarrow$  error = 5%

63.2%  
error = 36.8%

$$\frac{5}{100} = e^{-t/T}$$

$$\frac{36.8}{100} = e^{-t/T}$$

$$\rightarrow \text{fraction out} = \frac{y(t) - y_0}{y_0 - y_\infty} = e^{-\tau t}$$

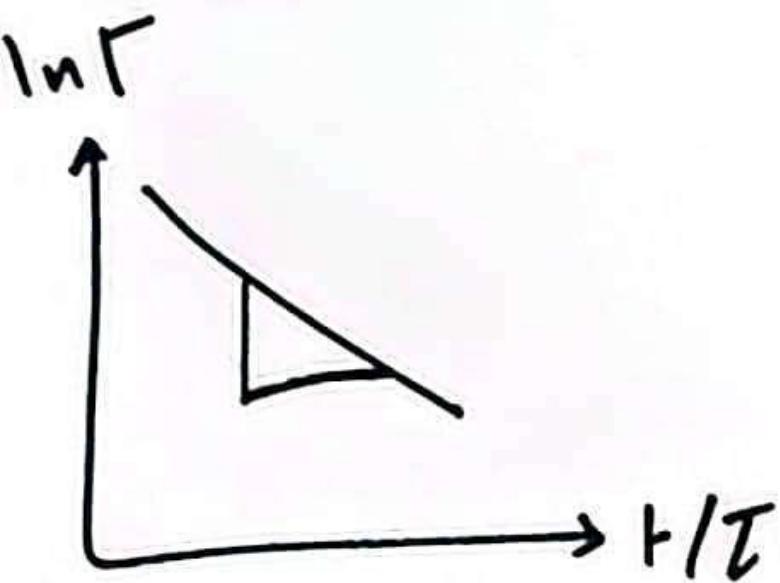
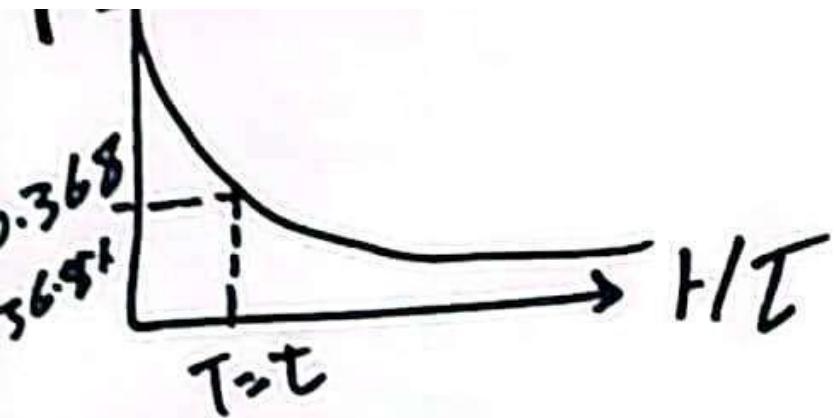
rise time  $\tau_r$   $\rightarrow$   $90\% \text{KA}$   
settling time  $\tau_s$   $\rightarrow$   $95\% \text{KA}$   
 $\tau_s \rightarrow$  response time  
75%, 25%, 6%,  $\rightarrow$  response time

$$90\% \text{ ka error} = 10\%$$

$$\frac{10}{100} = e^{-t/\tau} \rightarrow \boxed{t = 2.3\tau}$$

95% ka error = 5%  
error = 36.8%.

$$\frac{5}{100} = e^{-t/\tau}$$
$$\frac{36.8}{100} = e^{-t/\tau}$$
$$\frac{1}{e} = \frac{1}{2}$$
$$\boxed{t = \tau}$$



$$\text{slope} = \frac{-1}{T}$$

$$T(t) = T_{\infty} + (T_0 - T_{\infty}) e^{-t/T}$$

$$y(t) = y_{\infty} + (y_0 - y_{\infty}) e^{-t/T}$$

\* First order :-

$$a_1 \dot{y} + a_0 y = b_0 X \div a_0$$

$$\left( \frac{a_1}{a_0} \right) \dot{y} + y = \frac{b_0}{a_0} X$$

$\downarrow$   
 $\tau$

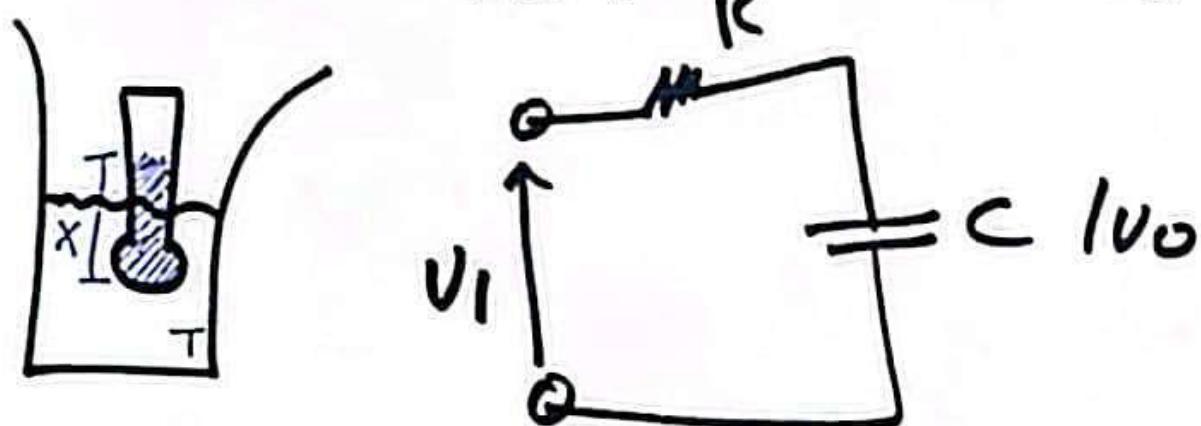
|  $\tau \downarrow$  speed ↑  
better

$$\underline{\tau} \dot{y} + y = \underline{k} X$$

$\rightarrow$  static sensitivity.

Time constant (s) from calibration  
(speed of response) static

\* Thermometer :-

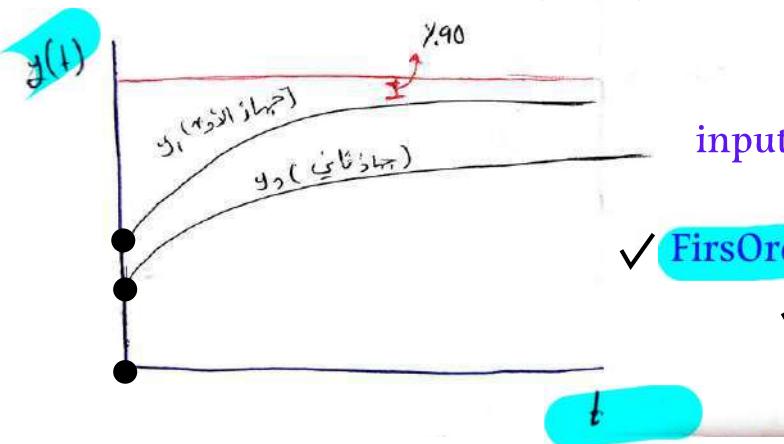


$$\tau = \underline{R C}$$

example :

no error ( zero)

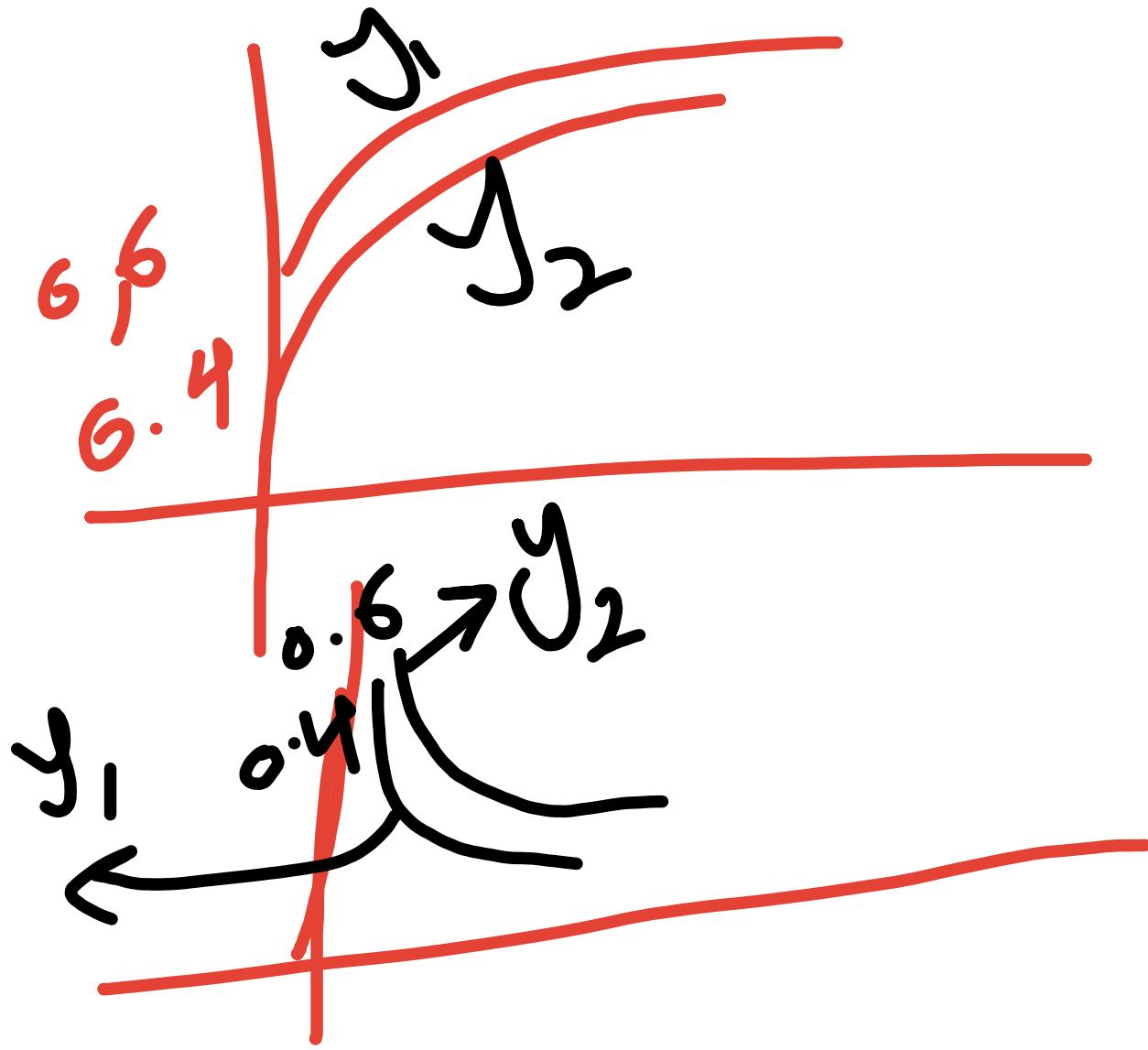
question



افرض عندي جهازين وهي رسماتهم  
فبدي اقارن بينهم

او الاشي بحدد نوع ال measurement وال  
من رسمة ال output

✓ First Order measurement فم السمة انا عاوف انها  
✓ step input هو input وال



one first order

اسئلة على رسمات ال zero-order

yes ideal ✓? ideal instrument to do

perfect ✓? Dynamic response to do  
داتا ال input بآخر ديركت بال output

time lag / phase shift ✓  
هل يوجد بين ال output وال input  
time lag / phase shift  
هذا يعني ما ياخذ في  
الوقت بين ال output وال input

جيئ بالتقريب الفكرة الكوركاسيل اخرين يجربون  
Time lag بعد فترة من

distortion ✓  
(وصل غير العزل قبل الموج)

where

$m$  = mass of liquid within thermometer

$c_v$  = specific heat of liquid within thermometer

$h$  = convection heat transfer coefficient between bulb and environment

$A_s$  = thermometer surface area

The term  $hA_s$  controls the rate at which energy can be transferred between a fluid and a body; it is analogous to electrical conductance. By comparison with Equation 3.3,  $a_0 = hA_s$ ,  $a_1 = mc_v$ , and  $b_0 = hA_s$ . Rewriting for  $t \geq 0^+$  and simplifying yields

$$\frac{mc_v dT(t)}{hA_s dt} + T(t) = T_\infty$$

From Equation 3.4, this implies that the time constant and static sensitivity are

$$\tau = \frac{mc_v}{hA_s} \quad K = \frac{hA_s}{hA_s} = 1$$

Direct comparison with Equation 3.5 yields this thermometer response:

$$T(t) = T_\infty + [T(0) - T_\infty]e^{-t/\tau} \\ = 37 - 17e^{-t/1} \text{ } [^\circ\text{C}]$$

mention

من قانون التأوه يعرف العوامل التي

يعتمد عليها

الهي المass(m) وال

heat (conductivity (cv  
(treatment (h

$$\tau = \frac{mc_v}{hA_s}$$

$$K = \frac{hA_s}{hA_s} = 1$$

اما اسمها الـ

Static sensitivity

steady state او

لأنها بترتبط بال input

السؤال عن الجهاز الاول في حالة first order و step

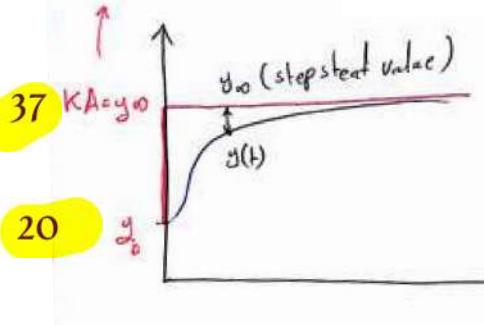
Also, it is significant that we found that the response of the temperature measurement system in this case depends on the environmental conditions of the measurement that control  $h$ , because the magnitude of  $h$  affects the magnitude of  $T$

explain

أيضاً، من المهم أننا وجدنا أن استجابة نظام قياس درجة الحرارة في هذه الحالة تعتمد على الظروف البيئية للقياس الذي يتحكم في  $h$ ، لأن حجم  $h$  يؤثر على حجم تأوه.

تكرار للي فوق بأنه النظام يعتمد عوامل تأوه ومنها  $h$

لأنه بالـ 37 غير معمول  
2nd order كار



هي انتقلت من 20 لـ 37  
المعادلة تاب الرسمة هي حل المعادلة الى فوق

$$T(t) = T_\infty + [T(0) - T_\infty]e^{-t/\tau}$$

$$= 37 - 17e^{-t/\tau} \text{ [°C]}$$

$$f(t) = 20 + 17u(t)$$

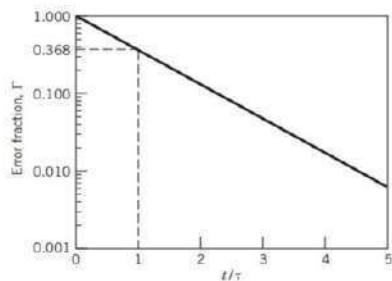


Figure 3.8 The error fraction played on semilog coordinates.

الشكل 3.8 جزء الخطأ الذي تم تشغيله على الإحداثيات  
شبه اللوغاريتمية.

خلصنا من الحالة الأولى لل FirstOrder وهي لما يكون ال

may be modeled using a first-order differential equation of the form

$$a_1\dot{y} + a_0y = F(t)$$

with  $\dot{y} = dy/dt$ . Dividing through by  $a_0$  gives

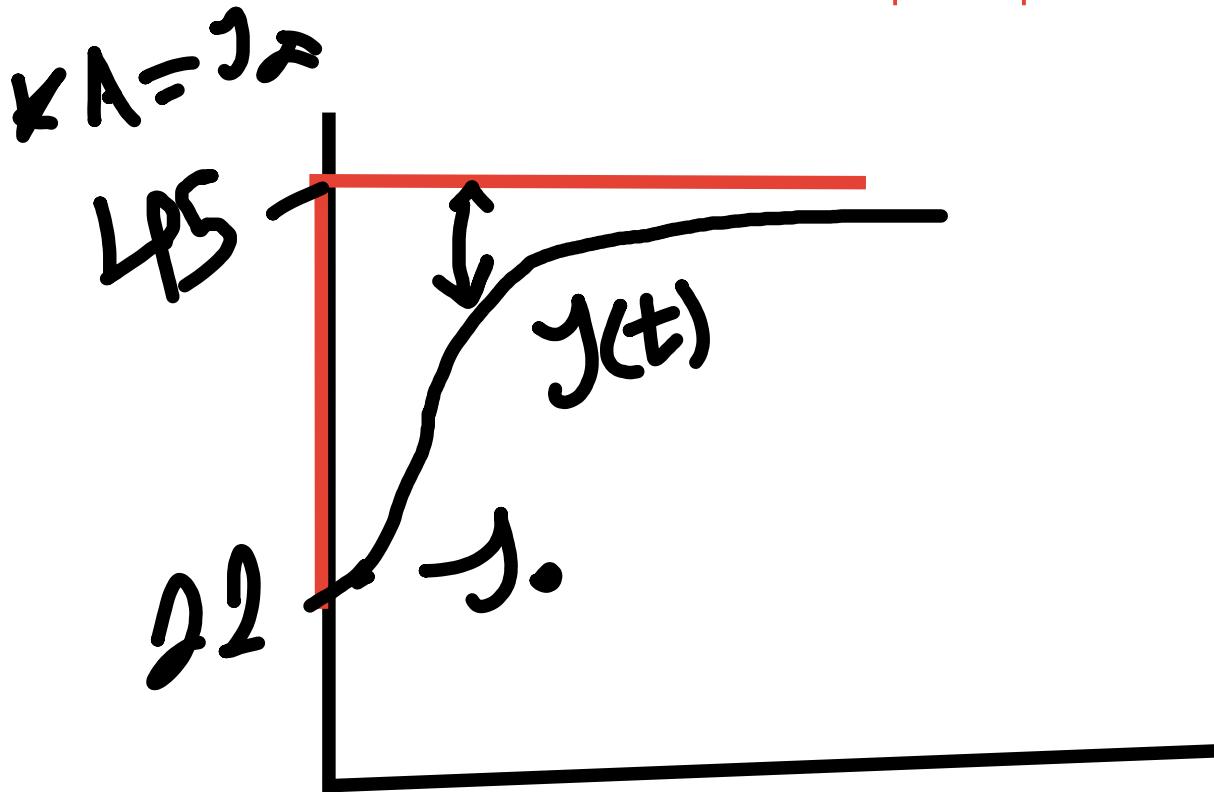
$$\tau y + y = KF(t)$$

بمعنى كنا نعرض بدل ال

simple periodic function input  $f(t) \leftarrow$  step input هسا بدنا ننتقل للحالة الثانية التي بدل ال

( $y(t)$ )  $\leftarrow$  step input و بشوف لو كانت ال ( $f(t)$ ) شكلها  $\sin$  او  $\cos$  كيف تكون شكل ال ( $y(t)$ ) output

Suppose a bulb thermometer originally indicating 22 C is suddenly exposed to a fluid temperature of 45 C. Develop a model that simulates the thermometer output response



$$T(t) = t\alpha + (t - t_0) \frac{e^{-\frac{t-t_0}{\tau}}}{KA}$$

$$T(t) = 45 - 23e^{-t/\tau}$$

## امثلة مخططين بس الدكتور ما شرح عنهم

### Example 3.4

For the thermometer in Example 3.3 subjected to a step change in input, calculate the 90% rise time in terms of  $t/\tau$ .

**KNOWN** Same as Example 3.3

**ASSUMPTIONS** Same as Example 3.3

**FIND** 90% response time in terms of  $t/\tau$

**SOLUTION** The percent response of the system is given by  $(1 - \Gamma) \times 100$  with the error fraction,  $\Gamma$ , defined by Equation 3.6. From Equation 3.5, we note that at  $t = \tau$ , the thermometer will indicate  $T(t) = 30.75^\circ\text{C}$ , which represents only 63.2% of the step change from  $20^\circ$  to  $37^\circ\text{C}$ . The 90% rise time represents the time required for  $\Gamma$  to drop to a value of 0.10. Then

$$\Gamma = 0.10 = e^{-t/\tau}$$

or  $t/\tau = 2.3$ .

**COMMENT** In general, a time equivalent to  $2.3\tau$  is required to achieve 90% of the applied step input for a first-order system.



### Example 3.5

A particular thermometer is subjected to a step change, such as in Example 3.3, in an experimental exercise to determine its time constant. The temperature data are recorded with time and presented in Figure 3.10. Determine the time constant for this thermometer. In the experiment, the heat transfer coefficient,  $h$ , is estimated to be  $6 \text{ W/m}^2 \cdot ^\circ\text{C}$  from engineering handbook correlations.

**KNOWN** Data of Figure 3.10

$$h = 6 \text{ W/m}^2 \cdot ^\circ\text{C}$$

**ASSUMPTIONS** First-order behavior using the model of Example 3.3, constant properties

**FIND**  $\tau$

**SOLUTION** According to Equation 3.7, the time constant should be the negative reciprocal of the slope of a line drawn through the data of Figure 3.10. Aside from the first few data points, the data appear to follow a linear trend, indicating a nearly first-order behavior and validating our model assumption. The data is fit to the first-order equation<sup>2</sup>

$$2.3 \log \Gamma = (-0.194)t + 0.00064$$

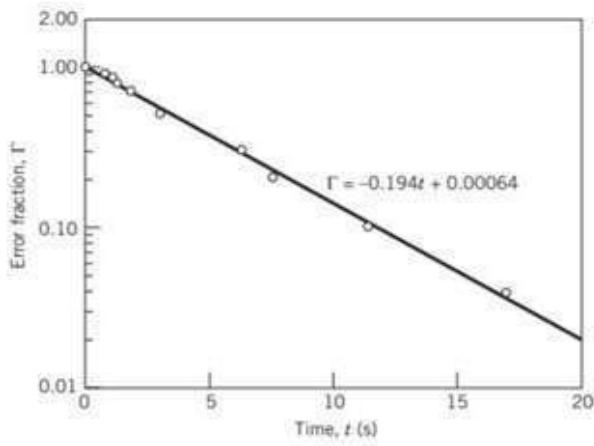


Figure 3.10 Temperature-time history of Example 3.5.

With  $m = -0.194 = -1/\tau$ , the time constant is calculated as  $\tau = 5.15$  seconds.

**COMMENT** If the experimental data were to deviate significantly from first-order behavior, this would be a clue that either our assumptions do not fit the real-problem physics or that the test conduct has control or execution problems.

may be modeled using a first-order differential equation of the form

$$a_1 \dot{y} + a_0 y = F(t)$$

with  $\dot{y} = dy/dt$ . Dividing through by  $a_0$  gives

$$\tau \dot{y} + y = KF(t) \quad \text{الأساسية}$$

$f(t) \leftarrow$  step input بدل ال

هسا بدننا ننتقل للحالة الثانية الي بدل ال  $(f(t) \leftarrow$  بعض simple periodic function input

وبشوف لو كانت ال  $f(t)$  شكلها sin او cos كيف تكون شكل ال  $y(t)$

measuring system to which an input of the form of a simple periodic function,  $F(t) = A \sin \omega t$ , is applied for  $t \geq 0^+$ :

هون ال  $f(t)$ . هي مدخلات ال periodic

$$F(t) = A \sin \omega t$$

بعضها في معادلة ال first order

$$\tau \dot{y} + y = KA \sin \omega t$$

حل المعادلة التفاضلية الي فوق

input in sin ( periodic )

$$f(t) = A \sin \omega t$$

Amplitude

frequency

angular frequency  
circular frequency  
(rad/s)

this differential equation yields the measurement system output signal, that is, the time response to the applied input,  $y(t)$ :

$$y(t) = Ce^{-t/\tau} + \frac{KA}{\sqrt{1+(\omega\tau)^2}} \sin(\omega t - \tan^{-1}\omega\tau) \quad (3.8)$$

where the value for  $C$  depends on the initial conditions.

sin  $\leftarrow$  periodic جا في

$$\tau \dot{y} + y = KA \sin \omega t$$

$$y(t) = Ce^{-t/\tau} + \frac{KA}{\sqrt{1+(\omega\tau)^2}} \sin(\omega t - \tan^{-1}\omega\tau)$$

Transient response

يعود تدريجياً مع الزمن

وهو يقترب من القيمة

final

هسا فوق حكينا انه  $A \sin(\omega t)$  هو ال input فمن وين اجا كل الباقي

$$\frac{K}{\sqrt{1+\omega^2\tau^2}}$$

أحمد مداد

الفاكتور  
( factor )

يكون ضعافاً

حيث  $\phi$  فاي

الرقم الي رفع بعلمه

scaling of Amplitude  
(A)

عرفنا ال periodic واخذنا مدخلاته وعوضناه بال FirsOrder هسا اكيد رح يخطرلك طب كيف حتصير

FirsOrder output لـ



**important**

نفس الرسمة ولكن زيجه كنه ضربتها خ  $k$

ولكن اع تأثيره في  $y(t)$  بين  $F(t)$  وال  $y(t)$

فال  $y(t) = kF(t)$

های هي رسمة ال periodic

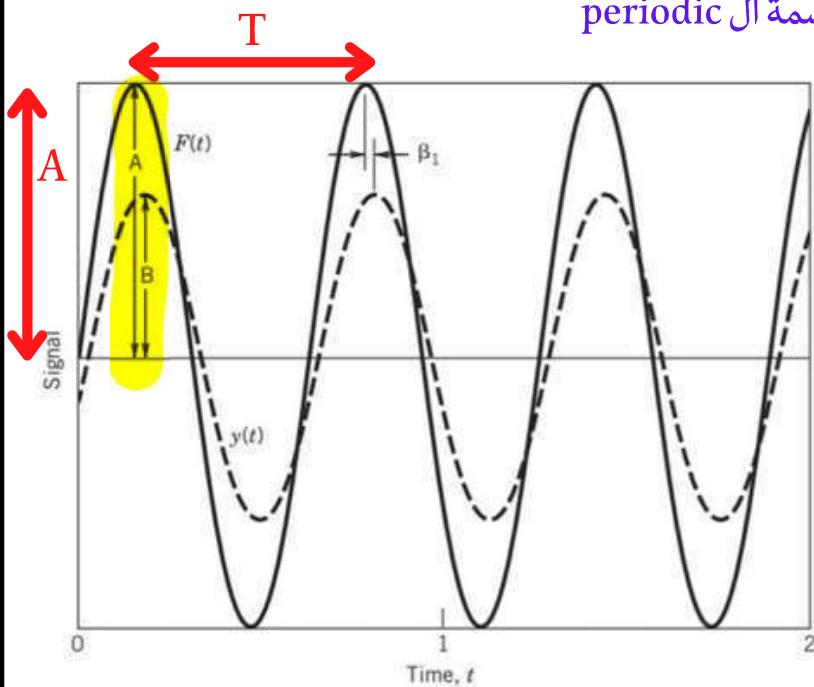


Figure 3.11 Relationship between a sinusoidal input and output: amplitude, frequency, and time delay.

هسا نتفق عاشي الخط المتصل هو ال inputs

$$F(t) = A \sin \omega t$$

والمنقط هو ال output

$$y(t) = Ce^{-t/\tau} + \frac{KA}{\sqrt{1 + (\omega\tau)^2}} \sin(\omega t - \tan^{-1} \omega\tau)$$

لو حبيت ابسطها بأنه اشيل ال factors

$$y(t) = B \sin(\omega t + \phi)$$

out put Amplitude  $B = \frac{KA}{\sqrt{1 + (\omega\tau)^2}}$

لود حلل الرسمة وشو عمل ال periodic بال

**explain**

**important**

$$T = \frac{1}{f} \text{ (sec)}$$

$$\text{(circular frequency)} \quad \omega = 2\pi f \text{ (rad/s)}$$

$$f = \frac{\omega}{2\pi} \text{ (Hz)}$$

(1) اعطي نفس كل ال input بـ  $\tau$  (2) صاربته ال output زبي gab

صغار التكبير متطابق (B =  $\frac{KA}{\sqrt{1 + (\omega\tau)^2}}$ )

Response scaling

$$A > B \rightarrow \text{input}$$

(3) لا خط المبدا في ال input  
ست نه بذاته الرسمة في ال output

applied is known as the frequency response of the system. The frequency affects the magnitude of amplitude  $B$  and also can bring about a time delay. This time delay,  $\beta_1$ , is seen in the phase shift,  $\Phi(\omega)$ , of the steady response. For a phase shift given in radians, the time delay in units of time is

$$\beta_1 = \frac{\Phi}{\omega}$$

that is, we can write

$$\sin(\omega t + \Phi) = \sin \left[ \omega \left( t + \frac{\Phi}{\omega} \right) \right] = \sin[\omega(t + \beta_1)]$$

\* Periodic Imp.

$$T\ddot{y} + y = K \ddot{x}$$

$$x = A \sin(\omega t)$$

$$T\ddot{y} + y = K A \sin(\omega t)$$

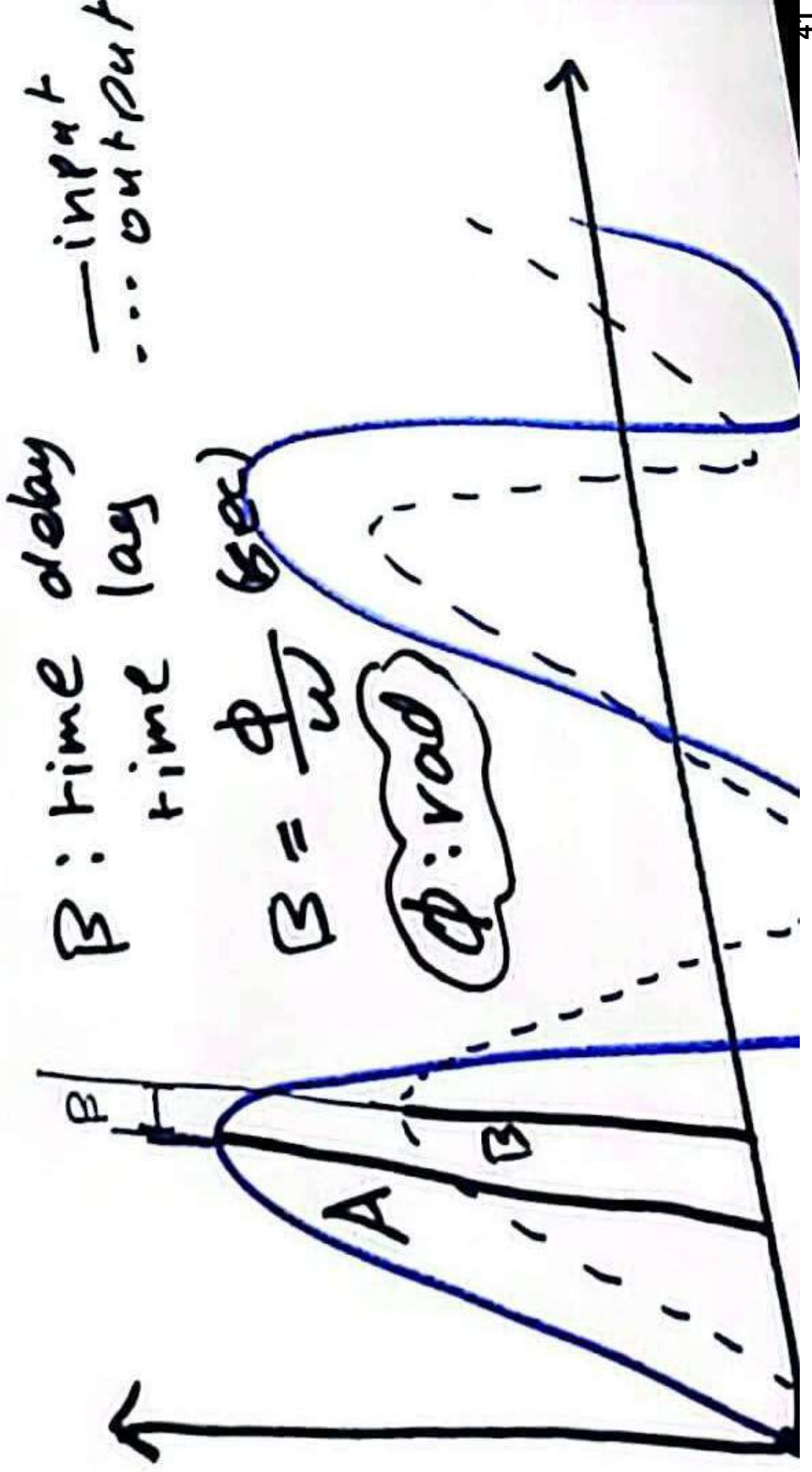
time response

$$y(t) = C e^{-t/T} + \sqrt{\frac{K A}{(1 + \omega^2 T^2)^{1/2}}} \sin(\omega t - \tan^{-1} \omega T)$$

depends  
on the  
initial  
conditions

Transient  
response

→  
Steady state  
response



Equation 3.8 can be rewritten in a general form:

$$y(t) = Ce^{-t/\tau} + B(\omega)\sin[\omega t + \Phi]$$

$$B(\omega) = \frac{KA}{\sqrt{1 + (\omega\tau)^2}}$$

$$\Phi(\omega) = -\tan^{-1}(\omega\tau)$$
(3.9)

where  $B(\omega)$  represents the amplitude of the steady response and the angle  $\Phi(\omega)$  represents the *phase shift*. A relative illustration between the input signal and the system output response is given in

هذا هو ملخص المقادير

$y(t) \rightarrow$  Time response

$$\sqrt{1 + (\omega\tau)^2} \rightarrow$$

جذر المقدار بين المدخل والمخرج

$B(\omega) \rightarrow$  Amplitude of output

$\Phi(\omega) \rightarrow$  phase shift

$\omega$ : input frequency (مقدار التردد)  
 $\tau$ : Time constant (مقدار الدوران)

يساوي  $\omega\tau$  اكبر



الصلة بين الـ  $A$  والـ  $B$

$$\sqrt{1 + (\omega\tau)^2}$$

مقدار

$$\frac{B_{\text{output}}}{kA_{\text{input}}} = \frac{M}{\sqrt{1 + (\omega\tau)^2}}$$

لذلك  $M = \sqrt{1 + (\omega\tau)^2}$

We define a magnitude ratio,  $M(\omega)$ , as the ratio of the output signal amplitude to the input signal amplitude,  $M(\omega) = B/KA$ . For a first-order system subjected to a simple periodic input, the magnitude ratio is

$$M(\omega) = \frac{B}{KA} = \frac{1}{\sqrt{1 + (\omega\tau)^2}}$$
magnitude
(3.10)

لذلك  $B_1 = \frac{\phi}{\omega}$

frequency response

$$M(\omega) = \frac{B}{KA} = \frac{1}{\sqrt{1 + (\omega\tau)^2}}$$

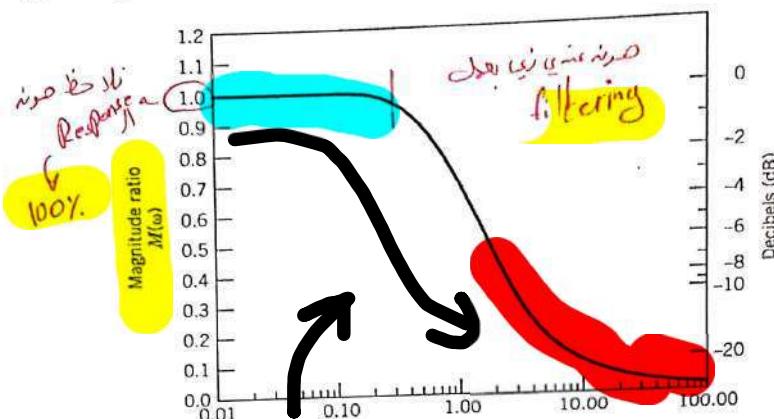


Figure 3.12 First-order system frequency response: magnitude ratio.

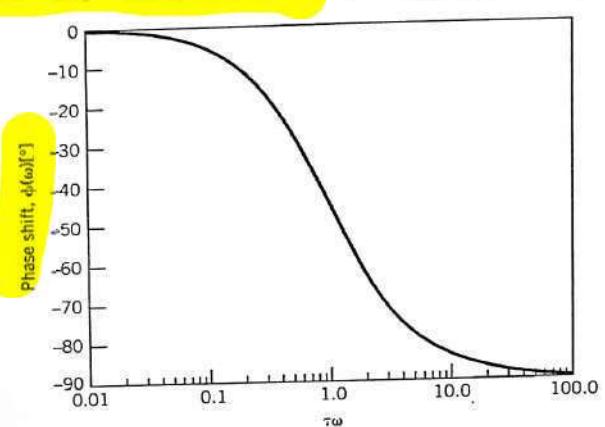


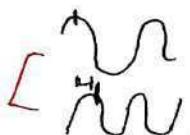
Figure 3.13 First-order system frequency response: phase shift.

$U = \frac{B}{KA}$   $\rightarrow$  output  $C$   $\rightarrow$   $A_w M$   $\rightarrow$   $\text{frequency Response}$   $\rightarrow$  مركب مع ال

عندما نندرج لذريعة الناس ناس استيعاب بغيره  $\rightarrow$  اخرين  
وناس استيعاب سرعه  $\rightarrow$  اوله الرسنه

رسنه او phase shift  $\rightarrow$  كما قالت ال رسنه ال طردية  
رسنه او phase shift  $\rightarrow$  كما قالت ال رسنه ال طردية

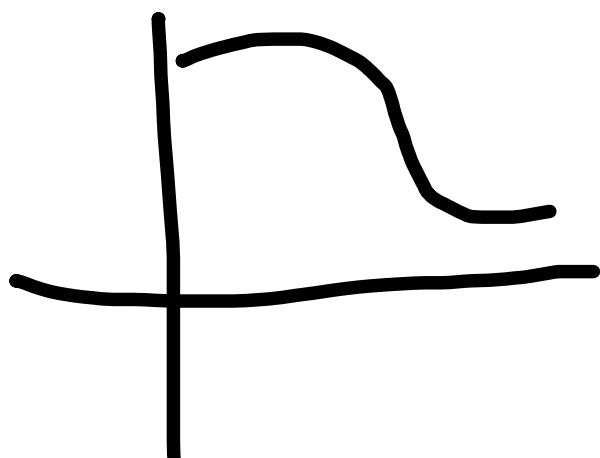
ادنا بالكتروبل بذكر  $M$  اعم من  
ف عادتا تكون التباين بين ال  
رسنه او input output خلص



اي معرفه الفرق تابع قبل الفرق تابع  
اي تعرفه يعني بضم φ

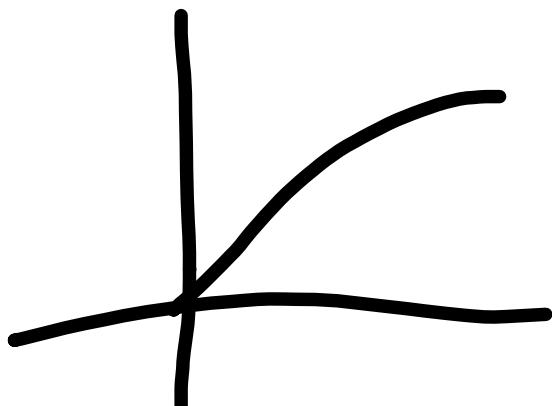
sin

الـ response يقل مع مرور الوقت  
ويكون الخطأ في البداية صفر وفي  
النهاية يصير واحد



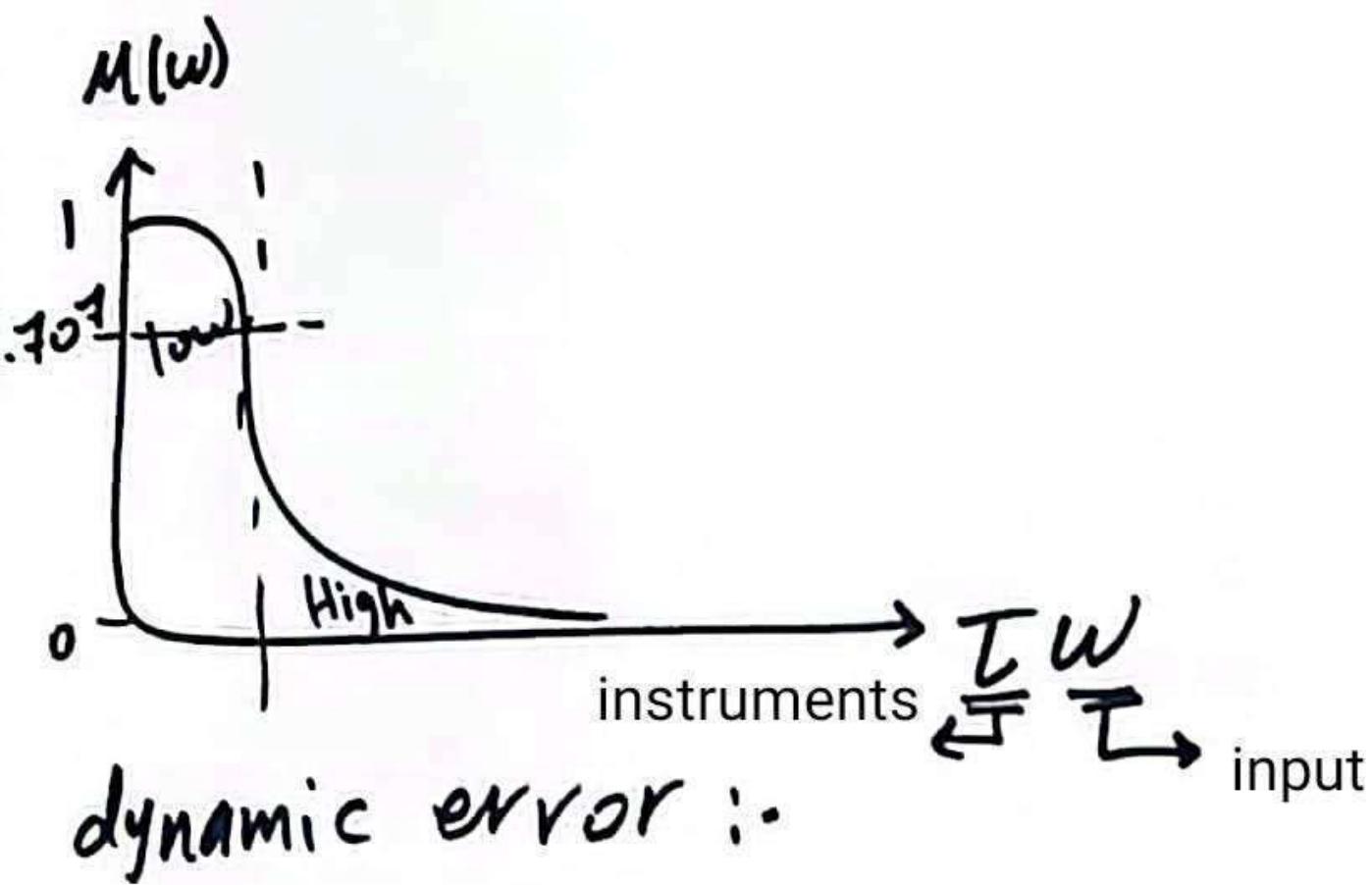
step

يكون في البداية الـ response صفر ويبلاش بصير زي الزيرو ، يعني يلاش يصير واحد ، في البداية الـ 1 صفر والخطأ response



- $M(\omega) \rightarrow$  magnitude
- $\phi(\omega) \rightarrow$  phase shift

$$\textcircled{1} M(\omega) = \frac{1}{\sqrt{1 + \omega^2 T^2}} = \frac{B}{KA}$$



$$\delta(\omega) \propto M(\omega) - 1 \quad \text{just in periodic}$$

\* Freq univ.  $\Rightarrow$

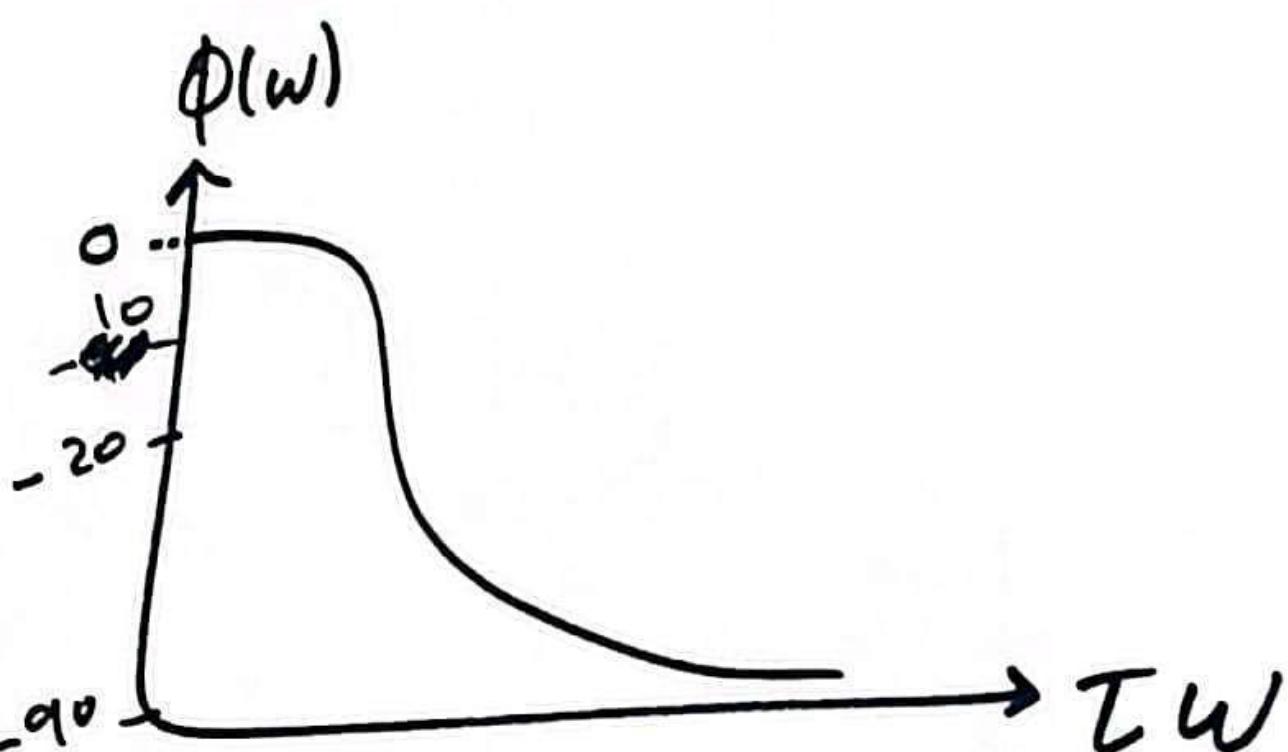
$$0.1 \leq M(\omega) \leq 1 \quad | \quad dB = 20 \log M$$

$$-3 \leq dB \leq 0 \quad |$$


---

② phase shift :

$$\phi(\omega) = \begin{cases} -\tan^{-1}(I\omega) & \text{if } I\omega > 0 \\ 0 & \text{if } I\omega = 0 \end{cases}$$



قبل های بصفحتين (( cap (3.9, 3.10) في الفصل السادس ))

the Dynamic error,  $\delta(\omega)$ , of the system is defined as ~~(3.10)~~

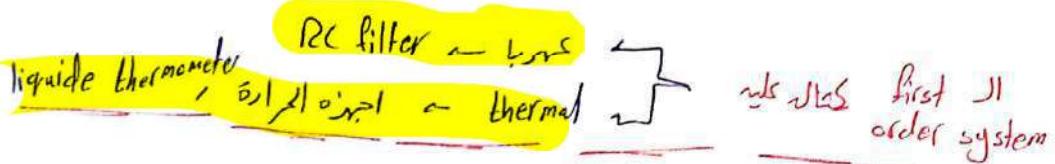
$$\delta(\omega) = M(\omega) - 1$$

For a first-order system, we define a *frequency bandwidth* as the frequency band over which  $M(\omega) \geq 0.707$ ; in terms of the decibel (plotted in Figure 3.12) defined as

$$dB = 20 \log M(\omega) \quad (3.11)$$

this is the band of frequencies within which  $M(\omega)$  remains above -3 dB.

**ex of 1st order**



## Second-order System

**important**

$$a_2 \ddot{y} + a_1 \dot{y} + a_0 y = b_0 X \quad \rightarrow \text{linear}$$

( $a_0$  ثابت)  $y$  رد فعل

$$\frac{d^2}{dt^2} y + \frac{a_1}{a_0} \frac{dy}{dt} + \frac{a_0}{a_0} y = \left( \frac{b_0}{a_0} \right) x \quad \begin{matrix} k \\ \downarrow \\ \text{input } (f(t)) \end{matrix}$$

لوكا نه اراده حين موجود است احادي

static calibration curve

pencil style pressure gage

second, first  
ثانية من ايزهي  
جهاز من ايزهي  
جهاز من ايزهي

2nd and 1st

frequency  
جهاز من ايزهي

break first order  
نقطة اندية او اول  
order

شارف طبقة اول ميل ١٠٠  
شارف طبقة اول ميل ١٠٠

B و A هذا هي العلاقة تابع M

Zero order

$$y = K X \rightarrow \text{input } (K)$$

static sensitivity

first order

$$2\dot{y} + y = K X \rightarrow \text{static sensitivity}$$

Time constant

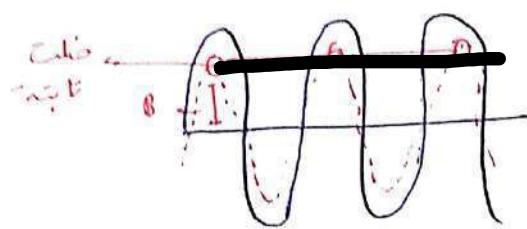
Second order

$$\frac{1}{\omega_n^2} \ddot{y} + 2\xi \dot{y} + y = K X \quad \begin{matrix} \text{damping ratio} \\ \downarrow \\ \omega_n \end{matrix}$$

Natural frequency (rad/s)

static  
sensitivity

هام خواص  
1st order



first order

second order

پہلی اور First order جسے پہلی پولہ کہا جاتا ہے

The functions  $M(\omega)$  and  $\Phi(\omega)$  represent the frequency response of the measurement system to periodic inputs. These equations and universal curves provide guidance in selecting measurement systems and system components.

response behavior. (We can predict the dynamic behavior if the time constant and static sensitivity of the system and the range of input frequencies are all known.)

$$f = \frac{\omega}{2\pi}$$

e.g.  
 $\omega = 7.17 \text{ rad/s}$   
 $\omega = 7.17 \text{ s}^{-1}$

$\omega = 120\pi \text{ rad/s}$   
 $f = ?$   
 $f = \frac{\omega}{2\pi} = \frac{120\pi}{2\pi} = 60 \text{ Hz}$

### Example 3.6

A temperature sensor is to be selected to measure temperature within a reaction vessel. It is suspected that the temperature will behave as a simple periodic waveform with a frequency somewhere between 1 and 5 Hz. Sensors of several sizes are available, each with a known time constant. (Based on time constant, select a suitable sensor, assuming that a dynamic error of  $\pm 2\%$  is acceptable.)

KNOWN

$$1 \leq f \leq 5 \text{ Hz}$$

$$|\delta(\omega)| \leq 0.02$$

ASSUMPTIONS First-order system

$$F(t) = A \sin \omega t$$

FIND Time constant,  $\tau$

SOLUTION With  $|\delta(\omega)| < 0.02$ , we would set the magnitude ratio between  $0.98 \leq M \leq 1.02$ . From Figure 3.12, we see that first-order systems never exceed  $M = 1$ . So the constraint becomes  $0.98 \leq M \leq 1$ . Then,

$$0.98 \leq M(\omega) = \frac{1}{\sqrt{1 + (\omega\tau)^2}} \leq 1$$

From Figure 3.12, this constraint is maintained over the range  $(0 \leq \omega\tau \leq 0.2)$ . We can also see in this figure that for a system of fixed time constant, the smallest value of  $M(\omega)$  will occur at the largest frequency. So with  $\omega = 2\pi f = 2\pi(5)\text{rad/s}$  and solving for  $M(\omega) = 0.98$  yields,  $\tau \leq 6.4 \text{ ms}$ . Accordingly, a sensor having a time constant of 6.4 ms or less will work.

قسمت على  $\omega$

# **Problems**

First order :- step input

$$* \mathcal{T} \dot{y} + y = K \underline{\underline{F(t)}} \quad \text{input}$$
$$\mathcal{T} = \frac{1}{\omega_n^2}$$

$$* \text{fraction error} \leftarrow \frac{y_t - y_\infty}{y_0 - y_\infty} = e^{-t/\mathcal{T}}$$

$$* T = T_0 + (T_0 - T_\infty) e^{-t/\mathcal{T}}$$

for periodic:

$$* \mathcal{T} \dot{y} + y = K A \sin \omega t \quad \text{"rad"}$$

$$* \phi(\omega) = -\tan^{-1}(\omega \mathcal{T}) \quad \text{phase shift}$$

$$* M(\omega) = \frac{1}{\sqrt{1 + \omega^2 \mathcal{T}^2}} \times \text{Mag}^+$$

dynamic error: } Time lag:

$$\Rightarrow S(\omega) = M(\omega) - 1 \quad \left\{ \begin{array}{l} \phi \\ M(\omega) = \frac{\phi}{\omega} \end{array} \right.$$

$$\underline{\underline{3.2}} \quad \underline{\underline{75\%}} \quad 190\% \quad \underline{\underline{95\%}} \\ \text{Tr} \qquad \qquad \qquad \frac{1}{T_s}$$

90% input  $\rightarrow$  10% error

$$[ = e^{-\frac{t}{T_s}} \quad \text{جذب:}$$

$$\textcircled{a} \quad 0.4 \frac{1}{T_s} + T_s = 4 u(t)$$

$$T_s \dot{y} + y = K u(t)$$

$$T_s = \underline{0.4} \quad // K = 4$$

$$\textcircled{b} \quad \frac{10}{100} = e^{-t/0.4} \rightarrow t = 0.9215$$

$$\textcircled{c} \quad \frac{25}{100} = e^{-t/0.4} \rightarrow t = 0.5545$$

$$\textcircled{d} \quad \frac{75}{100} = e^{-t/0.4}$$

$$\textcircled{e} \quad \frac{5}{100} = e^{-t/0.4} \rightarrow t = 1.198$$

3.3]

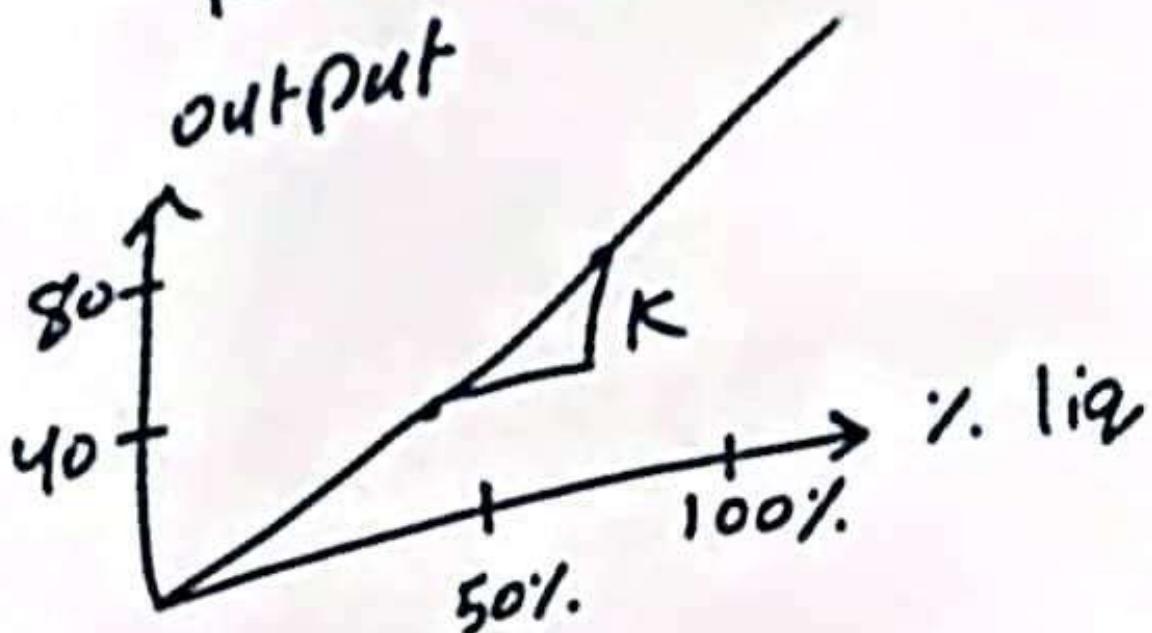
100% liq  $\rightarrow$  80 unit

0% liq  $\rightarrow$  0 unit

50% liq / 50% vapor  $\rightarrow$  40 unit

يطلب السؤال

$$K = ? ?$$



$$K = \frac{dy}{dx} = \frac{80 - 40}{100 - 50}$$

$$= 0.8 \text{ unit / \% liq}$$

$\rightsquigarrow K \uparrow$ , better

$$\textcircled{1} \quad 5\dot{y} + \frac{5y}{\tau} = u(t) \quad \div 5$$

Ta5%.

Ta0%.

$$\rightarrow \dot{y} + \frac{y}{\tau} = \frac{1}{5} u(t) \quad \tau = 75\%$$

$$\tau = 1 / K = \frac{1}{5}$$

→ 25%.

$$T75\% \rightarrow t = 1.3863 \text{ s}$$

$$\frac{25}{100} = e^{-t/1}$$

$$\rightarrow t = 1.3863 \text{ s}$$

$$T010\% \rightarrow 10\%$$

$$\frac{10}{100} = e^{-t/1} \rightarrow t = 2.3065$$

$$T015\% \rightarrow 5\%$$

$$\frac{5}{100} = e^{-t/1} \rightarrow t = 2.9957 \text{ s}$$

3.4

$$0.5 \dot{y} + y = F(t)$$

$$\tau = 0.5$$

$$K = 1$$

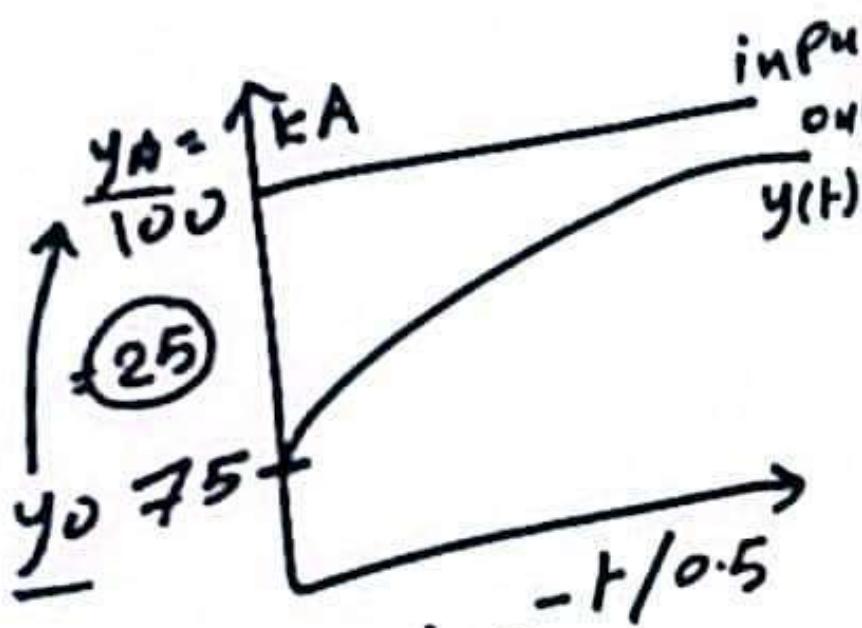
75 output

100 input

$$y(t) = KA + (y_0 - KA)e^{-t/\tau}$$

relationship of output and input

يطلب السؤال



$$y(t) = 100 + (75 - 100) e^{-t/0.5}$$

$$F(t) = 75 + 25 u(t)$$

3.9)  $\delta(\omega) = ?$

periodic  
first order

$$f = 2 \text{ Hz}$$

السؤال طالب dynamic error

$$\tau = 0.7 \text{ s}$$

$$\delta(\omega) = \frac{\underline{\mu(\omega)}}{1} - 1$$

من قانون f

$$\mu(\omega) = \frac{1}{\sqrt{1 + \tau^2 \omega^2}}$$

$$= \sqrt{1 + 0.7^2 + 4\pi^2}$$

$$\mu(\omega) = 0.11295$$

$$\omega = 2\pi f$$

$$\omega = 4\pi \text{ rad/s}$$

$$\delta(\omega) = 1 - 0.11295$$

$$= 0.88705$$

$$\boxed{\delta = 89 \%}$$

3.11) first  $\sigma^{10}$   
 $T = 2^s$   
 $S = \pm 2\%$   
 $S = M(\omega) - 1$   
 $\frac{\pm 0.2}{100} = M(\omega) - 1$

$\mu(\omega) = 1 \mp 0.02$   
 $0.98 \leq M(\omega) \leq 1.02$

$\mu(\omega) = \frac{1}{\sqrt{1 + 2^2 \omega_{max}^2}}$

$0.98 = \frac{1}{\sqrt{1 + 2^2 \omega_{max}^2}}$

$\omega_{max} = 0.101529 \text{ rad/s}$   
 $\omega = 2\pi f \rightarrow f = 0.01616 \text{ Hz}$

"B = ?!"

$\omega_{max} \rightarrow M_{min}$   
 $\omega_{min} \rightarrow M_{max}$

Fmax = ??

السؤال طالب ماكس

$M(\omega) = \frac{1}{\sqrt{1 + 2^2 \omega^2}} = 1$

$\omega_{min} = 0$

$f_{min} = 0$

$$B \rightarrow \frac{\phi(\omega)}{\omega}$$

$$= \frac{-0.2}{0.101529}$$

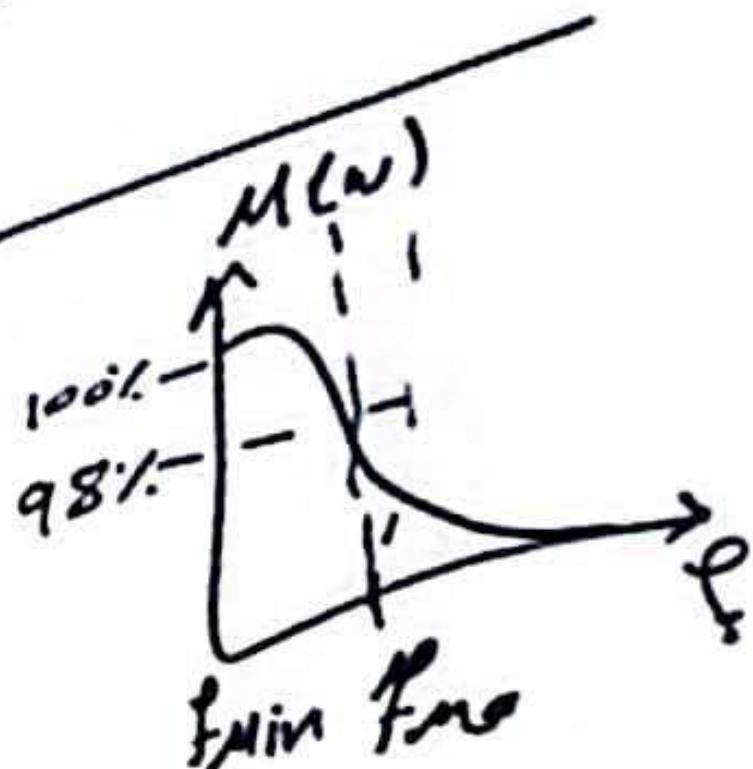
$$= \Theta 1.97^s \\ \downarrow \\ \text{time lag}$$

$$\delta_{\max} = 0.01616$$

$$\delta_{\min} = 0$$

$$\left| \begin{array}{l} \phi(\omega) = -\tan^{-1}(2 \times 10^{-10}) \\ \phi = -0.002 \text{ rad} \end{array} \right.$$

الفرع ب طالب  
lag



$$\phi(\omega) = -\tan^{-1}(2 \omega J)$$

$$3.13] \quad T = 0.15$$

$$T(t) = 115 + 12 \sin \frac{\pi}{2} t {}^{\circ}$$

$$K = 5 \text{ mV/}{}^{\circ} \quad \delta = ?$$

$$T(0) = 115 {}^{\circ} \quad \beta = ?$$

$$\boxed{\omega = 2}$$

$$\delta = 1 - M(\omega)$$

$$\begin{aligned} & 1 - 0.957826 \\ & = 0.042174 \end{aligned}$$

$$M(\omega) = \frac{1}{\sqrt{1 + 2^2 \omega^2}}$$

$$\begin{aligned} & = \frac{1}{\sqrt{1 + 0.15^2 \times 2^2}} \\ & = 0.957826 \end{aligned}$$

$$\delta = 4.2174\%$$

$$\phi = -\tan^{-1}(Tw)$$

$$= -0.2915$$

$$B = \frac{\phi}{\omega}$$

$$= \frac{-0.2915}{2}$$

$$\begin{aligned} & = -0.14575 \\ & = 0.14575 \text{ lag} \end{aligned}$$

طالب dynamic error and  
phase lag

3.15

$$R = 1 \times 10^6 \Omega$$

$$f = 1000 \text{ Hz}$$

$$\mu(\omega) = 50\%$$

$$\tau = L/R$$

$$L = ??$$

$$\omega = 2\pi f$$

$$= 2000\pi \text{ rad/s}$$

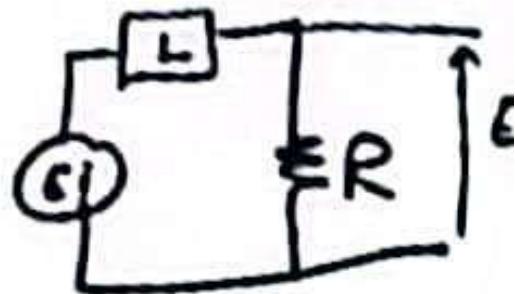
$$\underline{\mu}(\omega) = \frac{1}{\sqrt{1 + \underline{\zeta}^2 \frac{\omega}{\underline{\omega}}^2}}$$

$$\frac{50}{100} = \frac{1}{\sqrt{1 + \underline{\zeta}^2 + (2000\pi)^2}}$$

$$\tau = 0.00276$$

$$\tau = \frac{L}{R} \rightarrow 0.00276 = \frac{L}{1 \times 10^6}$$

$$L = 276 \text{ HZ}$$



$$L \frac{dI}{dt} + RI = E(t)$$

$$\textcircled{P} L \dot{I} + RI = E(t) \leftarrow \tau \dot{y} + y = k u(t)$$

$$\boxed{\frac{L}{R}} \dot{I} + I = \frac{1}{R} E(t)$$

$$\tau = \frac{L}{R} \Leftrightarrow$$

***second order***

***second order***

***second order***

## e.g of 2nd order devices

second order

1-

2-

system. Examples of second-order instruments include accelerometers and pressure transducers (including microphones and loudspeakers.)

In general, a second-order measurement system subjected to an arbitrary input,  $F(t)$ , can be described by an equation of the form

$$a_2\ddot{y} + a_1\dot{y} + a_0y = F(t) \quad (3.12)$$

where  $a_0$ ,  $a_1$ , and  $a_2$  are physical parameters used to describe the system and  $\ddot{y} = d^2y/dt^2$ . This equation can be rewritten as

$$\frac{1}{\omega_n^2}\ddot{y} + \frac{2\xi}{\omega_n}\dot{y} + y = KF(t) \quad (3.13)$$

where

$$\omega_n = \sqrt{\frac{a_0}{a_2}} = \text{(natural frequency of the system)}$$

$$\xi = \frac{a_1}{2\sqrt{a_0 a_2}} = \text{(damping ratio of the system)}$$

قبل بصفحتين

ما يجب ملء اهتمام وجد امثلة على  
الذرة في المقدمة

$0 \leq \xi < 1$  (underdamped system solution)

$$y_h(t) = Ce^{-\xi\omega_n t} \sin(\omega_n \sqrt{1 - \xi^2} t + \Theta) \quad (3.14a)$$

$\xi = 1$  (critically damped system solution)

$$y_h(t) = C_1 e^{\lambda_1 t} + C_2 t e^{\lambda_1 t} \quad (3.14b)$$

$\xi > 1$  (overdamped system solution)

$$y_h(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \quad (3.14c)$$

The homogeneous solution determines the transient response of a system. (The damping ratio,  $\xi$ , is a measure of system damping, a property of a system that enables it to dissipate energy internally.)

(2 input)  $\rightarrow$  متغير ثابت للمعادلة  $\rightarrow$  First order

$f(t) \rightarrow$  step input

$\theta(t) \rightarrow$  simple periodic function input

④ second-order  $\rightarrow$  [Microphones and loud speakers]  
 → Example: Microphones and loud speakers  
 include → accelerometers and  
 pressure transducers.

$$* \underline{a_2} \ddot{y} + \underline{a_1} \dot{y} + \underline{a_0} \textcircled{y} = \underline{b_0} X$$

$$\frac{1}{w_n^2} \ddot{y} + \frac{2\zeta}{w_n} \dot{y} + 1 y = K f(x)$$

↳ natural frequency [rad/s]

$$w_n = \sqrt{\frac{a_0}{a_2}}$$

$\zeta \rightarrow$  damping ratio

$$\zeta = \frac{a_1}{[2 \sqrt{a_0 a_2}]} \rightarrow \text{critical damping coeff}$$

$$CC = 2 \sqrt{a_0 a_2}$$

$$\zeta = \frac{a_1}{CC}$$

((Step input function))

## Step Function Input

Again, the step function input is applied to determine the general behavior and speed at which the system will respond to a change in input. The response of a second-order measurement system to a step function input is found from the solution of Equation 3.13, with  $F(t) = AU(t)$ , to be

**Underdamping**  $y(t) = KA - KAE^{-\zeta \omega_n t} \left[ \frac{\zeta}{\sqrt{1-\zeta^2}} \sin(\omega_n t \sqrt{1-\zeta^2}) + \cos(\omega_n t \sqrt{1-\zeta^2}) \right] \quad 0 \leq \zeta < 1 \quad (3.15a)$  **under damped**

**critical Damping**  $y(t) = KA - KA(1 + \omega_n t)e^{-\omega_n t}$   $\zeta = 1$  (3.15b)

System

over Damped

$$y(t) = KA - KA \left[ \frac{\zeta + \sqrt{\zeta^2 - 1}}{2\sqrt{\zeta^2 - 1}} e^{(-\zeta + \sqrt{\zeta^2 - 1})\omega_n t} + \frac{\zeta - \sqrt{\zeta^2 - 1}}{2\sqrt{\zeta^2 - 1}} e^{(-\zeta - \sqrt{\zeta^2 - 1})\omega_n t} \right] \quad \zeta > 1 \quad (3.15c)$$

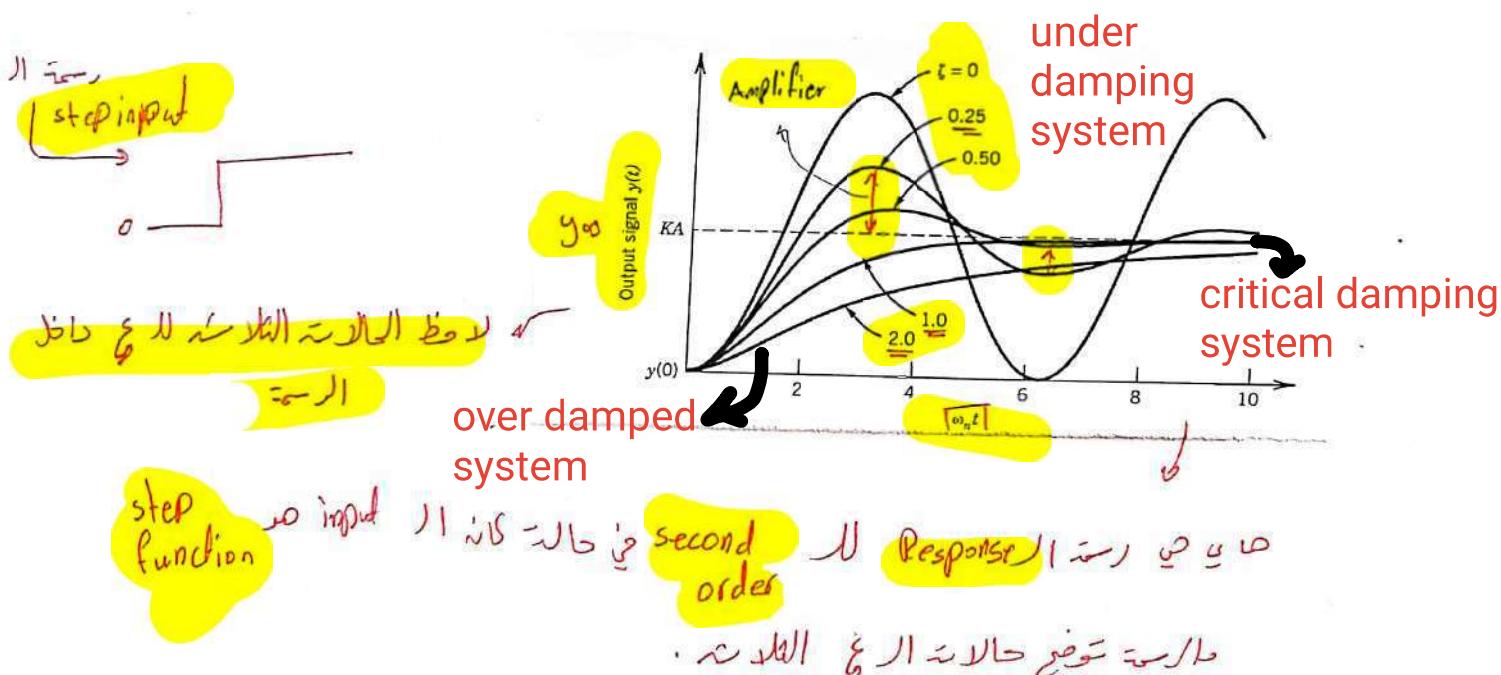
where we set the initial conditions,  $y(0) = \dot{y}(0) = 0$  for convenience.

Equations 3.15a-c are plotted in Figure 3.14 for several values of  $\zeta$ . The interesting feature is the transient response. (For under-damped systems, the transient response is oscillatory about the steady value and occurs with a period.)

$$T_d = \frac{2\pi}{\omega_d} = \frac{1}{f_d} \quad (3.16)$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} \quad (3.17)$$

where  $\omega_d$  is called the *ringing frequency*. In instruments, this oscillatory behavior is called "ringing." (The ringing phenomenon and the associated ringing frequency are properties of the measurement system and are independent of the input signal. It is the free oscillation frequency of a system displaced from its equilibrium.)



د ط کلمات دنیا ر (zeta) حجیر ؟!

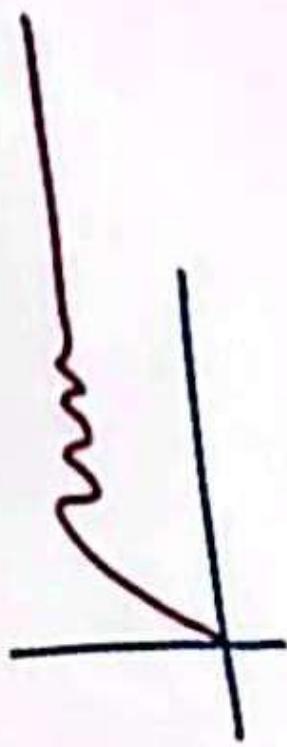
(١) کلاردنالر damping میاں system نیل خٹ کئے اتھ output

٢) **الـ Amplifier** هو الأداة التي تزيد من القوة وتحافظ على الصورة.

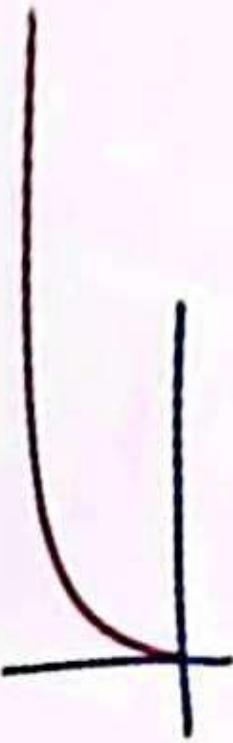
Amplitude  $\|x\|$  is  $\zeta \sqrt{1 - \zeta^2}$  (for  $\zeta < 1$ )

values for  $\rho$  :-

- ④  $0 \leq \rho \leq 1$       [under damped system]
- ⑤  $\rho = 1$       [critically damped system]
- ⑥  $\rho > 1$       [over damped system]



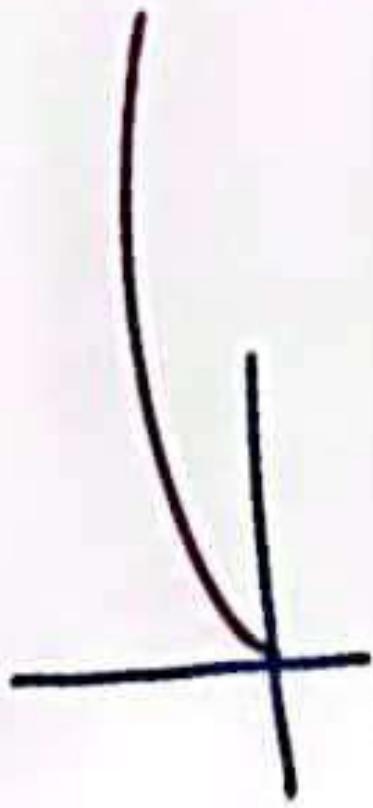
- ④  $\rho = 1$       [critically damped system]



- ⑤  $\rho > 1$       [over damped system]

③  $\omega > 1$

[over damped system]



$\zeta = 0$  [undamped system]

$$\omega_0 t = \theta$$



$$\textcircled{1} \quad 0 \leq \xi \leq 1$$

$$y(t) = kA - kA e^{-\xi \omega_n t} \left[ \frac{\xi}{\sqrt{1-\xi^2}} \sin(\omega_n t \sqrt{1-\xi^2}) + \cos(\omega_n t \sqrt{1-\xi^2}) \right]$$

$\sin \cos + \underline{\text{حوله}}$

$$\textcircled{2} \quad \xi = 1$$

$$y(t) = kA - kA (1 - \omega_n t) e^{-\omega_n t}$$

$\sin \cos + \underline{\text{غيره}}$

$$\textcircled{3} \quad \xi > 1$$

$\sin \cos + \underline{\text{حوله}}$

$$y(t) = kA - kA \left[ \frac{\xi + \sqrt{\xi^2 - 1}}{2\sqrt{\xi^2 - 1}} e^{(-\xi + \sqrt{\xi^2 - 1})\omega_n t} + \frac{\xi - \sqrt{\xi^2 - 1}}{2\sqrt{\xi^2 - 1}} e^{(-\xi - \sqrt{\xi^2 - 1})\omega_n t} \right] +$$

$\oplus$   $\zeta \rightarrow$  is a measure of system damping.

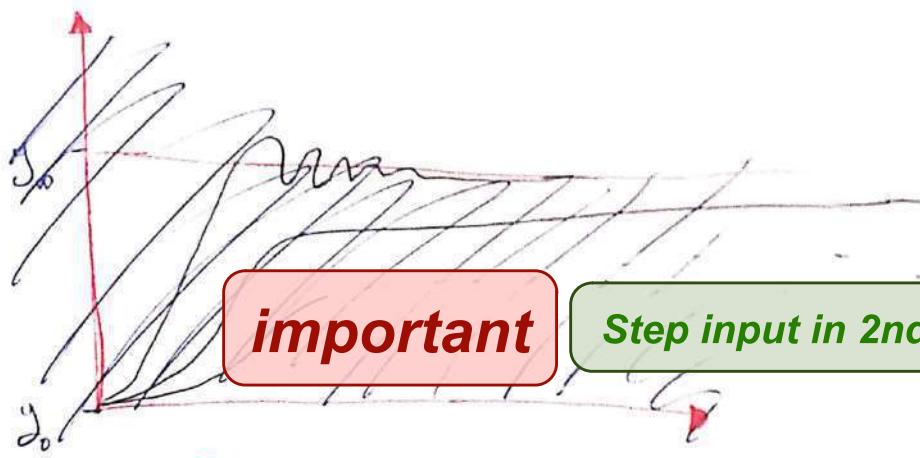
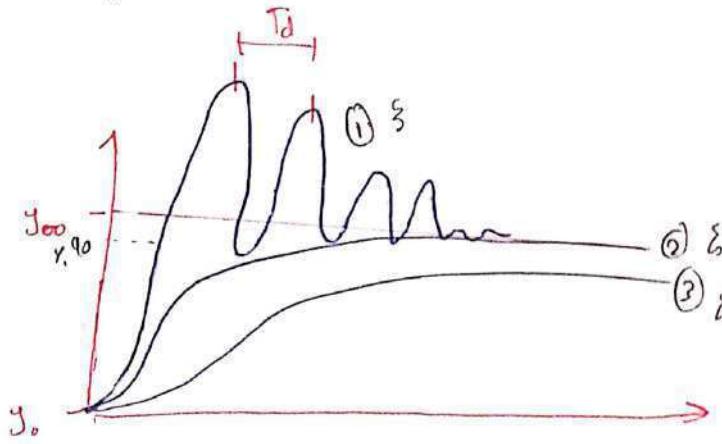
~~@property~~ of system that enable it to dissipate energy internally.

function input:

step function general behavior in response to change in input

→ determine the general response which is a function of time  $F(t) = Au(t)$

→ For under damped system  
Transient response is oscillatory  
① Transient response occurs with  
about steady value & occurs with  
a period

**important****Step input in 2nd order**under damp  $\xi < 1$ critically damp  $\xi = 1$ over damp  $\xi > 1$ 

(1) → under damp

(2) → critically damp

(3) → over damp

نحو ٩٠٪ من خط الباقي في الموجة؟ Response

under damp &gt; critically damp &gt; over damp

مشكلة الـ under damping

الـ oscillations  $\xi < 1 \rightarrow$  under damp

مشكلة الـ critical damping

 $\xi > 1 \rightarrow$  over damp

صافي كهونج معادلتنا إلى الموجة

w\_d, f\_d: ringing frequency

$$w_d = w_0 \sqrt{1 - \xi^2}$$

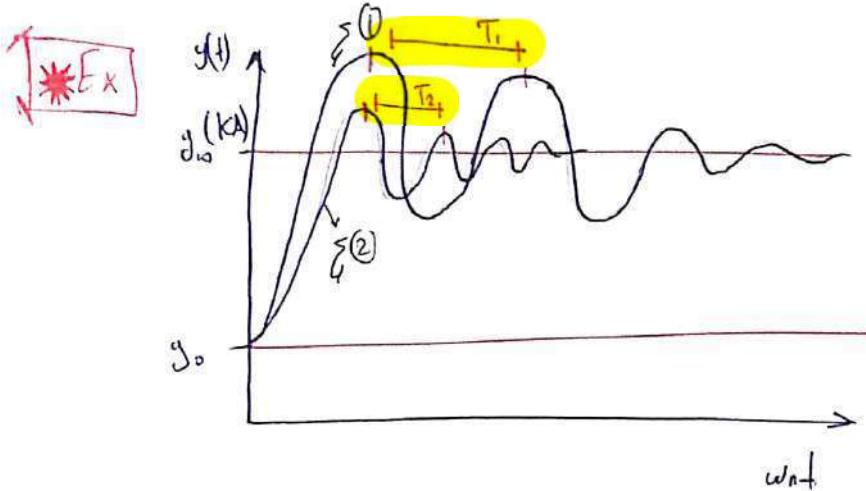
the rise time is the time required to first achieve 90% of  $(KA - y_0)$ .

w\_d &gt; 1, ξ &lt; 1

rise time or settling

$$T_d = \frac{2\pi}{w_d} = \frac{1}{f_d}$$

Figure 3.14. With this in mind, the time required for a measurement system's oscillations to settle to within  $\pm 10\%$  of the steady value,  $KA$ , is defined as its *settling time*. The constant  $k$  is approximately the ratio of the initial overshoot to the steady-state error.



حالة

$$Zeta = 0$$

rising frequency = natural frequency

$$f = \omega_n$$

أولاً الـ oscillation  $\sim$   $f_1 \sim \zeta(1) \sim 500$  هـ 80 هـ صـ المـ (1)

$f_2 > f_1$  لأن  $f_1 \sim \zeta(1) \sim 500$  هـ 80 هـ صـ المـ (2)

$$\text{معنـ} T_1 = \frac{1}{f_1}$$

$$f_1 \sim \zeta(1) \quad f_2 \sim \zeta(2) \quad \text{معنـ} T_1$$

$f_2 > f_1$  because  $t_1 > t_2$

$$\text{معنـ} f_1 \sim \zeta(1) T_1$$

$$\text{معنـ} f_2 \sim \zeta(2) T_2$$

ملاحظات

(rise and settling times). Example 3.8 describes such a test! Typically, measurement systems suitable for dynamic signal measurements have specifications that include 90% rise time and settling time. Adjust Second Order Parameters.vi explores system values and response)

### Example 3.7

Determine the physical parameters that affect the natural frequency and damping ratio of the accelerometer of Example 3.1.

**KNOWN** Accelerometer shown in Figure 3.3

**ASSUMPTIONS** Second-order system as modeled in Example 3.1

**FIND**  $\omega_n$ ,  $\zeta$

من خلال القوانين فوق

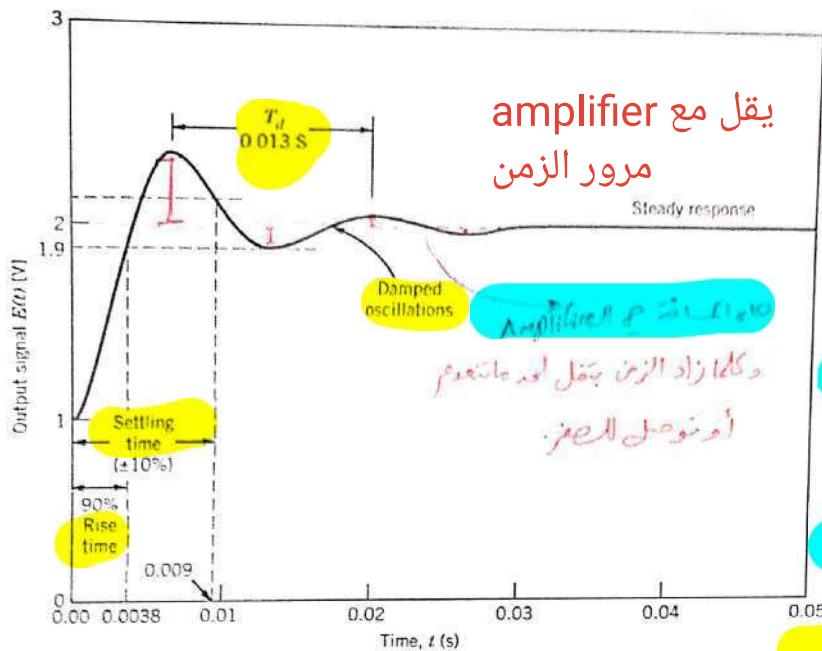


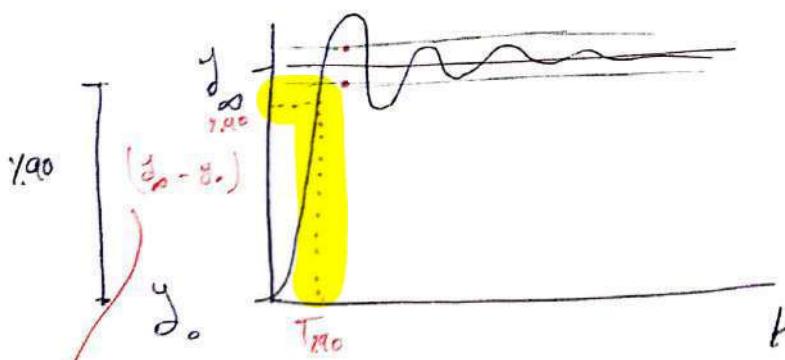
Figure 3.15 Pressure transducer time response to a step input for Example 3.8.

**SOLUTION** A comparison between the governing equation for the accelerometer in Example 3.1 and Equation 3.13 gives

$$q: \text{frictional damping}, \text{spring stiffness}, \text{parameter}$$

Accordingly, the physical parameters of mass, spring stiffness, and frictional damping control the natural frequency and damping ratio of this measurement system.

## ***important***



$$y_{\infty} \rightarrow y_{\text{infinity}} / y_{\text{steady state}}$$

$\text{Co} \rightarrow \text{giant/giant}$

مكالمة اتصالات ملحوظة في بروتوكول time Base

٢٠١٥ = error میانگین از ۰.۱۵ نهاد

هي وقته ما يكون الفرق بين العناصر تامٌ فالـ

- settling time

$\pm 10\%$  ممکن یا خوب  $\pm 5\%$

↑ period of ringing

$$\textcircled{1} \quad T_d = \frac{2\pi}{\omega_d} = \frac{1}{f_d}$$

*w<sub>d</sub>* ← *w<sub>n</sub>*

*w<sub>d</sub>* = *w<sub>n</sub>*  $\sqrt{1 - \rho^2}$

*w<sub>n</sub>* natural frequency      *ρ* damping ratio

$f = \frac{1}{T_d}$

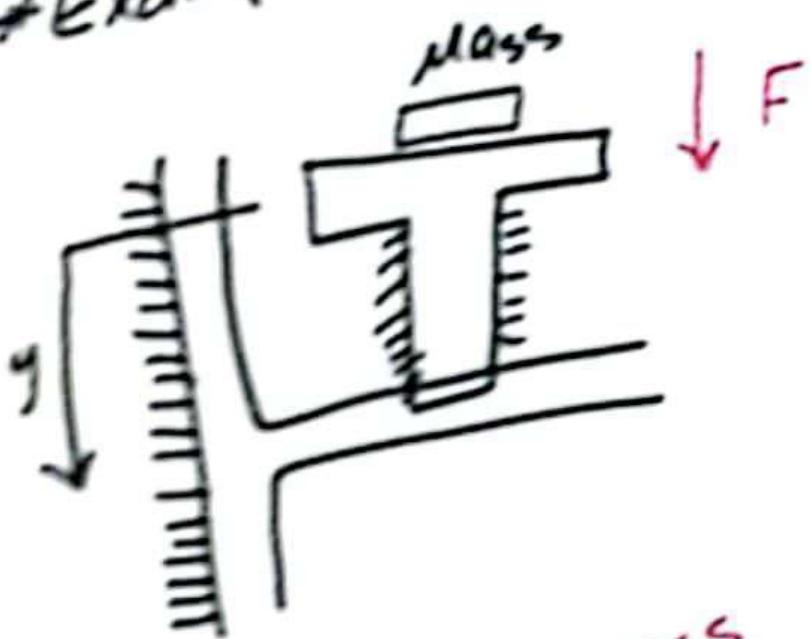
→ Ringing frequency : ringing phenomena and associated ringing frequency are properties of the measurement system and are independent of input signal

→ It's free oscillation freq for system display from its equilibrium.

→ Transient is controlled by *g, w<sub>n</sub>*

→ Time constant in 2<sup>nd</sup> define →  $\tau = \frac{1}{\zeta w_n}$

+ Example: Force



input → <sup>mass</sup>  
output → displacement

$$m\ddot{y} + c\dot{y} + Ks y = F(t)$$

$$\frac{F(t)}{k_s}$$

$$y + \textcircled{y} =$$

$$\sqrt{k_s}$$

$$-\frac{1}{k_s}$$

$$= \sqrt{\frac{k_s}{m}}$$

$$\sqrt{\frac{q_0}{\alpha_2}}$$

$$\omega n =$$

$$\sqrt{k_s}$$

$$\rightarrow k$$

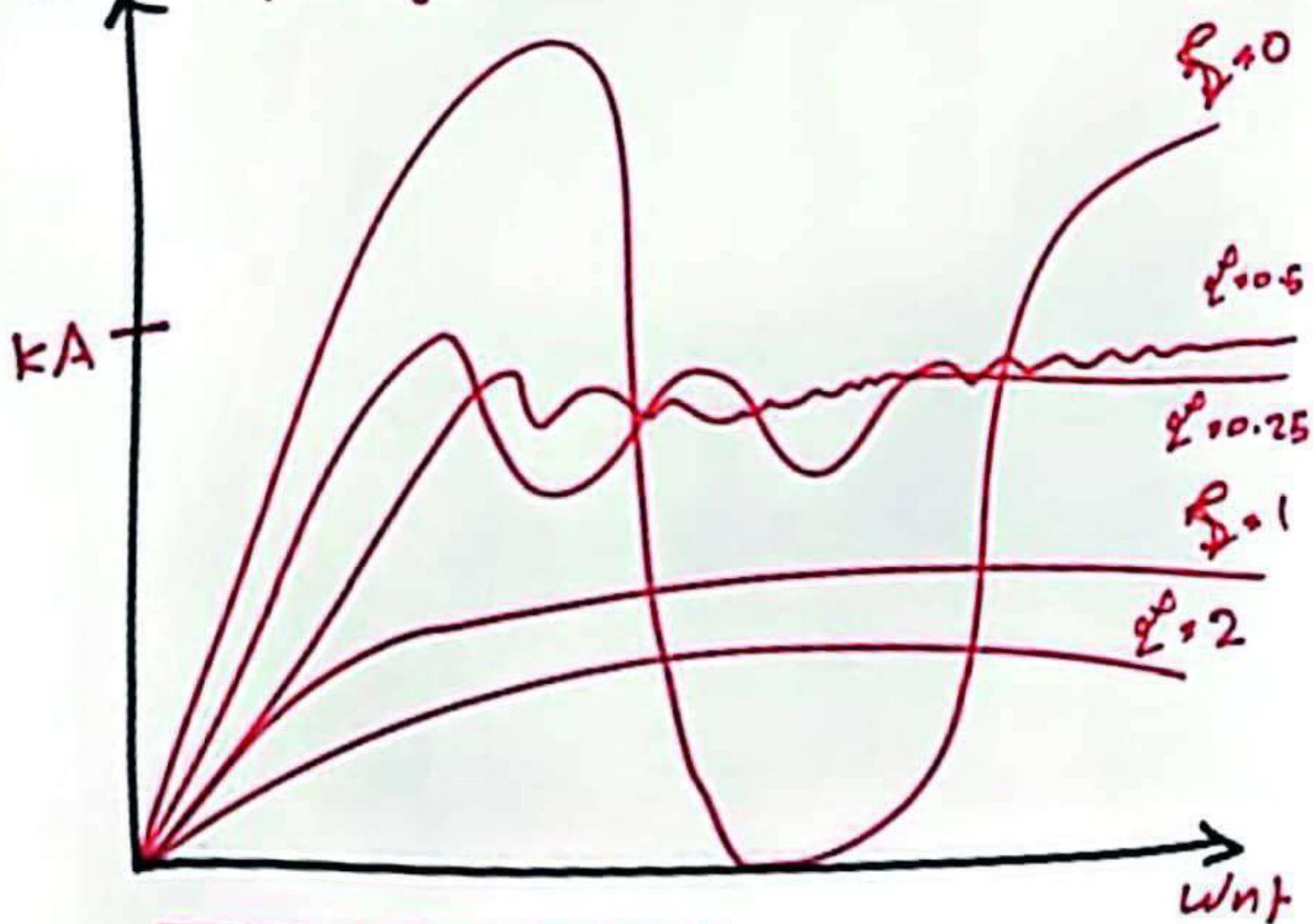
$$f = \frac{a_1}{2\sqrt{a_0 a_2}}$$

$$cc = 2\sqrt{1 + \frac{\pi}{ka}}$$
$$\times 2\sqrt{\frac{\pi}{Ea}}$$

$$f = \frac{c}{2\sqrt{\frac{\pi}{ka}}}$$

\* 2nd order time response to step input:-

$y(t)$  output signal



$$F(t) = A \cdot u(t)$$

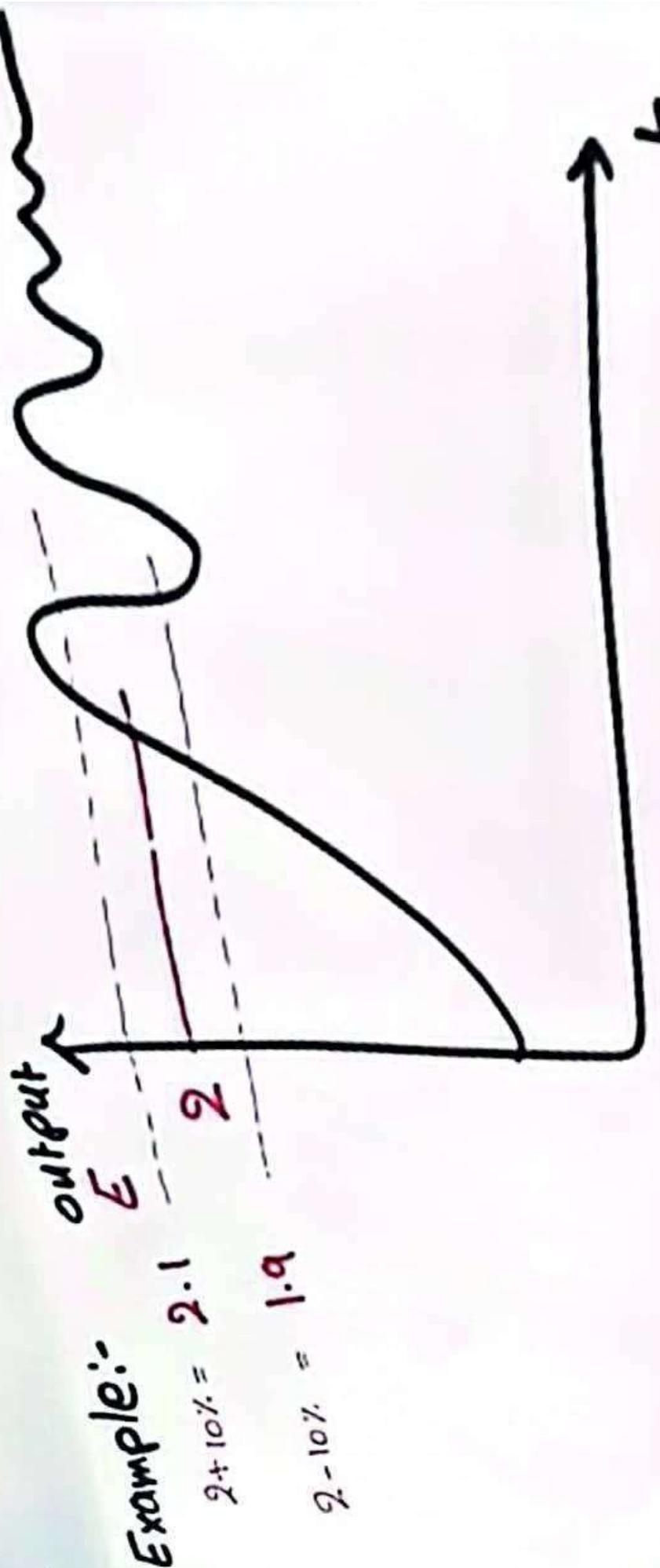
- rise time : time to reach 90% of input
- settling time : time required for measurement system's oscillation to settle with  $\pm 1\%$  of steady value [KA]
- typical  $\zeta = 0.6 - 0.8$

- \* increase  $L$ 
  - reduce oscillation
  - system response becomes slower
  
- \* increase  $w_n$ 
  - system response becomes better <sup>(Fast)</sup>
  
- \* decreasing  $L$ 
  - Rise time is reduced

## ② Time response

Transient response + steady state

$$y(t) = \frac{y_1 - (y_{\max})_1 - y_0}{1 + \frac{\ln(y_1/y_2)}{(2\pi)^2}}$$
$$\therefore$$
$$y_1 = (y_{\max})_1 - y_0$$



$$\tau_1 = K P(0) = 1 \text{ ms}$$

$$\tau_2 = K P(\infty) = 2 \text{ ms}$$

$$K = P_0 / P_{\infty} = 1.05$$

المطلب هو  $\text{rise time and settling time}$

steady state =  $K P_{\infty}$

$K A$

$$P_0 = 1 \text{ atm} \rightarrow E_0 = K_P(0) = 1 \text{ V}$$

$$P_{\infty} = 2 \text{ atm} \rightarrow E_{\infty} = K_P(\infty) = \underline{\underline{2 \text{ V}}}$$

$$\left| \zeta \right| = 1 \text{ V/atm}$$

$$\boxed{\text{steady} = \cancel{K_P} E_0}$$

$$\boxed{1.9 \leq E(R) \leq 2.1}$$

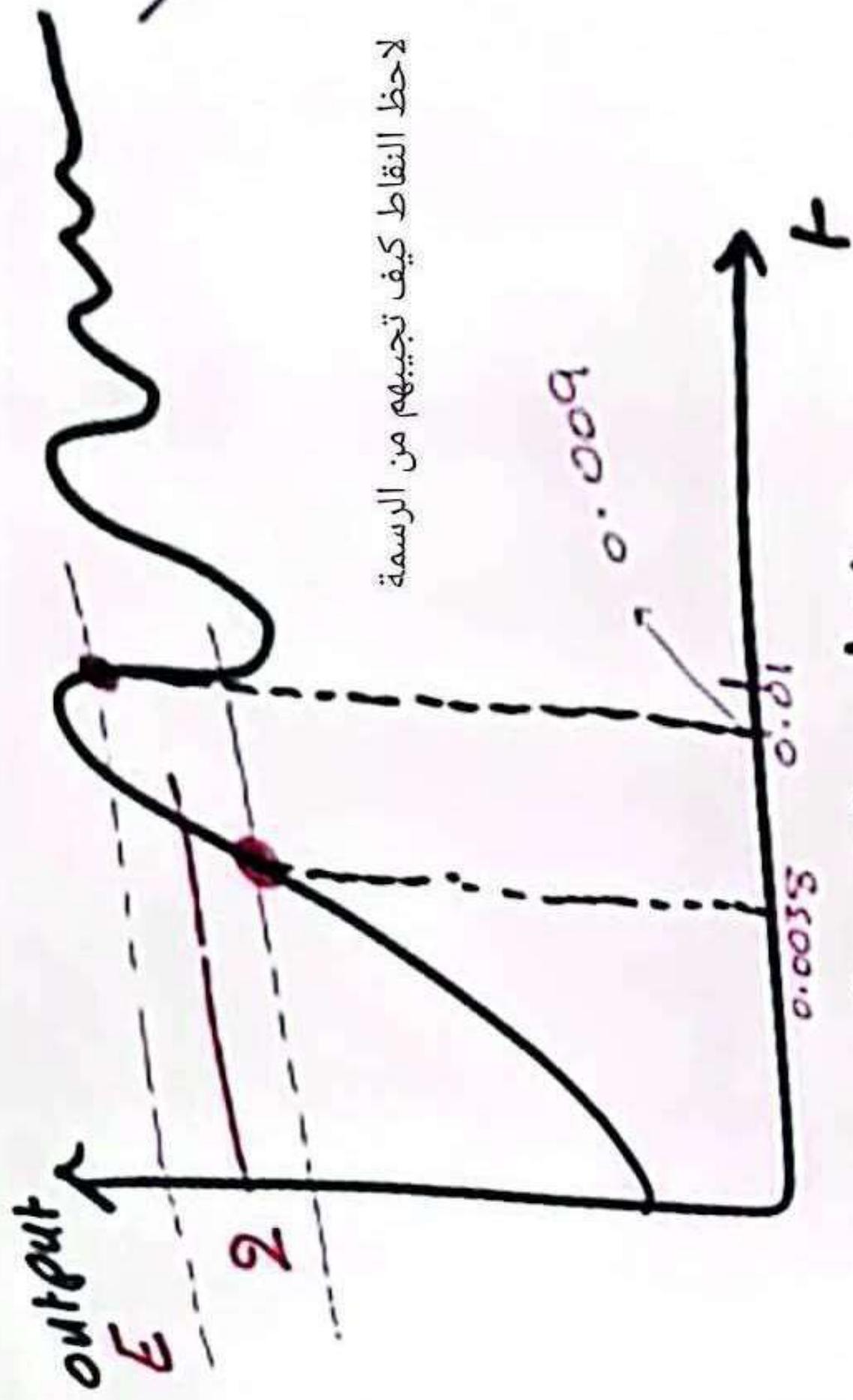
$$\boxed{1 \text{ mS}}$$

$$4 \times 10^{-3} \text{ s}$$

$$\begin{aligned} \text{rise time} &\rightarrow T_r = \boxed{4 \text{ ms}} \\ \text{settling time} &\rightarrow T_s = \boxed{9 \text{ ms}} \end{aligned}$$

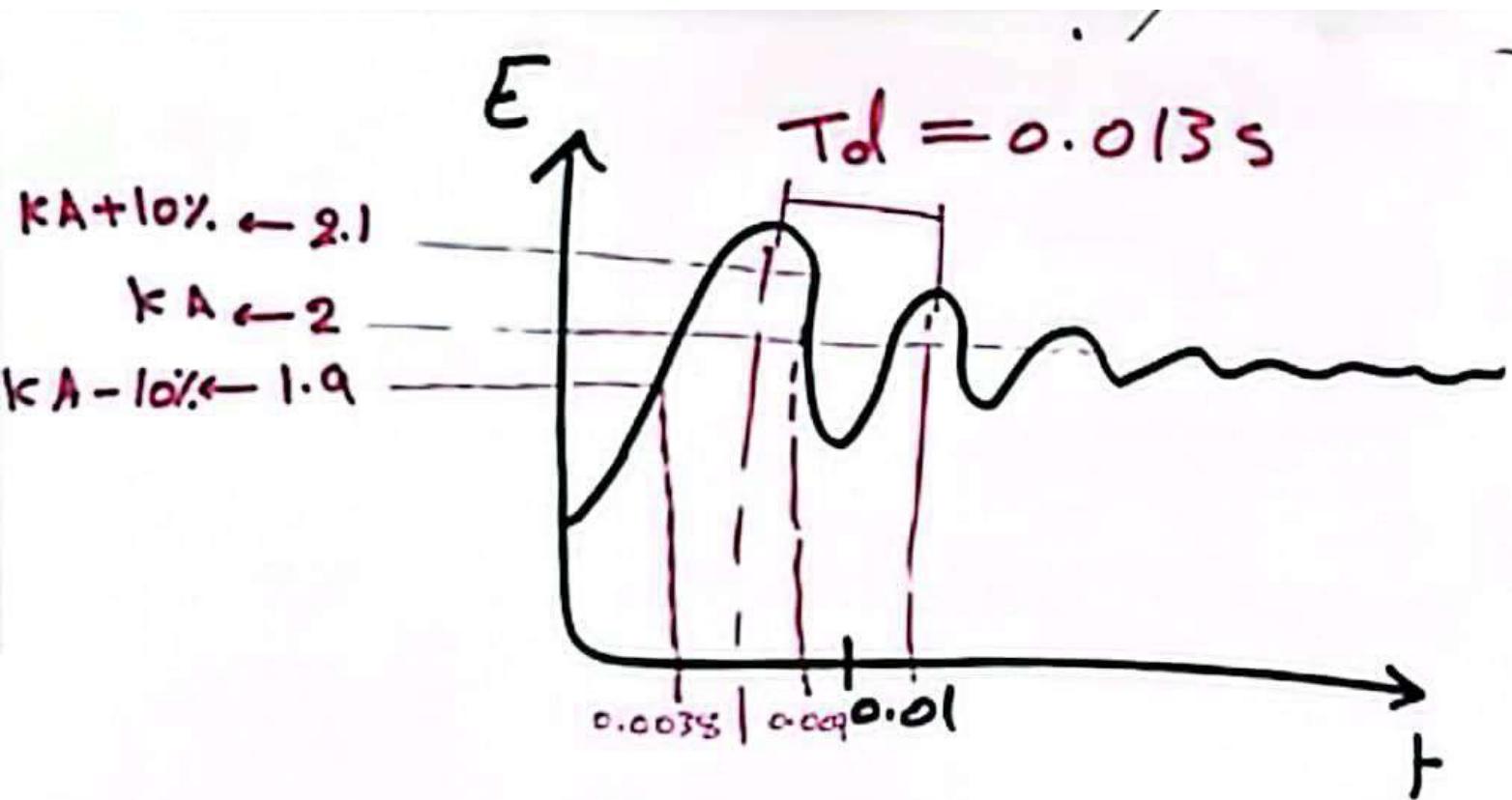
KA + ERROR = Settling time

KA - Error = rise time



$$\psi_0 = \kappa \rho(0) = 20$$

$$\psi \rightarrow \psi_0 = \kappa \rho(\infty) = 20$$



$\rightarrow w_d$

المطلب الثالث هو حساب  
wd / raining frequency

$$T_d = \frac{2\pi}{w_d}$$

$$0.013 = \frac{2\pi}{w_d}$$

$$\boxed{w_d = 483 \text{ rad/s}}$$

\* simple periodic function input

→ response of periodic input

$y(t) = \text{transient response} + \text{steady response}$

Time response

→ response 2<sup>nd</sup> for periodic In

$$F(t) = A \sin \omega t$$

$$y(t) = y_h + \frac{KA \sin [\omega t + \phi(\omega)]}{[1 - (\omega/\omega_n)^2]^2 + [2\{\omega/\omega_n\}]^2} \quad \text{1/1}$$

time response/ response

→ frequency - phase shift \*

$$\phi(\omega) = \tan^{-1} \left( \frac{-2\{\omega/\omega_n\}}{1 - (\omega/\omega_n)^2} \right)$$

\* Frequency response

→ magnitude ratio \*

$$M(\omega) = \frac{B(\omega)}{KA} = \frac{1}{[1 - (\omega/\omega_n)^2]^2 + [2\{\omega/\omega_n\}]^2}$$

$\frac{Y_2}{Y_1}$

### Example 3.8

(c) The curve shown in Figure 3.15 is the recorded voltage output signal of a diaphragm pressure transducer subjected to a step change in input. From a static calibration, the pressure–voltage relationship was found to be linear over the range 1 atmosphere (atm) to 4 atm with a static sensitivity of 1 V/atm. For the step test, the initial pressure was atmospheric pressure,  $p_a$ , and the final pressure was  $2p_a$ . Estimate the rise time, the settling time, and the ringing frequency of this measurement system.)

*KNOWN*  $p(0) = 1 \text{ atm}$

$$p_\infty = 2 \text{ atm}$$

$$K = 1 \text{ V/atm}$$

#### **ASSUMPTIONS** Second-order system behavior

**FIND** Rise and settling times;  $\omega_n$

**SOLUTION** The ringing behavior of the system noted on the trace supports the assumption that the transducer can be described as having a second-order behavior. From the given information

$$E(0) = Kp(0) = 1 \text{ V}$$

$$E_{\infty} = Kp_{\infty} = 2 \text{ V}$$

so that the step change observed on the trace should appear as a magnitude of 1 V. The 90% rise time occurs when the output first achieves a value of 1.9 V. The 90% settling time will occur when the output settles between  $1.9 < E(t) < 2.1$  V. From Figure 3.15 (the rise occurs in about 4 ms) and the settling time is about 9 ms. (The period of the ringing behavior,  $T_d$ , is judged to be about 13 ms for an  $\omega_d \approx 485$  rad/s.

## Simple periodic function Input

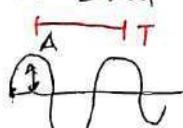
second order system رع سرچو لفنت مکانیکال

$$\frac{1}{\omega_n^2} \ddot{y} + \frac{2\xi}{\omega_n^2} \dot{y} + y = k f(t)$$

Amplitude  $\omega = A \sin \omega t$   $\leftarrow$  periodic waves  $f(t)$  have same frequency

$$T = \frac{1}{f}$$

$$\omega = 2\pi f$$



مقدمة الى الـ input و output

النسبة المئوية Magnitude (مدى) رجوع اقصى

من الأسباب أوجهة صنفه:-

### 1) ~~Pass~~ Response time

out put  $\propto \beta$ ,  $\phi$   
 Amplitude phase shift

$$\mu(\omega) = \frac{B(\omega)}{KA}$$

function  
is also  
frequency

static sensitivity

Amplitude  
of input

### Simple Periodic Function Input

The response of a second-order system to a simple periodic function input of the form  $F(t) = A \sin \omega t$  is given by

$$\text{Periodic in second} \quad y(t) = y_h + \frac{KA \sin[\omega t + \Phi(\omega)]}{\left\{ [1 - (\omega/\omega_n)^2]^2 + [2\zeta\omega/\omega_n]^2 \right\}^{1/2}} \quad (3.19)$$

with frequency-dependent phase shift

$$\Phi(\omega) = \tan^{-1} \left( -\frac{2\zeta\omega/\omega_n}{1 - (\omega/\omega_n)^2} \right) \quad (3.20)$$

The exact form for  $y_h$  is found from Equations 3.14a-c and depends on the value of  $\zeta$ . The steady response, the second term on the right side, has the general form

$$y_{\text{steady}}(t) = y(t \rightarrow \infty) = B(\omega) \sin[\omega t + \Phi(\omega)] \quad (3.21)$$

with amplitude  $B(\omega)$ . Comparing Equations 3.19 and 3.21 shows that the amplitude of the steady response of a second-order system subjected to a sinusoidal input is also dependent on  $\omega$ . So the amplitude of the output signal is frequency dependent. In general, we can define the magnitude ratio,  $M(\omega)$ , for a second-order system as

$$\text{ZAP} \quad M(\omega) = \frac{B(\omega)}{KA} = \frac{1}{\left\{ [1 - (\omega/\omega_n)^2]^2 + [2\zeta\omega/\omega_n]^2 \right\}^{1/2}} \quad (3.22)$$

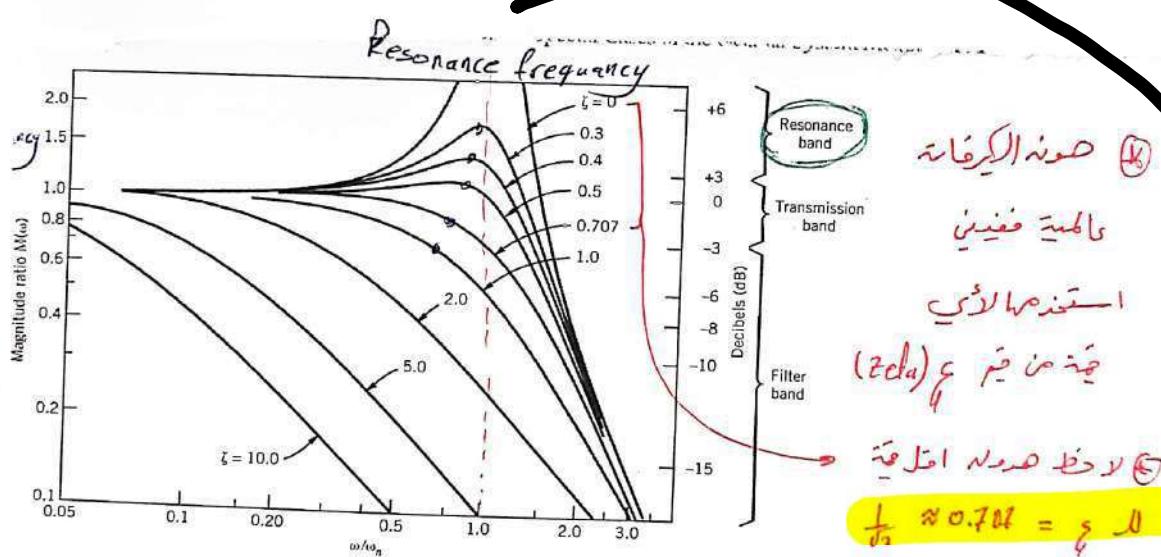
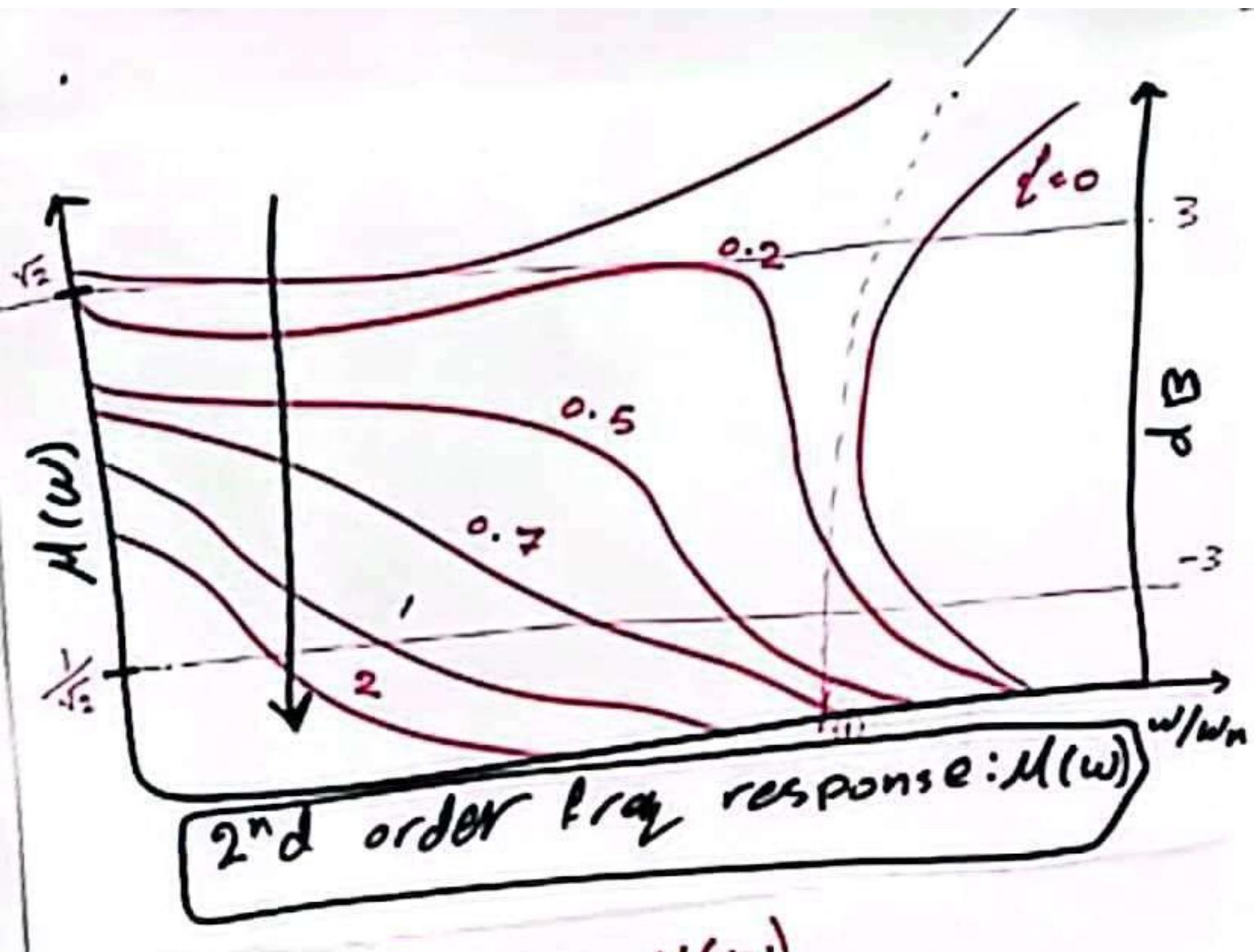


Figure 3.16 Second-order system frequency response: magnitude ratio.



\* لو كانت  $\mu = 0$  فإن المغناطيسية سوار  $m$  متساوية  $\rightarrow \frac{m}{m} = 1$  حيث أن المغناطيسية في صنف المغناطيسية تكون ذاتية بمعنى أنه المغناطيسية التي تنشأ عن نفسها

المغناطيسية ذاتية طبعاً صادراً المغناطيسية يمكن بوضوح أنه حجم المغناطيسية متساوية في كل مكان  $\rightarrow$  المغناطيسية ذاتية



$\text{dB} \rightarrow 20 \log M(\omega)$   
 بتزيد  $\zeta$  و بتقل  $M$  وبقل المرنين  
 كلما نزلنا تحت

كلما طلعننا لفوق وزادت ال  $M$  بتقل ال  $\zeta$

الافضل دائمًا اكون ما بين -3 و 3 الاحسن اكون فوق منطقة

filter band

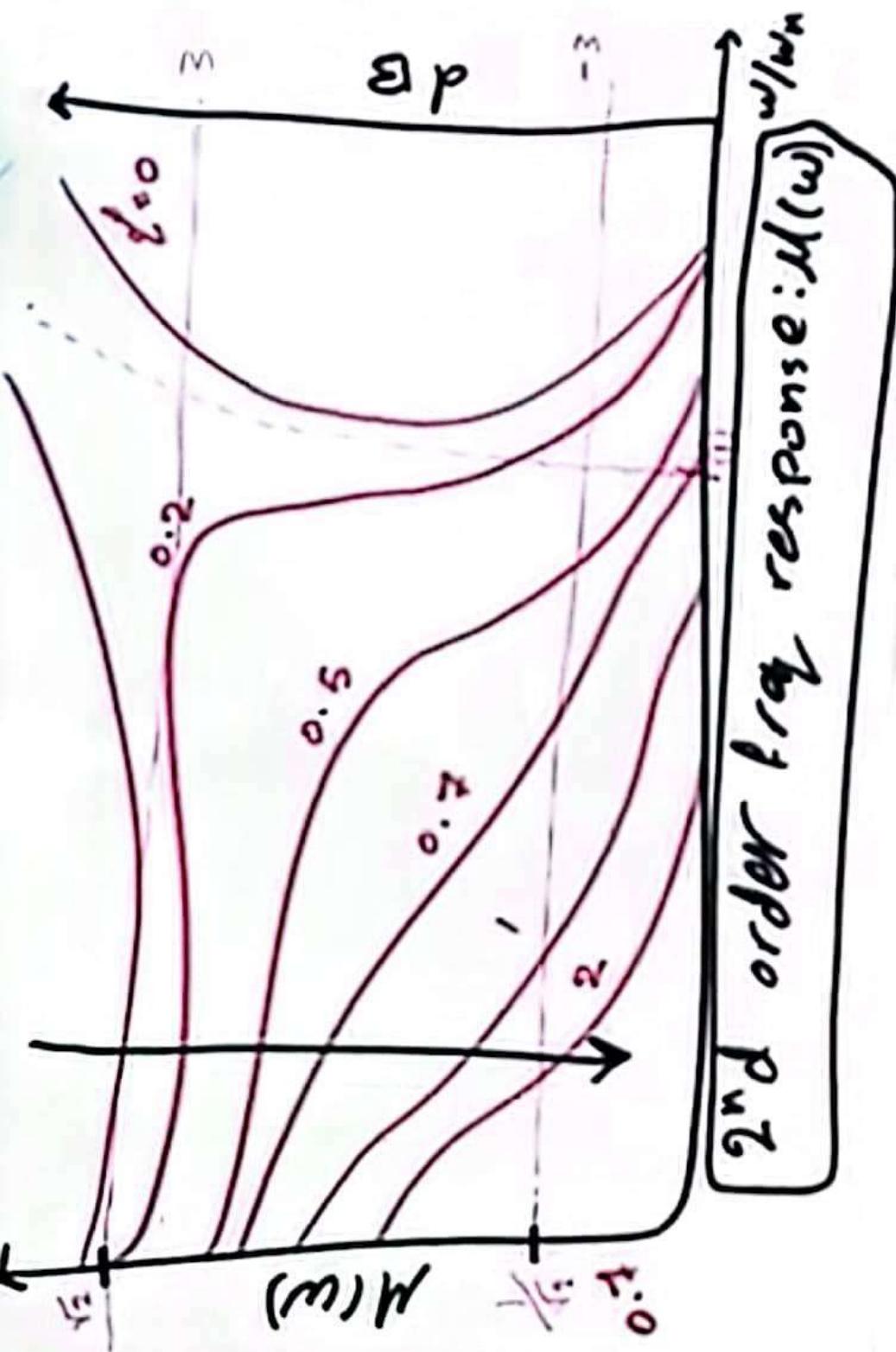
وتحت ال resonance



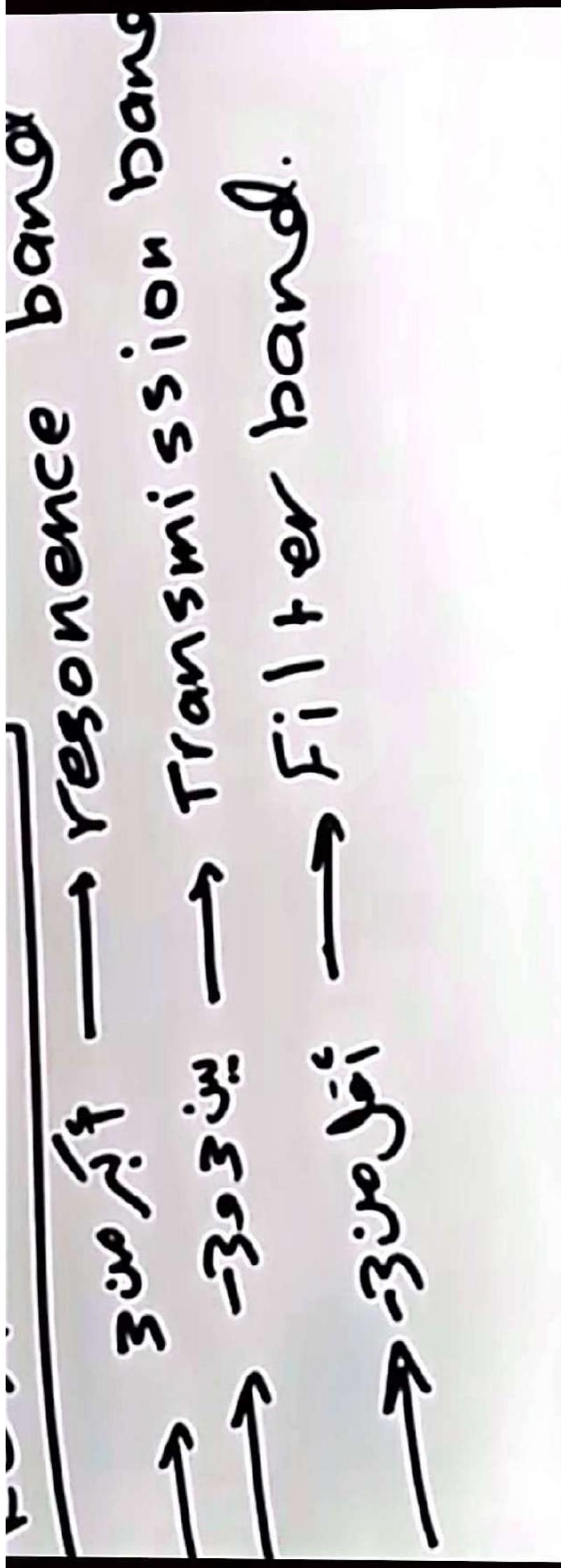
$$\text{dB} \rightarrow 20 \log M(\omega)$$

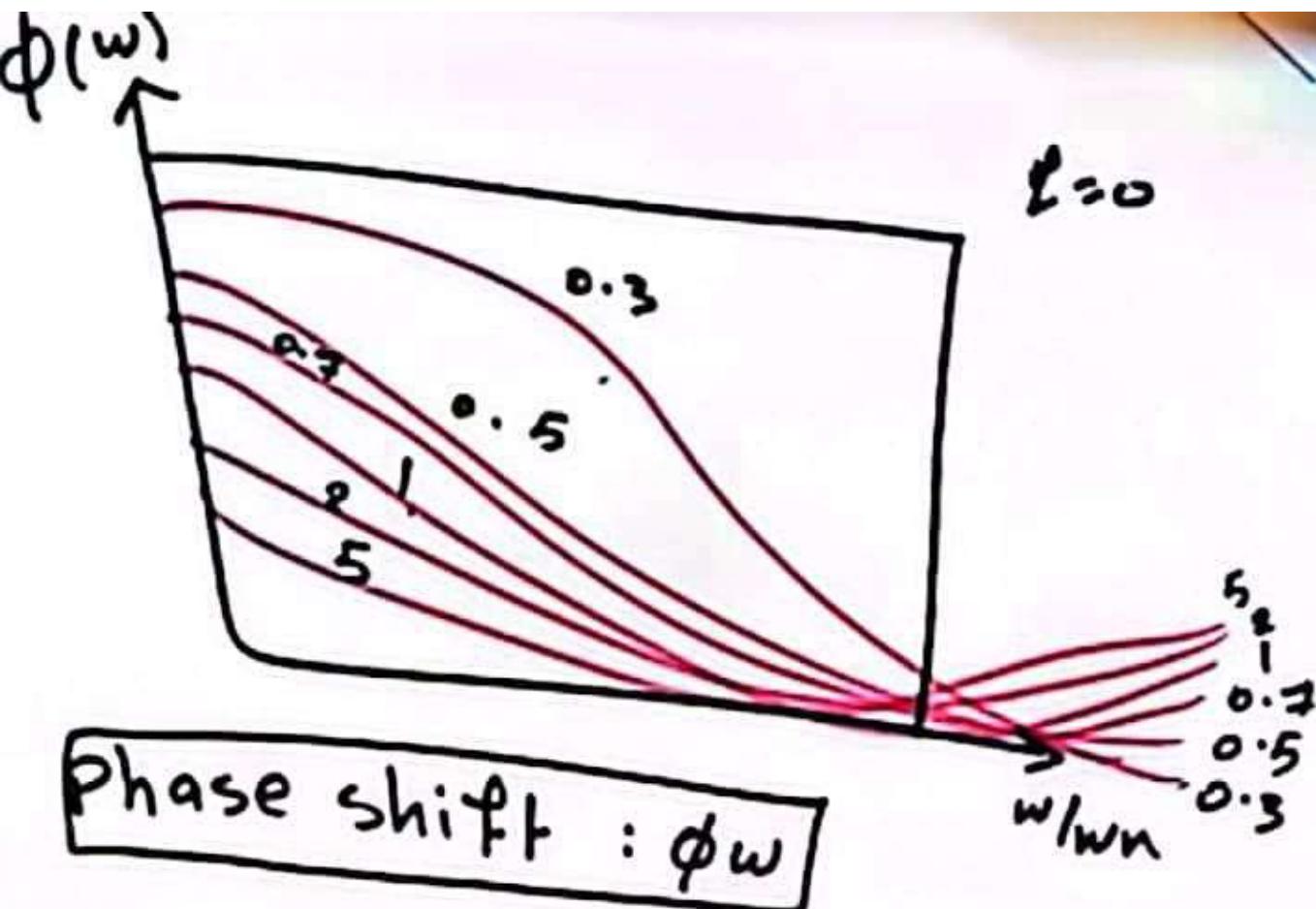
عند قيمة  $M$  مقدارها 0.7 وفوق بطلع هذا ال resonance وبكون ال Output اصغر من input

Resonance  $\rightarrow$  input  $\rightarrow$  output



وأعلى قيمة نون dB هي 3 واقل قيمة هي -3 لهيكل الرسمة تتشتت لثلاثة مناطق أعلى قيمة في ال magnitude هي  $\sqrt{2}$  وأقل قيمة هي 0.7





→ Resonance freq in under damped

هون لو اكتر من 0.7 رح يكون فش ريزونس ف في زيت معي لازم التزم فيه :

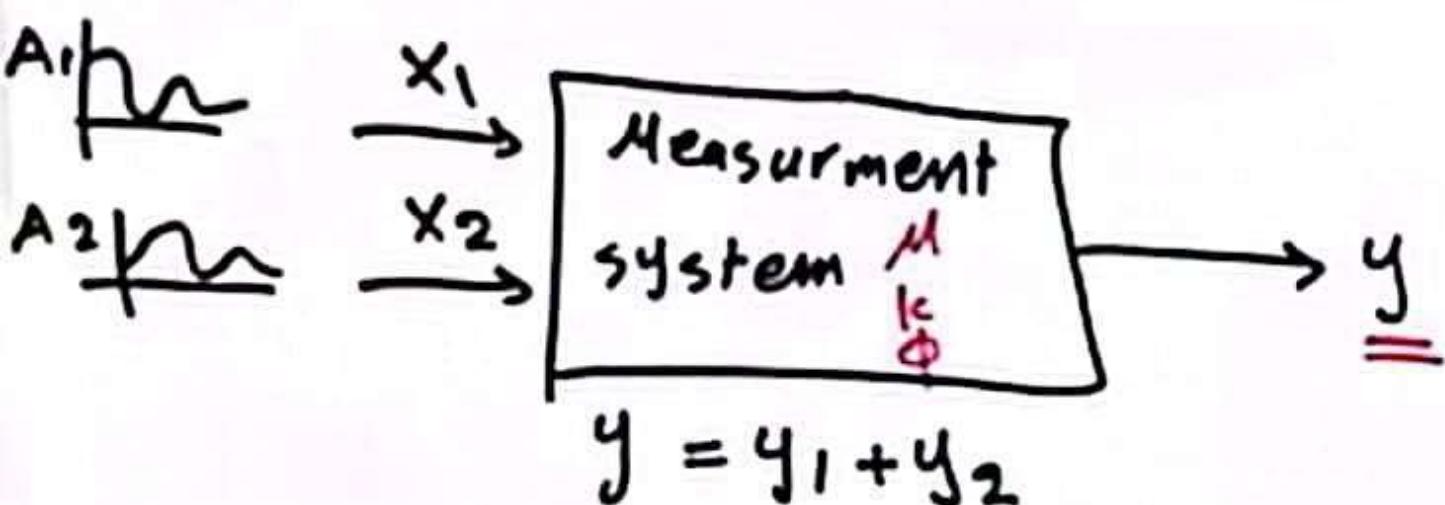
$$\omega_R = \omega_n \sqrt{1 - 2\zeta^2}$$

$$0 < \zeta < \frac{1}{\sqrt{2}} \quad (0.7)$$

when  $\zeta = 0 \quad \therefore \quad \boxed{\omega_R = \omega_n}$

frequency لا يجوز وضع ترددات في منطقة فيها  $\rightarrow$  frequencies within the resonance band is undesirable and damaging the sensors.

# Multipie Function Inputs:-



$$\rightarrow \text{Amplitude (B)} = M(\omega) \cdot k \cdot A_1 + M(\omega) \cdot k \cdot A_2$$

*output*

## • Coupled system :-



$$k_{eq} = K_1 * K_2$$

$$M(\omega)_{eq} = M_1(\omega) * M_2(\omega)$$

$$\Phi(\omega)_{eq} = \Phi_1(\omega) + \Phi_2(\omega)$$

$$\rightarrow \text{Amplitude (B)} = M_{eq} \cdot k_{eq} \cdot A$$

*output*

*problems*

### Example :- 3.12

$$2^{\text{nd}} \rightarrow k=1 // L=2 // \omega_n = 628 \text{ rad/s}$$

$$F(t) = 5 + 10 \sin 25t + 20 \sin 400t$$

$$y(t) = ??$$

المعادلة المعطاة هنا هي معادلة ال input  
لقيمة معينة وهي طريقة تحويلها

$$\textcircled{1} \quad y(t) = k \cdot \ell(t)$$

$$y(t) = 5k + \underline{10k \sin 25t} + \underline{20k \sin 400t}$$

\textcircled{2} \quad اضرب معاملات  $\sin$  بـ

\textcircled{3} \quad ازدیق  $\phi(\omega)$  مقدمة داخل  $\sin$

$$\mu(25) =$$

$$\mu(25) = \frac{1}{\sqrt{\left( \left( 1 - \left( 25/628 \right)^2 \right)^2 + \left( \frac{2 * 2 * 25}{628} \right)^2 \right)^2}}$$

$$\mu(25) = 0.989 \approx \boxed{0.99}$$

$$\mu(400) = \frac{1}{\sqrt{\left( \left( 1 - \left( 400/628 \right)^2 \right)^2 + \left( \frac{2 * 2 * 400}{628} \right)^2 \right)^2}}$$

$$\mu(400) = 0.382 \approx 0.39$$

$$\phi(25) = -9.1^\circ$$

$$\phi(400) = -77^\circ$$

$$y(t) = 5 + 10K \sin 25t + 20K \sin 400t$$

$$M(25) = 0.99$$

$$\phi(25) = -9.1$$

$$M(400) = 0.39 \quad \phi(400) = -77$$

$$y(t) = 5 + (10 * 1 * 0.99) \sin [25t + -9.1]$$

$$+ (20 * 1 * 0.39) * \sin [400t + -77]$$

$$y(t) = 5 + 9.9 \sin(25t - 9.1) + 7.8 \sin(400t - 77)$$

Resonance band :

هـاي منطـقة موجودـة فـقط في الـ second اـحـنا بالـfirـst ما كـنا نـرـتفـع فوقـ %100

Tr ab 90°

3.2 b)  $\ddot{y} + 2\dot{y} + 4y = u(t)$

$$\therefore u \rightarrow \frac{1}{4}\ddot{y} + \frac{1}{2}\dot{y} + y = \frac{1}{4}u(t)$$

$$w_n^2 = 4 \rightarrow w_n = 2$$

$$\therefore \frac{2\ell}{w_n} = \frac{1}{2} \rightarrow \frac{\ell}{\pi} = \frac{1}{2} \rightarrow \ell = 0.5$$

$$KA = \frac{1}{4}$$

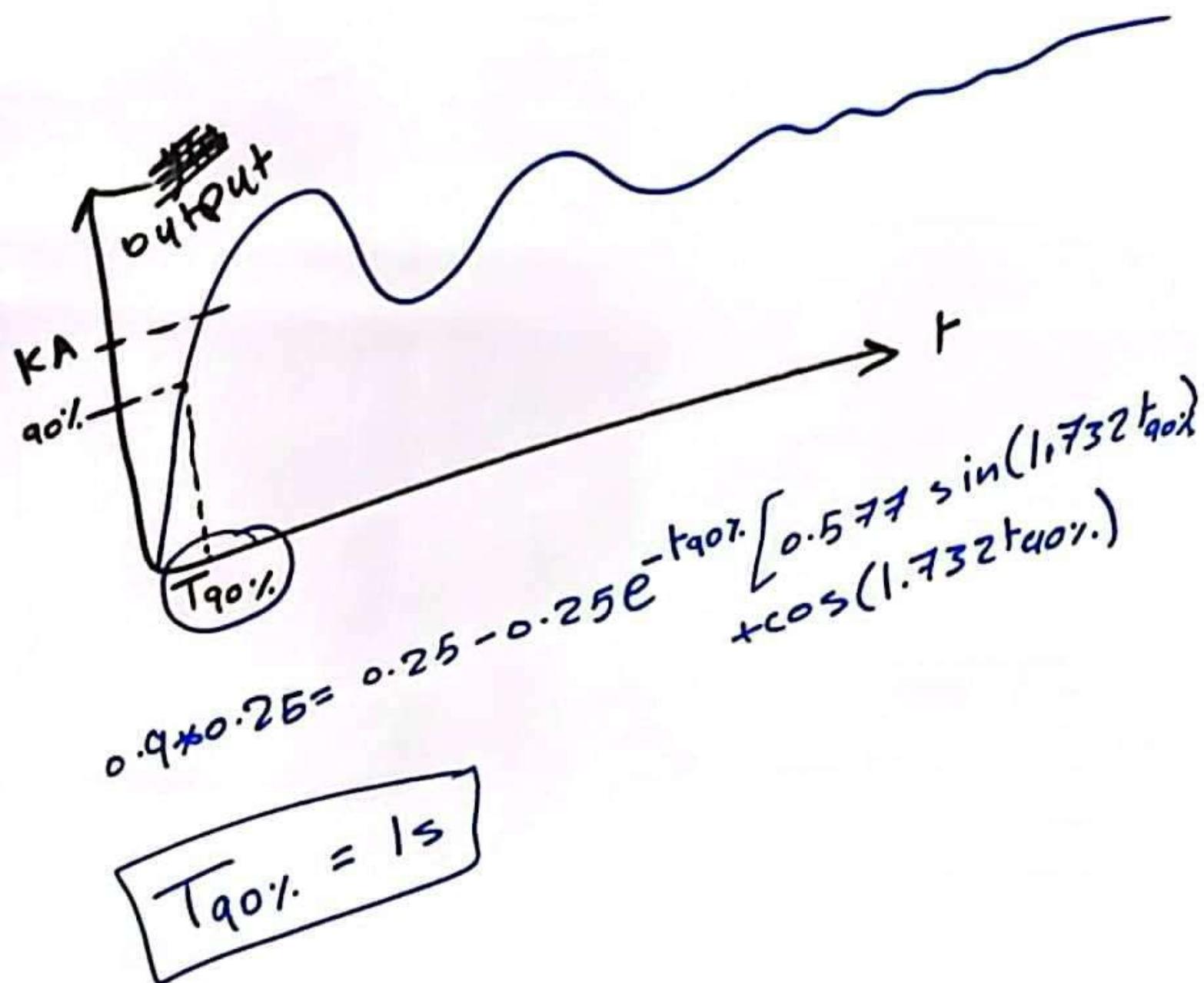
$$y(t) = KA - KAE^{-\ell w_n t} \left[ \frac{\ell}{\sqrt{1-\ell^2}} \sin(w_n t \sqrt{1-\ell^2}) + \cos(w_n t \sqrt{1-\ell^2}) \right]$$

$$y(t) = 0.25 - 0.25 e^{-0.5 \times 2t} \left[ \frac{0.5}{\sqrt{1-0.5^2}} \sin(2t \sqrt{1-0.5^2}) + \cos(2t \sqrt{1-0.5^2}) \right]$$

$$y(t) = 0.25 - 0.25 e^{-t} \left[ 0.5 \sin(1.732t) + \cos(1.732t) \right]$$

المطلوب معطيك معادلة ال input  
وطالب معادلة ال output

المعادلة المعطاة هنا يا المعادلة العامة لل input والمطلوب قيمة output



المطلب الثاني كان : حساب قيمة  $T_{rise}$  ، من خلال المعادلة يلي طلعتها بتعوض قيمة ال  $M$  يلي هي عبارة عن  $KA \times$  النسبة يلي طالبك اياها

3.17

$$\ell = 0.6 \quad \ell = 0.9 \quad \ell = 2$$

$$\text{dynamic error} = \pm 5\%$$

3.22

$$1 - 5\% \leq M(\omega) \leq 1 + 5\%$$

$$0.95 \leq M(\omega) \leq 1.05$$

$$M(\omega) = \frac{1}{\sqrt{[1 - (\omega/\omega_n)^2]^2 + [2\zeta\ell \cdot \omega/\omega_n]^2}}$$

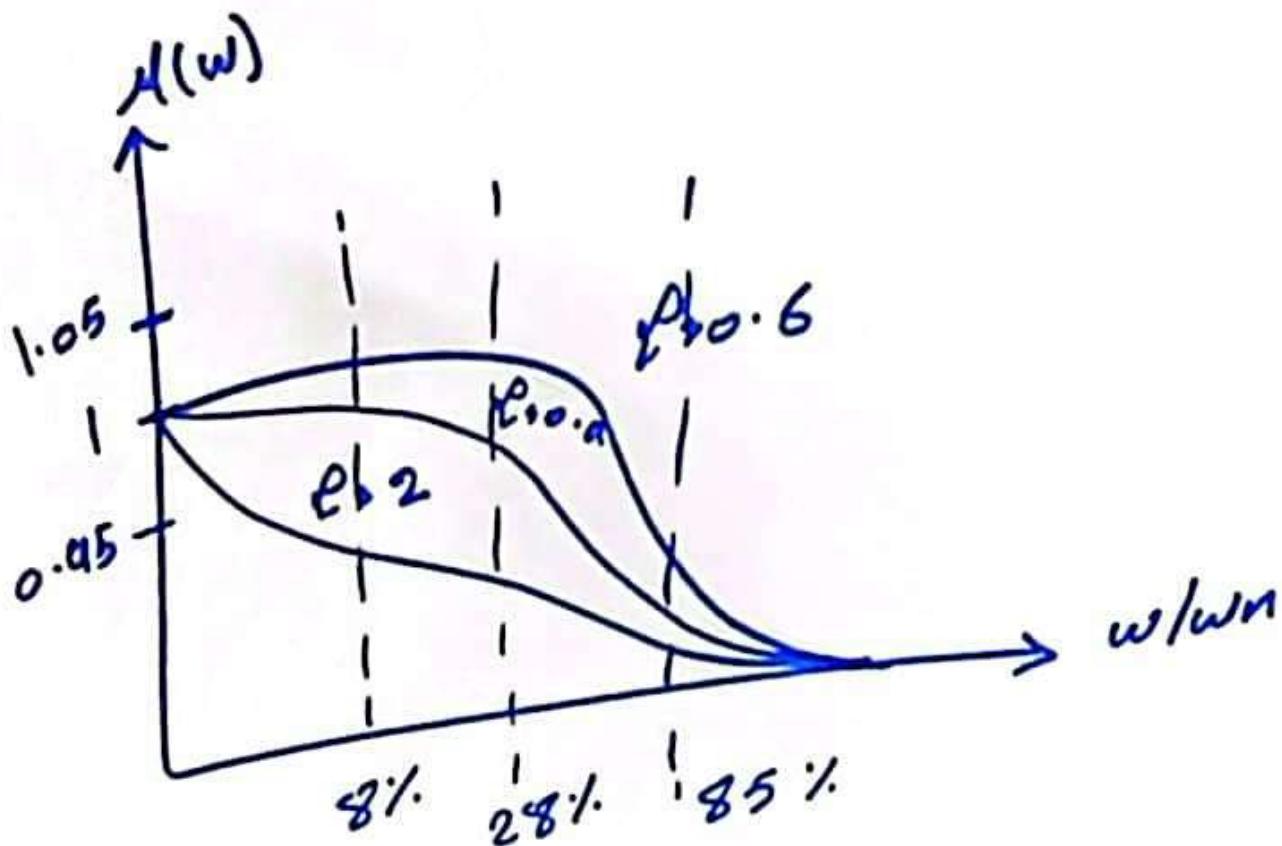
$$1.05 = \frac{1}{\sqrt{[1 - (\omega/\omega_n)^2]^2 + [2\zeta\ell \cdot \omega/\omega_n]^2}} \quad \ell = 0.6$$

$$0 \leq \frac{\omega}{\omega_n} \leq 0.85$$

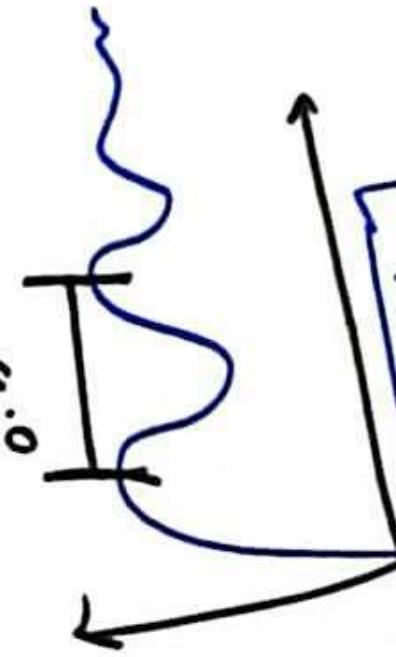
$$0 \leq \frac{\omega}{\omega_n} \leq 0.28 \quad \ell = 0.9$$

$$0 \leq \frac{\omega}{\omega_n} \leq 0.08 \quad \ell = 2$$

1



$$\text{period of oscillation} = 0.577 \text{ ms}$$



$$\omega_0 = ?$$

3.20

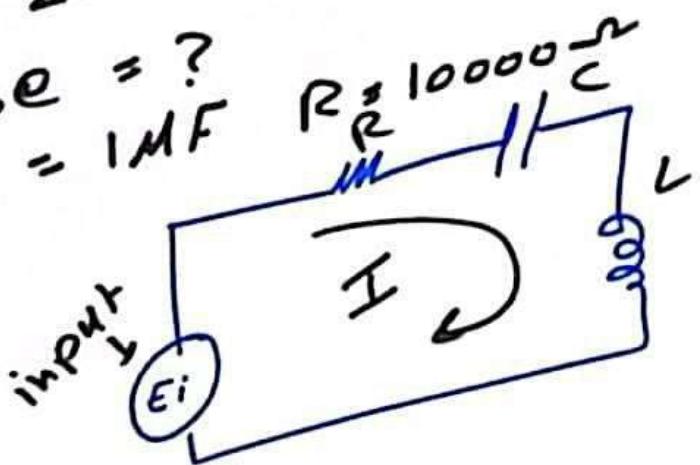
$$\omega_0 = \frac{2\pi}{T_d} = \frac{2\pi}{0.577} = 10.889 \text{ rad/s}$$

$$f = \frac{1}{T_d} = \frac{1}{0.577} = 1.735 \text{ Hz}$$

$$3.24 \quad L\ddot{I} + RI + \frac{I}{C} = E(t) \quad \omega_n = ? \quad \zeta = ?$$

$$E(t) = 1 + 0.5 \sin 2000t \quad "v"$$

\*  $E(t) = 1 + 0.5 \sin 2000t$  response = ?  
 steady state  
 when  $L = 2H$   $C = 1MF$



$$L\ddot{I} + RI + \frac{I}{C} = E(t)$$

المطالب طالبك أول شي تحسبله

$\omega_n$

$\zeta$

ومن ثم طالبك تحسبله  $\theta M$  and steady response يلي هما هون

وكمان طالبك بالأخير تكتبه معادلة output

$$\frac{1}{\omega_n^2} = 2 \times 10^{-6}$$

$$\omega_n^2 = \frac{1}{2 \times 10^{-6}}$$

$$\boxed{\omega_n = 707 \text{ rad/s}}$$

$$\frac{2\rho}{\omega_n} = 1 \times 10^{-6} \times 10000$$

$$\frac{2\rho}{\omega_n} = 0.01 \times 707$$

$$\boxed{\rho = 3.535}$$

السؤال معطيني معاالتين معاادة عامة و معاالة عند قيمة معينة انا من خلال المعاالة العامة بجىب قيمة  $\omega_n$  و  $\rho$

$$E(t) = 1 + 0.5 \sin(2000t)$$

steady response for periodic  
 $\therefore M(\omega) \phi(\omega)$

من هون بحصل  $M$  و  $\phi$  من خلال قوانينها

$$M(\omega) = \frac{1}{\left( \left( 1 - \left( \frac{2000}{707} \right)^2 \right)^2 + \left( 2 * 2 * \frac{2000}{707} \right)^2 \right)^{1/2}}$$

$$M(\omega) = 0.05$$

$$\phi = \tan^{-1} \left( \frac{-2 * 3.5 * \frac{2000}{707}}{1 - \left( \frac{2000}{707} \right)} \right) \text{ rad}$$

$$\boxed{\phi = -1.23 \text{ rad}}$$

خطوات حل المعادلة لما تكون input  
عند قيمة معينة طبقها هو:

$$I(R) = 1 + [0.5 * 0.05 * 1 + 0.23] * \sin(200\pi R) + [0.025 * 0.05 * 1 - 0.23] * \sin(200\pi R)$$

عندی ثلث مطالب

$$3 \cdot 31 \quad \mu = 0.4$$

$$\omega_n = 18000 \text{ Hz}$$

$$\omega = 4500 \text{ Hz}$$

$$\delta(\omega) = ? ?$$

$$\Phi(\omega) = ? ?$$

resonance frequency = ??

$$\omega_n = (18000 * 2\pi) \text{ rad/s}$$

$$\omega = (4500 * 2\pi) \text{ rad/s}$$

$\rightarrow \mu = 0.4 \rightarrow$  under damped.

$$\delta(\omega) = \mu(\omega) - 1$$

$$\mu(\omega) = \left( \left( 1 - \frac{4500 * 2\pi}{18000 * 2\pi} \right)^2 \right)^2 + \left( 2 * 0.4 * \frac{4500 * 2\pi}{18000 * 2\pi} \right)^2$$

$$\mu(\omega) = 1.04 \rightarrow \boxed{\mu(4500 * 2\pi) = 1.04}$$

$$\begin{aligned}\delta(\omega) &= \mu(\omega) - 1 \\ \delta(4500 * 2\pi) &= \mu(4500 * 2\pi) - 1 \\ &= 1.04 - 1 = 0.04 = \pm 4\% \end{aligned}$$

$$\phi(4500\pi) = \tan^{-1} \left( \frac{-2 + 0.4 + \left( \frac{4500\pi}{18000} \right)^2}{1 - \left( \frac{4500\pi}{18000} \right)^2} \right)$$

$$= \boxed{-0.21}$$

$$\omega_R = \omega n \sqrt{1 - 2f^2}$$

$$f_R = f_n \sqrt{1 - 2f^2}$$

$$= 18000 \times \sqrt{1 - 2 \times 0.4} r \\ = 14.8 \text{ Hz}$$

$$\begin{aligned}
 \text{Time lag} &= \frac{\phi(s)}{n} \\
 &= \frac{0.21}{4500 + 2\pi} \\
 &= \boxed{7.4367 \times 10^{-6} \text{ s}}
 \end{aligned}$$

في حالة طلب

تابع لاغ

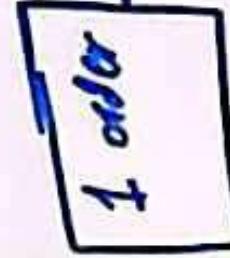
**3.34**

first order

$$T = 1.4 \text{ ms}$$

$$K_1 = 2.0 / C$$

$$T(H) = 10 + 50 \sin 6281$$



$$10 + 50 \sin 6281 T = 1.4 + 10^{-3} s$$

$$K_1 = 2$$

$$\begin{aligned} K_2 &= 1 \\ L &= 0.9 \\ \omega_n &= 5000 \text{ rad/s} \end{aligned}$$

second order

$$K = 1$$

$$L = 0.9$$

$$\omega_n = 5000 \text{ rad/s}$$

$$K_1 = 2.0 / C$$

$$T(H) = 10 + 50 \sin 6281$$

$$K_2 = 1$$

$$L = 0.9$$

$$\omega_n = 5000 \text{ rad/s}$$

$$\mu_1(\omega) = \sqrt{1 + \omega^2 \times L^2}$$

$$= \sqrt{1 + 628^2 \times (1.4 \times 10^{-3})^2}$$

$$= 0.75$$

$$\mu_2(\omega) = \frac{1}{((1 - \frac{628}{5000 \times 2\pi})^2 + (2 \times 0.9 + \frac{628}{5000 \times 2\pi})^2)^{0.5}}$$

$$\mu_2(\omega) = 1$$

$$M_{eq} = (1)(0.75) = \boxed{0.75}$$

$$\phi_1(\omega) = -\tan^{-1}(\omega L)$$

$$\phi_1(\omega) = -\tan^{-1}(\omega \tau)$$

$$\phi_1(\omega) = -\tan^{-1}(-62821 \cdot 4 \times 10^{-3}) = -0.781 \text{ rad}$$

$$\phi_2(\omega) = \tan^{-1} \left( \frac{-240.91 \times \left( \frac{6283}{5000 \times 2\pi} \right)}{1 - \left( \frac{628}{5000 \times 2\pi} \right)^2} \right)$$

$$\phi_2(\omega) = -0.036 \text{ rad}$$

$$\phi_{eq} = -0.721 + \frac{-0.036}{-0.757 \text{ rad}}$$

$$\begin{aligned}\kappa_{eq} &= K_1 * K_2 \\ &= 1 * 2 = \boxed{2} \\ \mu_{eq}(\omega) &= 0.75 \\ \phi_{eq}(\omega) &= -0.75\pi\end{aligned}$$

$$\mu_{eq}(\omega) = 0.75$$

$$\phi_{eq}(\omega) = -0.75\pi$$

$$k_{eq} = 2$$

$$y(H) = 10 + (50 * 0.75 * 2) * \sin(628f - 0.75\pi)$$

$$y(H) = 10 + 75 \sin(628f - 0.75\pi)$$

ch.4

# Probability and statistics

study random variations.

① Types of error :-

1 Baise error (fixed error)  
"systematic"

→ removed : by calibration

2 Precision (random error)

→ minimized by :- repetitions  
replication

# probability and statistics

Engineering Measurement مفاهيم القياس Measurement statistic

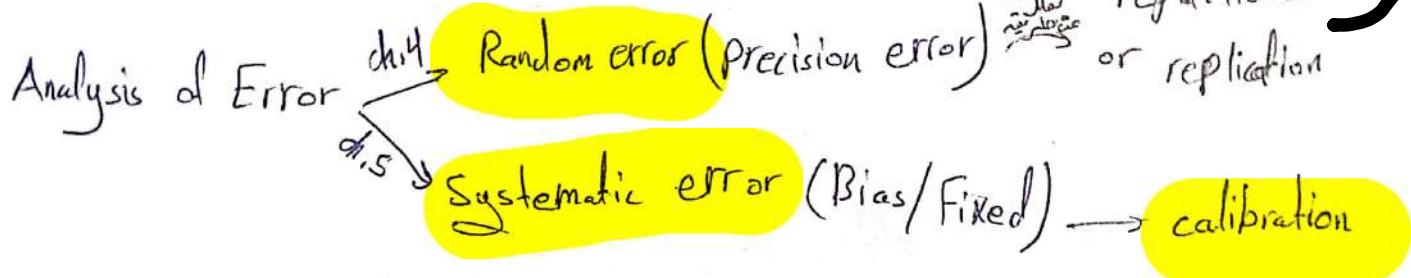
(C)

Repetition /  
replication

there are 2 types of error

(أخطاء نوعية ونوعية ثالثة)

Sol: repetitions



stats

بيانات統計データ

بيانات測定結果

Data  $\rightarrow X_1, X_2, \dots, X_N$  Measurements.

our goal of statistics

1) Find Representative value (مقدار تمثيلى)

$$\text{Ex} \quad \text{Mean} \rightarrow \bar{x} = \frac{x_1 + x_2 + \dots + x_n}{N}$$

results  
Point estimate  
مقدار التقدير

بيانات المعاينات هي بيانات المعاينة التي تم الحصول عليها من خلال

ادارة البيانات

2) Measure of variability / uncertainty

(scattered

)

Point estimate  
Measure of variability  
مقدار التقدير  
مقدار التغير

→ ex → standard deviation  $s_x$

3. { precision interval → random error جمله ۱.۴  
confidence interval → all type of error جمله ۱.۵

Types of error (۱) random error

(۲) Based error

(۳) Fased error

Error Margin

$$\bar{x} \pm u_x (\text{p.v.})$$

precisitive value  
Mean Value

uncertainty

confidence level  
probability  
significat level

→ ex 42 ± 1 (unit) ۹۵%

لهما ایلی ایلی  $u_x$  گوییم  $s_x$  ج ۱ ۲

$u_x$  (widder ۹۵%) نیز ۹۵% probability ایلی بک ج ۲

question

این کار تجربه بتغیر کنید  $\rightarrow$  random experiment چه بک

## \* Statistics and Probability.

---

If  $x_1, x_2, x_3 \dots x_n$  taken under fixed conditions (bias = zero)  
∴ we have random error only.

$$x' = \bar{x} \pm u_{\bar{x}} \quad (\text{P}\%)$$

Point estimate (avg)

True value

Uncertainty interval

Possible range of error (precision interval)

→  $\bar{x}$  → representative value

"most probable estimate of  $x'$  based on available data"

→  $u_{\bar{x}}$  → measure of variability

→ P% → precision interval at given P%

For a given set of measurements, we want to be able to quantify (1) a single representative value that best characterizes the average of the measured data set; (2) a representative value that provides a measure of the variation in the measured data set; and (3) how well the average of the data set represents the true average value of the entire population of the measured variable. The value of item (1) can vary with repeated data sets and so the difference between it and the true average value of the population is a type of random error. Item (3) requires establishing an interval within which the true average value of the population is expected to lie. This interval quantifies the probable range of this random error and it is called a random uncertainty.

## 4.2 STATISTICAL MEASUREMENT THEORY

**4.2 STATISTICAL MEASUREMENT THEORY**

Sampling refers to obtaining a set of data through repeated measurements of a variable under fixed operating conditions. This variable is known as the *measured variable* or, in statistical terms, the *measurand*. By fixed operating conditions, we mean that the external conditions that control the process are held at constant values. In actual engineering practice, truly fixed conditions are difficult to attain and the term "fixed operating conditions" is used in a nominal sense. That is, we consider the process conditions to be maintained as closely as possible and deviations from these conditions will show up in the data set variations.

From a statistical analysis of the data set and an analysis of sources of error that influence these data, we can estimate  $x'$  as  $\underline{x}$ , with uncertainty  $\sigma_x$ .

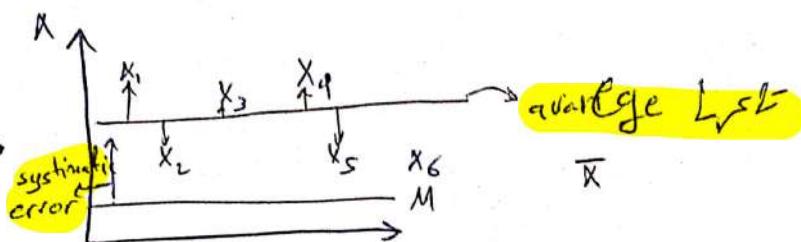
$$\bar{x}' = \bar{x} \pm u_{\bar{x}} \quad (P\%) \quad \text{uncertainty} \quad (4.1)$$

where  $\bar{x}$  represents the most probable estimate of  $x'$  based on the available data and  $\pm u_x$  represents the uncertainty interval in that estimate at some probability level,  $P\%$ . Uncertainties are numbers that quantify the possible range of the effects of errors. The uncertainty interval is the range about  $\bar{x}$

**synthetic Error** : there is a bias between values

random Error : random values

٤٥) الارتكاز نبی رج سوچن الفرضیه مابینه ای دارای systematic error و دارای random error



- real - all day & X = ( Trails

\* لم يتم الالتفاف حول نتائج القياسات المختبرية

(True Value)

$$X' = M$$

اگر  $\mu$  کے مقابلے میں  $\bar{x}$  کا مکانیکی ایکسپریسٹیو فلچر = Random error + Systematic error  
True Value

وهي انتظامية  $\rightarrow$  systematic error  $\neq 0$

وهي عشوائية  $\rightarrow$  random error

(probability density function))

خطة ادلة في المنهجية

120 Chapter 4 Probability and Statistics

Table 4.1 Sample of Random Variable  $x$

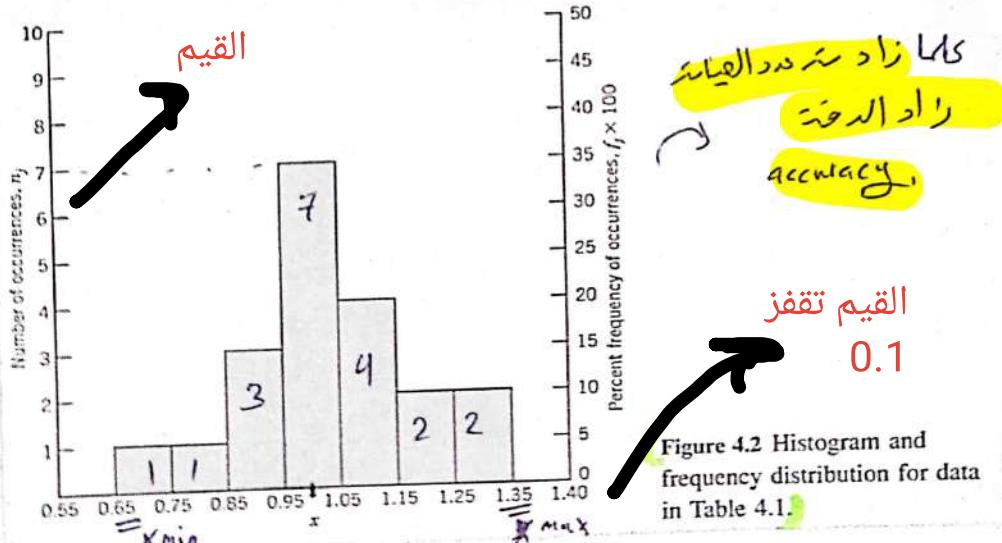
$i$	$x_i$	$i$	$x_i$
1	0.98	11	1.02
2	1.07	12	1.26
3	0.86	13	1.08
4	1.16	14	1.02
5	0.96	15	0.94
6	0.68	16	1.11
7	1.34	17	0.99
8	1.04	18	0.78
9	1.21	19	1.06
10	0.86	20	0.96

لرسم الرسم

Histogram:

Frequency distribution

In Figure 4.1, there exists a region on the axis where the data points tend to clump; this region contains the central value. Such behavior is typical of most engineering variables. We might expect that the true mean value of  $x$  is contained somewhere in this clump.



لما زاد عدد العينات  
زاد الدقة  
accuracy

القيم تتفاوت  
0.1

Figure 4.2 Histogram and frequency distribution for data in Table 4.1.

(1) بحث في تأثير  
الإختصار  
طريق العينة  
istogramme  
الخاصة هنا  
ما يكتفي به 20  
حول ملء فجوة

كلما زادت العينة حوت تتواءل  $\rightarrow$  إن اهتمي في 0.65 حالي في 1.35 الاتجاهات في المرة  
والآخر تكرارها في المطردة غير صحيحة كلما اتجه العينة زادت الدقة ويعمل في الاتجاه

# Histogram رسم خطوات

Histogram لارنامہ سرایی مدد اسکیاں جو

1.  $N \geq 20$  عدد العینات اکبر من ۲۰

2. If I have  $20 < N < 40 \rightarrow$  نیچلے کافی نہیں  
جیسا کہ

$$n_i \geq 5$$

$K \leq 7$  کا نتیجہ  $(0.95 - 1.05)$  میں افرازہ میں خودہ نسبت میں اسکے Histogram کیا جائے

3. If  $20 < N < 40$  نیچلے  $K \geq 4$



Histogram کیا جائے اسکا

This description for variable  $x$  can be extended. Suppose we plot the data of Table 4.1 in a different way. The abscissa will be divided between the maximum and minimum measured values

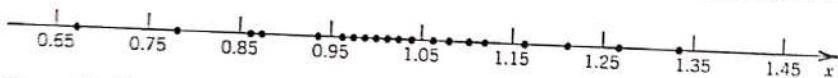


Figure 4.1 Concept of density in reference to a measured variable (from Ex. 4.1).

**SOLUTION** To develop the histogram, compute a reasonable number of intervals for this data set. For  $N = 20$ , a convenient estimate of  $K$  is found from Equation 4.2 to be

$$K = 1.87(N - 1)^{0.40} + 1 = 7$$

Next, determine the maximum and minimum values of the data set and divide this range into  $K$  intervals. For a minimum of 0.68 and a maximum of 1.34, a value of  $\delta x = 0.05$  is chosen. The intervals are as follows:

$j$	Interval	$n_j$	$f_j = n_j/N$
1	$0.65 \leq x_i < 0.75$	1	0.05
2	$0.75 \leq x_i < 0.85$	1	0.05
3	$0.85 \leq x_i < 0.95$	3	0.15
4	$0.95 \leq x_i < 1.05$	7	0.35
5	$1.05 \leq x_i < 1.15$	4	0.20
6	$1.15 \leq x_i < 1.25$	2	0.10
7	$1.25 \leq x_i < 1.35$	2	0.10

$$\begin{array}{c} \boxed{0.6} \approx \boxed{0.68} \\ \boxed{1.4} \approx \boxed{1.34} \end{array} \quad \begin{array}{l} \text{أصغر رقم} \rightarrow \text{صغر} \\ \text{أكبر رقم} \rightarrow \text{كبير} \end{array}$$

$$k = 1.87(N-1)^{40} + 1 = 7$$

K الوجهات  
نخدم المترادفات

$$0.11 = \frac{1.4 - 0.6}{7} \quad \cancel{\frac{0.74}{7}} \quad \text{لكل جدول} \quad \rightarrow$$

1)  $(0.6 - 0.71)$

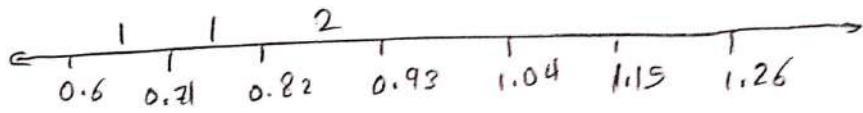
لكل جدول

2)  $(0.71 - 0.82)$

بـ 0.11

3)  $(0.82 - 0.93)$

4)  $(0.93 - 1.04)$



5)  $(1.04 - 1.15)$

6)  $(1.15 - 1.26)$

7)  $(1.26 - 1.37)$

وزن الأرقام بالمتراوحة مُعطى عند كل الجدول

Histogramme avec les poids

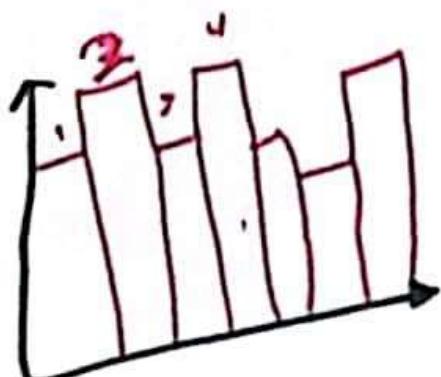
Histogram :-  
→ conditions :-

①  $N > 20$

②  $20 < N < 40, n_i \geq 5$

③ # of intervals  $K \geq 4$

④  $N > 40 \rightarrow K = 1.87(N-1)^{0.4} + 1$



data = 30  
6G  
5. data

for example

①  $N$   
②  $K$   
③  $\frac{n_1 - n_7}{K} = 7$

The standard deviation,  $\sigma$ , a commonly used statistical parameter, is defined as the square root of the variance, that is  $\sigma = \sqrt{\sigma^2}$ .

لـ  $x_i$  اداه نوح

$$x_i = \bar{x} \pm z_{\alpha} \sigma (\%)$$

Position  
Value      Z-value      Probability

لـ  $x_i$  اداه نوح

2.  $x_1, x_2, \dots, x_n$

لفرض اخذنا

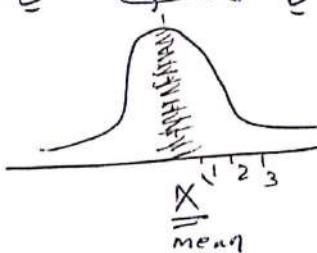
$$N(\bar{x}, \sigma^2)$$

Known mean      Variance

(لـ  $x_i$  اداه نوح)  $x_i = \bar{x} \pm z \sigma$

Known mean

position اداه نوح معاي اداه نوح interval اداه نوح



لـ  $z = 1$  اداه نوح

لـ  $z = 2$  اداه نوح

لـ  $z = 3$  اداه نوح

لـ  $z = 1$  اداه نوح  $\approx 68\%$

$$\bar{x} \pm 1\sigma \rightarrow (68\%)$$

$$\bar{x} \pm 2\sigma \rightarrow (95\%)$$

$$\bar{x} \pm 3\sigma \rightarrow (99\%)$$

لـ  $z = 3$  اداه نوح  $\approx 99\%$

لـ  $z = 2$  اداه نوح  $\approx 95\%$

لـ  $z = 1$  اداه نوح  $\approx 68\%$

- ٢) Normal distributed لیکن  $\sqrt{2}$  نویسید
- ٣)  $n > 30$

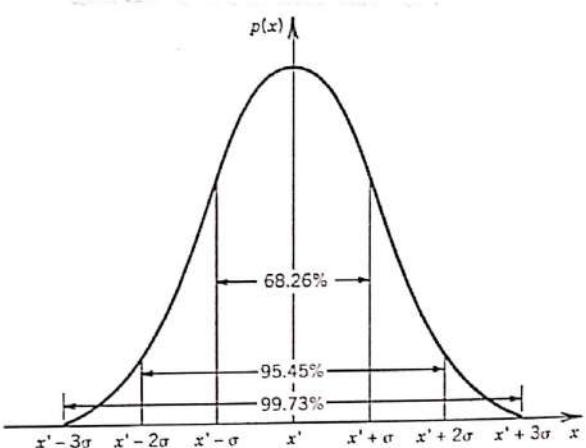


Figure 4.4 Relationship between the probability density function and its statistical parameters,  $x'$  and  $\sigma$ , for a normal (gaussian) distribution.

$$x' - z_1\sigma \leq x \leq x' + z_1\sigma,$$

- $z_1 = 1$ , 68.26% of the area under  $p(x)$  lies within  $\pm z_1\sigma$  of  $x'$ .
- $z_1 = 2$ , 95.45% of the area under  $p(x)$  lies within  $\pm z_1\sigma$  of  $x'$ .
- $z_1 = 3$ , 99.73% of the area under  $p(x)$  lies within  $\pm z_1\sigma$  of  $x'$ .

$$x_i = x' \pm z_1 \sigma \quad (\%)$$

#### Example 4.2

Using the probability values in Table 4.3, show that the probability that a measurement will yield a value within  $x' \pm \sigma$  is 0.6826 or 68.26%.

**KNOWN** Table 4.3

$$z_1 = 1$$

**ASSUMPTIONS** Data follow a normal distribution.

$$\text{FIND } P(x' - \sigma \leq x \leq x' + \sigma)$$

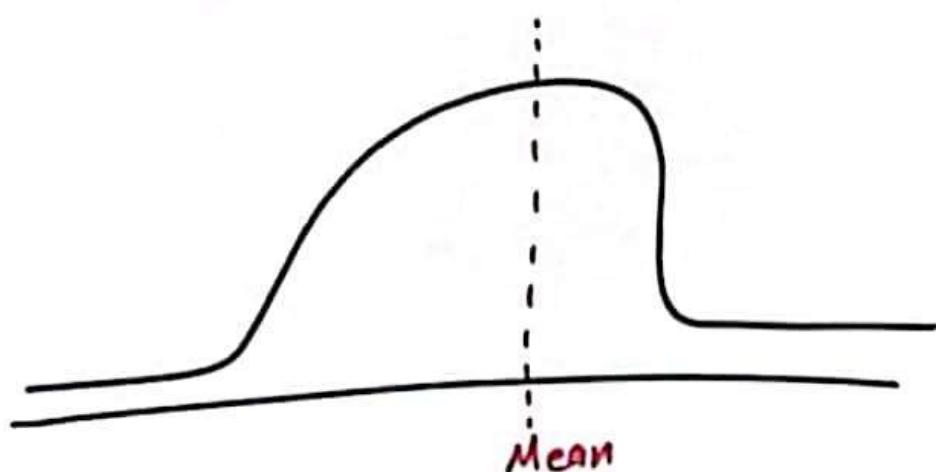
2  $x_1, x_2, x_3 \dots x_n$  (normal)

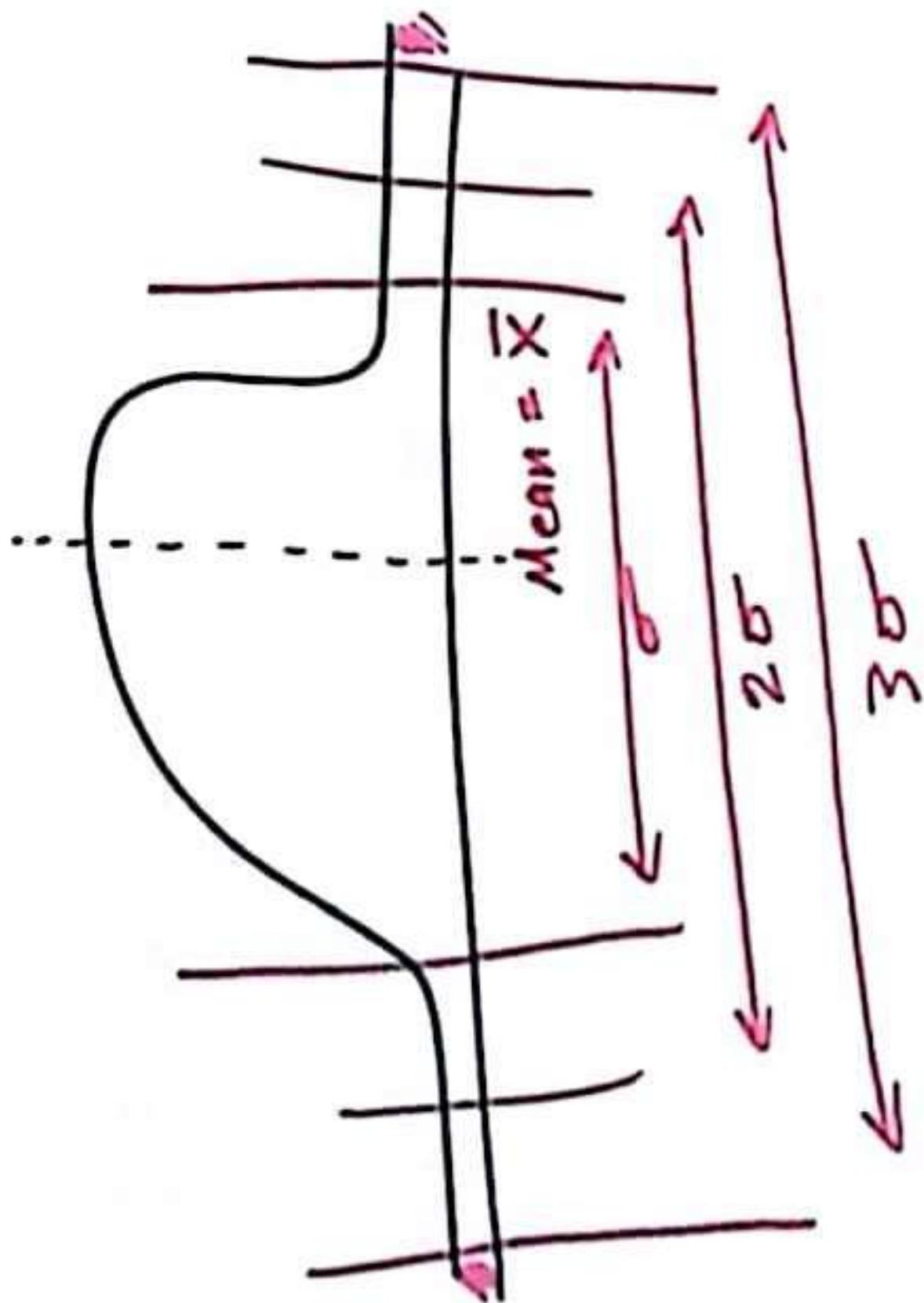
$$\rightarrow \bar{x} = x' \pm z\sigma \quad (P\%)$$

- True value  
- estimate  $\bar{x}$

normal distribution

$x' \pm 1\sigma$	(68.27%)	precision interval
$x' \pm 2\sigma$	(95.45%)	
$x' \pm 3\sigma$	(99.73%)	
$x' \pm 0.674\sigma$	(50%)	





**SOLUTION** To estimate the probability that a single measurement will have a value within some interval, we need to solve the integral

$$\frac{1}{\sqrt{2\pi}} \int_0^{z_1=1} e^{-\beta^2/2} d\beta$$

over the interval defined by  $z_1$ . Table 4.3 lists the solutions for this integral. Using Table 4.3 for  $z_1 = 1$ , we find  $P(z_1) = 0.3413$ . However, since  $z_1 = (x_1 - x')/\sigma$ , the probability that any measurement of  $x$  will produce a value within the interval  $0 \leq x \leq x' + \sigma$  is 34.13%. Since the normal distribution is symmetric about  $x'$ , the probability that  $x$  will fall within the interval defined between  $-z_1\sigma$  and  $+z_1\sigma$  for  $z_1 = 1$  is  $(2)(0.3413) = 0.6826$  or 68.26%. Accordingly, if we made a single measurement of  $x$ , the probability that the value found would lie within the interval  $x' - \sigma \leq x \leq x' + \sigma$  would be 68.26%.

**COMMENT** Similarly, for  $z_1 = 1.96$ , the probability would be 95.0%.

### Example 4.3

The statistics of a well-defined varying voltage signal are given by  $x' = 8.5$  V and  $\sigma^2 = 2.25$  V<sup>2</sup>. If a single measurement of the voltage signal is made, determine the probability that the measured value indicated will be between 10.0 and 11.5 V.

**KNOWN**  $x' = 8.5$  V  
 $\sigma^2 = 2.25$  V<sup>2</sup>

**ASSUMPTIONS** Signal has a normal distribution about  $x'$ .

**FIND**  $P(10.0 \leq x \leq 11.5)$

**SOLUTION** To find the probability that  $x$  will fall into the interval  $10.0 \leq x \leq 11.5$  requires finding the area under  $p(x)$  bounded by this interval. The standard deviation of the variable is  $\sigma = \sqrt{\sigma^2} = 1.5$  V, so our interval falls under the portion of the  $p(x)$  curve bounded by  $z_1 = (10.0 - 8.5)/1.5 = 1$  and  $z_2 = (11.5 - 8.5)/1.5 = 2$ . From Table 4.3, the probability that a value will fall between  $8.5 \leq x \leq 10.0$  is  $P(8.5 \leq x \leq 10.0) = P(z_1 = 1) = 0.3413$ . For the interval defined by  $8.5 \leq x \leq 11.5$ ,  $P(8.5 \leq x \leq 11.5) = P(z_2 = 2) = 0.4772$ . The area we need is just the overlap of these two intervals, so

$$P(10.0 \leq x \leq 11.5) = P(8.5 \leq x \leq 11.5) - P(8.5 \leq x \leq 10.0) \\ = 0.4772 - 0.3413 = 0.1359$$

So there is a 13.59% probability that the measurement will yield a value between 10.0 and 11.5 V.

**COMMENT** In general, the probability that a measured value will lie within an interval defined by any two values of  $z_1$ , such as  $z_a$  and  $z_b$ , is found by integrating  $p(x)$  between  $z_a$  and  $z_b$ . For a normal density function, this probability is identical to the operation,  $P(z_b) - P(z_a)$ .

3. If  $x - XN$  finite data set

$s_x$  is  $\bar{x}$  per

Finite-sized data sets provide the statistical estimates known as the *sample mean value* ( $\bar{x}$ ), the *sample variance* ( $s_x^2$ ), and its outcome, the *sample standard deviation* ( $s_x$ ), defined by

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i \quad (4.14a)$$

$$s_x^2 = \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2 \quad (4.14b)$$

$$s_x = \sqrt{s_x^2} = \left( \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2 \right)^{1/2} \quad (4.14c)$$

the finite statistical estimates and the statistics based on an assumed  $p(x)$ . For a normal distribution of  $x$  about some sample mean value,  $\bar{x}$ , we can state that statistically

$x_t = \bar{x} \pm t_{v,p} s_x$  ( $P\%$ )  
 where the variable  $t_{v,p}$  provides a coverage factor used for finite data sets and which replaces the  $z$  variable. This new variable is referred to as the *Student's t variable*,

$$t = \frac{\bar{x} - x'}{s_x / \sqrt{N}} \quad (4.16)$$

The interval  $\pm t_{v,p} s_x$  represents a precision interval, given at probability  $P\%$ , within which one should expect any measured value to fall.

$$x_t = \bar{x} \pm t_{v,p} \cdot s_x$$

$v$ : decrease of freedom =  $N - 1$   
 $p$ : probability,

Table 4.4 Student's *t* Distribution

$v$	$t_{50}$	$t_{90}$	$t_{95}$	$t_{99}$
1	1.000	6.314	12.706	63.657
2	0.816	2.920	4.303	9.925
3	0.765	2.353	3.182	5.841
4	0.741	2.132	2.770	4.604
5	0.727	2.015	2.571	4.032
6	0.718	1.943	2.447	3.707
7	0.711	1.895	2.365	3.499
8	0.706	1.860	2.306	3.355
9	0.703	1.833	2.262	3.250
10	0.700	1.812	2.228	3.169
11	0.697	1.796	2.201	3.106
12	0.695	1.782	2.179	3.055
13	0.694	1.771	2.160	3.012
14	0.692	1.761	2.145	2.977
15	0.691	1.753	2.131	2.947
16	0.690	1.746	2.120	2.921
17	0.689	1.740	2.110	2.898
18	0.688	1.734	2.101	2.878
19	0.688	1.729	2.093	2.861
20	0.687	1.725	2.086	2.845
21	0.686	1.721	2.080	2.831
30	0.683	1.697	2.042	2.750
40	0.681	1.684	2.021	2.704
50	0.680	1.679	2.010	2.679
60	0.679	1.671	2.000	2.660
$\infty$	0.674	1.645	1.960	2.576

The value for the *t* estimator provides a coverage factor that is a function of the probability,  $P$ , and the degrees of freedom in the data set,  $v = N - 1$ . These *t* values can be obtained from Table 4.4, William S. Gosset<sup>5</sup> (1876)

Table 4.3 Probability Values for Normal Error Function. One-sided integral solutions for  $P(Z < z)$   $\int_{-\infty}^z e^{-t^2/2} dt$ 

$z$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0110	0.0140	0.0160	0.0170	0.0179	0.0189	0.0199
0.1	0.0318	0.0348	0.0378	0.0397	0.0417	0.0436	0.0456	0.0473	0.0481	0.0493
0.2	0.0593	0.0632	0.0671	0.0690	0.0708	0.0727	0.0746	0.0764	0.0781	0.0797
0.3	0.1310	0.1317	0.1325	0.1329	0.1331	0.1333	0.1336	0.1339	0.1341	0.1347
0.4	0.1954	0.1991	0.1998	0.1995	0.1991	0.1986	0.1980	0.1973	0.1964	0.1959
0.5	0.2518	0.2590	0.2665	0.2709	0.2734	0.2756	0.2773	0.2789	0.2799	0.2804
0.6	0.2982	0.3291	0.3424	0.3557	0.3689	0.3822	0.3954	0.4086	0.4217	0.4349
0.7	0.3380	0.3611	0.3842	0.3823	0.3794	0.3734	0.3761	0.3791	0.3821	0.3852
0.8	0.3881	0.3910	0.3939	0.3967	0.3995	0.4023	0.4051	0.4078	0.4106	0.4133
0.9	0.4389	0.4388	0.3312	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.4713	0.3458	0.3481	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.4953	0.3003	0.3080	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.5189	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.5333	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.5482	0.4292	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319	
1.5	0.5633	0.4388	0.4387	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.5783	0.4483	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.5931	0.4584	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.6071	0.4689	0.4699	0.4684	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.6213	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4758	0.4761	0.4767
2.0	0.6353	0.4778	0.4803	0.4788	0.4793	0.4799	0.4803	0.4808	0.4812	0.4817
2.1	0.6482	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.6601	0.4868	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.6713	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.6818	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.6918	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.6993	0.4955	0.4955	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.7065	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.7134	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.7191	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.7245	0.4987	0.4987	0.4988	0.4988	0.4988	0.4989	0.4989	0.4990	

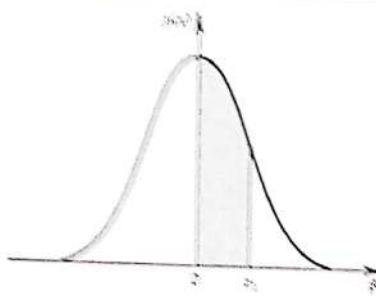


Figure 4.3 Integration terminology for the normal error function and Table 4.3.

(مکالمہ) لو جندي انه اچھاں ای مرادہ ہے اور خدا کے طالع ہے

$X + 16 \quad (68\%)$

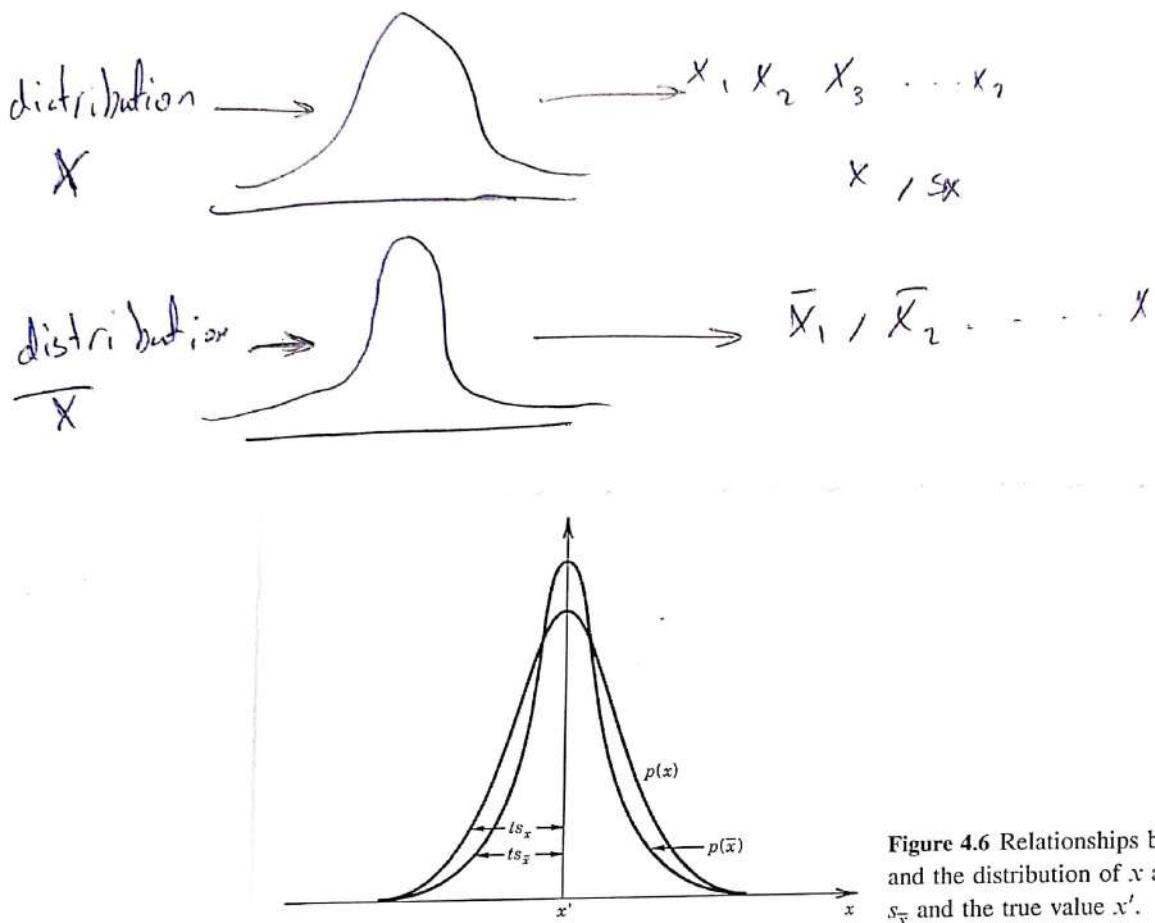
لو بھی اچھا چالہ اسے سمجھو

$$1 - \frac{1}{2} e^{-\frac{1}{2}} = \frac{1}{2} \rightarrow \text{بھی مفہومی}$$

$$\frac{1}{2} \rightarrow$$

normal distribution of the sample mean values about the true mean. The variance of the distribution of mean values that could be expected can be estimated from a single finite data set through the standard deviation of the means,  $s_{\bar{x}}$ :

$$s_{\bar{x}} = \frac{s_x}{\sqrt{N}}$$



**Figure 4.6** Relationships between  $s_x$  and the distribution of  $x$  and between  $s_{\bar{x}}$  and the true value  $x'$ .

So how good is the estimate of the true mean of a variable based on a finite-sized sample? The standard deviation of the means represents a measure of how well a measured mean value represents the true mean of the population. The range over which the possible values of the true mean value might lie at some probability level  $P$  based on the information from a sample data set is given as

$$\hat{x} = \bar{x} \pm t_{v,p} s_{\bar{x}} \quad (P\%) \quad (4.18)$$

- ③  $x_1, x_2, x_3 \dots x_n$  finite dataset

$$x' \rightarrow \bar{x} = \frac{1}{N} \sum_{i=1}^N x_i$$

$$\sigma^2 \rightarrow s_x^2 = \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2$$

$$s_x = \sqrt{s_x^2}$$

Variance ←  $\sigma^2$   
standard deviation ←  $s_x$

$$x_i = \bar{x} \pm t_{(v, p)} \cdot s_x \quad (P\%)$$

degree of freedom  
 $v = [N-1]$

### Example 4.4

Consider the data of Table 4.1. (a) Compute the sample statistics for this data set. (b) Estimate the interval of values over which 95% of the measurements of  $x$  should be expected to lie. (c) Estimate the true mean value of  $x$  at 95% probability based on this finite data set.

**KNOWN** Table 4.1

$N = 20$

**ASSUMPTIONS** Data set follows a normal distribution.

No systematic errors.

**FIND**  $\bar{x}, \bar{x} \pm t_{v, 95} s_x$ , and  $\bar{x} \pm t_{v, 95} s_{\bar{x}}$

**SOLUTION** The sample mean value is computed for the  $N = 20$  values by the relation

$$\bar{x} = \frac{1}{20} \sum_{i=1}^{20} x_i = 1.02$$

This, in turn, is used to compute the sample standard deviation

$$s_x = \left[ \frac{1}{19} \sum_{i=1}^{20} (x_i - \bar{x})^2 \right]^{1/2} = 0.16$$

The degrees of freedom in the data set is  $v = N - 1 = 19$ . From Table 4.4 at 95% probability,  $t_{19, 95} = 2.093$ . Then, the interval of values in which 95% of the measurements of  $x$  should lie is given by Equation 4.15:

$$x_i = 1.02 \pm (2.093 \times 0.16) = 1.02 \pm 0.33 \quad (95\%)$$

Accordingly, if a 21st data point were taken, there is a 95% probability that its value would lie between 0.69 and 1.35.

The true mean value is estimated by the sample mean value. However, the random uncertainty at 95% probability for this estimate is  $t_{19, 95} s_{\bar{x}}$ , where

$$s_{\bar{x}} = \frac{s_x}{\sqrt{N}} = \frac{0.16}{\sqrt{20}} = 0.036 \approx 0.04$$

Then, in the absence of systematic errors, we write, from Equation 4.19,

$$x' = \bar{x} \pm t_{19, 95} s_{\bar{x}} = 1.02 \pm 0.08 \quad (95\%)$$

So at 95% confidence, the true mean value lies between 0.94 and 1.10. Program *Finite-population.vi* demonstrates the effect of sample size on the histogram and the statistics of the data set.

4.

$$\boxed{\bar{x} \pm t_{v, p} \cdot s_{\bar{x}}}$$

$$s_{\bar{x}} = \frac{s_x}{\sqrt{N}}$$

## 5. Pooled statistics

Replacility

$$\begin{cases}
 R \rightarrow X_1, X_2, X_3, \dots, X_N \\
 B \rightarrow X_1, X_2, \dots, X_N \\
 M \rightarrow X_1, \dots, \dots, X_N \\
 \vdots \\
 N
 \end{cases}$$

Repetitions

$$\left( \bar{\bar{X}} \pm t \cdot S_{\bar{\bar{X}}} \right) \quad (P\%)$$

pooled mean  $\bar{\bar{X}}$   
 pooled standard deviation  $S_{\bar{\bar{X}}}$

إذا تم تكرار التجربة  $(P\%)$  على  $(N)$  مجموعات

$$\bar{\bar{X}} = \frac{\sum X_{i,j}}{MN}$$

لذلك فإن المقدمة المترادفة من المقدمة المترادفة للتجربة كلها هي

$$\begin{aligned}
 1) &\rightarrow S \\
 2) &\rightarrow S \\
 3) &\rightarrow S
 \end{aligned}$$

يمكن حل ذلك دالياً كالتالي

$$\frac{\text{مجموع المترادفات}}{\text{عدد المترادفات}} = \bar{\bar{X}}$$

## Pooled Statistics

The pooled standard deviation of  $\bar{x}$  is defined by

$$\langle s_x \rangle = \sqrt{\frac{1}{M(N-1)} \sum_{j=1}^M \sum_{i=1}^N (x_{ij} - \bar{x}_j)^2} = \sqrt{\frac{1}{M} \sum_{j=1}^M s_{x_j}^2}$$

with degrees of freedom  $v = \sum_{j=1}^M v_j = \sum_{j=1}^M (N_j - 1)$ , and the pooled standard deviation of the means is given by

$$(s_{\bar{x}}) = \frac{\langle s_x \rangle}{\sqrt{\frac{1}{M} \sum_{j=1}^M N_j}}$$

(4.25)

$$s_{\bar{x}} = \frac{s_x}{\sqrt{M \mu}}$$

$$V = M(\mu - 1)$$

$$S\bar{x} = \frac{Sx}{\sqrt{n}} \quad \text{standard error of Means}$$

$$\bar{x}' = \bar{x} \pm t_{v,p} \cdot S\bar{x} \quad (P\%)$$

↓  
 Point estimate      ↓  
 Precision  
 interval  
 of mean

### 5 Pooled stats.

$\bar{x}_1$	$\bar{x}_2$	$\bar{x}_3$
$N_1$	$N_2$	$N_3$
$s_x$	$s_x$	$s_x$
20	10	20

Pooled

$$\rightarrow \bar{\bar{x}} = \frac{1}{MN} \sum_{j=1}^M \sum_{i=1}^N x_{ij}$$

↓  
 Pooled mean      ↓  
 Repetitions      ↓  
 Replications      →  $\sqrt{s_x^2}$

$$\rightarrow S\bar{x} = \frac{Sx}{\sqrt{MN}}$$

Pooled standard

$$\rightarrow Sx = \sqrt{\frac{1}{M} \sum s_x^2}$$

④ cont Pooled stats :-

$$X_i = \bar{\bar{X}} \pm t_{V, P} (\bar{s}_{\bar{X}_i})$$

$\hookrightarrow V = N(N-1)$

---

5 precision interval for sample variance:

$$V \cdot \frac{s^2}{\chi^2_{97.5\%}} \leq \sigma^2 \leq V \cdot \frac{s^2}{\chi^2_{99.5\%}}$$

(95%)

$\downarrow$                              $\downarrow$

$N-1$                              $N-1$

## ② data outlier detection

↳ How to reject a measurement reading

$$① \bar{x} \pm t_{v, 99.8\%} \cdot s_x = x_i$$

→ if  $x_i$  outside  $\boxed{PI}$  → reject this reading

$$② z_0 = \left| \frac{x_i - \bar{x}}{s_x} \right|$$

$$\text{③ } P(z_0) = P(0 < z < z_0) \quad \text{outside test}$$

$$\text{④ } p_a = 1 - \frac{1}{2N}$$

→ If  $P(z_0) > p_a$  → reject reading

∴

$$\textcircled{1} \bar{x} \pm t_{v, 99.8\%} \cdot s_x = \dots$$

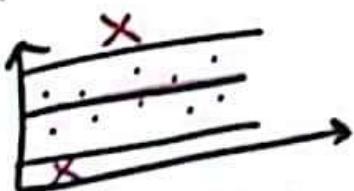
→ if  $x_i$  outside  $\boxed{PI}$  → reject this reading

$$\textcircled{2} z_0 = \left| \frac{x_i - \bar{x}}{s_x} \right| \quad \boxed{z_0}$$

$$\textcircled{3} p(z_0) = P(0 < z < z_0) \quad \text{outside test}$$

$$\textcircled{4} \underline{p_a} = 1 - \frac{1}{2N}$$

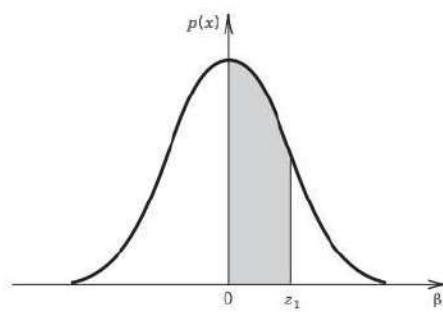
→ If  $p(z_0) > p_a \rightsquigarrow$  reject reading



*problems*

**Table 4.3** Probability Values for Normal Error Function: One-Sided Integral Solutions for  $p(z_1) = \frac{1}{(2\pi)^{1/2}} \int_0^{z_1} e^{-\beta^2/2} d\beta$ 

$z_1 = \frac{x_1 - x'}{\sigma}$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1809	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2794	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4292	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4758	0.4761	0.4767
2.0	0.4772	0.4778	0.4803	0.4788	0.4793	0.4799	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.49865	0.4987	0.4987	0.4988	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990

**Figure 4.3** Integration terminology for the normal error function and Table 4.3.

4.1

$$X_i = \bar{X} \pm \frac{\sigma}{\sqrt{n}} \quad (P\%)$$

حکای Large data فوراً بعکی های normal

$X \rightarrow \text{normal}$

50%

$$\bar{X} = 9.2$$

السؤال طالب ال range ومعطيك هاي المعطيات

$$\sigma = 1.1$$

مجرد ما السؤال حکای normal بتضل رایح على های العلاقة

$$X_i = \bar{X} \pm z\sigma \quad (P\%)$$

$$= 9.2 \pm [0.674 * 1.1] \quad (50\%)$$

نسبة 50% حافظينها

$$X_i = 9.2 \pm \frac{0.7414}{u\bar{x}} \quad (50\%)$$

لاحظ كلما زاد precision زاد u

$$X_i = 9.2 \pm [2 * 1.1] \quad (95\%)$$

$$X_i = 9.2 \pm \frac{2.2}{u\bar{x}} \quad (95\%)$$

Case 2

4.2

$$x \rightarrow \text{normal} \sim$$
$$\bar{x} = 192$$
$$\sigma = 10$$

~~192~~

$$x_i = \bar{x} \pm z \sigma$$
$$= 192 \pm [ *10 ]$$

(90%) (90%)

$$\frac{90\%}{2} = 45\%$$

0.45

كمان طالب

case 2

4.2

$$X_i = \bar{X} + \sigma \quad (\alpha\%)$$

$$X_i = 192 \pm [1.645 * 10] \quad (\alpha\%)$$

$$X_i = 192 \pm 16.45 \frac{\alpha\%}{2}$$

$X_i = 192 \pm 0.45$

area  $0.45$

area  $0.45$

$$\begin{aligned} & \text{area } 0.4495 \rightarrow 1.64 \\ & \text{area } 0.4505 \rightarrow 1.64 \\ & \frac{0.4505 - 0.4495}{0.4505 - 0.4495} = \frac{1.64 - 1.64}{1.64 - X} \\ & X = 1.645 \end{aligned}$$

إذا طلب السؤال وحالياً  
فهذا large sample size  
يبرهن على سكشن ٢ تابع النورمل

case 2

case 7.2

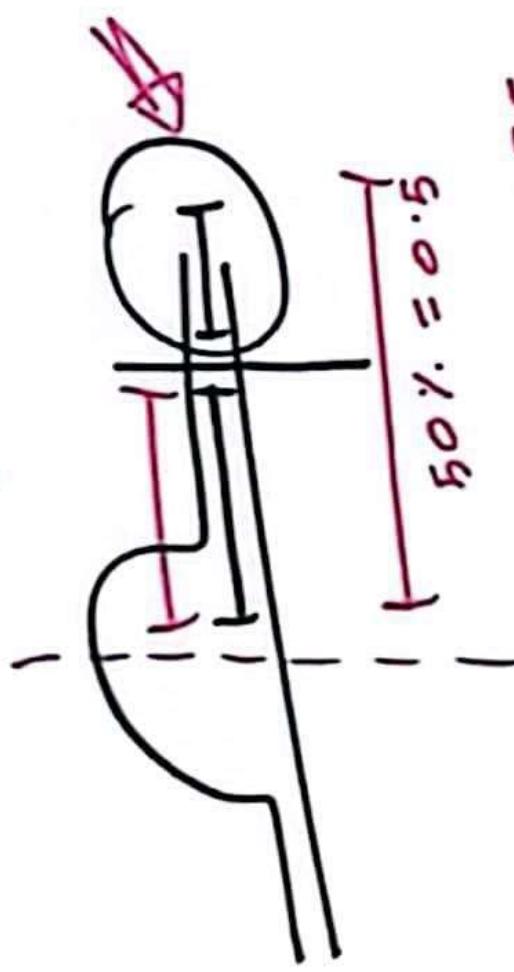
طالب هان الاحتمالية الأكبر من 150

$$\begin{aligned}
 \bar{x} &= 121 \\
 \text{U.3} &= 150 \\
 \rho(x_i) &= \frac{\bar{x}_i - \bar{x}}{\sigma} = 2.02857
 \end{aligned}$$

تابع الفرع السابق

$$P(2.024857) = 0.47855$$

one side 50%



$$0.5 - 0.47855$$

$$= 0.0213$$

لأنها upper بطرحها  
one side من 0.5 و هون  
عشان هييك هتش من 1

U.a

$\bar{x}_1$	$s_x^1$	$F_2$	$\bar{x}_2$	$s_x^2$	$F_3$	$\bar{x}_3$	$s_x^3$
$\bar{x} = 50.465$	$s_x = 0.972$	$\bar{x} = 50.68$	$s_x = 1.17679$	$s\bar{x} = 0.2631$		$\bar{x} = 50.640$	$s_x = 0.9817$
$s\bar{x} = \frac{s_x}{\sqrt{n}}$ $= \frac{0.972}{\sqrt{20}}$						$s\bar{x} = 0.219$	
						$s\bar{x} = 0.2173$	
						$\checkmark = N - 1$	$= 19$
						$= 20 - 1$	

هذا السؤال كان معطى قيم بـ  $s$  و  $N$  و طالب تحسبي  $\bar{x}$  و  $s_x$  وسيجتمع الـ  $d.f$  وكل من الحاسبة وكمان طالب  $d.f$  واللي هي هون 19

5.8  
error within samples and between  
samples [precision and random  
error]

الفرع الثاني من السؤال السابق يقتلي لو  
عملت Histogram هل رج يأثر الاجابة فوق

ملاحظة اذا كان هان  $n \leq 20$  نستخدم  $\sigma$   
واذا كان  $n > 20$  نستخدم normal sample

اذا حكالي السؤال عينات كبيرة  
اذا قلي قليل او  $n \leq 20$  اذن finite

**Table 4.4** Student's *t* Distribution

<i>v</i>	<i>t</i> <sub>50</sub>	<i>t</i> <sub>90</sub>	<i>t</i> <sub>95</sub>	<i>t</i> <sub>99</sub>
1	1.000	6.314	12.706	63.657
2	0.816	2.920	4.303	9.925
3	0.765	2.353	3.182	5.841
4	0.741	2.132	2.770	4.604
5	0.727	2.015	2.571	4.032
6	0.718	1.943	2.447	3.707
7	0.711	1.895	2.365	3.499
8	0.706	1.860	2.306	3.355
9	0.703	1.833	2.262	3.250
10	0.700	1.812	2.228	3.169
11	0.697	1.796	2.201	3.106
12	0.695	1.782	2.179	3.055
13	0.694	1.771	2.160	3.012
14	0.692	1.761	2.145	2.977
15	0.691	1.753	2.131	2.947
16	0.690	1.746	2.120	2.921
17	0.689	1.740	2.110	2.898
18	0.688	1.734	2.101	2.878
19	0.688	1.729	2.093	2.861
20	0.687	1.725	2.086	2.845
21	0.686	1.721	2.080	2.831
30	0.683	1.697	2.042	2.750
40	0.681	1.684	2.021	2.704
50	0.680	1.679	2.010	2.679
60	0.679	1.671	2.000	2.660
∞	0.674	1.645	1.960	2.576

The value for the *t* estimator provides a coverage factor that is a function of the probability, *P*, and the degrees of freedom in the data set, *v* = *N* – 1. These *t* values can be obtained from Table 4.4, which is a tabulation from the *Student's t distribution* as developed by William S. Gosset<sup>5</sup> (1876–1937). Gossett recognized that the use of the *z* variable with *s<sub>x</sub>* in place of *σ* did not yield accurate estimates of the precision interval, particularly at small degrees of freedom. Careful inspection of Table 4.4 shows that the *t* value inflates the size of the interval required to attain a percent probability, *P*%. That is, it has the effect of increasing the magnitude of *t<sub>v,P</sub>s<sub>x</sub>* relative to *z<sub>1σ</sub>* at a desired probability. As the value of *N* increases, *t* approaches those values given by the *z* variable just as the value of *s<sub>x</sub>* must approach *σ*. It should be understood that for very small sample sizes (*N* ≤ 10), sample statistics can be misleading. In that situation other information regarding the measurement may be required, including additional measurements.

<sup>5</sup> At the time, Gosset was employed as a brewer and statistician by a well-known Irish brewery. You might pause to reflect on his multifarious contributions.

4.11

$$P = 95\%$$

$$x_i = \bar{x} \pm t_{v, P} \cdot s_x$$

$\sigma_x$  ✓  $| \approx x$   
 $s_x \checkmark$   $(95\%)$

$$F_1 \rightarrow x_i = 50.465 \pm [t_{19, 95\%} \cdot 0.972]$$

$$= 50.465 \pm [2.093 \cdot 0.972]$$

$$x_{i_{f_1}} = 50.465 \pm 2.034 \quad (95\%)$$

$$F_2 \rightarrow x_i = 50.680 \pm 2.093 \cdot 1.1768$$

بناءً كمان على الفرع السابق طالب مني الـ  
Range الـ  $n \leq 20$  لهيك يعتبر انه finite

$$F_3 \rightarrow x_i = 50.640 \pm 2.093 \cdot 0.9917$$

Case 3

$$4.12 \quad x^1 = \bar{x} \pm t_{19,95\%} \cdot s_{\bar{x}}$$

CI →

$$F_1 \rightarrow x^1 = 50.465 \pm (2.093 \times 0.2173)$$

$$F_2 \rightarrow x^1 = 50.680 \pm (2.093 \times 0.2631)$$

$$F_3 \rightarrow x^1 = 50.640 \pm (2.093 \times 0.219)$$

اذا جابلك سيرة CI او قلك point estimate بتشتغل على هذا  
وهذا تابع للفرع السابق طالب

Case 4

$\downarrow$  Variability  $\downarrow$   $S_x$ ,  $\uparrow N$   $\leftarrow$   
 $\downarrow$   $S_x$   $\sim$  CI  $\leftarrow$

case 4  
اذا طلب CI او estimate فورا بقول هاي

الا في حالة ذكر fall بقول فورا هاي كالصفحة التالية

4.13

case 7.1

$$\begin{aligned}x_i &= \bar{x} = k_{v,p} \cdot s_x \\&= 50.640 \pm 2.055 * 0.9817 \quad (95\%) \\&= 50.640 \pm 2.055\end{aligned}$$



case 7.1

pooled

4.14'

$$x_i = \bar{\bar{x}} \pm t_{v,p} s_{\bar{x}}$$
$$\bar{\bar{x}} = \frac{\bar{x}_{F_1} + \bar{x}_{F_2} + \bar{x}_{F_3}}{3}$$

pooled  
mean

$$= \frac{50 \cdot 465 + 50 \cdot 658 + 50 \cdot 64}{3}$$

$$\bar{\bar{x}} = 50 \cdot 595$$

$$s_{\bar{x}} = \frac{0.972 + 1.176 + 0.981}{3}$$

$$s_{\bar{x}} = \frac{s_x}{\sqrt{MN}}$$

case 5

$$\bar{x} = 50.595$$

$$Sx = \frac{0.072 + 1.176 + 0.981}{3}$$

$$Sx = 1.043$$

$$S\bar{x} = \frac{Sx}{\sqrt{MN}}$$

$$S\bar{x} = \frac{1.043}{\sqrt{3 * 20}}$$

$$S\bar{x} = 0.1346 \\ \approx 0.135$$

$$\bar{x} = 50.595$$

$$S_x = \frac{0.072 + 1.176 + 0.981}{3}$$

$$S_x = 1.043$$

$$S\bar{x} = \frac{1.043}{\sqrt{3 * 20}}$$

$$S\bar{x} = 0.1346$$

$$S\bar{x} \approx 0.135$$

$$x_i = \bar{x} \pm t_{v,p} + S\bar{x}$$

$$x_i = 50.595 \pm 2.093 * 0.135$$

(95%)

CI

$$x_i = \bar{x} \pm t_{v, \rho} * s_{\bar{x}}$$

١٩, ٩٥٪.

$$x_i = 50.595 \pm 2.093 * 0.135 \quad (95\%)$$

CI

في حالة طلبني مقدار CI و Range  
في هذا السؤال نفس الفكرة

1	923
2	932
3	908
4	932
5	919
6	916
7	927
8	931
9	926
10	923

$$X_i = \bar{X} \pm t_{v, p} * S_x \rightarrow 95\%.$$

$\downarrow 10 - 1 = 9$

$$\bar{X} = 923.7$$

$$S_x = 7.718$$

$$X_i = 923.7 \pm (2.262) (7.718)$$

$$X_i = 923.7 \pm 17.458$$

$$X_i = 906.242 \leq X_i \leq 941.158$$

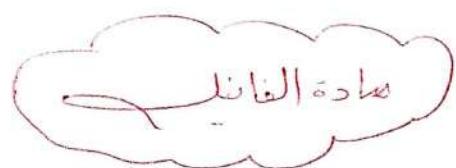
✓

السؤال طالب اوجدي قيمة Range + حكالي

انه اطرد القيم من الجدول بلي براته ، هسا انا بحسب العناصر واخترت t

لأنه هي أقل من ٢٠ ، بعد هيك جبت الرينج الان اي قيمة بالجدول

مش من ضمن الفترة بحذفها



## (ch 5) Uncertainty analysis

### 5.1 (introduction)

the results! Uncertainty analysis provides a methodical approach to estimating the quality of the results from an anticipated test or from a completed test. This chapter focuses on how to estimate the "± what?" in a planned test or in a stated test result.)

يعظر تحليل عدم المطابق نظرية مترقبة لتقدير جودة النتائج من غيرها المنشورة أو ماحتها،  
الإنتشار ويرجع هنا النتائج على كثافة قدر "ماذا؟" في اختبار المعيار أو في نتائجه

اختبار معلنة :

- error are property of the measurement.
- Uncertainty is a property of the result.
- error are effects and uncertainty are number

### 5.2 (Measurement error)

$\bar{x}$  average is called: Random error ( $\rightarrow$  Error is independent of the value of the measured quantity)

ويمكن يكونه موجب أو سالب ويقتصر على حالات

Replication (B)      Repetition (A)

النوع الثاني هو ادى المقدار بعيدة عن القيمة المقصودة systematic error ( $\rightarrow$  error is dependent on the value of the measured quantity)

A) caliperation -> وحدات المقدار متحدة الاختلاف وينتج عن طرق غير ملائمة

B) dominant method

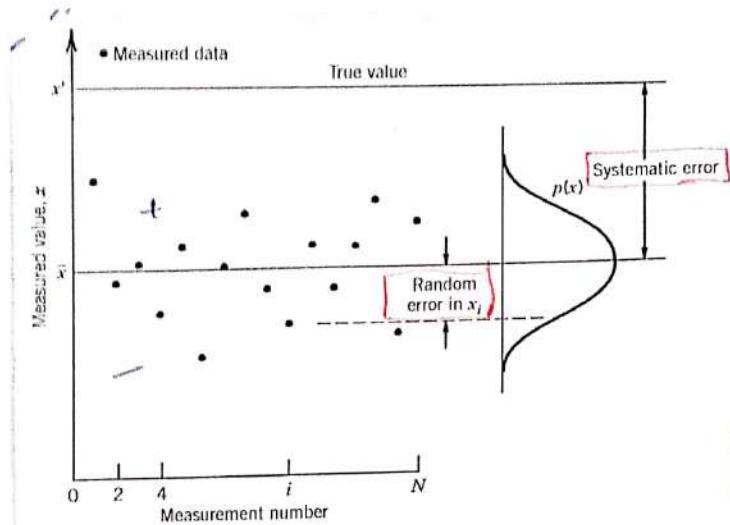


Figure 5.1 Distribution of errors on repeated measurements.

the uncertainty in that value,

$$(x' = \bar{x} \pm u_x \quad (P\%)) \quad (4.1)$$

في كل التجاری تكون داخير اسواره ای کوئنہ حاصل کرے تو جو داد دیجی اس تجارتی میں

$$x' = \bar{x} \pm u_x \quad (P\%) \quad \rightarrow \text{مذکور}$$

$x'$  → ~~precise~~ predictive value

$u_x$  → uncertainty.

### (5.3) design-stage Uncertainty analysis

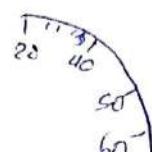
$u_d$   $\rightarrow$   $u_0$  zero-order uncertainty  
 uncertainty design  $\rightarrow$   $u_c$  catalogue uncertainty.

resolve the information provided by the instrument (This zero-order uncertainty of the instrument,  $u_0$ , assumes that the variation expected in the measured values will be only that amount due to instrument resolution and that all other aspects of the measurement are perfectly controlled.)

يمكن أن يكون الخطأ في المترادفات مقداراً ملحوظاً في المترادفات

$$U_0 = \pm \frac{1}{2} \text{ resolution (95%)} \\ \text{Resolution} = 1$$

$$\text{Resolution} = \frac{40 - 20}{4} = 5$$



Resolution II

$$U_0 = \pm \frac{1}{2} 5 \text{ (95%)} \\ U_0 = \pm 2.5 \text{ (95%)}$$

$$U_0 = \pm 2.5 \text{ (95%)}$$

$$\text{Resolution} = 0.01$$

20.20

$$U_0 = \pm 0.005 \text{ (95%)}$$

(The second piece of information that is usually available is the manufacturer's statement concerning instrument error. We can assign this stated value as the *instrument uncertainty*,  $u_c$ . Essentially,  $u_c$  is an estimate of the expected systematic uncertainty due to the instrument. If no probability level is provided with such information, a 95% level can be assumed.)

الخطوة الثانية هي الحصول على قراءة ملحوظة لبيان ارتكبة الخطأ  
حيث يطلب بخطاب المترادفات (بـ 95%)، ذلك يعني أنه يجب أن تكون القراءة ملحوظة

(9-10) هي الخطوة الثالثة لبيان ارتكبة الخطأ

حيث القراءة تكون ملحوظة

$$\underline{9.5} \approx 9.3 \text{ cm}$$

Resolution II needs to be higher

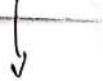
$$\underline{10} \approx 9.7$$

## Catalogue errors

Combining Element Error : RSS Method (Uncertainty propagation)

$$u_x = \sqrt{u_1^2 + u_2^2 + \dots + u_k^2}$$

$$= \sqrt{\sum_{k=1}^K u_k^2} \quad (P\%)$$



(95%)

is to 15% of

$u_c$  II

Catalogue error

## Design-Stage Uncertainty

The design-stage uncertainty,  $u_d$ , for an instrument or measurement method is an interval found by combining the instrument uncertainty with the zero-order uncertainty.

$$u_d = \sqrt{u_0^2 + u_c^2} \quad (P\%) \rightsquigarrow 95\% \quad (5.3)$$

نهاية منتهى كنه اوجي ال  $u_d$  وال  $u_c$  مثل اوجي  $u_0$

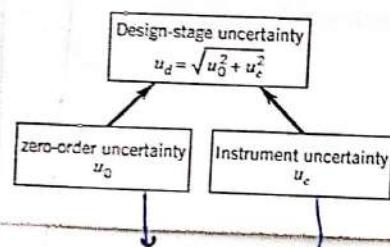


Figure 5.2 Design-stage uncertainty procedure in combining uncertainties.

أخطاء للشخص منه لا يدرك في  
القياس

أخطاء من  
الأداة صالح  
القياس

### Example 5.1

Consider the force measuring instrument described by the following catalog data. Provide an estimate of the uncertainty attributable to this instrument and the instrument design-stage uncertainty.

**U<sub>0</sub>**

( Resolution:	0.25 N
Range:	0 to 100 N
Linearity error:	within 0.20 N over range
Hysteresis error:	within 0.30 N over range

**U<sub>1</sub>**

*Instrument  
Design Stage*

**KNOWN** Catalog specifications

**U<sub>2</sub>**

**ASSUMPTIONS** Instrument uncertainty at 95% level; normal distribution

**FIND**  $u_c$ ,  $u_d$

**SOLUTION** We follow the procedure outlined in Figure 5.2. An estimate of the instrument uncertainty depends on the uncertainty assigned to each of the contributing elemental errors of linearity,  $e_1$ , and hysteresis,  $e_2$ , respectively assigned as

$$u_1 = 0.20 \text{ N} \quad u_2 = 0.30 \text{ N}$$

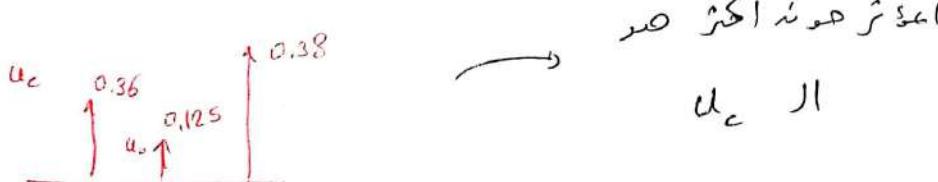
Then using Equation 5.2 with  $K = 2$  yields

$$\begin{aligned} u_e &= \sqrt{(0.20)^2 + (0.30)^2} \\ &= 0.36 \text{ N} \end{aligned}$$

The instrument resolution is given as 0.25 N, from which we assume  $u_0 = 0.125 \text{ N}$ . From Equation 5.3, the design-stage uncertainty of this instrument would be

$$\left( u_d = \sqrt{u_0^2 + u_e^2} = \sqrt{(0.36)^2 + (0.125)^2} \right) \\ = \pm 0.38 \text{ N (95\%)}$$

**COMMENT** The design-stage uncertainty for this instrument is simply an estimate based on the "experience" on hand, in this case the manufacturer's specifications. Additional information might justify modifying these numbers or including additional known elemental errors into the analysis.



### Example 5.2

A voltmeter is used to measure the electrical output signal from a pressure transducer. The nominal pressure is expected to be about 3 psi ( $3 \text{ lb/in.}^2 = 0.2 \text{ bar}$ ). Estimate the design-stage uncertainty in this combination. The following information is available:

**U<sub>C</sub>**

**Voltmeter**

**U<sub>0</sub>**

Resolution:  $10 \mu\text{V}$   
Accuracy: within 0.001% of reading

**Transducer**

Range:  $\pm 5 \text{ psi} (\sim \pm 0.35 \text{ bar})$   
Sensitivity:  $1 \text{ V/psi}$   
Input power:  $10 \text{ VDC} \pm 1\%$   
Output:  $\pm 5 \text{ V}$   
Linearity error: within  $2.5 \text{ mV/psi}$  over range  
Sensitivity error: within  $2 \text{ mV/psi}$  over range  
Resolution: negligible

**KNOWN** Instrument specifications

**ASSUMPTIONS** Values at 95% probability; normal distribution of errors

**FIND**  $u_c$  for each device and  $u_d$  for the measurement system

**SOLUTION** The procedure in Figure 5.2 is used for both instruments to estimate the design-stage uncertainty in each. The resulting uncertainties are then combined using the RSS approximation to estimate the system  $u_d$ .

The uncertainty in the voltmeter at the design stage is given by Equation 5.3 as

$$(u_d)_E = \sqrt{(u_o)_E^2 + (u_c)_E^2}$$

From the information available,

$$(u_0)_E = 5 \mu\text{V}$$

For a nominal pressure of 3 psi, we expect to measure an output of 3 V. Then,

$$(u_c)_E = (3 \text{ V} \times 0.00001) = 30 \mu\text{V}$$

so that the design-stage uncertainty in the voltmeter is

$$(u_d)_E = 30.4 \mu\text{V}$$

The uncertainty in the pressure transducer output at the design stage is also found using Equation 5.2. Assuming that we operate within the input power range specified, the instrument output uncertainty can be estimated by considering the uncertainty in each of the instrument elemental errors of linearity,  $e_1$ , and sensitivity,  $e_2$ :

$$\begin{aligned}(u_c)_p &= \sqrt{u_1^2 + u_2^2} \\ &= \sqrt{(2.5 \text{ mV/psi} \times 3 \text{ psi})^2 + (2 \text{ mV/psi} \times 3 \text{ psi})^2} \\ &= 9.61 \text{ mV}\end{aligned}$$

Since  $(u_0) \approx 0 \text{ V/psi}$ , the design-stage uncertainty in the transducer in terms of indicated voltage is  $(u_d)_p = 9.61 \text{ mV}$ .

Finally,  $u_d$  for the combined system is found by using the RSS method for the design-stage uncertainties of the two devices. The design-stage uncertainty in pressure as indicated by this measurement system is estimated to be

$$\begin{aligned}u_d &= \sqrt{(u_d)_E^2 + (u_d)_p^2} \\ &= \sqrt{(0.030 \text{ mV})^2 + (9.61 \text{ mV})^2} \\ &= \pm 9.61 \text{ mV} \quad (95\%)\end{aligned}$$

But since the sensitivity is 1 V/psi, the uncertainty in pressure can be stated as

$$u_d = \pm 0.0096 \text{ psi} \quad (95\%)$$

**COMMENT** Note that essentially all of the uncertainty is due to the transducer. Design-stage uncertainty analysis shows us that a better transducer, not a better voltmeter, is needed if we must improve the uncertainty in this measurement!

#### 5.4 Identifying error sources

acquisition, and data reduction. Errors that enter during each of these steps can be grouped under their respective error source heading: (1) calibration errors, (2) data-acquisition errors, and (3) data-reduction errors. Within each of these three error source groups, list the types of errors encountered.

الخطوة الأولى - التسجيل - (بيانات)  
الخطوة الثانية - (بيانات)  
الخطوة الثالثة - (بيانات)

## calibration Errors

رَعَيْتَ

(1) the standard or reference value used in the calibration, (2) the instrument or system under calibration, and (3) the calibration process.) For example, the laboratory standard used for

**Table 5.1** Calibration Error Source Group

Element	Error Source <sup>a</sup>
1	Standard or reference value errors
2	Instrument or system errors
3	Calibration process errors
4	Calibration curve fit (or see Table 5.3)
etc.	

<sup>a</sup>Systematic error or random error in each element

reduction requires calibration in some units.

$$E_1 = 0.1 \rightarrow E_2 = 0.01$$

۳- ایجاد کنترل مکانیزم برای این اتفاقات

جہاز

جامعة العمارنة  
الطباطبائى

1

### Data-Acquisition Errors

An error that arises during the act of measurement is listed as a data-acquisition error. These

## Data-Reduction Errors

#### Curve fits and correlations with their associated unknowns (,

**Table 5.2** Data-Acquisition Error Source Group

Element	Error Source <sup>a</sup>
1	Measurement system operating conditions
2	Sensor-transducer stage (instrument error)
3	Signal conditioning stage (instrument error)
4	Output stage (instrument error)
5	Process operating conditions
6	Sensor <u>installation</u> effects
7	Environmental effects
8	Spatial variation error
9	Temporal variation error
etc.	

<sup>a</sup>Systematic error or random error in each element.

**Note:** A total input-to-output measurement system calibration combines elements 2, 3, 4, and possibly 1 within this error source group.

spatial variation error :

لـ مثلاً بيـ اقـيـ درـجـةـ المـراـدـ سـيـارـةـ →  
لو اجيـ دـ حـصـيـنـ اـ كـعـنـاـسـ حـيـ اـمـرـكـيـ  
رـجـ بـ يـخـلـعـ هـ نـطـقـهـ طـفـلـةـ اـ كـرـاءـ  
حـيـ اـمـرـكـيـ .

*bio* ~~theo~~ ( $\rightarrow$ ) *bio* *theo* ②

الآن، وستنفق فقط من رصي

زنگنه فناوری افق

وال歇 جزء من صغر.

下

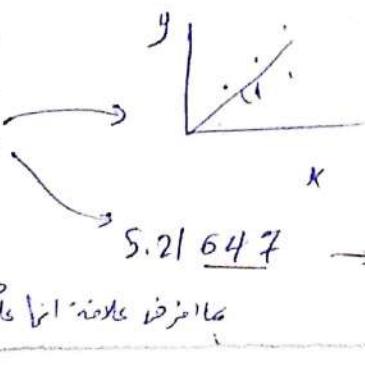
→ Temporal ~~Fogged~~ Variations ↗  
error

Qualities مهارات و مميزات HR Manager (HR)

Table 5.3 Data-Reduction Error Source Group

Element	Error Source <sup>a</sup>
1	Curve fit error
2	Truncation error
3	Modeling error
etc.	

<sup>a</sup>Systematic error or random error in each element.



$$5.21 \underline{647} \rightarrow 5.21 \text{ المتر} \rightarrow 647 \text{ Truncation error = 647}$$

$$y = a_0 + a_1 K_1 + a_2 K_2$$

موديل خطى لبيانات

### (5.5) Systematic and Random error

#### Systematic Error

الخطأ المترافق

A systematic error<sup>2</sup> remains constant in repeated measurements under fixed operating conditions.

equivalent to probability level of 68% for normal distribution. The systematic uncertainty at any confidence level is given by  $t_{v,pb}$ , or simply  $t_b$ . The interval defined by the systematic uncertainty at the 95% probability level is written as

$$\pm B = \pm 2b \quad (95\%) \quad (5.4)$$

الخطأ المترافق هو مترافق في كل المعاينات. ونرى أن التقدير الصعب للخطأ المترافق من دون المقارنة، لذا يجب اتخاذ إجراءات لتقديره. هناك عدة طرق: (1) التسليمة، (2) طريقة المعاينات المترافق، (3) المعاينات بين المختبرات، أو (4) تقييم الخبرة. مع ذلك، يمكن التسليمة باستخدام التسليمة.

التجربة / الخبرة

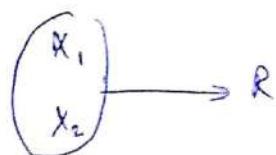
التجربة / الخبرة

#### Random Error

The estimate of the probable range of a random error is given by its random uncertainty. The random standard uncertainty,  $s_{\bar{x}}$ , is defined by the interval given by  $\pm s_{\bar{x}}$ , where

$$s_{\bar{x}} = s_x / \sqrt{N} \quad (5.5)$$

5.6 ~~Uncertainty~~ Uncertainty Analysis: [EPR ~~Propagation~~ Propagation]

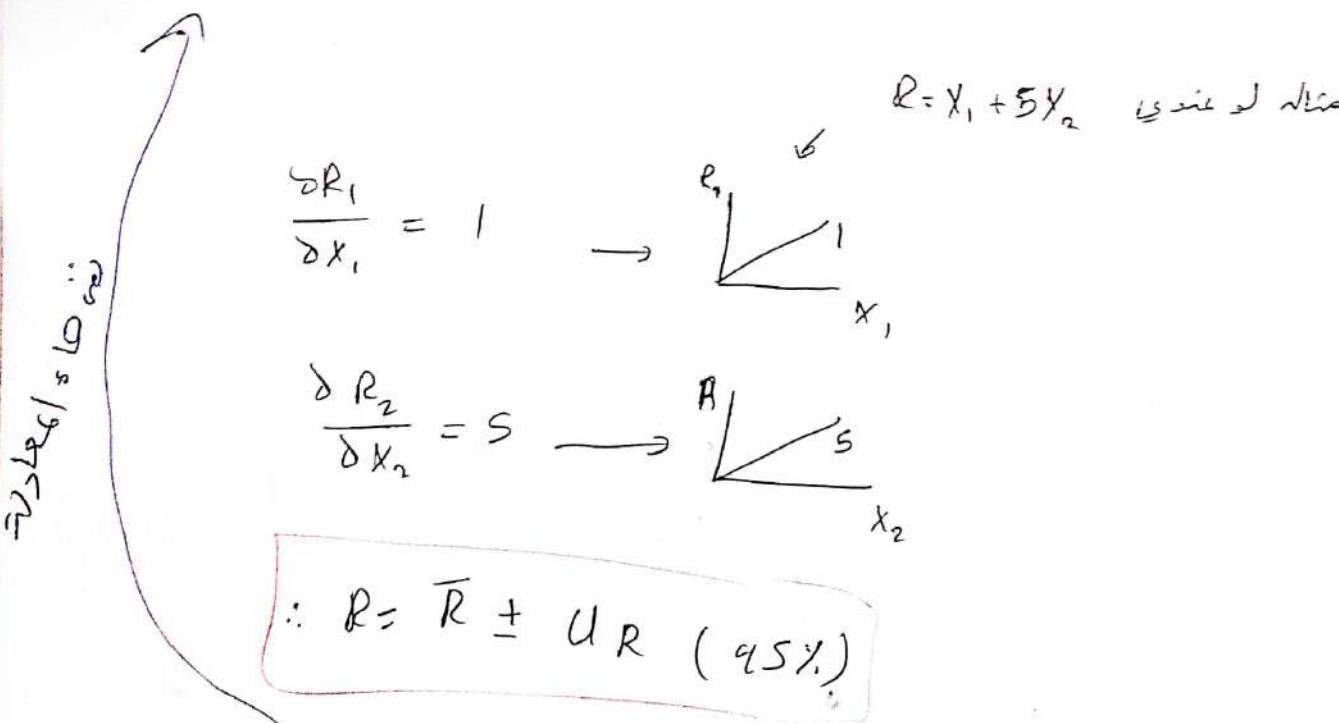


عوامل x<sub>i</sub> يدخل في حساب R  $\rightarrow$   
كل عوامل حفظها، وتحتها خطأي  
 $R \rightarrow$

$$R = f(x_1, x_2, \dots, x_n)$$

$$R = f(x_i) \rightarrow i \sim (1-n)$$

$$\frac{u_R^2}{\text{Total uncertainty}} = \sum_{i=1}^n \left( \frac{\partial R}{\partial x_i} \right)^2 \cdot \frac{(u_{x_i})^2}{\text{sensitivity uncertainty}}$$



The contribution of the uncertainty in  $x$  to the result  $R$  is estimated by the term  $\theta_i u_{x_i}$ . The most probable estimate of  $u_R$  is generally accepted as that value given by the second power relation (4), which is the square root of the sum of the squares (RSS). The propagation of uncertainty in the variables to the result is by

$$u_R = \left[ \sum_{i=1}^L (\theta_i u_{x_i})^2 \right]^{1/2} \quad (5.15)$$

حالات حكينا جمل

ناتيجه العدائية

$$R = X_1 \pm X_2 \dots \pm X_n$$

$$\sigma_R = \sqrt{\sum_{i=1}^n \left( \frac{\partial R}{\partial X_i} \right)^2 \cdot (\sigma_{X_i})^2}$$

$$R = \bar{R} \pm \sigma_R (\text{asy.})$$

$$R = X_1 \pm \bar{R} X_2 \quad -\text{ناتيجه}$$

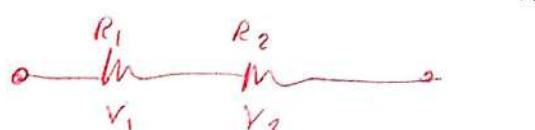
$(\sigma_{X_1})^2$  استبعدهم ونوعهم بعدد ص امر بـ ✓

$$\left( \frac{\sigma_R}{\sigma_{X_1}} \right)^2 = (1)^2 = 1, \quad \left( \frac{\sigma_R}{\sigma_{X_2}} \right)^2 = (1)^2 = 1 * (\sigma_{X_2})^2 = (\sigma_{X_2})^2$$

$$1 * (\sigma_{X_1})^2 = (\sigma_{X_1})^2$$

$$\sigma_R = \sqrt{(\sigma_{X_1})^2 + (\sigma_{X_2})^2}$$

$$R = \underbrace{X_1 \pm X_2}_{\bar{R}} \quad \pm \sqrt{(\sigma_{X_1})^2 + (\sigma_{X_2})^2} \quad (\text{asy.})$$



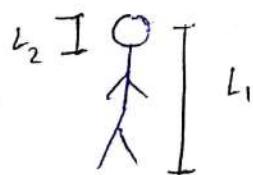
حالات الواقع :-

$$(الحالات) R_1 + R_2 \approx \text{total resistance} \Rightarrow$$

$$(الحالات) V_1 + V_2 \approx V_{eq} \text{ مرضي للجهة}$$

الحالات الالكترونية بـ  $V_{eq}$   $\approx$

$$V_{eq} = L_1 - L_2$$



فإذن الخطأ المعيارى يعطى كالت

$$R = \frac{X_1 X_2}{\sqrt{X_1^2 + X_2^2}}$$

$$u_R^2 = \sum_{i=1}^2 \left( \frac{\partial R}{\partial x_i} \right)^2 \times (u_{x_i})^2$$

لخطوة انا احسب الجذر  
العدي

$$u_R^2 = \left( \frac{\partial R}{\partial x_1} \right) \circ (u_{x_1})^2 + \left( \frac{\partial R}{\partial x_2} \right) \circ (u_{x_2})^2$$

$$u_R^2 = \frac{(X_2)^2 \cdot (u_{x_1})^2}{X_1^2} + \frac{(X_1)^2 \cdot (u_{x_2})^2}{X_2^2}$$

$$\begin{cases} X_1^2 u_{x_1}^2 = \text{خطأ معياري} \\ X_2^2 u_{x_2}^2 = \text{خطأ معياري} \end{cases}$$

$$u_R^2 = R^2 \frac{(u_{x_1})^2}{X_1^2} + R^2 \frac{(u_{x_2})^2}{X_2^2}$$

$$\begin{aligned} R &= X_1 \cdot X_2 \\ R^2 &= X_1^2 + X_2^2 \end{aligned}$$

$$\frac{u_R^2}{R^2} = \frac{(u_{x_1})^2}{X_1^2} + \frac{(u_{x_2})^2}{X_2^2}$$

نسبة الخطأ المعياري لـ  $R$

error 1 نسبة  
الخطأ المعياري  
الثانوي

error 2 نسبة  
الخطأ المعياري  
الثانوي

$$\begin{aligned} u_{x_1} &= 1 - 8 \approx 1 \\ X_1 &= 100 \end{aligned}$$

$$\frac{u_{x_1}}{X_1} = 1 \%$$

نسبة الخطأ المعياري لـ  $x_1$

$$u_R = \sqrt{\left( \frac{(u_{x_1})^2}{X_1^2} + \frac{(u_{x_2})^2}{X_2^2} \right) \cdot R^2}$$

$$R = \sqrt{X_1 \cdot X_2} \pm \sqrt{\left( \frac{(u_{x_1})^2}{X_1^2} + \frac{(u_{x_2})^2}{X_2^2} \right) \cdot R^2} \quad (\% \text{ of } R)$$

Zero order

**for instrument resolution alone** The value  $u_0$  estimates the extent of variation expected in the measured value when all influencing effects are controlled and is found using Equation 5.1. By itself, a zero-order uncertainty is inadequate for the reporting of test results.

ch6

## Analog Electrical Devices and Measurement

المفاهيم الـ ٤ في المتر

- 1) Basic principles of electric measurement.
- 2) Bridge circuits
- 3) Null mode / deflection mode
- 4) signal conditioning.
- 5) grounding / shielding.

- understand the principles behind common analog voltage and current measuring devices,
- understand the operation of balanced and unbalanced resistance bridge circuits,
- define, identify, and minimize loading errors,
- understand the basic principles involved in signal conditioning, especially filtering and amplification, and
- apply proper grounding and shielding techniques in measuring system hookups.

موجود داخل سياق كهربائي رسموا فيه قوة نجت  
وجود مجال مغناطيسي داخل سياق conductor

target Torque  $\propto$  Torque due to current  $\propto$  deflection

④ Analog devices (current measurement)  $\rightarrow$  direct current

Force  $\propto$  IAB  $F = IIB$

$\rightarrow$  L: length of conductor  
B: magnetic field strength

$$T_u = N/A B \sin \alpha$$

A = cross section Area

B = magnetic field strength

I = current

$\alpha$  = angle between cross section Area and magnetic field

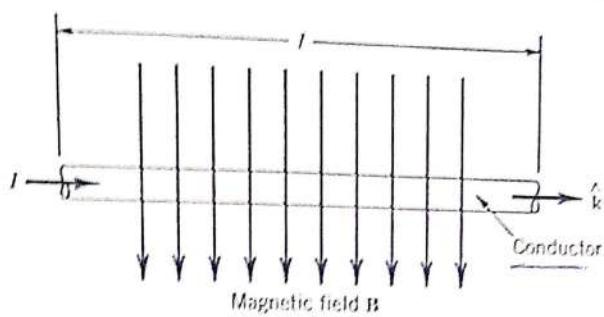


Figure 6.1 Current-carrying conductor in a magnetic field.

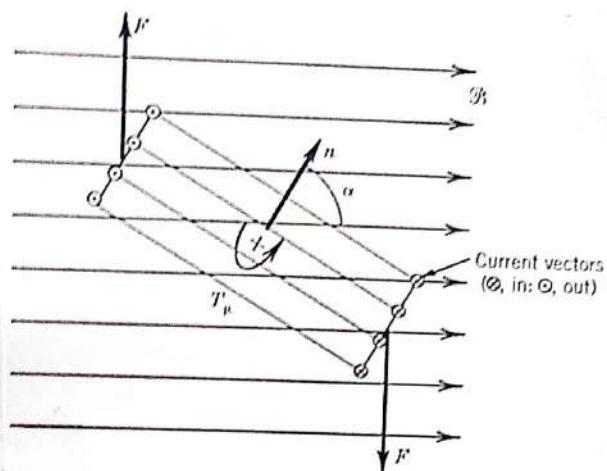


Figure 6.2: Forces and resulting torque on a current loop in magnetic field

نحوه موجي حوزه ?  
Voltage ( $E$ )  $\rightarrow$   $I$  (current)  $\rightarrow$   $T$  (Torque)

$\rightarrow \theta$   $\rightarrow$  deflection mode

كتاب مع عينة اليار ادجعه في تفاصيل  
جنب

null mode

روح تاحد على املاك و تغير

1) Bridge circuit , 2) galvanometer

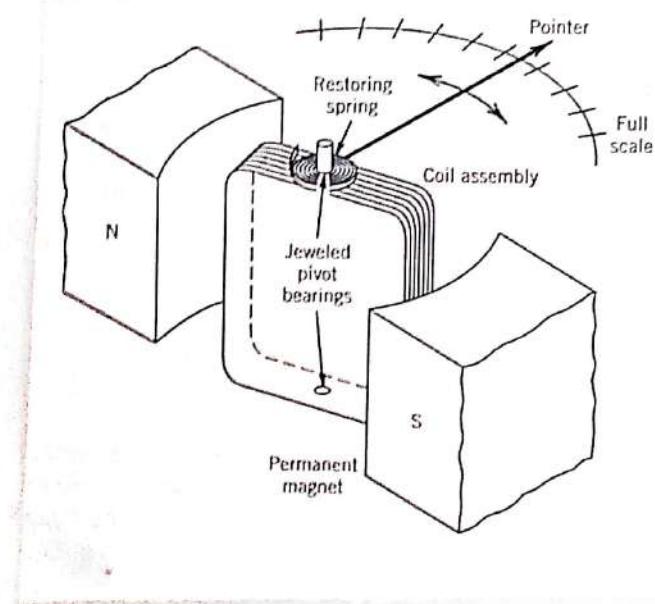


Figure 6.3 Basic D'Arsonval meter movement.

galvanometer از صوحه  
تساس فله موجيه تيار روح تغير  
الى داخله فلارغ اير بيرت  
 $I_g = 0$  بجي

We apply Newton's second law with the mechanical free-body diagram shown in Figure 6.5a.

second order

$$\text{ميكانيكا ثانية} \quad J \frac{d^2\theta}{dt^2} + c \frac{d\theta}{dt} + k\theta = T_{\mu} \quad T_{\mu} \text{ متر} \quad (6.4)$$

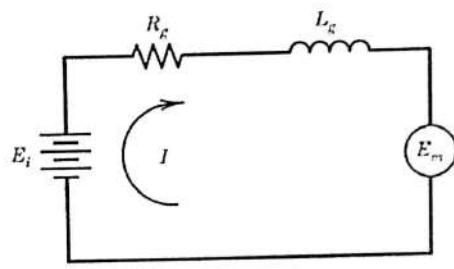
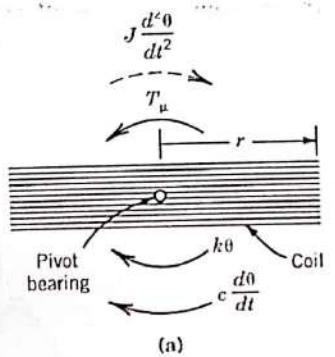
As a current passes through the coil, a force is developed that produces a torque on the coil:

$$\text{متر} \rightarrow \text{معادلة ثانية} \rightarrow T_{\mu} = 2NBlrI \quad (6.5)$$

the electrical free body of Figure 6.5b gives

$$L_g \frac{dI}{dt} + R_g I = E_i - E_m \quad R \parallel L \text{ متر} \quad (6.7)$$

جهاز قياس المتر



جهاز قياس المتر

Figure 6.5 Circuit and free-body diagram for Example 6.1.

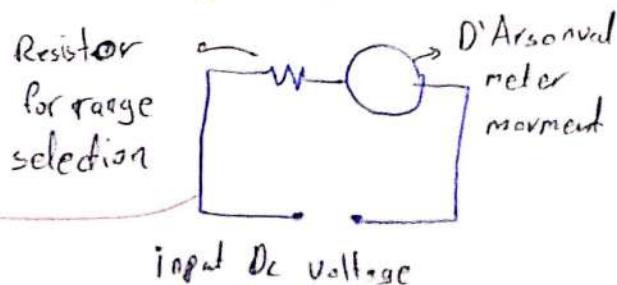
التيار المتر ينبع من التيار المتر المترافق مع حركة المتر (أ) التيار المتر ينبع من التيار المتر المترافق مع حركة المتر المترافق (ب)

التيار المتر ينبع من التيار المتر المترافق مع حركة المتر المترافق (أ) التيار المتر ينبع من التيار المتر المترافق مع حركة المتر المترافق (ب)

### Analog Voltage Meter

An AC voltage can be measured through rectification or through the use of an electromagnet either in an electrodynamometer or with a movable iron vane.

A DC voltmeter circuit



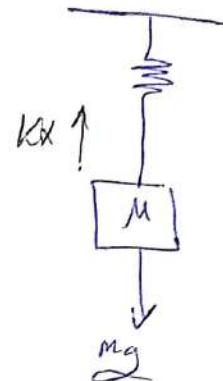
deflictive or Null

\* فحص زن و مراجعة لفترة \*

أمثلة على اسفله ملحوظة

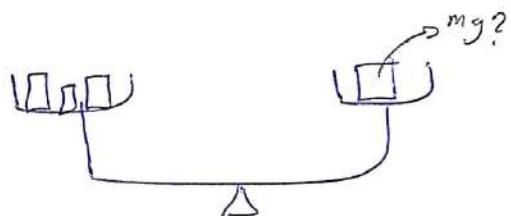
الورقة

((deflictive mode ))



$$E \rightarrow I \rightarrow T \rightarrow \theta$$

بذر اصطدام اوزانه يغير المبارز



(( Null mode ))

ما يغير وزنها في اطراد هفرست = وزنة الافتقال

## Oscilloscope

The Oscilloscope is a partial graphical device providing an analog representation of a measured signal. It is used to measure and to visually display voltage magnitude versus time for dynamic signal for dynamic signals over a wide range of frequencies with a signal bandwidth extending commonly into the megahertz range and, with some units, into the gigahertz range.

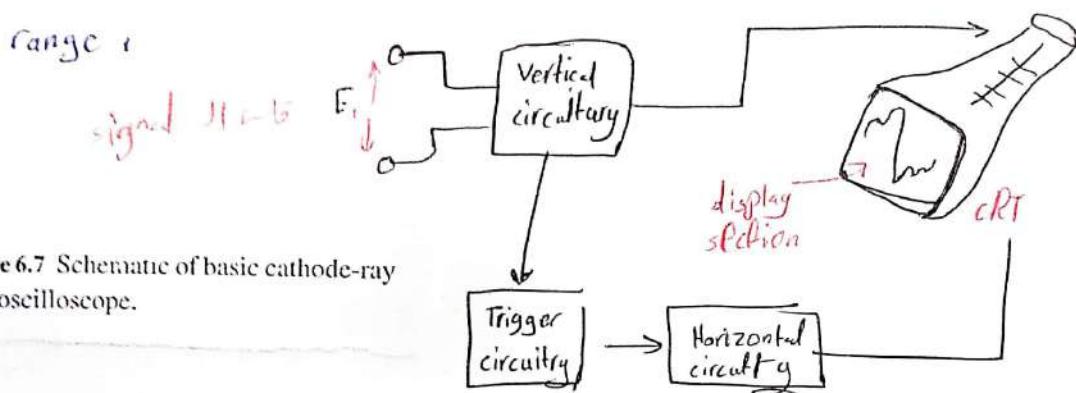
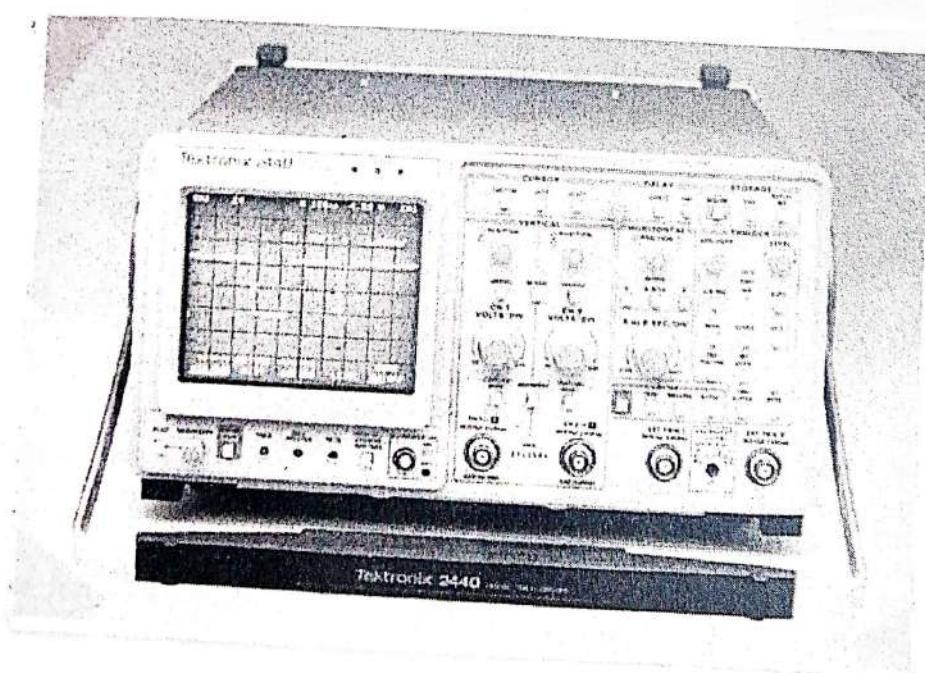
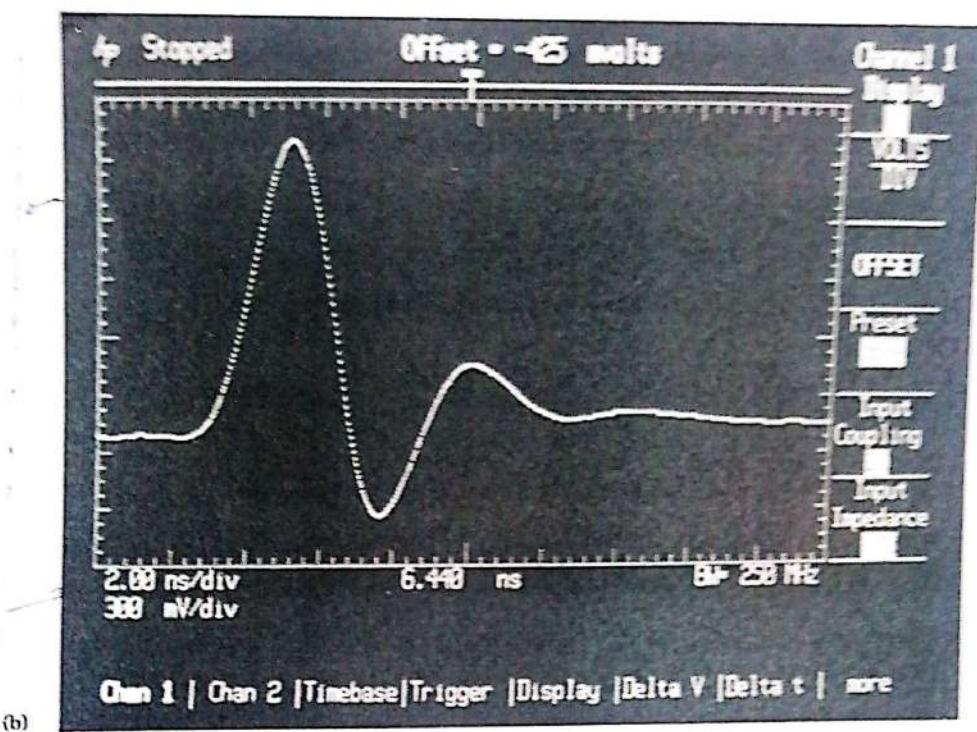


Figure 6.7 Schematic of basic cathode-ray tube oscilloscope.

Analog Electrical Devices and Measurements



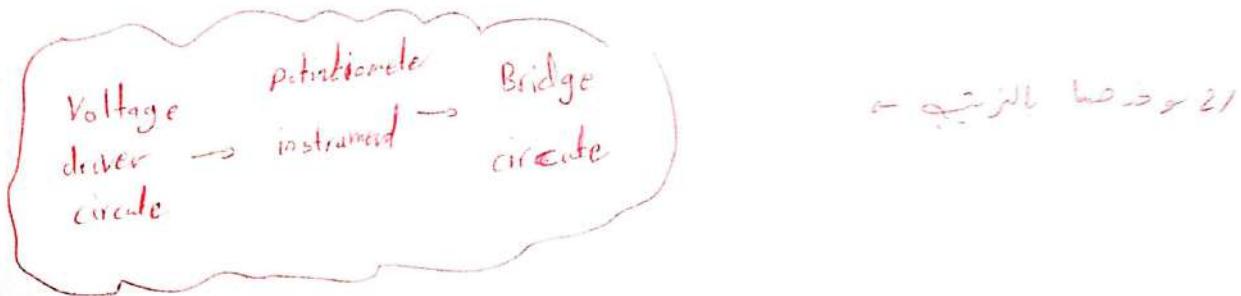


(b)

Chan 1 | Chan 2 | Timebase | Trigger | Display | Delta V | Delta t | more

Figure 6.8 (a) Digital oscilloscope. (Photograph courtesy of Tektronix, Inc.) (b) Oscilloscope output. (Photograph courtesy of Hewlett-Packard Company.)

what is in this menu electrical measurement جدول المعاينات لـ sensor II  
بعض المعاينات المهمة



Voltage divider circuit

مثال توصيف

$$V_{AB} = \frac{R_3}{R_T} \cdot E$$

وهو V



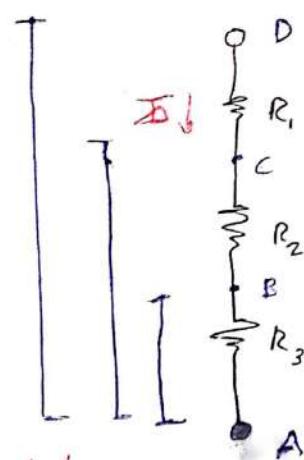
$$V_{AC} = \frac{R_2 + R_3}{R_T} \cdot E$$

$$E = V_{AP}$$

الكتل

$$V_{AD} = E$$

branching is available for D with 0 load



## Chapter 6 Analog Electrical Devices and Measurements

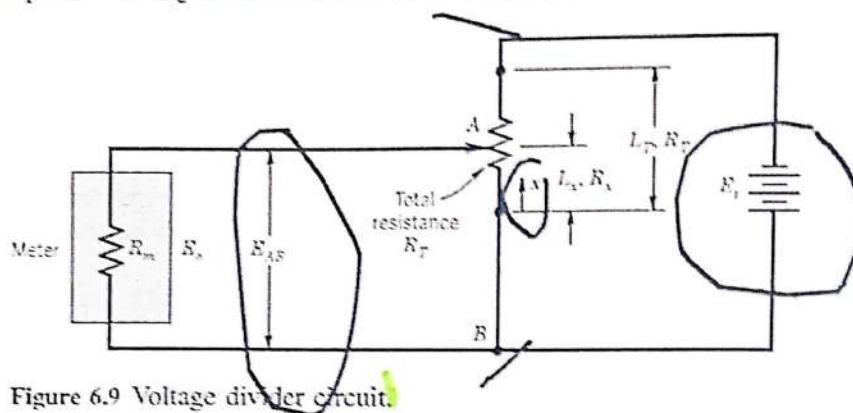


Figure 6.9 Voltage divider circuit.

$$\text{output voltage } E_o = \frac{R_{BA}}{R_T} \cdot E_i$$

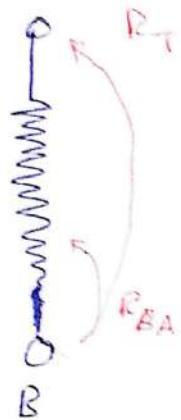
مدة إنتاج  
متغير

which holds so long as the internal resistance

$\approx E_i$  بحسب

of the meter,  $R_m$ , is very large relative to  $R_T$ .

$$E_o = \frac{R_x}{R_T} \cdot E_i$$



فقط اجزاء

$\approx \approx$  المتر  $R_m$   $\ll$   $R_T$   $\therefore 0 = (\text{أجزاء المتر})_b$   $E_i$   $\approx$   $E_o$  ) اجزاء

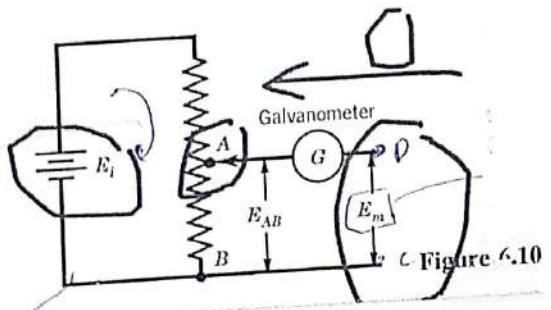
active  $\Rightarrow$  loading error  $\approx$    
 transducer

$$E_{\text{total}} = E_{\text{total}}' + \text{جزء المتر}$$

## Potentiometer instrument

balanced condition, indicated by a zero current flow through the galvanometer. A null balance, corresponding to zero current flow through  $G$ , occurs only when  $E_m = E_{AB}$ . With a known and

non measurement is based on voltage divider (1)



Basic potentiometer circuit.

مدى الايام  
يتبعها اصحاب الماء  
نقطة في الدارة وصفرها  
0 ماء

~~جهاز~~ جهاز galvanometer

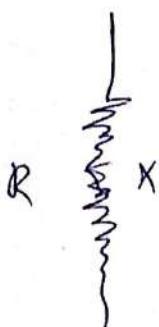
$$E_m = E_{AB} \quad \text{لأن } I_g = 0$$

galvanometer  
تيار اعماق

فلا يمر في الماء

$$E_{AB} = E_m \quad \text{لأن } E_m = E_{AB}$$

$$E_{AB} = \frac{R_{AB}}{R_T} \cdot E_i$$



حيث أن  $R$  تناسب مع الطول طردياً

$R$  يتناسب مع

$$R = \frac{\rho L}{A}$$

$\rho$ : resistivity

L: Length

A: cross section Area

## Ohmmeter Circuits

A simple way to measure resistance is by imposing a voltage across the unknown resistance and measuring the resulting current flow where  $I = \frac{V}{R}$  is the formula for resistance.

$$R = E/L$$

**Bridge Circuits** → *null method*  
→ *deflection method*

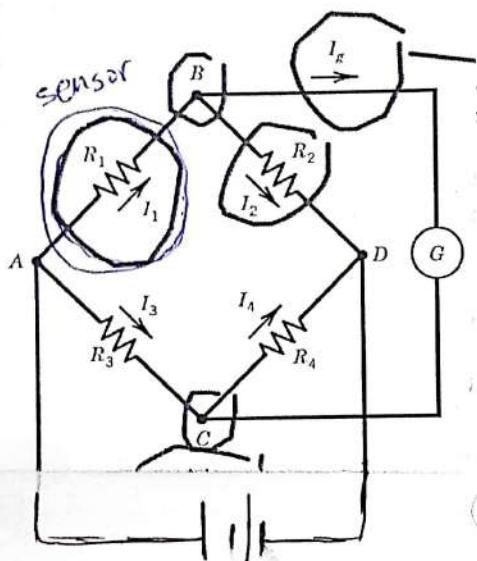


Figure  $E_1$   
Basic current-sensitive wheatstone bridge circuit (G, galvanometer)

صورة الفك رى اطیار اند طلے پیز سے حفظ sensor لازم اور لہائی بارہ  
پر جمع فری (سازنہ) palance

وكانه اى  $R_1$  و  $R_3$  ثابت بغير لازم اغير  $R_2$  بحسب يرجع بالانسحاب

$R_2 \uparrow$  لازم از  $\rightarrow \uparrow \text{sensor}(R_1)$

$R_1 \downarrow$  ملائم اقل من   $\downarrow$  sensar  $R_1$

Balance II As ظاہر نہیں

فی الحال نیز با Balance اینکه در میان دویست کمیتی  
 $I_{g=0}$  از گذشته داشتند  $I_1 = I_2$  است اما پس از

اگر ~~Balance~~ باید  $I_1 < I_2$  باشد

1)  $I_{g=0}$       2)  $I_1 = I_2$       3)  $I_3 = I_4$

4)  $V_B = I_c$       5) ~~A, D~~  $A, D$  نیز نیستند

(و)  $E_1 =$

6)  $R_1 \cdot R_4 = R_2 \cdot R_3$        $\frac{R_2}{R_1} = \frac{R_4}{R_3}$   
 (مزایا و معایب)

Balance نیز اینکه  $I_{g=0}$  دارای پیشنهاد شده است اینکه

$R_1 R_3 - R_2 R_4 = 0$        $\text{که درینجا } \boxed{V_B - V_C = 0} \text{ است}$

$\left\{ \begin{array}{l} I_1 R_1 - I_2 R_3 = 0 \rightarrow I_1 = I_2 \\ I_2 R_2 - I_3 R_4 = 0 \rightarrow I_2 = I_3 \end{array} \right. \quad \left\{ \begin{array}{l} I_1 = I_2 \\ I_3 = I_4 \end{array} \right.$

(که نیز داشته باشد) ایضاً  $I_1 = I_2$  است

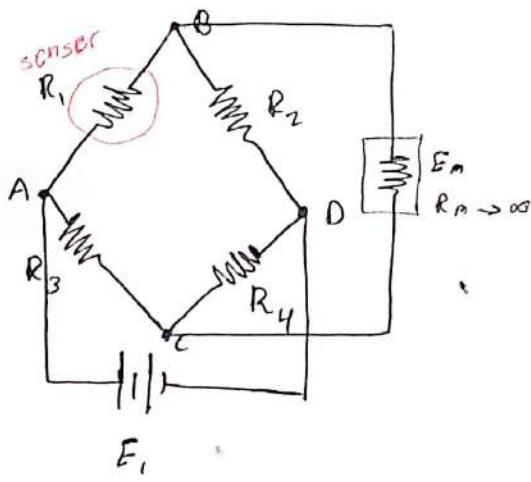
$$\frac{R_2}{R_1} = \frac{R_4}{R_3}$$

where  $R_2$  is an adjustable variable resistance.

through a closed-loop controller circuit. An advantage of the null method is that the applied input voltage need not be known, and changes in the input voltage do not affect the accuracy of the measurement. In addition, the current detector or controller need only detect if there is a flow of

## Deflection method

In an unbalanced condition, the magnitude of the current or voltage drop for the meter portion of the bridge circuit is a direct indication of the change in resistance of one or more of the arms of bridge. Consider first the case where the voltage drop from node  $B$  to node  $C$  in the basic bridge is



Balance is given by

$$R_1 R_4 = R_2 R_3$$

$$V_{BC} = 0$$

(sensor)  $R_s$  will be zero

$$R_1 R_4 \neq R_2 R_3$$

( $R_s$ ,  $B$ ) will be non-zero

one active arm is

$R_s$  will be non-zero

for

(Bridge output)  $V_{BC} = V_B - V_C$

$$= \frac{R_1}{R_1 + R_2} \cdot E_i - \frac{R_3}{R_3 + R_4} \cdot E_i$$

(+)

$\rightarrow$

$$V_{BC} = \frac{R_2}{R_1 + R_2} \cdot E_i - \frac{R_4}{R_3 + R_4} \cdot E_i$$

(-)

will be zero for  $E_i = 0$

In contrast to the null method of operation of a Wheatstone bridge, the deflection bridge requires a meter capable of accurately indicating the output voltage, as well as a stable and known

### Example 6.3

A certain temperature sensor experiences a change in electrical resistance with temperature according to the equation

$$R = R_o[1 + \alpha(T - T_o)] \quad (6.29)$$

where

$R$  = sensor resistance ( $\Omega$ )

$R_o$  = sensor resistance at the reference temperature,  $T_o$  ( $\Omega$ )

$T$  = temperature ( $^{\circ}\text{C}$ )

$T_o$  = reference temperature ( $0^{\circ}\text{C}$ )

$\alpha$  = the constant  $0.00395^{\circ}\text{C}^{-1}$

This temperature sensor is connected in a Wheatstone bridge like the one shown in Figure 6.13, where the sensor occupies the  $R_1$  location, and  $R_2$  is a calibrated variable resistance. The bridge is operated using the null method. The fixed resistances  $R_3$  and  $R_4$  are each equal to  $500\ \Omega$ . If the

temperature sensor has a resistance of  $100\ \Omega$  at  $0^{\circ}\text{C}$ , determine the value of  $R_2$  that would balance the bridge at  $0^{\circ}\text{C}$ .

KNOWN  $R_1 = 100\ \Omega$   
 $R_3 = R_4 = 500\ \Omega$

FIND  $R_2$  for null balance condition

$$\frac{R_2}{R_1} = \frac{R_4}{R_3}$$

SOLUTION From Equation 6.11, a balanced condition for this bridge would be achieved when  $R_2 = R_1 R_4 / R_3$  or  $R_2 = 100\ \Omega$ . Notice that to be a useful circuit,  $R_2$  must be adjustable and provide an indication of its resistance value at any setting.

## Strain Measurement

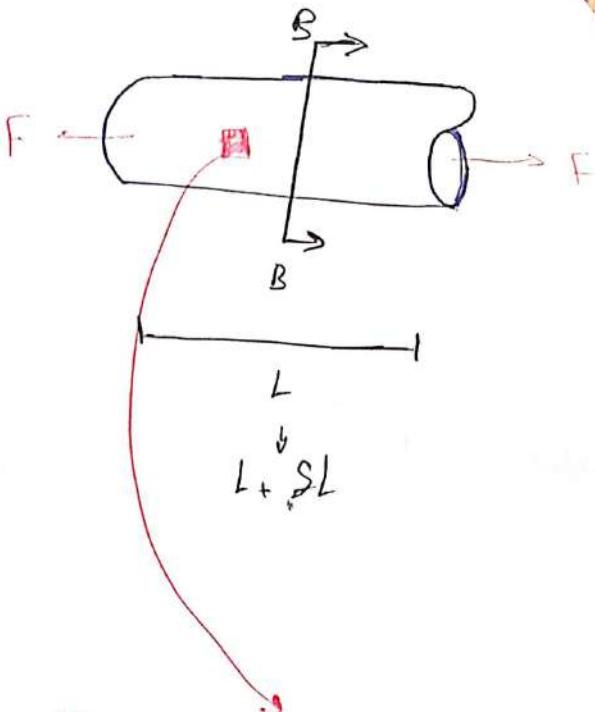
Tension  $\sigma \propto \epsilon$

عسانا ملحوظة مفعول قوى درجة

section II



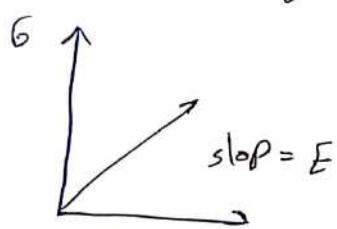
الحالات ~ المقادير



\* stress  $\sigma = F/A$

\* strain  $\epsilon = \frac{\Delta L}{L}$

\* Modulus of elasticity  $E = \frac{\sigma}{\epsilon} \rightarrow \sigma = E\epsilon$



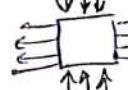
\* Poisson ratio ( $\nu$ )  $\nu = \frac{\epsilon_L}{\epsilon_a} = \frac{\frac{\Delta L}{L}}{\frac{\Delta d}{d}} = \frac{\epsilon_{lateral}}{\epsilon_{axial}}$

mechanical  $\nu = \frac{E_L}{E_a}$

ويجب أن يكون  $\nu < 0.5$  لعدم قيامه للقوة والجهد واحفاظه

واحدة واحدة في اتجاه

( $\sigma_y$ ) compression



( $\sigma_x$ ) tension

stresses in اتجاه لونية compression و tension

$\sigma_x$  tension  $\rightarrow$  طول

$\sigma_y$  compression  $\rightarrow$  عرض

(axial)  $\sigma_x$  vs (lateral)  $\sigma_y$

The design of load-carrying components for machines and structures requires information concerning the distribution of forces within the particular component. Proper design of devices such as shafts, pressure vessels, and support structures must consider load-carrying capacity and allowable deflections. Mechanics of materials provides a basis for predicting these essential characteristics of a mechanical design, and provides the fundamental understanding of the behavior of load-carrying parts. However, theoretical analysis is often not sufficient, and experimental measurements are required to achieve a final design.

يتطلب تصميم اجزاء تأمينها عن توزيع القوة داخل الكونفرنر  
 ① يجب ان يأخذ المعلم المتبقي بعين الاعتبار  
 ② يجب ان يأخذ المعلم المتبقي بعين الاعتبار  
 المكونات اما في المجموع

such changes in length or shape of a material can be measured. This chapter discusses the measurement of physical displacements in engineering components. The stress is calculated from these measured deflections.

Upon completion of this chapter, the reader will be able to ~~as per text~~

- define strain and delineate the difficulty in measuring stress,
- state the physical principles underlying mechanical strain gauges,
- analyze strain gauge bridge circuits, and
- describe methods for optical strain measurement.

## stress and strain

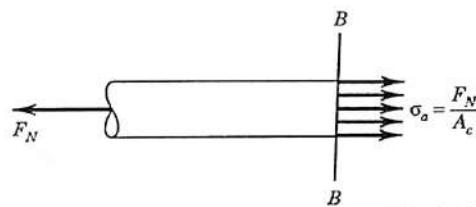
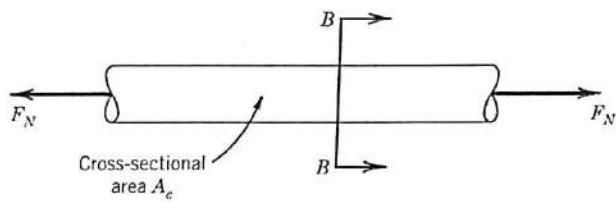


Figure 11.1 Free-body diagram illustrating internal forces for a rod in uniaxial tension.

the normal stress is defined as :-

$$\sigma_a = F_N / A_c$$

the ratio of the change in ~~height~~ length of the rod (which results from applying the load) to the original length is the axial strain defined as :-



$$\epsilon_a = \Delta L / L$$

the relationship between axial stress and strain for this elastic behaviour is expressed as:

$$\alpha_a = E_m \epsilon_a$$

$E_m$ : is modulus of elasticity, or young's modulus, and the relationship is called Hooke's law.

## Lateral strain

This property is called poisson ratio, defined as:

$$\nu_p = \frac{|\text{Lateral strain}|}{|\text{axial strain}|} = \frac{\epsilon_t}{\epsilon_a}$$

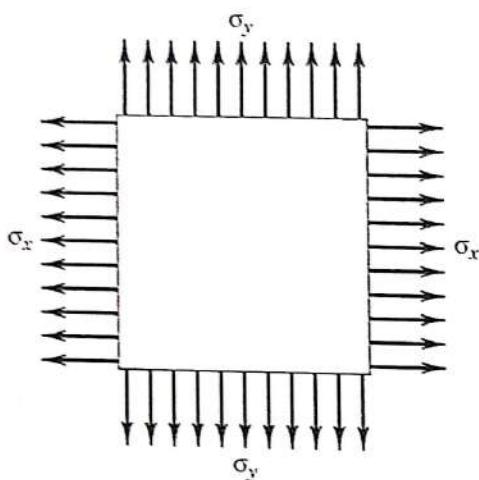


Figure 11.3 Biaxial state of stress.

عکس جذب احتیاط از اراده

الصيغة الاتية تسمى معادلة

أول الات من المدخلات الميكانيكية بطرفة بطيئة مثل:-

(1) مراقبة التغير في المسافة بين دلائل ناتجة عن سطح عالي و سطح منخفض

(2) مقدمة ملحوظة في حجم الجسم

عوادي طار طاره امسح انتشار اعندهي لينا سلاسلها دسخون

(3) تتحقق بقدرة ملحوظة حساسة

(4) لا يزيد انتشاره باطرافه اشكال

الردود لقياسات ايجاد

الاستنفافية

### 11.3 RESISTANCE STRAIN GAUGES

الى يعتمد لقياس ارتفاع اور ابعاد اور الخلافة

The measurement of the small displacements that occur in a material or object under mechanical load can be accomplished by methods as simple as observing the change in the distance between two scribe marks on the surface of a load-carrying member, or as advanced as optical holography. In any case, the ideal sensor for the measurement of strain would (1) have good spatial resolution, implying that the sensor would measure strain at a point; (2) be unaffected by changes in ambient conditions; and (3) have a high-frequency response for dynamic (time-resolved) strain measurements. A sensor that closely meets these characteristics is the bonded resistance strain gauge.

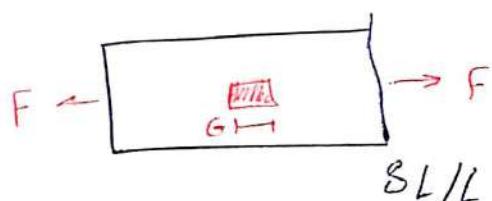
نوع المتر

دبي جهاز يقيس ال strain فار diffusivity

strain خواص

strain GAUGES

لو الواقعه الصيغه اي بلا حمر صا، اما اسفله  
عندها سطالتنه العظمه الكرة



كالكرة اسفله 0.01% الصيغه دع سهل 0.1%

غير الطول للجسم (غير مواد راعي هنا (رصاص صر صار العيابس)) ومقدار التغيره ضئيل جداً

## Metallic Gauges

\* the Resistance  $R$  is given by:-

$$R = \rho L / A_c$$

$$\left. \begin{array}{l} R = \rho L / A \\ L = \text{length} \\ A = \text{area} \\ \rho = \text{resistivity} \end{array} \right\}$$

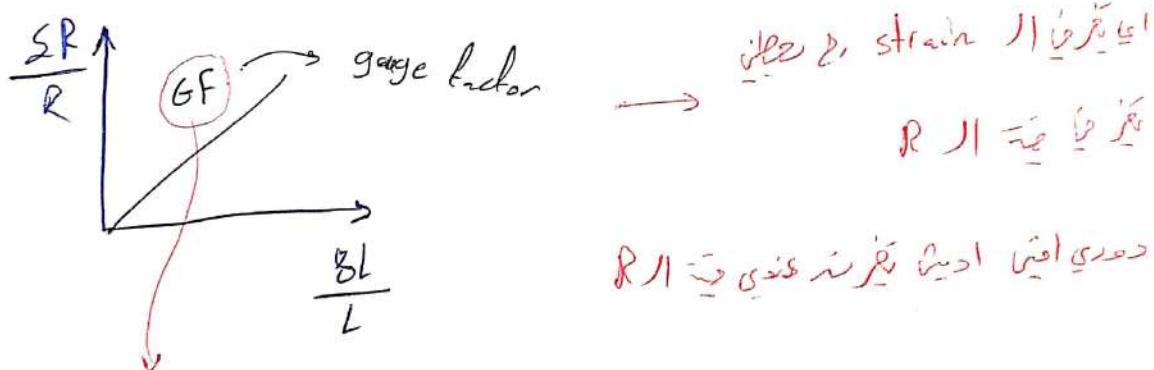
\* the total change in  $R$  is due to several effect, as illustrated in total differential:

$$dR = A_c (\sigma_e dt + L d\sigma_e) - \rho L dA_c$$

$$= \frac{\rho^2 L^2}{A_e^2} d\sigma_e$$

\* which may be expressed in term of poisson's ratio as:-

$$\frac{dR}{R} = \frac{dL}{L} (1 + 2\nu_p) d\frac{\sigma_e}{\rho_e}$$



$$\frac{\delta R}{R} = GF \cdot \frac{\delta L}{L}$$

(الخطوة العاشرة) علامة الـ  $\Delta$  على المقادير



Bonding

$$GF = \frac{\partial R / R}{\partial L / L}$$

وهو يعادل  $\frac{1}{\rho} \times \frac{1}{A_c} \times \frac{1}{L}$  كونه مدار من مجموعات المقادير  
دخل بعدها إلى آخر تمارين رياضية من  $\Delta$

$$= 1 + 2\nu + \pi E_m$$

• كل العوامل التي تؤثر على احتبار اجهزة الارزنج جانر اور  
strain gauge

(ا) ان يكون حساس لذبذبات strain مترتبة عليه انتشار

(ب) بختار مادة بحيث تكون سطحها سطح اقل انتشار حرارة يكون

لذلك يرجع تأثير الغرسته للمرارة كائنة في الماء (نواتره) الى العلاقة  
(output)  $\rightarrow$  input بين انتشار حرارة  $\rightarrow$  interfacing input

• output  $\rightarrow$  work needed  $\rightarrow$  modulating input

### example 11.1

Determine the total resistance of a copper wire having a diameter of 1mm and a length of 5cm the resistivity of copper is  $1.7 \times 10^{-8} \Omega \text{ m}$

$$\text{Known } D = 1\text{ mm} \rightarrow 10^{-3}\text{ m}$$

$$L = 5\text{ cm} \rightarrow 5 \times 10^{-2}\text{ m}$$

$$\rho_e = 1.7 \times 10^{-8} \Omega \text{ m}$$

Find the total electrical resistance

$$\text{Resistance from equation} \rightarrow R = \rho_e L / A_c$$

where

$$A_c = \frac{\pi}{4} D^2 = \frac{\pi}{4} (1 \times 10^{-3})^2 = 7.85 \times 10^{-7} \text{ m}^2$$

The resistance is then

$$R = \frac{(1.7 \times 10^{-8} \Omega \text{ m})(5 \times 10^{-2} \text{ m})}{7.85 \times 10^{-7} \text{ m}^2} = 1.08 \times 10^{-3} \Omega$$

**COMMENT** If the material were nickel ( $\rho_e = 7.8 \times 10^{-8} \Omega \text{ m}$ ) instead of copper, the resistance would be  $5 \times 10^{-3} \Omega$  for the same diameter and length of wire.

### Example 11.2

A very common material for the construction of strain gauges is the alloy constantan (55% copper with 45% nickel), having a resistivity of  $49 \times 10^{-8} \Omega \text{ m}$ . A typical strain gauge might have a resistance of  $120 \Omega$ . What length of constantan wire of diameter 0.025 mm would yield a resistance of  $120 \Omega$ ?

**KNOWN** The resistivity of constantan is  $49 \times 10^{-8} \Omega \text{ m}$ .

**FIND** The length of constantan wire needed to produce a total resistance of  $120 \Omega$

**SOLUTION** From Equation 11.6, we may solve for the length, which yields in this case

$$L = \frac{RA_c}{\rho_e} = \frac{(120 \Omega)(4.91 \times 10^{-10} \text{ m}^2)}{49 \times 10^{-8} \Omega \text{ m}} = \underline{\underline{0.12 \text{ m}}}$$

The wire would then be 12 cm in length to achieve a resistance of  $120 \Omega$ .

**COMMENT** As shown by this example, a single straight conductor is normally not practical for a local strain measurement with meaningful resolution. Instead, a simple solution is to bend the wire conductor so that several lengths of wire are oriented along the axis of the strain gauge, as shown in Figure 11.4.

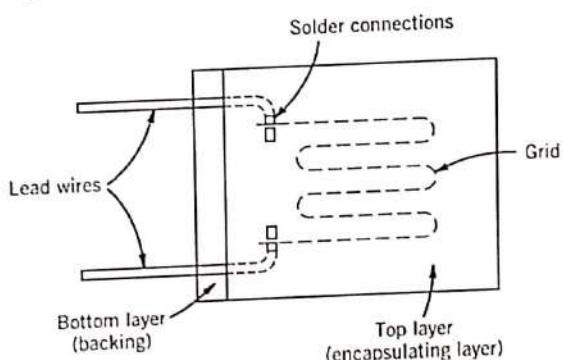


Figure 11.4 Detail of a basic strain gauge construction. (Courtesy of Micro-Measurements Division, Measurements Group, Inc., Raleigh, NC.)

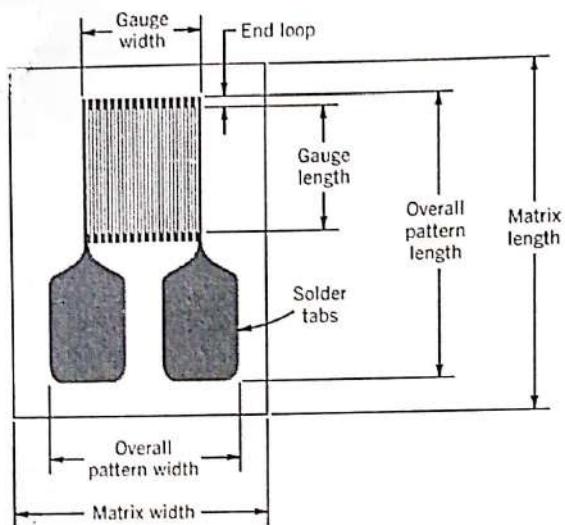
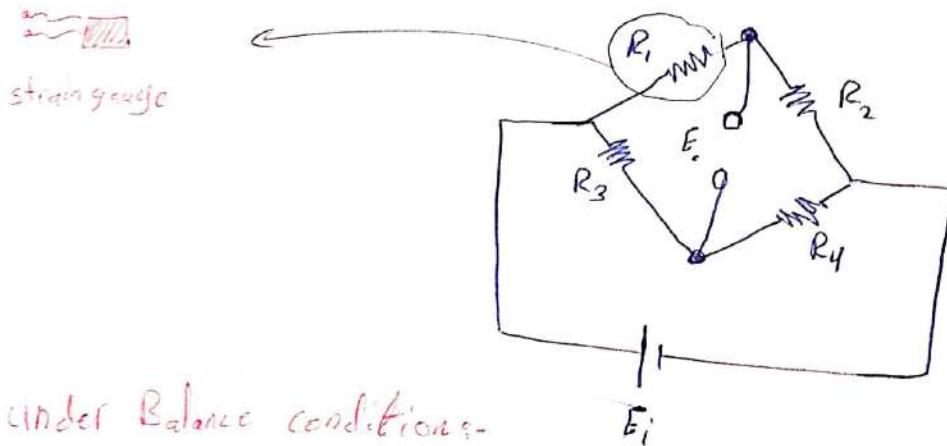


Figure 11.5 Construction of a typical metallic foil strain gauge. (Courtesy of Micro-Measurements Division, Measurements Group, Inc., Raleigh, NC.)

## 11.4 STRAIN GAUGE ELECTRICAL CIRCUITS

↪ strain Gauge Bridge circuits



$$R_1 \cdot R_4 = R_2 \cdot R_3 \quad , \quad E_o = 0$$

مترافق

$$R_1 = R_2 = R_3 = R_4$$

عند تطبيق مقدمة على الأذرع  
سيكون المعاين بالذات بمحادر

$E_o$  will be Voltage if strain is applied

$F \rightarrow \left( \frac{F}{A} \right) \xrightarrow{\alpha} \text{strain} \xrightarrow{\alpha/E} \left( \frac{\delta L}{L} \right) \xrightarrow{\text{SR}} \left( \frac{\delta R}{R} \right) \xrightarrow{\delta E_o} \text{Bridge output}$

متغيرات  
strain  
stress

strain will increase  
Bridge output will increase

Bridge output ( $E_o$ )

$$E_o = \frac{1}{4} \frac{\text{SR}}{R} \cdot E_i$$

$$E_o = \frac{GF \cdot \epsilon}{4}$$

عوامل التكبير

$$GF = \frac{SR/R}{\epsilon L/L}$$

$$\boxed{SR/R = GF \cdot (\epsilon L/L)}$$

the Bridge output under these condition

$$E_o + SE_o = E_i \frac{(R_1 + SR) R_4 - R_3 R_2}{(R_1 + SR + R_2)(R_3 + R_4)} \Rightarrow \text{جواب مطلوب}$$

مطلب مطلوب

جواب مطلوب مطلب مطلوب ↓

reduces to

$$\frac{\delta E_o}{E_i} = \frac{\delta R/R}{4 + 2(\delta R/R)} \approx \frac{\delta R/R}{4} \quad (11.14)$$

under the assumption that  $\delta R/R \ll 1$ . This simplified form of Equation 6.15 is suitable for all but those measurements that demand the highest accuracy, and is valid for values of  $\delta R/R \ll 1$ . Using the relationship from Equation 11.11 that  $\delta R/R = GF\epsilon$ ,

$$\frac{\delta E_o}{E_i} = \frac{GF\epsilon}{4 + 2GF\epsilon} \approx \frac{GF\epsilon}{4} \quad (11.15)$$

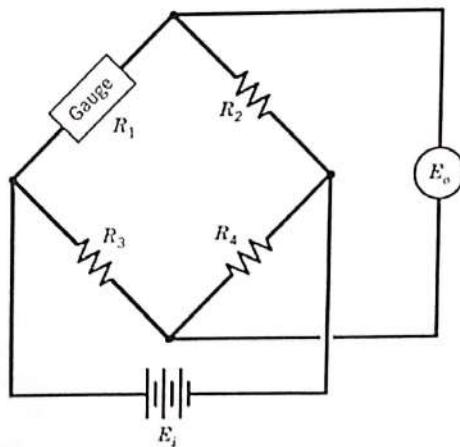


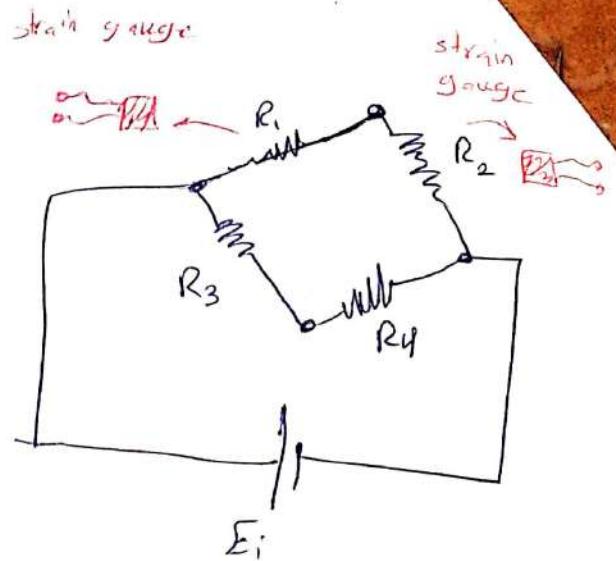
Figure 11.9 Basic strain gauge Wheatstone bridge circuit.

ـ تـ اـ حـسـنـ الـ فـرـادـةـ تـابـعـ

ـ عـلـىـ اـسـمـاـتـ بـرـدـلـيـ بـرـدـلـيـ

ـ لـفـعـاـدـيـنـ بـرـدـلـيـ

ـ 2 active arm ـ قـبـلـهـ فـيـهـ



$$SE_0 = \frac{1}{4} GF \cdot E \cdot E_i$$

ـ الـ فـرـادـةـ تـابـعـ

ـ one active arm

$$SE_0 = \frac{1}{2} \cdot GF \cdot G \cdot E_i$$

ـ two active arm

ـ لـمـاـكـيـنـ اـنـ يـقـوـيـ اـنـ اـنـ يـكـوـنـ اـصـحـ

ـ arm

### 11.3 example

A strain gauge, having a gage factor of 2, is mounted on a rectangular steel bar ( $E_m = 200 \times 10^5 \text{ kN/m}^2$ ), as shown in figure 11.11. The bar is 3 cm wide and 1 cm high, and is subjected.

ـ مـاـنـ اـنـ يـقـوـيـ اـنـ اـنـ يـكـوـنـ اـصـحـ  
ـ مـاـنـ اـنـ يـقـوـيـ اـنـ اـنـ يـكـوـنـ اـصـحـ

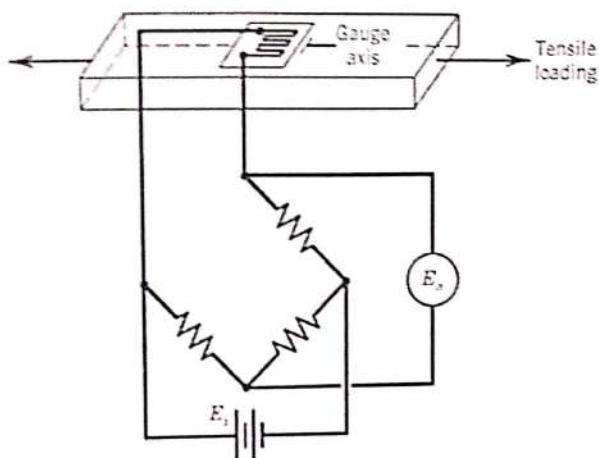


Figure 11.11 Strain gauge circuit subject to uniaxial tension.

to a tensile force of 30 kN. Determine the resistance change of the strain gauge if the resistance of the gauge was  $120\Omega$  in the absence of the axial load.

**KNOWN**  $GF = 2$     $E_m = 200 \times 10^6 \text{ kN/m}^2$     $F_N = 30 \text{ kN}$   
 $R = 120\Omega$     $A_c = 0.03 \text{ m} \times 0.01 \text{ m}$

**FIND** The resistance change of the strain gauge for a tensile force of 30 kN

**SOLUTION** The stress in the bar under this loading condition is

$$\sigma_a = \frac{F_N}{A_c} = \frac{30 \text{ kN}}{(0.03 \text{ m})(0.01 \text{ m})} = 1 \times 10^5 \text{ kN/m}^2$$

and the resulting strain is

$$\varepsilon_a = \frac{\sigma_a}{E_m} = \frac{1 \times 10^5 \text{ kN/m}^2}{200 \times 10^6 \text{ kN/m}^2} = 5 \times 10^{-4} \text{ m/m} \quad (11.16)$$

For strain along the axis of the strain gauge, the change in resistance from Equation 11.11 is

$$\delta R/R = \varepsilon GF$$

or

$$\delta R = R\varepsilon GF = (120\Omega)(5 \times 10^{-4})(2) = 0.12\Omega$$

#### Example 11.4

Suppose the strain gauge described in Example 11.3 is to be connected to a measurement device capable of determining a change in resistance with a stated uncertainty of  $\pm 0.005\Omega$  (95%). This stated uncertainty includes a resolution of  $0.001\Omega$ . What uncertainty in stress would result when using this resistance measurement device?

**KNOWN** A stress is to be inferred from a strain measurement using a strain gauge having a gauge factor of 2 and a zero load resistance of  $120\Omega$ . The measurement of resistance has a stated uncertainty of  $\pm 0.005\Omega$  (95%).

FIND The design-stage uncertainty in stress

SOLUTION The design-stage uncertainty in stress,  $(u_d)_\sigma$ , is given by

$$(u_d)_\sigma = \frac{\partial \sigma}{\partial (SR)} (u_d)_{SR}$$

with

$$\sigma = \varepsilon E_m = \frac{\delta R / R}{GF} E_m$$

Then with  $(u_d)_{SR} = 0.005 \Omega$  and

$$\frac{\partial \sigma}{\partial (SR)} = \frac{E_m}{R(GF)}$$

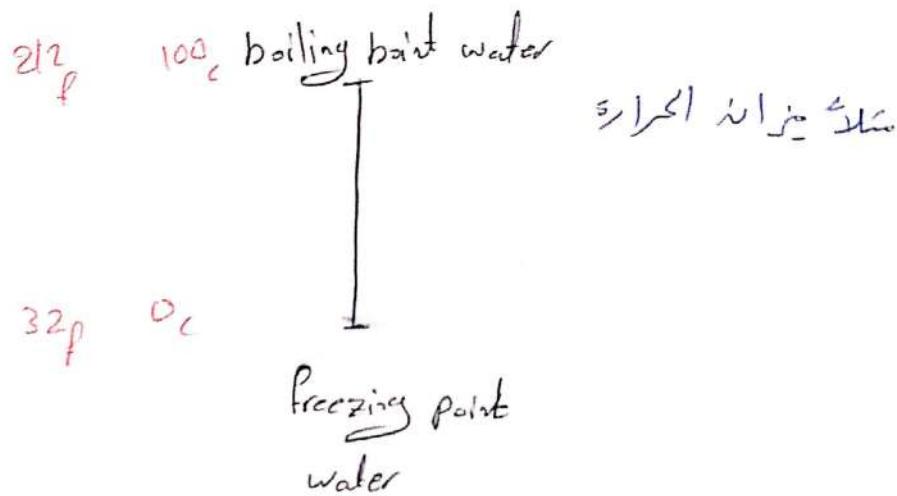
We can express the uncertainty as

$$(u_d)_\sigma = \frac{E_m}{R(GF)} (u_d)_{SR} = \frac{200 \times 10^6 \text{ kN/m}^2}{120 \Omega (2)} (0.005 \Omega)$$

This results in a design-stage uncertainty in stress of  $(u_d)_\sigma = \pm 4.17 \times 10^3 \text{ kN/m}^2$  (95%) or ~2.4% of the expected stress.

## Temperature Measurements

- scales →
- 1) Fixed Reference point
  - 2) Deg. size
  - 3) Interpolation results



scale مقياس اخذ 100 درجة مئوية على scale اعتبر 212 درجة فارسية

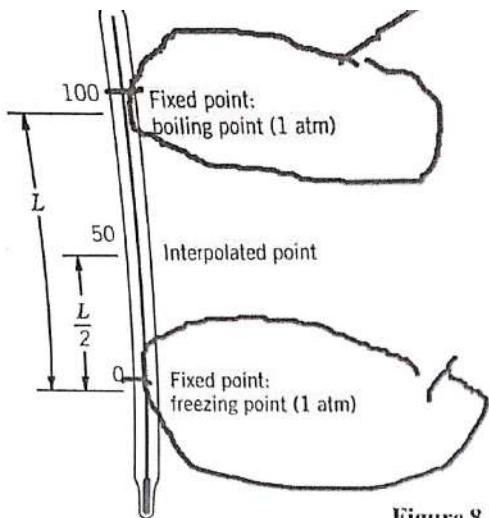
size of درجة فارسية 212 - 32 = 180 درجة مئوية  
degree

بالتالي درجة مئوية = 180 درجة فارسية

### 8.2 TEMPERATURE STANDARDS AND DEFINITION

Temperature can be loosely described as the property of an object that describes its hotness or coldness, concepts that are clearly relative. Our experiences indicate that heat transfer tends

A temperature scale provides for three essential aspects of temperature measurement: (1) the definition of the size of the degree, (2) fixed reference points for establishing known temperatures, and (3) a means for interpolating between these fixed temperature points. These provisions are



the process of establishing  
50°C without a fixed point  
calibration is called  
interpolation.

Figure 8.1 Calibration and interpolation for a liquid-in-glass thermometer.

1) thermal expansion

↳ Liquid  
↳ metal

2) electrical  
resistance

3) thermo-electric  
measurement

### 8.3 THERMOMETRY BASED ON THERMAL EXPANSION

1) liquid in glass thermometer

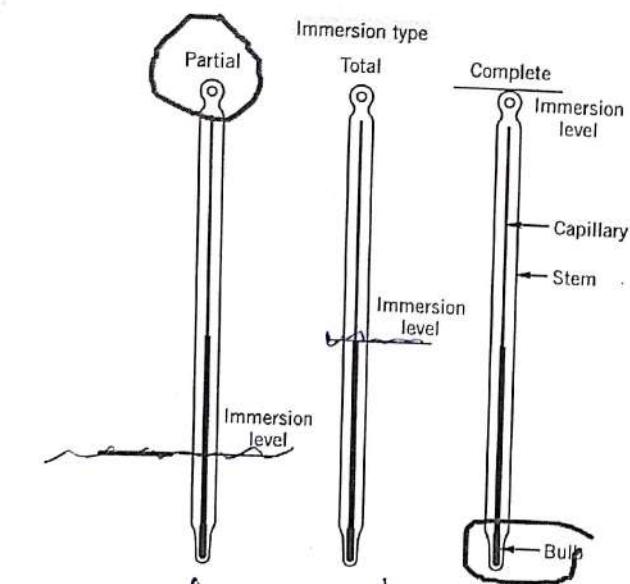


Figure 8.2 Liquid-in-glass thermometers.

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الجهاز المائي  
الجهاز المائي

During calibration, such a thermometer is subject to one of three measuring environments:

1. For a *complete immersion thermometer*, the entire thermometer is immersed in the calibrating temperature environment or fluid.
2. For a *total immersion thermometer*, the thermometer is immersed in the calibrating temperature environment up to the liquid level in the capillary. *M.Mars*
3. For a *partial immersion thermometer*, the thermometer is immersed to a predetermined level in the calibrating environment. *J.S.*

For the most accurate temperature measurements, the thermometer should be immersed in the same manner in use as it was during calibration.<sup>2</sup>

## ② Bimetallic thermometers

- \* the physical phenomena employed in bimetallic temperature sensor is the differential thermal expansion of two metals.
- \* the physical basis for the relationship between the radius of curvature and temperature is given as:-

$$r_c \propto \frac{d}{[(\alpha_u)_A - (\alpha_u)_B] (T_2 - T_1)}$$

where  $r_c$  = radius of curvature

$\alpha_u$  = mutual thermal expansion coefficient

$T$  = Temperature       $d$  = thickness,

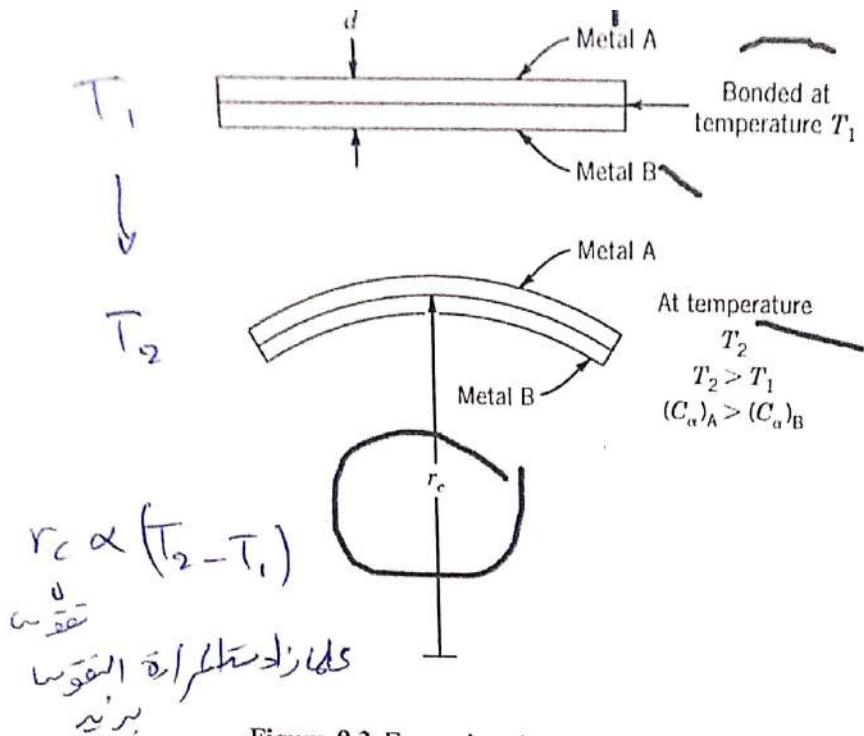


Figure 8.3 Expansion thermometry using bimetallic materials: strip, spiral, and helix forms.

Bimetallic strips employ one metal having a high coefficient of thermal expansion with another having a low coefficient, providing increased sensitivity. Invar is often used as one of the metals.

Dial thermometers using a bimetallic strip as their sensing element typically provide temperature measurements with uncertainties of  $\pm 1^\circ C$ .

#### 8.4 ELECTRICAL RESISTANCE THERMOMETRY

$$\frac{\Delta R}{R} \propto \frac{1}{k} \Delta T$$

Resistors تيار مدار

- Resistance temperature detectors (RTDs) may be formed from a solid metal wire that exhibits an increase in electrical resistance with temperature. Depending on the materials selected, the resistance may increase or decrease with temperature. As a first-order approximation, the resistance change of a thermistor may be expressed as

$$R - R_0 = k(T - T_0) \quad (8.2)$$

The relationship between the resistance of a metal conductor and its temperature may also be expressed as the polynomial expansion:

$$R = R_0 [1 + \alpha(T - T_0) + \beta(T - T_0)^2 + \dots] \quad (8.4)$$

This approximation can be expressed

$$R = R_0 [1 + \alpha(T - T_0)] \quad (8.5)$$

where  $\alpha$  is the temperature coefficient of resistivity. For example, for platinum conductors the linear material constant

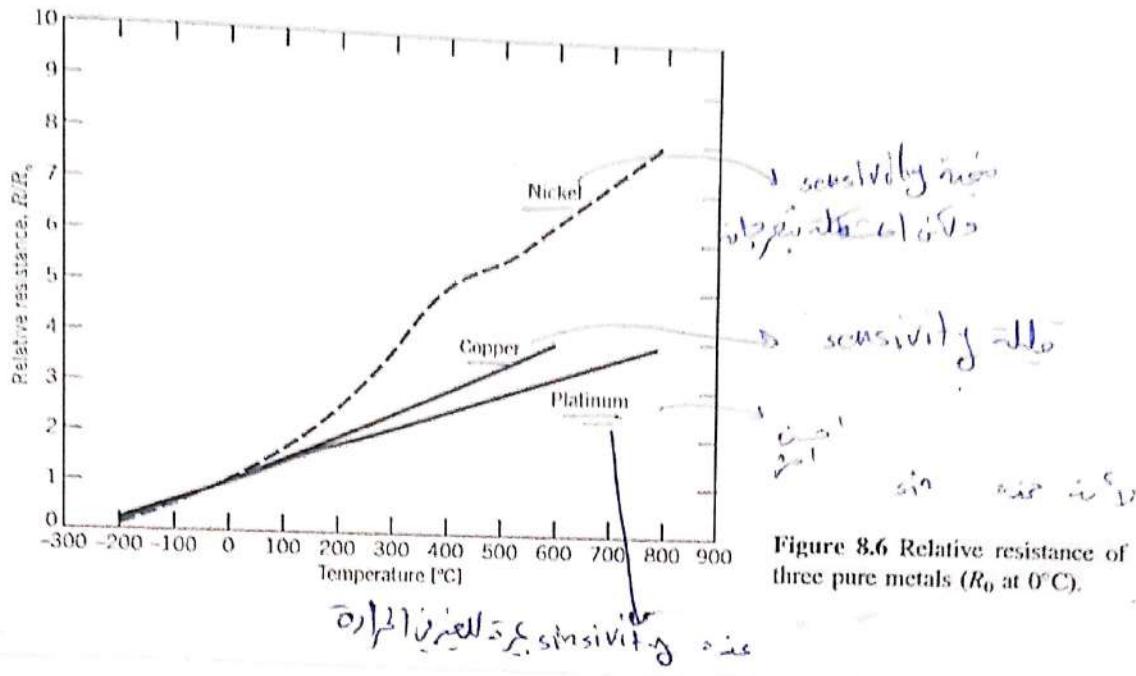
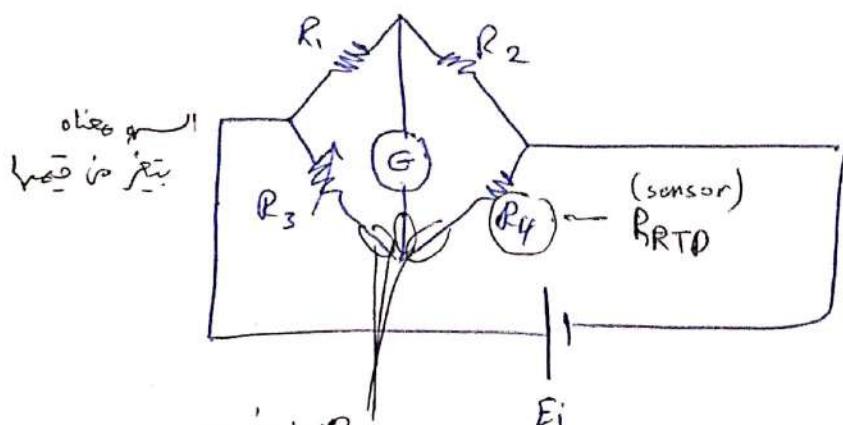


Figure 8.6 Relative resistance of three pure metals ( $R_0$  at  $0^\circ\text{C}$ ).

$$1) \text{ RTD} \rightarrow \Delta R \propto \Delta T$$

$$2) \text{ thermistor} \rightarrow \Delta R \propto \frac{1}{\Delta T}$$

### Resistance Temperature Device Resistance Measurement



Bridge circuit مفهوم هو

$$R_1 = R_2 \quad (\text{من المنشورة})$$

مقدمة  
متر  
سوار

$$R_1 R_4 = R_2 R_3$$

$$O = G + I$$

$$T_1 \rightarrow T_2$$

$$R_{RTD} = \frac{R_3 - R_1}{R_3} \times T_2$$

$$R_{RTD} = R_3 \rightarrow T_1$$

$$R_{RTD}^{\text{new}} = R_3^{\text{new}} \rightarrow T_2$$

\* Wheatstone bridge circuits are commonly used for these measurements.

\* At balance condition neglected lead wire effects  $r_1, r_2, r_3, r_4$ .

$$\frac{R_1}{R_2} = \frac{R_3}{R_{RTD}} \quad (8.6)$$

but with lead wire resistance included in the circuit analysis.

$$\frac{R_1}{R_2} = \frac{R_3 + r_1}{R_{RTD} + r_3} \quad (8.7)$$

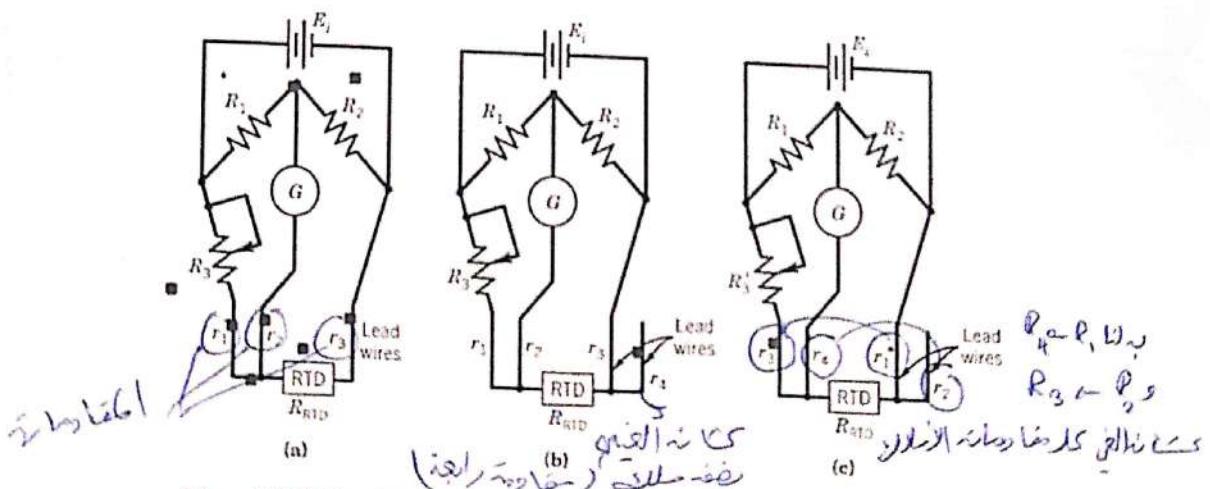


Figure 8.7 Bridge circuits. (a) Callendar-Griffiths 3-wire bridge; (b) and (c) Mueller 4-wire bridge. An average of the readings in (b) and (c) eliminates the effect of lead wire resistances.

$$R_{RTD} + r_3 = R_3 + r_1$$

$$R_{RTD} = R_3 + r_1 - r_3 \rightarrow (8.8)$$

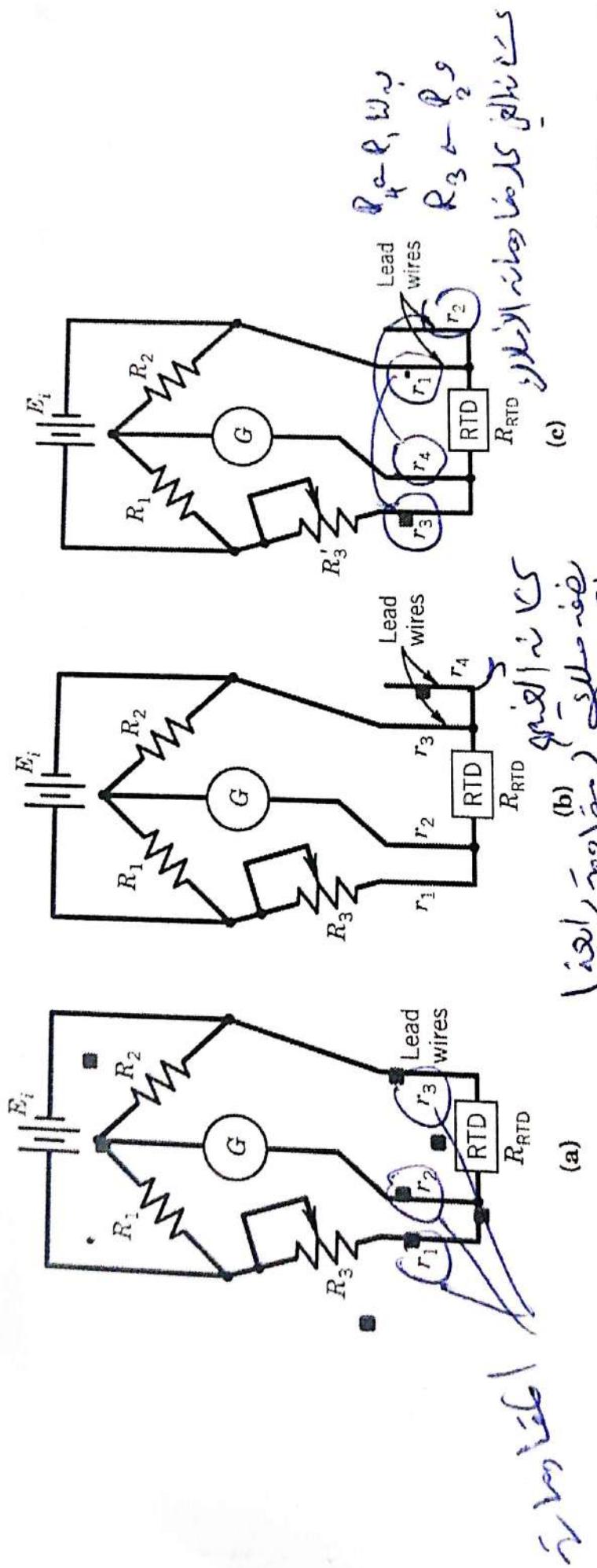


Figure 8.7 Bridge circuits. (a) Callendar-Gough 3-wire bridge; (b) and (c) Mueller 4-wire bridge. An average of the readings in (b) and (c) eliminates the effect of lead wire resistances.

$$R_{\text{RTD}} + r_3 = R_3 + r_1 \quad (8.9)$$

and in the second measurement configuration, Figure 8.7c, yields

$$R_{\text{RTD}} + r_1 = R'_3 + r_3 \quad (8.10)$$

where  $R_3$  and  $R'_3$  represent the indicated values of resistance in the first and second configurations, respectively. Adding Equations 8.8 and 8.9 results in an expression for the resistance of the RTD in terms of the indicated values for the two measurements:

$$R_{\text{RTD}} = \frac{R_3 + R'_3}{2} \quad (8.11)$$

With this approach, the effect of variations in lead wire resistances is essentially eliminated.

### Example 8.1

An RTD forms one arm of an equal-arm Wheatstone bridge, as shown in Figure 8.8. The fixed resistances,  $R_2$  and  $R_3$  are equal to  $25 \Omega$ . The RTD has a resistance of  $25 \Omega$  at a temperature of  $0^\circ\text{C}$  and is used to measure a temperature that is steady in time.

The resistance of the RTD over a small temperature range may be expressed, as in Equation 8.5:

$$R_{\text{RTD}} = R_0[1 + \alpha(T - T_0)]$$

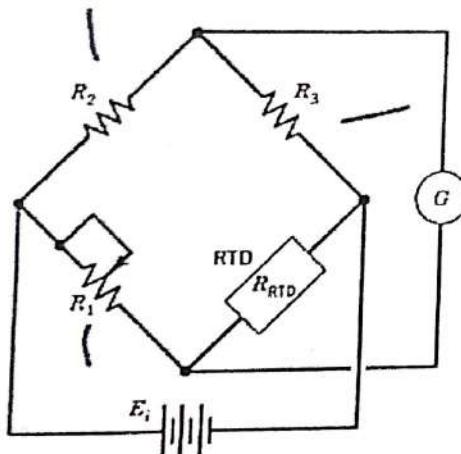


Figure 8.8 RTD Wheatstone bridge arrangement.

Suppose the coefficient of resistance for this RTD is  $0.003925^\circ\text{C}^{-1}$ . A temperature measurement is made by placing the RTD in the measuring environment and balancing the bridge by adjusting  $R_1$ . The value of  $R_1$  required to balance the bridge is  $37.36 \Omega$ . Determine the temperature of the RTD.

**KNOWN**  $R(0^\circ\text{C}) = 25 \Omega$

$$\alpha = 0.003925^\circ\text{C}^{-1}$$

$$R_1 = 37.36 \Omega \text{ (when bridge is balanced)}$$

**FIND** The temperature of the RTD.

**SOLUTION** The resistance of the RTD is measured by balancing the bridge; recall that in a balanced condition

$$R_{\text{RTD}} = R_1 \frac{R_3}{R_2}$$

The resistance of the RTD is measured to be  $37.36 \Omega$ . With  $R_0 = 25 \Omega$  at  $T = 0^\circ\text{C}$  and  $\alpha = 0.003925^\circ\text{C}^{-1}$ , Equation 8.5 becomes

$$37.36 \Omega = 25(1 + \alpha T) \Omega$$

The temperature of the RTD is  $126^\circ\text{C}$ .

### Example 8.2

Consider the bridge circuit and RTD of Example 8.1. To select or design a bridge circuit for measuring the resistance of the RTD in this example, the required uncertainty in temperature would be specified. If the required uncertainty in the measured temperature is  $\leq 0.5^{\circ}\text{C}$ , would a 1% total uncertainty in each of the resistors that make up the bridge be acceptable? Neglect the effects of lead-wire resistances for this example.

**KNOWN** A required uncertainty in temperature of  $\pm 0.5^{\circ}\text{C}$ , measured with the RTD and bridge circuit of Example 8.1.

**FIND** The uncertainty in the measured temperature for a 1% total uncertainty in each of the resistors that make up the bridge circuit.

**ASSUMPTION** All uncertainties are provided and evaluated at the 95% confidence level.

**SOLUTION** Perform a design-stage uncertainty analysis. Assuming at the design stage that the total uncertainty in the resistances is 1%, then with initial values of the resistances in the bridge equal to  $25\ \Omega$ , the design-stage uncertainties are set at

$$u_{R_1} = u_{R_2} = u_{R_3} = (0.01)(25) = 0.25\ \Omega$$

The root-sum-squares (RSS) method is used to estimate the propagation of uncertainty in each resistor to the uncertainty in determining the RTD resistance by

$$u_{\text{RTD}} = \sqrt{\left[\frac{\partial R}{\partial R_1} u_{R_1}\right]^2 + \left[\frac{\partial R}{\partial R_2} u_{R_2}\right]^2 + \left[\frac{\partial R}{\partial R_3} u_{R_3}\right]^2}$$

where

$$R = R_{\text{RTD}} = \frac{R_1 R_3}{R_2}$$

and we assume that the uncertainties are not correlated. Then, the design-stage uncertainty in the resistance of the RTD is

$$\begin{aligned} u_{\text{RTD}} &= \sqrt{\left[\frac{R_3}{R_2} u_{R_1}\right]^2 + \left[\frac{-R_1 R_3}{R_2^2} u_{R_2}\right]^2 + \left[\frac{R_1}{R_2} u_{R_3}\right]^2} \\ u_{\text{RTD}} &= \sqrt{(1 \times 0.25)^2 + (1 \times -0.25)^2 + (1 \times 0.25)^2} = 0.433\ \Omega \end{aligned}$$

To determine the uncertainty in temperature, we know

$$R = R_{\text{RTD}} = R_0[1 + \alpha(T - T_0)]$$

and

$$u_T = \sqrt{\left(\frac{\partial T}{\partial R} u_{\text{RTD}}\right)^2}$$

Setting  $T_0 = 0^{\circ}\text{C}$  with  $R_0 = 25\ \Omega$ , and neglecting uncertainties in  $T_0$ ,  $\alpha$ , and  $R_0$ , we have

$$\begin{aligned} \frac{\partial T}{\partial R} &= \frac{1}{\alpha R_0} \\ \frac{1}{\alpha R_0} &= \frac{1}{(0.003925^{\circ}\text{C}^{-1})(25\ \Omega)} \end{aligned}$$

Then the design-stage uncertainty in temperature is

$$u_T = u_{\text{RTD}} \left( \frac{\partial T}{\partial R} \right) = \frac{0.433\ \Omega}{0.098\ \Omega/\text{ }^{\circ}\text{C}} = 4.4^{\circ}\text{C}$$

The desired uncertainty in temperature is not achieved with the specified levels of uncertainty in the pertinent variables.

**COMMENT** Uncertainty analysis, in this case, would have prevented performing a measurement that would not provide acceptable results.

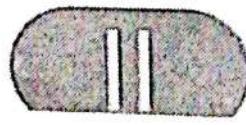
### Example 8.3

Suppose the total uncertainty in the bridge resistances of Example 8.1 was reduced to 0.1%. Would the required level of uncertainty in temperature be achieved?

**KNOWN** The uncertainty in each of the resistors in the bridge circuit for temperature measurement from Example 8.1 is  $\pm 0.1\%$ .

**FIND** The resulting uncertainty in temperature.

**SOLUTION** The uncertainty analysis from the previous example may be directly applied, with the uncertainty values for the resistances appropriately reduced. The uncertainties for the resistances



are reduced from 0.25 to 0.025, yielding

$$t_{\text{RTD}} = \sqrt{(1 \times 0.025)^2 + (1 \times -0.025)^2 + (1 \times 0.025)^2} = 0.0433 \Omega$$

and the resulting 95% uncertainty interval in temperature is  $\pm 0.44^\circ\text{C}$ , which satisfies the design constraint.

**COMMENT** This result provides confidence that the effect of the resistors' uncertainties will not cause the uncertainty in temperature to exceed the target value. However, the uncertainty in temperature also depends on other aspects of the measurement system. The design-stage uncertainty analysis performed in this example may be viewed as ensuring that the factors considered do not produce a higher than acceptable uncertainty level.

## Thermistors

$$\Delta R \propto \frac{1}{\Delta T}$$

الترميستور ( Thermistor ) هو مكون إلكتروني ( Semiconductor ) يحول الحرارة إلى إشارات كهربائية.

Thermistors (from thermally sensitivity resistors) are ceramics-like semiconductor devices, the most common thermistors are NTC, and the resistance of these thermistors decreases rapidly with temperature, which is contrast to the small increases of resistance with temperature of RTDs.

الترميستور ( Thermistor ) هو مكون إلكتروني ( Semiconductor ) يحول الحرارة إلى إشارات كهربائية، وهو مكون شائع في الأجهزة الكهربائية مثل الملاعق والثلاجات، ويعمل على تغيير مقاومته مع درجة الحرارة.

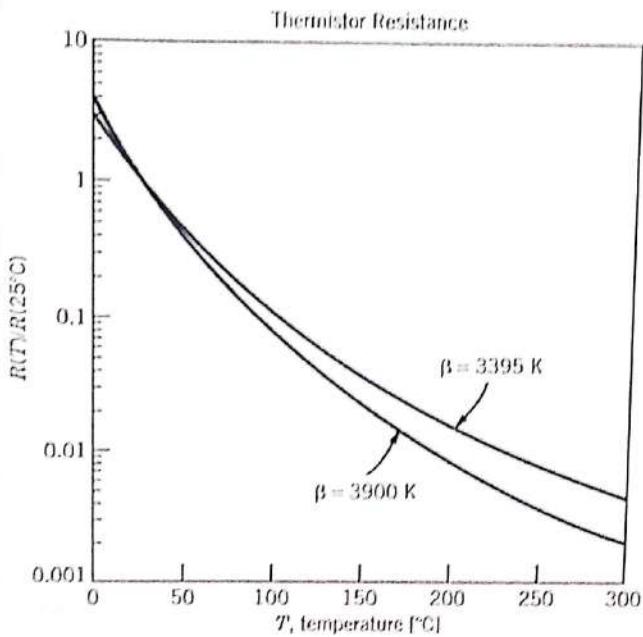
a more accurate functional relationship between resistance and temperature for thermistor is generally assumed to be of the form:-

$$R = R_0 e^{\beta(1/T - 1/T_0)}$$

$$R_0 \rightarrow T_0$$

$$R \rightarrow T$$

the Parameter  $\beta$  ranges from 3500 to 4600 K.



18.9  
18.10

Figure 8.9 Representative thermistor resistance variations with temperature.

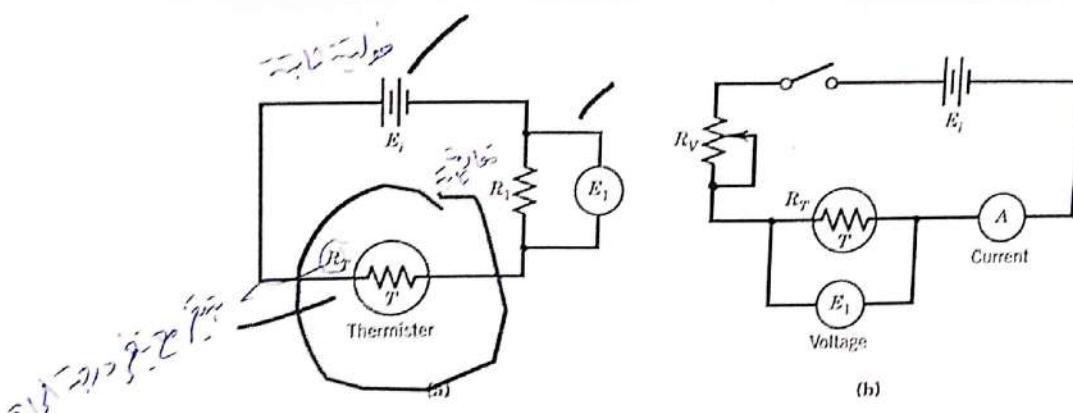


Figure 8.10 Circuits for determining  $\beta$  for thermistors. (a) Voltage divider method:  
 $R_T = R_1(E_i/E_1 - 1)$ . Note: Both  $R_1$  and  $E_1$  must be known values. The value of  $R_1$  may be varied to achieve appropriate values of thermistor current. (b) Volt-ammeter method.  
Note: Both current and voltage are measured.

$$\frac{R_T}{R_1} = \frac{E_i}{E_1} - 1 \quad \xrightarrow{\text{Rearrange}} \quad \frac{E_i}{R_1 + R_2} = \frac{E_1}{R_1}$$

جواب ایکھا - [8.4], [8.5], [8.6] جلو پڑو

Thermistors are generally used when high sensitivity, ruggedness, or fast response times are required (9). Thermistors are often encapsulated in glass, and thus can be used in corrosive or abrasive environments. The resistance characteristics of the semiconductor material may change at elevated temperatures, and some aging of a thermistor occurs at temperatures above 200°C. The high resistance of a thermistor, compared to that of an RTD, eliminates the problems of lead wire resistance compensation.

A commonly reported specification of a thermistor is the zero-power resistance and dissipation constant. The zero-power resistance of a thermistor is the resistance value of the thermistor with no flow of electric current. The zero power resistance should be measured such that a decrease in the current flow to the thermistor results in not more than a 0.1% change in resistance. The dissipation constant for a thermistor is defined at a given ambient temperature as

$$\delta = \frac{P}{T - T_\infty} \quad (8.13)$$

where

$\delta$  = dissipation constant

$P$  = power supplied to the thermistor

$T, T_\infty$  = thermistor and ambient temperatures, respectively

### Example 8.4

The output of a thermistor is highly nonlinear with temperature, and there is often a benefit to linearizing the output through appropriate circuit, whether active or passive. In this example we examine the output of an initially balanced bridge circuit where one of the arms contains a thermistor. Consider a Wheatstone bridge as shown in Figure 8.8, but replace the RTD with a thermistor having a value of  $R_0 = 10,000\Omega$  with  $\beta = 3680\text{K}$ . Here we examine the output of the circuit over two temperature ranges: (a) 25–325°C, and (b) 25–75°C.

**KNOWN** A Wheatstone bridge where  $R_2 = R_3 = R_4 = 10,000\Omega$  and where  $R_1$  is a thermistor.

**FIND** The output of the bridge circuit as a function of temperature.

**SOLUTION** The fundamental relationship between resistances in a Wheatstone bridge and the normalized output voltage is provided in Equation 6.14:

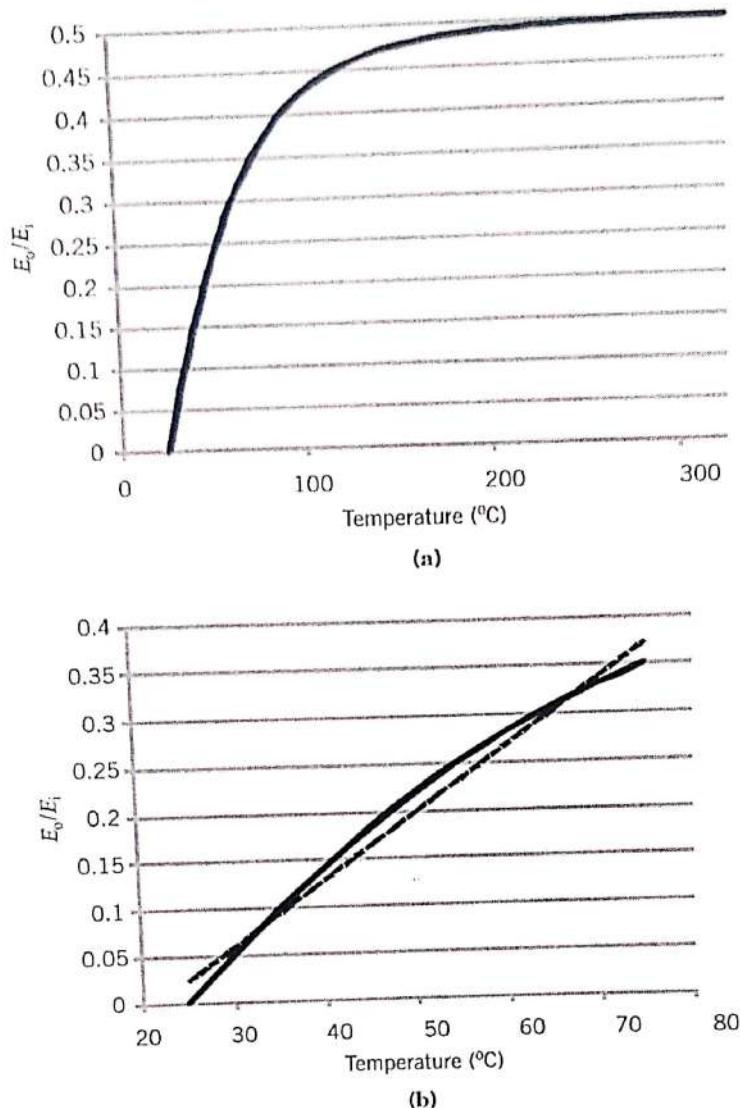
$$\frac{E_o}{E_i} = \left( \frac{R_1}{R_1 + R_2} - \frac{R_3}{R_3 + R_4} \right) \quad (6.14)$$

And the resistance of the thermistor is

$$R = R_0 e^{\beta(1/T - 1/T_0)}$$

Substituting in Equation 6.14 for  $R_1$  yields

$$\frac{E_o}{E_i} = \left( \frac{R_0 e^{\beta(1/T - 1/T_0)}}{R_0 e^{\beta(1/T - 1/T_0)} + R_2} - \frac{R_3}{R_3 + R_4} \right)$$



**Figure 8.11** Normalized bridge output voltage as a function of temperature with a thermistor as the temperature sensor: (a) 25° to 325°C, (b) 25° to 75°C.

Figure 8.11a is a plot of this function over the range 25° to 325°C. Clearly the sensitivity of the circuit to changes in temperature greatly diminishes as the temperature increases above 100°C, with an asymptotic value of 0.5. Figure 8.11b shows the behavior over the restricted range 25° to 75°C; a linear curve fit is also shown for comparison. Over this range of temperature, assuming a linear relationship between normalized output and temperature would be suitable for many applications provided that the accompanying linearity error is acceptable.

**Example 8.5**

The material constant  $\beta$  is to be determined for a particular thermistor using the circuit shown in Figure 8.10a. The thermistor has a resistance of  $60 \text{ k}\Omega$  at  $25^\circ\text{C}$ . The reference resistor in the circuit,  $R_1$ , has a resistance of  $130.5 \text{ k}\Omega$ . The dissipation constant  $\delta$  is  $0.09 \text{ mW}/^\circ\text{C}$ . The voltage source used for the measurement is constant at  $1.564 \text{ V}$ . The thermistor is to be used at temperatures ranging from  $100$  to  $150^\circ\text{C}$ . Determine the value of  $\beta$ .

**KNOWN** The temperature range of interest is from  $100^\circ$  to  $150^\circ\text{C}$ .

$$R_0 = 60,000 \Omega, \quad T_0 = 25^\circ\text{C}$$

$$E_i = 1.564 \text{ V}, \quad \delta = 0.09 \text{ mW}/^\circ\text{C}, \quad R_1 = 130.5 \text{ k}\Omega$$

**FIND** The value of  $\beta$  over the temperature range from  $100^\circ$  to  $150^\circ\text{C}$ .

**SOLUTION** The voltage drop across the fixed resistor is measured for three known values of thermistor temperature. The thermistor temperature is controlled and determined by placing the thermistor in a laboratory oven and measuring the temperature of the oven. For each measured voltage across the reference resistor, the thermistor resistance  $R_T$  is determined from

$$R_T = R_1 \left( \frac{E_i}{E_1} - 1 \right)$$

The results of these measurements are as follows:

Temperature ( $^\circ\text{C}$ )	$R_1$ Voltage (V)	$R_T$ ( $\Omega$ )
100	1.501	5477.4
125	1.531	2812.9
150	1.545	1604.9

Equation 8.12 can be expressed in the form of a linear equation as

$$\ln \frac{R_T}{R_0} = \beta \left( \frac{1}{T} - \frac{1}{T_0} \right) \quad (8.14)$$

Applying this equation to the measured data, with  $R_0 = 60,000 \Omega$ , the three data points above yield the following:

$\ln(R_T/R_0)$	$(1/T - 1/T_0)[\text{K}^{-1}]$	$\beta(\text{K})$
-2.394	$-6.75 \times 10^{-4}$	3546.7
-3.060	$-8.43 \times 10^{-4}$	3629.9
-3.621	$-9.92 \times 10^{-4}$	3650.2

**COMMENT** These results are for constant  $\beta$  and are based on the behavior described by Equation 8.12, over the temperature range from  $T_0$  to the temperature  $T$ . The significance of the measured differences in  $\beta$  is examined further.

The measured values of  $\beta$  in Example 8.5 are different at each value of temperature. If  $\beta$  were truly a temperature-independent constant, and these measurements had negligible uncertainty, all three measurements would yield the same value of  $\beta$ . The variation in  $\beta$  may be due to a physical effect of temperature, or may be attributable to the uncertainty in the measured values.

Are the measured differences significant, and if so, what value of  $\beta$  best represents the behavior of the thermistor over this temperature range? To perform the necessary uncertainty analysis, additional information must be provided concerning the instruments and procedures used in the measurement.

### Example 8.6

Perform an uncertainty analysis to determine the uncertainty in each measured value of  $\beta$  in Example 8.5, and evaluate a single best estimate of  $\beta$  for this temperature range. The measurement of  $\beta$  involves the measurement of voltages, temperatures, and resistances. For temperature there is a random error associated with spatial and temporal variations in the oven temperature with a random standard uncertainty of  $s_T = 0.19^\circ\text{C}$  for 20 measurements. In addition, based on a manufacturer's specification, there is a known measurement systematic uncertainty for temperature of  $0.36^\circ\text{C}$  (95%) in the thermocouple.

The systematic errors in measuring resistance and voltage are negligible, and estimates of the instrument repeatability, which are based on the manufacturer's specifications in the measured values and assumed to be at a 95% confidence level, are assigned systematic uncertainties of 1.5% for resistance and 0.002 V for the voltage.

**KNOWN** Standard deviation of the means for oven temperature,  $s_T = 0.19^\circ\text{C}$ ,  $N = 20$ . The remaining errors are assigned systematic uncertainties at 95% confidence assuming large degrees of freedom, such that  $B_x = 2b_x$ :

$$B_T = 2b_T = 0.36^\circ\text{C}$$

$$B_R/R = (2b_R)/R = 1.5\%$$

$$B_E = 2b_E = 0.002 \text{ V}$$

**FIND** The uncertainty in  $\beta$  at each measured temperature, and a best estimate for  $\beta$  over the measured temperature range.

**SOLUTION** Consider the problem of providing a single best estimate of  $\beta$ . One method of estimation might be to average the three measured values. This results in a value of 3609 K. However, since the relationship between  $(R_T/R_0)$  and  $(1/T - 1/T_0)$  is expected to be linear, a least-squares fit can be performed on the three data points, and include the point  $(0, 0)$ . The resulting value of  $\beta$  is 3638 K. Is this difference significant, and which value best represents the behavior of the thermistor? To answer these questions, an uncertainty analysis is performed for  $\beta$ .

For each measured value,

$$\beta = \frac{\ln(R_T/R_0)}{1/T - 1/T_0}$$

Uncertainties in voltage, temperature, and resistance are propagated into the resulting value of  $\beta$  for each measurement.

Consider first the sensitivity indices  $\theta_i$  for each of the variables  $R_T$ ,  $R_0$ ,  $T$ , and  $T_0$ . These may be tabulated by computing the appropriate partial derivatives of  $\beta$ , evaluated at each of the three temperatures, as follows:

$T$ (°C)	$\theta_{R_T}$ (K/Ω)	$\theta_{R_0}$ (K/Ω)	$\theta_T$	$\theta_{T_0}$
100	-0.270	0.0247	-37.77	59.17
125	-0.422	0.0198	-27.18	48.48
150	-0.628	0.0168	-20.57	41.45

The determination of the uncertainty in  $\beta$ ,  $u_\beta$ , requires the uncertainty in the measured value of resistance for the thermistor,  $u_{R_T}$ . But  $R_T$  is determined from the expression

$$R_T = R_1[(E_i/E_1) - 1]$$

and thus requires an analysis of the uncertainty in the resulting value of  $R_T$ , from measured values of  $R_1$ ,  $E_i$ , and  $E_1$ . All errors in  $R_T$  are treated as uncorrelated systematic errors, yielding

$$b_{R_T} = \sqrt{\left[\frac{\partial R_T}{\partial R_1} b_{R_1}\right]^2 + \left[\frac{\partial R_T}{\partial E_i} b_{E_i}\right]^2 + \left[\frac{\partial R_T}{\partial E_1} b_{E_1}\right]^2} = \sqrt{[\theta_{R_1} b_{R_1}]^2 + [\theta_{E_i} b_{E_i}]^2 + [\theta_{E_1} b_{E_1}]^2}$$

To arrive at a representative value, we compute  $b_{R_T}$  at 125°C. The systematic standard uncertainty in  $R_1$  is 0.75% of 130.5 kΩ, or 978 Ω, and in  $R_0$  is 450 Ω. The systematic standard uncertainties in  $E_i$  and  $E_1$  are each 0.001 V, and  $\theta_{R_1} = 0.022$ ,  $\theta_{E_i} = 85238$ , and  $\theta_{E_1} = 87076$ . This gives  $b_{R_T} = 123.7$  Ω.

An uncertainty for  $\beta$  is determined for each of the measured temperatures. The propagation of the measurement systematic errors for temperature and resistance is found as

$$b_\beta = \sqrt{[0_T b_T]^2 + [0_{T_0} b_{T_0}]^2 + [0_{R_T} b_{R_T}]^2 + [0_{R_0} b_{R_0}]^2}$$

where

$$\begin{aligned} b_T &= 0.18^\circ\text{C} & b_{R_T} &= 123.7 \Omega \\ b_{T_0} &= 0.18^\circ\text{C} & b_{R_0} &= 450 \Omega \end{aligned}$$

The random standard uncertainty for  $\beta$  contains contributions only from the statistically determined oven temperature characteristics and is found from

$$s_{\bar{\beta}} = \sqrt{(\theta_T s_T)^2 + (\theta_{T_0} s_{T_0})^2}$$

where both  $s_T$  and  $s_{T_0}$  are 0.19, as determined with  $v = 19$ .

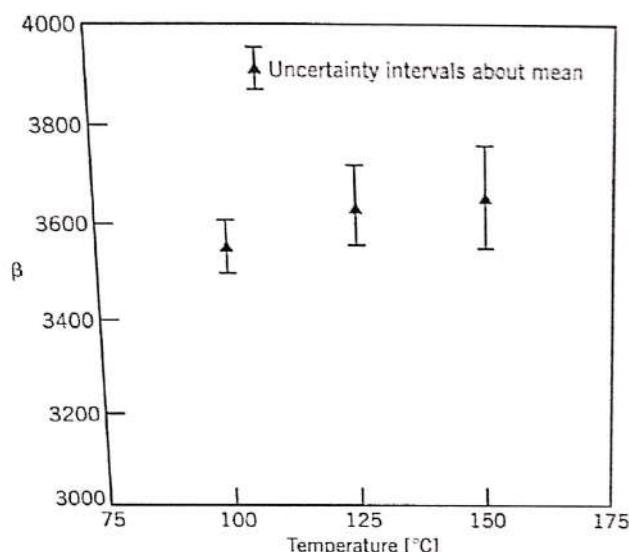
The resulting values of uncertainty in  $\beta$  are found from

$$u_\beta = t_{v,95} \sqrt{b_\beta^2 + s_{\bar{\beta}}^2}$$

where  $v_\beta$  is sufficiently large (see Eq. 5.39) so that  $t_{v,95} \approx 2$ . At each temperature the uncertainty in  $\beta$  is determined as shown in Table 8.3. The effect of increases in the sensitivity indices  $\theta_i$  on the total uncertainty is to cause increased uncertainty in  $\beta$  as the temperature increases.

**Table 8.3** Uncertainties in  $\beta$ 

$T$ [°C]	Uncertainty		
	Random $s_{\bar{\beta}}$ [K]	Systematic $b_{\bar{\beta}}$ [K]	Total $\pm u_{\bar{\beta}} 95\%$ [K]
100	13.3	37.4	79.4
125	10.6	53.9	109.9
150	8.8	78.4	157.8

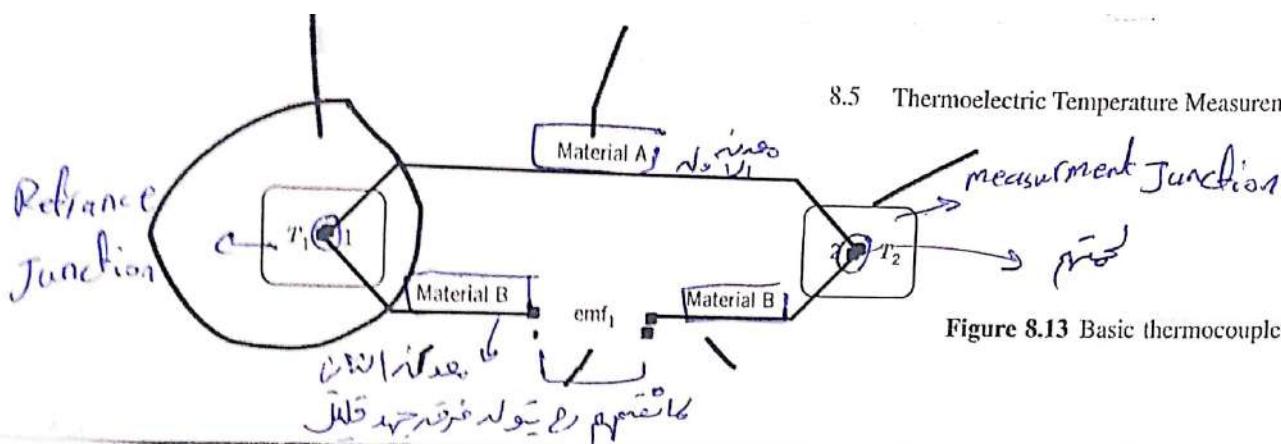


**Figure 8.12** Measured values of  $\beta$  and associated uncertainties for three temperatures. Each data point represents  $\bar{\beta} \pm u_{\bar{\beta}}$ .

The original results of the measured values of  $\beta$  must now be reexamined. The results from Table 8.3 are plotted as a function of temperature in Figure 8.12, with 95% uncertainty limits on each data point. Clearly, there is no justification for assuming that the measured values indicate a trend of changes with temperature, and it would be appropriate to use either the average value of  $\beta$  or the value determined from the linear least-squares curve fit.

### 8.5 THERMOELECTRIC TEMPERATURE MEASUREMENT

The most common method of measuring and controlling temperature uses an electrical circuit called a thermocouple. A *thermocouple* consists of two electrical conductors that are made of dissimilar metals and have at least one electrical connection. This electrical connection is referred to as a *junction*. A thermocouple junction may be created by welding, soldering, or by any method that provides good electrical contact between the two conductors, such as twisting the wires around one another. The output of a thermocouple circuit is a voltage, and there is a definite relationship between this voltage and the temperatures of the junctions that make up the thermocouple circuit. We will examine the causes of this voltage, and develop the basis for using thermocouples to make engineering measurements of temperature.



$\text{emf}_1$  نه ؟ 1)  $\propto (T_2 - T_1)$  3) Reference Junction .  
2) Type of material .

( ایجاد این emf کیفیت چگونه است )

- 1) Seebeck Effect
- 2) Peltier effect
- 3) Thomson effect

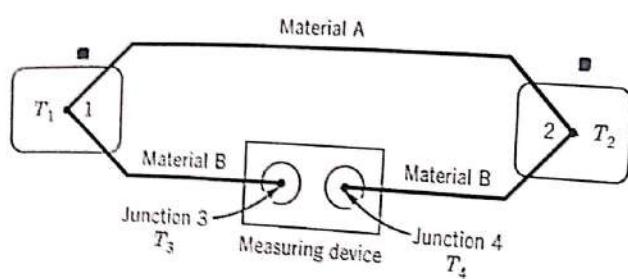


Figure 8.16 Typical thermocouple measuring circuit.

emf نمیتواند دو جایی قرار گیرد

A thermocouple junction is the source of an *electromotive force* (emf), which gives rise to the potential difference in a thermocouple circuit. It is the basis for temperature measurement using thermocouples. The circuit shown in Figure 8.13 is the most common form of a thermocouple circuit used for measuring temperature.

Reference junction :-  $0^\circ\text{C}$  → Because of the ease with which it can be obtained

In two basic ways :? 1) creating a reference point by electric circuit,  
2) ice bath ~~sus~~ served.

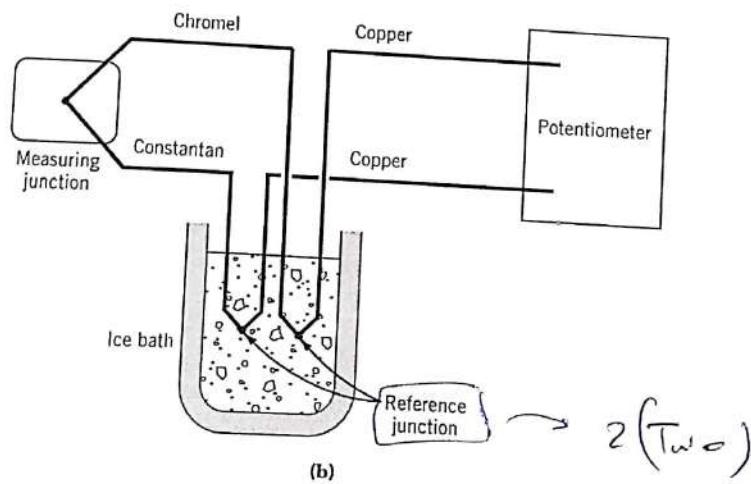
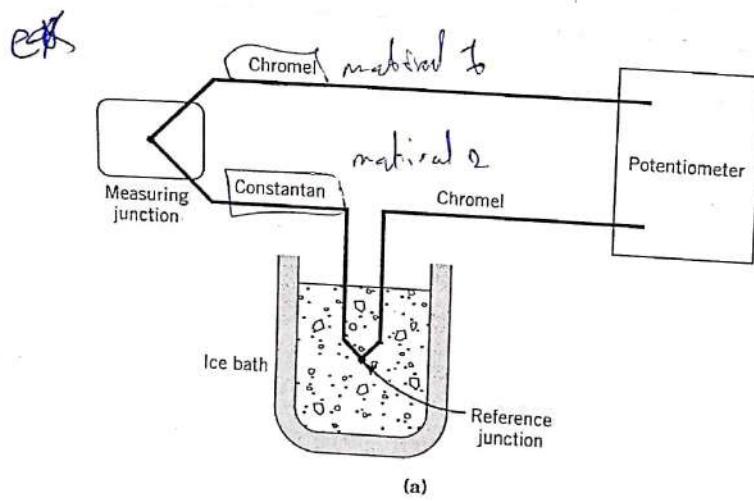


Table 8.5 Standard Thermocouple Compositions<sup>a</sup>

Type	Wire		Expected Systematic Uncertainty <sup>b</sup>
	Positive	Negative	
S	Platinum	Platinum/10% rhodium	±1.5°C or 0.25%
R	Platinum	Platinum/13% rhodium	±1.5°C
B	Platinum/30% rhodium	Platinum/6% rhodium	±0.5%
T	Copper	Constantan	±1.0°C or 0.75%
J	Iron	Constantan	±2.2°C or 0.75%
K	Chromel	Alumel	±2.2°C or 0.75%
E	Chromel	Constantan	±1.7°C or 0.5%

*Alloy Designations*

Constantan: 55% copper with 45% nickel

Chromel: 90% nickel with 10% chromium

Alumel: 94% nickel with 3% manganese, 2% aluminum, and 1% silicon

<sup>a</sup>From Temperature Measurements ANSI PTC 19.3-1974.

<sup>b</sup>Use greater value; these limits of error do not include installation errors.

Table 8.6 Thermocouple Reference Table for Type-J Thermocouple<sup>a</sup>

Temperature (°C)	Thermocouple emf (mV)									
	0	-1	-2	-3	-4	-5	-6	-7	-8	-9
-210	-8.095									
-200	-7.890	-7.912	-7.934	-7.955	-7.976	-7.996	-8.017	-8.037	-8.057	-8.076
-190	-7.659	-7.683	-7.707	-7.731	-7.755	-7.778	-7.801	-7.824	-7.846	-7.868
-180	-7.403	-7.429	-7.456	-7.482	-7.508	-7.534	-7.559	-7.585	-7.610	-7.634
-170	-7.123	-7.152	-7.181	-7.209	-7.237	-7.265	-7.293	-7.321	-7.348	-7.376
-160	-6.821	-6.853	-6.883	-6.914	-6.944	-6.975	-7.005	-7.035	-7.064	-7.094
-150	-6.500	-6.533	-6.566	-6.598	-6.631	-6.663	-6.695	-6.727	-6.759	-6.790
-140	-6.159	-6.194	-6.229	-6.263	-6.298	-6.332	-6.366	-6.400	-6.433	-6.467
-130	-5.801	-5.838	-5.874	-5.910	-5.946	-5.982	-6.018	-6.054	-6.089	-6.124
-120	-5.426	-5.465	-5.503	-5.541	-5.578	-5.616	-5.653	-5.690	-5.727	-5.764
-110	-5.037	-5.076	-5.116	-5.155	-5.194	-5.233	-5.272	-5.311	-5.350	-5.388
-100	-4.633	-4.674	-4.714	-4.755	-4.796	-4.836	-4.877	-4.917	-4.957	-4.997
-90	-4.215	-4.257	-4.300	-4.342	-4.384	-4.425	-4.467	-4.509	-4.550	-4.591
-80	-3.786	-3.829	-3.872	-3.916	-3.959	-4.002	-4.045	-4.088	-4.130	-4.173
-70	-3.344	-3.389	-3.434	-3.478	-3.522	-3.566	-3.610	-3.654	-3.698	-3.742
-60	-2.893	-2.938	-2.984	-3.029	-3.075	-3.120	-3.165	-3.210	-3.255	-3.300
-50	-2.431	-2.478	-2.524	-2.571	-2.617	-2.663	-2.709	-2.755	-2.801	-2.847
-40	-1.961	-2.008	-2.055	-2.103	-2.150	-2.197	-2.244	-2.291	-2.338	-2.385
-30	-1.482	-1.530	-1.578	-1.626	-1.674	-1.722	-1.770	-1.818	-1.865	-1.913
-20	-0.995	-1.044	-1.093	-1.142	-1.190	-1.239	-1.288	-1.336	-1.385	-1.433
-10	-0.501	-0.550	-0.600	-0.650	-0.699	-0.749	-0.798	-0.847	-0.896	-0.946
0	0.000	-0.050	-0.101	-0.151	-0.201	-0.251	-0.301	-0.351	-0.401	-0.451

(continued)

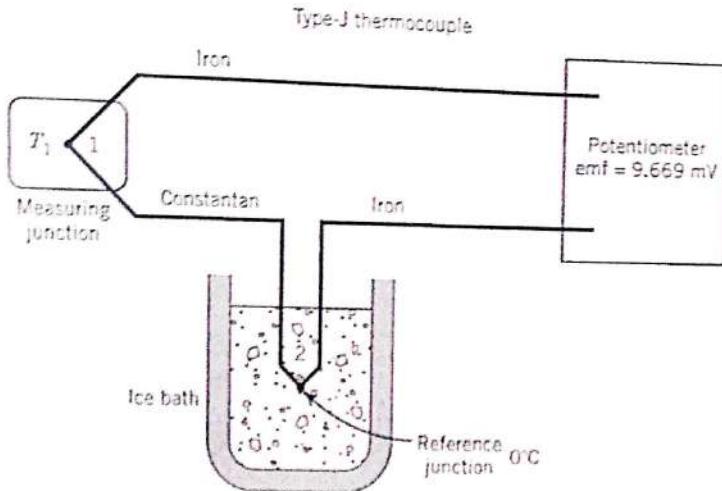


Figure 8.21 Thermocouple circuit for Example 8.7.

high-impedance voltmeters have been incorporated into commercially available temperature indicators, temperature controllers, and digital data-acquisition systems (DAS).

### Example 8.7

The thermocouple circuit shown in Figure 8.21 is used to measure the temperature  $T_1$ . The thermocouple reference junction labeled 2 is at a temperature of  $0^\circ\text{C}$ , maintained by an ice-point bath. The voltage output is measured using a potentiometer and found to be  $9.669 \text{ mV}$ . What is  $T_1$ ?

**KNOWN** A thermocouple circuit having one junction at  $0^\circ\text{C}$  and a second junction at an unknown temperature. The circuit produces an emf of  $9.669 \text{ mV}$ .

**FIND** The temperature  $T_1$ .

**ASSUMPTION** Thermocouple follows NIST standard.

**SOLUTION** Standard thermocouple tables such as Table 8.6 are referenced to  $0^\circ\text{C}$ . The temperature of the reference junction for this case is  $0^\circ\text{C}$ . Therefore, the temperature corresponding to the output voltage may simply be determined from Table 8.6, in this case as  $180.0^\circ\text{C}$ .

**COMMENT** Because of the law of intermediate metals, the junctions formed at the potentiometer do not affect the voltage measured for the thermocouple circuit, and the voltage output reflects accurately the temperature difference between junctions 1 and 2.

### Example 8.8

Suppose the thermocouple circuit in the previous example (Ex. 8.7) now has junction 2 maintained at a temperature of  $30^\circ\text{C}$ , and produces an output voltage of  $8.132 \text{ mV}$ . What temperature is sensed by the measuring junction?

**KNOWN** The value of  $T_2$  is  $30^\circ\text{C}$ , and the output emf is  $8.132 \text{ mV}$ .

**ASSUMPTION** Thermocouple follows NIST standard emf behavior.

**FIND** The temperature of the measuring junction.

**SOLUTION** By the law of intermediate temperatures the output emf for a thermocouple circuit having two junctions, one at  $0^\circ\text{C}$  and the other at  $T_1$ , would be the sum of the emfs for a thermocouple circuit between  $0^\circ$  and  $30^\circ\text{C}$  and between  $30^\circ\text{C}$  and  $T_1$ . Thus,

$$\text{emf}_{0-30} + \text{emf}_{30-T_1} = \text{emf}_{0-T_1}$$

This relationship allows the voltage reading from the nonstandard reference temperature to be converted to a  $0^\circ\text{C}$  reference temperature by adding  $\text{emf}_{0-30} = 1.537$  to the existing reading. This results in an equivalent output voltage, referenced to  $0^\circ\text{C}$  as

$$1.537 + 8.132 = 9.669 \text{ mV}$$

Clearly, this thermocouple is sensing the same temperature as in the previous example,  $180.0^\circ\text{C}$ . This value is determined from Table 8.6.

**COMMENT** Note that the effect of raising the reference junction temperature is to lower the output voltage of the thermocouple circuit. Negative values of voltage, as compared with the polarity listed in Table 8.4, indicate that the measured temperature is less than the reference junction temperature.

### Example 8.9

A J-type thermocouple measures a temperature of  $100^\circ\text{C}$  and is referenced to  $0^\circ\text{C}$ . The thermocouple is AWG 30 (American wire gauge [AWG]; AWG 30 is 0.010-in. wire diameter) and is arranged in a circuit as shown in Figure 8.17a. The length of the thermocouple wire is 10 ft in order to run from the measurement point to the ice bath and to a potentiometer. The resolution of the potentiometer is 0.005 mV. If the thermocouple wire has a resistance per unit length, as specified by the manufacturer, of  $5.6 \Omega/\text{ft}$ , estimate the residual current in the thermocouple when the circuit is balanced within the resolution of the potentiometer.

**KNOWN** A potentiometer having a resolution of 0.005 mV is used to measure the emf of a J-type thermocouple that is 10 ft long.

**FIND** The residual current in the thermocouple circuit.

**SOLUTION** The total resistance of the thermocouple circuit is  $56 \Omega$  for 10 ft of thermocouple wire. The residual current is then found from Ohm's law as

$$I = \frac{E}{R} = \frac{0.005 \text{ mV}}{56 \Omega} = 8.9 \times 10^{-8} \text{ A}$$

**COMMENT** The loading error due to this current flow is  $\sim 0.005 \text{ mV}/54.3 \mu\text{V}/^\circ\text{C} \approx 0.09^\circ\text{C}$ .

### Example 8.10

Suppose a high-impedance voltmeter is used in place of the potentiometer in Example 8.9. Determine the minimum input impedance required for the voltmeter that will limit the loading error to the same level as the potentiometer.

**KNOWN** Loading error should be less than  $8.9 \times 10^{-8} \text{ A}$ .

**FIND** Input impedance for a voltmeter that would produce the same current flow or loading error.

**SOLUTION** At  $100^\circ\text{C}$  a J-type thermocouple referenced to  $0^\circ\text{C}$  has a Seebeck voltage of  $E_s = 5.269 \text{ mV}$ . At this temperature, the required voltmeter impedance to limit the current flow to  $8.9 \times 10^{-8} \text{ A}$  is found from Ohm's law:

$$\frac{E_s}{I} = 5.269 \times 10^{-3} \text{ V} / 8.9 \times 10^{-8} \text{ A} = 59.2 \text{ k}\Omega$$

**COMMENT** This input impedance is not at all high for a modern microvoltmeter and such a voltmeter would be a reasonable choice in this situation. As always, the allowable loading error should be determined based on the required uncertainty in the measured temperature.

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Table 8.4 Thermocouple Designations

Type	Material Combination		Applications
	Positive	Negative	
E	Chromel(+) Constantan(-)		Highest sensitivity ( $<1000^\circ\text{C}$ )
J	Iron(+) Constantan(-)		Nonoxidizing environment ( $<760^\circ\text{C}$ )
K	Chromel(+) Alumel(-)		High temperature ( $<1372^\circ\text{C}$ )
S	Platinum/10% rhodium	Platinum(-)	Long-term stability
T	Copper(+) Constantan(-)		high temperature ( $<1768^\circ\text{C}$ ) Reducing or vacuum environments ( $<400^\circ\text{C}$ )

## Multiple measurement system

-  $\sigma_{\text{total}}$  =  $\sqrt{\sigma_{\text{true}}^2 + \sigma_{\text{random}}^2}$   
 uncertainty =  $\sigma_{\text{true}}$   
 uncertainty

advantages:

- ① well known measurement procedure
- ② enough ~~repetitions~~ replications

# Determinants

- ① overall precision
- ② bias

$$P = \sqrt{P_1^2 + P_2^2 + P_3^2 + \dots} \quad B = \sqrt{B_1^2 + B_2^2 + B_3^2 + \dots}$$

$$U_x^2 = B^2 + \left[ t_{n-1, 95\%} P^2 \right]$$

$P_{xt} \rightarrow$   $P_{\text{precision}}$   
 غير دقيقة  
 غير موثوقة

$B_{xt} \rightarrow$   $B_{\text{bias}}$   
 موجي  
 مسلي  
 مسلي

$$U_x = \sqrt{B^2 + \left[ t_{n-1, 95\%} P^2 \right] (95\%)}$$

$$R = x_1 + 2x_2$$

2.

$$u_R^2 = \sum \left( \frac{\partial R}{\partial x_i} \cdot u_{x_i} \right)^2$$

$$u_R^2 = (1 \cdot u_{x_1})^2 + (2 \cdot u_{x_2})^2$$

$$u_R = \sqrt{(1 \cdot u_{x_1})^2 + (2 \cdot u_{x_2})^2}$$

2)

$$R = x_1 - 9x_2$$

$$u_R^2 = (1 \cdot u_{x_1})^2 + (9 \cdot u_{x_2})^2 \quad \begin{cases} \frac{\partial R}{\partial x_2} = -9 \\ \frac{\partial R}{\partial x_1} = 1 \end{cases}$$

$$u_R = \sqrt{(u_{x_1})^2 - (9 \cdot u_{x_2})^2}$$

Q3

$$R = (x_1, x_2)$$

$$u_R^2 = \left( \frac{\partial R}{\partial x_1} \times u_{x_1} \right)^2 + \left( \frac{\partial R}{\partial x_2} \times u_{x_2} \right)^2 \rightarrow$$

$$u_R^2 = (x_2 \times u_{x_1})^2 \frac{x_1^2}{x_1^2} \quad (x_1 \times u_{x_2})^2 \times \frac{x_2^2}{x_2^2} \quad (Q.S.)$$

$$u_R^2 = \frac{R^2 u_{x_1}^2}{x_1^2} + \frac{R^2 u_{x_2}^2}{x_2^2} = \boxed{\left( \frac{u_{x_1}^2}{x_1^2} + \frac{u_{x_2}^2}{x_2^2} \right) R^2}$$

1

$$R = \frac{y_1}{y_2}$$

لـ

$$\frac{U_R^2}{R^2} = \frac{U_{y_1}^2}{y_1^2} + \frac{U_{y_2}^2}{y_2^2}$$

$$U_R = \sqrt{R^2 \left( \frac{U_{y_1}^2}{y_1^2} \right) + \left( \frac{U_{y_2}^2}{y_2^2} \right)}$$

$$R = \frac{x_1 * x_2 * x_3}{y_1 y_2 y_3} \Rightarrow$$

لـ

$$\frac{U_R^2}{R^2} = \frac{U_{x_1}^2}{x_1^2} + \frac{U_{x_2}^2}{x_2^2} + \frac{U_{x_3}^2}{x_3^2} + \frac{U_{y_1}^2}{y_1^2} + \frac{U_{y_2}^2}{y_2^2} + \frac{U_{y_3}^2}{y_3^2}$$

$$R = x_1 x_2 y_1$$

لـ

$$\frac{U_R^2}{R^2} = \frac{U_{x_1}^2}{x_1^2} + \frac{U_{x_2}^2}{x_2^2} + \frac{U_{y_1}^2}{y_1^2}$$

لـ

$$Q_1: u_o \pm \frac{1}{2} (\text{Resolution}) \quad | \quad \begin{aligned} \text{Resolution} &= 0.0014 \\ &= 0.625 \text{ mm} \end{aligned}$$

$$u_o = 10.000 \sin(95\%)$$

س: في حالة طلب السؤال ud tot

فكرة الاقتراح

Q2: 5.2: same

Resolution = 0.5 rpm

$$u_c) \text{ accuracy} = \% \text{ reading} \rightarrow u_c = \frac{1}{100} \times \text{reading}$$

	$u_o$	$u_c$	$u_d$
50	2.5	$\frac{1}{100} \times 50 = 0.5$	$\sqrt{2.5^2 + 0.5^2} = 2.54$
500	2.5	$\frac{1}{100} \times 500 = 5$	$\sqrt{2.5^2 + 5^2} = 5.5$
5000	2.5	$\frac{1}{100} \times 5000 = 50$	$\sqrt{2.5^2 + 50^2} = 50$

$$\text{total: } u_d = \sqrt{u_{d1}^2 + u_{d2}^2 + u_{d3}^2} = \sqrt{(2.54)^2 + (5.5)^2 + (50)^2} = 54.2 \text{ (لعمانوا)}$$

$$P_1: 50 \pm 2.54 \text{ (95\%)}$$

"precision interval"

$$P_2: 500 \pm 5 \text{ (95\%)}$$

أمثلة على

$$P_3: 5000 \pm 50 \text{ (95\%)}$$

accuracy = uc

Q3  
 resolution = 5  
 accuracy = 4% reading  
 reading = 90 kmph

uncertainty  
 (us)  $\frac{1}{100} \times 90 = \pm 0.9$   
 وحدة القيمة المئوية  
 مقدار القيمة المئوية

$$U_{d,r} = \pm 2.5$$

$$U_c = \frac{1}{100} \times 90 = \pm 0.9$$

$$U_d = \sqrt{\dots} = \pm 4.38$$

precision interval  $\Rightarrow 90 \pm 4.38$  (95%)

24  $\approx$  5.2 sequential processes Q : ud tot = uncertainty design stage



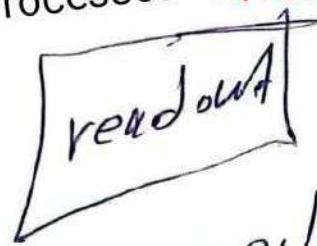
$$\text{accuracy} = 0.5\%$$

Resolution  $\approx 0.5$

$$U_d = 0$$

$$U_c = \pm 0.5$$

$$U_{ds} = \pm 0.05$$



$$\text{Res} = 0.1$$

$$acc = \pm 0.8$$

$$U_d = \pm 0.05$$

$$U_c = \pm 0.8$$

$$U_{dr} = \pm 0.08$$

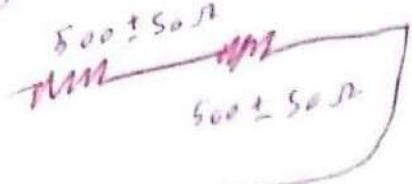
$$U_d = \sqrt{U_{ds}^2 + U_{dr}^2} = \pm 0.94$$

transducer  $\approx 100$  mV/volt  $\approx 5.2$  VDC السؤال

readout  $\approx 5$  فولت  $\approx 0.05$  فولت

# find the accuracy of circuits

Q5



$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

solt:

$$R_{eq} = R_1 + R_2$$

$$U_R^2 = \sum \left( \frac{\partial R}{\partial x_i} \times U_{xi} \right)^2$$

$$U_R^2 = U_{R1}^2 + U_{R2}^2$$

plD Ø

سیگنال

$$U_R = \pm \sqrt{U_{R1}^2 + U_{R2}^2}$$

$$U_R = \sqrt{S_o^2 + \zeta_o^2} = \pm 71. \sqrt{2}$$

أخطاء

Ø

$$U_R = \sqrt{\left( \frac{R_2^2}{(R_1+R_2)^2} U_{R1} \right)^2 + \left( \frac{R_1^2}{(R_1+R_2)^2} U_{R2} \right)^2}$$

$$U_R = \pm 35 \text{ } \Omega$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\Delta R_1 = \frac{R_2 (R_1+R_2) - R_1 R_2}{(R_1+R_2)^2}$$

$$\Delta R_2 = \frac{R_1 (R_1+R_2) - R_1 R_2}{(R_1+R_2)^2}$$

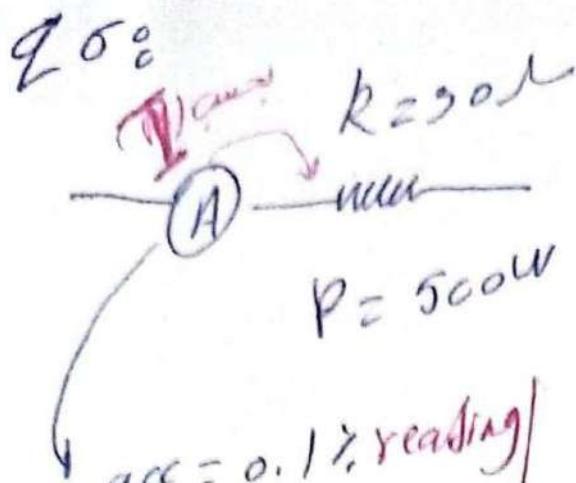
$$\frac{\partial R_1}{\partial R_2} = \frac{R_2^2}{(R_1+R_2)^2}$$

$$\frac{\partial R_2}{\partial R_1} = \frac{R_1^2}{(R_1+R_2)^2}$$

$U_R \downarrow \rightarrow$  accuracy ↑  
better ↑

Parallel > Series

the better is parallel because the  $U_R$  is smaller here



Sequential

$\text{acc} = 0.1\% \text{ reading}$

$$R = 100 \text{ mA}$$

$$U_0 = \pm 50 \text{ mA}$$

$$U_C = 0.1\% \times 100$$

$$U_C = 4.08 \times 10^{-3} \text{ A}$$

$$U_d = \sqrt{(100 \times 10^{-3})^2 + (4.08 \times 10^{-3})^2} \text{ A}$$

$$\boxed{U_d = 50 \text{ mA}}$$

ohmmeter

$\text{acc} = 0.5\% \text{ reading}$

$$R_{\text{res}} = 1 \Omega$$

$$U_0 = \pm 0.5$$

~~$$U_C = 0.5 \text{ V}$$~~

$$U_C = 0.5\% \times 30 = \pm 1.5$$

$$U_d = \pm 0.522$$

$$\text{Ammeter "I"} \rightarrow P = IV \quad | \quad V = IR$$

Currents  
in  
series  
parallel  
 $\text{A}$

$$P = I^2 R$$

$$\sqrt{\frac{P}{R}} = I \rightarrow I = \sqrt{\frac{500}{30}}$$

$$I = 4.08 \text{ A}$$

Ans!

$$P_1: 4.08 \pm 50 \text{ mA} \quad P_2: 30 \pm 0.522$$

abgleich

- ⊗ design stage uncertainty
  - ↳ sequential  $\rightarrow U_d \rightarrow U_{dtot} \rightarrow \text{Precision}$
  - ↳  $U_d \rightarrow \text{Precision}$

ch 5<sup>o</sup>

$$U_d = \sqrt{(U_o)^2 + (U_c)^2} \quad [U_X = U_c]$$

Random error  
Instrument bias  
Systematic error  
 $\rightarrow \sqrt{e_i^2 + e_s^2}$

$$U_o = \pm \frac{1}{2} r (\%)$$

Q<sub>1</sub>: 10, 20, 30, 40 --- (readings)  
required "design stage uncertainty"

resolution = A or "d<sub>90%</sub>"

$$\textcircled{1} \quad U_o = 0$$

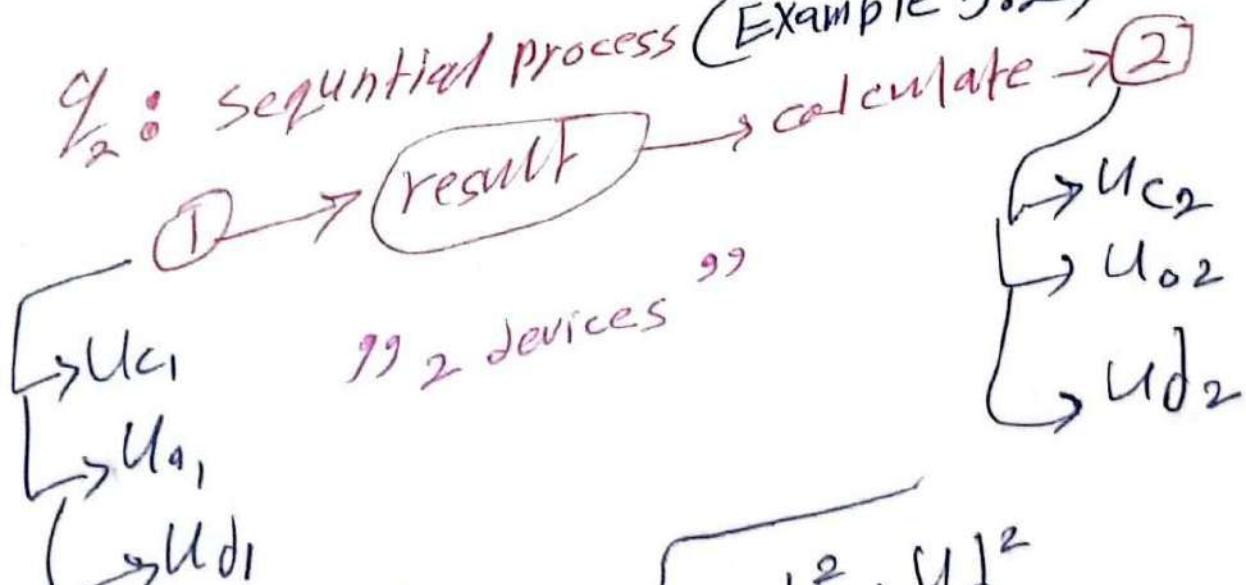
$$\textcircled{2} \quad U_c = \text{accuracy} = 4\% \text{ reading}$$

$$U_c \geq 4\% * 10$$

$$U_d \rightarrow \sqrt{U_c^2 + U_o^2}$$

$$10 \pm U_d (95\%)$$

Q<sub>2</sub>: sequential process (Example 5.2)



$$\Rightarrow U_{d\text{tot}} = \sqrt{U_{d1}^2 + U_{d2}^2}$$

## Q3 Identifying Error source --

  $U_R = \sqrt{\left( \sum \frac{\partial R}{\partial x_i} U_{x_i} \right)^2} \quad (95\%)$

$\otimes R = x_1 x_2$ $U_R = \pm \sqrt{(U_{x_1} x_2)^2 + (U_{x_2} x_1)^2}$	$\otimes R = x_1 + x_2$ $U_R = \pm \sqrt{(U_{x_1})^2 + (U_{x_2})^2} \quad 95\%$
$\otimes R = x_1 - x_2$ $U_R = \pm \sqrt{(U_{x_1})^2 - (U_{x_2})^2} \quad 95\%$	

$$R = \frac{x_1 x_3}{x_2 y_1}$$

$$U_R = \pm \sqrt{\left(\frac{U_{x_1}}{x_1}\right)^2 + \left(\frac{U_{x_2}}{x_2}\right)^2 + \left(\frac{U_{x_3}}{x_3}\right)^2 + \left(\frac{U_{y_1}}{y_1}\right)^2}$$

Q4 accuracy  $\rightarrow$  resistors فرق التوازن  $\Delta$   
 $U_R$  والتوافر  $\Delta$

$$(U_R)_{\text{series}} \rightarrow U_R = \sqrt{(U_{R_1})^2 + (U_{R_2})^2}$$

$$(U_R)_{\text{parallel}} \rightarrow U_R = \sqrt{\left(\frac{R_1^2}{(R_1+R_2)^2} U_{R_1}\right)^2 + \left(\frac{R_2^2}{(R_1+R_2)^2} U_{R_2}\right)^2}$$

$U_R \downarrow \Rightarrow \text{accuracy better}$

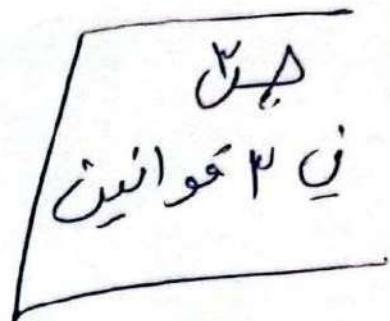
CH 6:

$$F = I * L * B$$

T  
torque

$$= I \times N \times A \times B \times \sin\alpha$$

Q1 Hour 1 can get ACP  
3 Methods ch 6



3 Resistors

$$V_1 = V_i \times \frac{R_1}{R_1 + R_2 + R_3}$$

$$R = \frac{E}{I} \quad | \quad R = \frac{P}{L} \quad | \quad A$$

$$Q_2 (R = \frac{PL}{A}) \rightarrow (\text{diameter})$$

Q3:  $\Delta d$  balance I\_h null Ans  $\rightarrow$  6 points

CH 11:

$$\sigma = F/A$$

$$\varepsilon = \frac{\Delta L}{L}$$

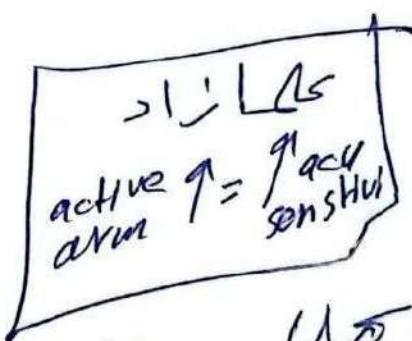
$$\sigma = E \varepsilon$$

$$V_2 = \frac{\Delta d}{\frac{d_0}{\Delta L}} \cdot L_0$$

$$GF = \frac{\Delta R/R}{\Delta L/L} \xrightarrow{\text{for } L_0, d_0}$$

CVV

$$Q_1 \Delta R \rightarrow \sigma = \frac{F}{A} / \sigma = E \varepsilon$$



$$Q_2 U_0 = \sum \frac{\partial U}{\partial R} \cdot U_{\Delta R} \quad | \quad \rightarrow GF = \frac{\Delta R}{R} / \varepsilon$$

$$Q_2 \quad U_{\Delta R} = \sum \dots \\ \Rightarrow \sigma = gE \rightarrow \frac{\sigma}{\Delta R} = \frac{Em}{RG_F}$$

$$\checkmark U = \frac{Em}{RG_F} \cdot x U_{\Delta R} (95\%)$$

ch 8:

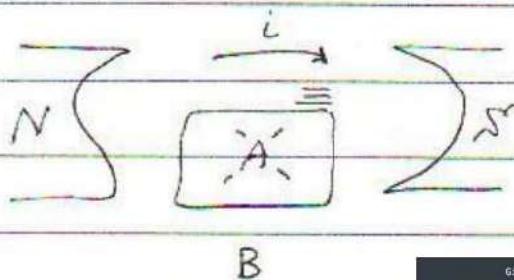
6.1

Current Loop,  $N = 20$

$$A = 1 \text{ inch}^2 \quad 0.0254 \text{ meters}$$

$$i = 0.02 \text{ A}$$

$$B = 0.4 \text{ wb/m}^2$$

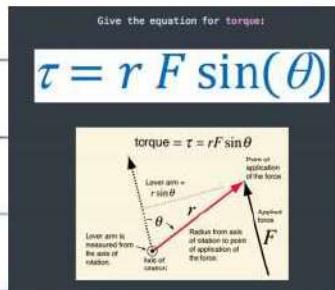


Find the max torque  $T$

$$T = NIAB \sin \alpha$$

$$T_{\max} = (20)(0.02)(2.54 \times 10^{-2})^2(0.4)$$

$$= 1.032256 \times 10^{-4} \text{ N.m}$$



ch 6

$\emptyset$ : between cross sectional area and magnetic field

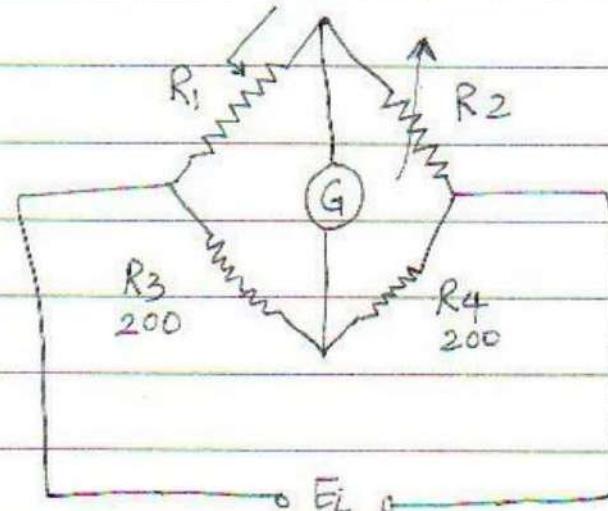
6.4

$$R_1 = 40x + 100$$

Determine  $R_2$

that balances this

bridge circuit.



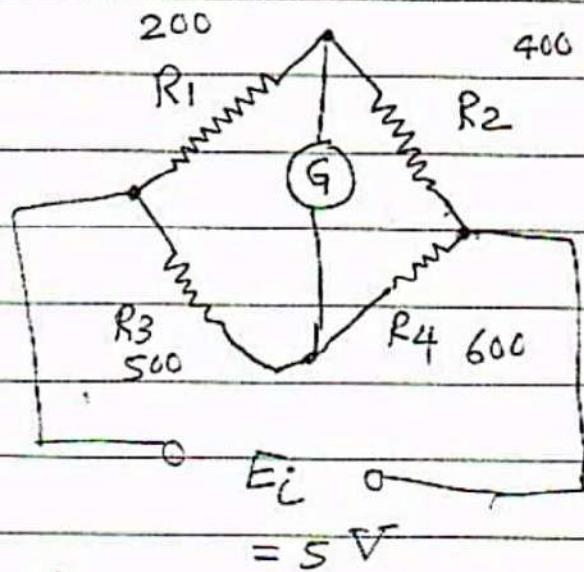
$$\cancel{R_1 \cdot R_4 = R_2 \cdot R_3} \quad (6.11)$$

$$\cancel{R_2 = 40x + 100}$$

$$\text{where } x=0, R_2 = 100 \Omega$$

ch 6

6.9



1- Is this bridge balanced?

$$R_1 R_4 \neq R_2 R_3$$

(No)

E<sub>o</sub>:  
(6.14)

2- Determine E<sub>o</sub>.

$$E_o = \left[ \frac{200}{600} - \frac{500}{1100} \right] \cdot (5) \xrightarrow{\leftrightarrow} 0.60606 \text{ V}$$

$$= \left[ \frac{400}{600} - \frac{600}{1100} \right] \cdot (5) = (+) 0.60606 \text{ V}$$

3- if R<sub>1</sub> → 250 Ω determine E<sub>o</sub>.

$$E_o = \left[ \frac{400}{650} - \frac{600}{1100} \right] \cdot (5) = +0.34965 \text{ V}$$

OH  
(Bridge output)  $V_{BC} = V_B - V_C$

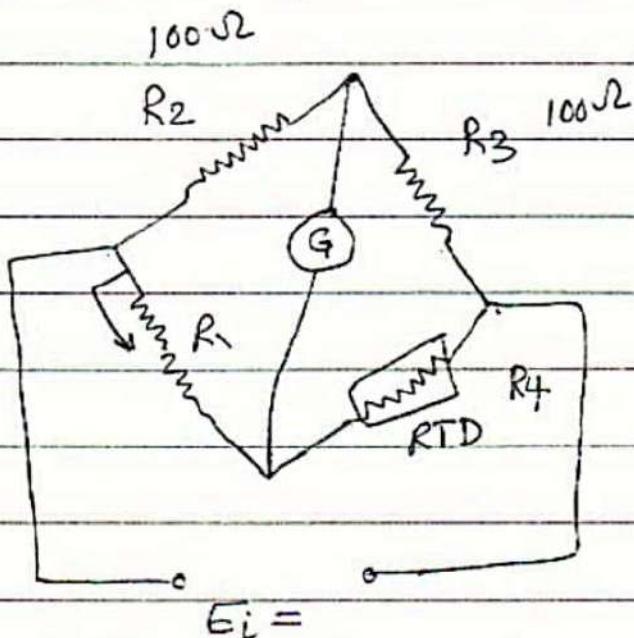
$$= \underbrace{\frac{R_1}{R_1+R_2} \cdot E_i}_{U_{BA}} - \underbrace{\frac{R_3}{R_3+R_4} \cdot E_i}_{U_{AC}}$$

ch 6

$$\underline{V_{BC}} = \underbrace{\frac{R_2}{R_1+R_2} \cdot E_i}_{U_{BD}} - \underbrace{\frac{R_4}{R_3+R_4} \cdot E_i}_{U_{DC}}$$

8.4

ch 8



$$R = R_0 [1 + \alpha(T - T_0)] \quad (8.4)$$

$$R_0 = 25\Omega \text{ at } T_0 = 0^\circ C \quad (-)$$

$$\alpha = 0.003925/\text{ }^\circ C$$

if  $R_1 = 41.485\Omega$  is used to balance the bridge  
determine the new temperature.

$$\cancel{R_2} \cdot \cancel{R_4} = \cancel{R_1} \cdot \cancel{R_3}$$

$$R_{RTD} = 41.485\Omega \text{ (new)}$$

$$41.485 = 25 \left[ 1 + (0.003925)(T - \underline{\underline{0}}) \right]$$

$$\underline{\underline{T}} = 168^\circ C$$

$T_f = \left( \frac{R_{RTD}}{R_0} - 1 \right) + T_0$

$\alpha$

## 11.8 Strain Gauges

ch 11

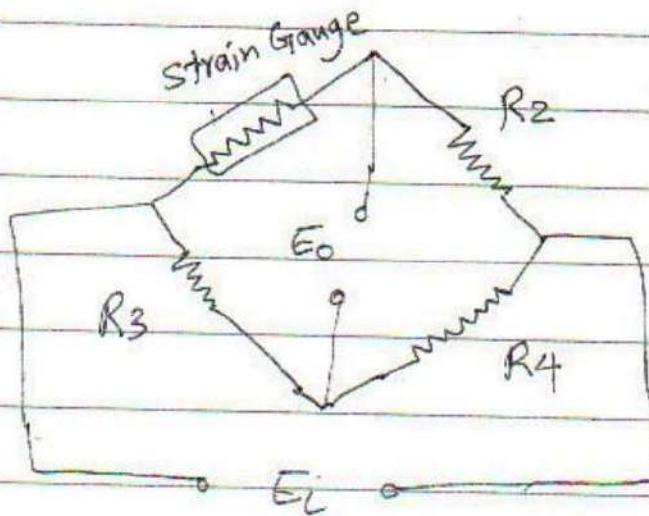
$$\frac{SE_0}{E_i} = \frac{SR/R}{4+2 SR/R} \quad (11.14)$$

$$SR/R = GF \cdot \epsilon \quad (11.11)$$

one active strain gauge

Equal Arms

$$R = 350 \Omega$$



$$SE_0 = 1 \text{ mV}$$

$$E_i = 5 \text{ V}$$

$$GF = 1.8$$

$$E_m = 70 \text{ GPa}$$

$$A_{\text{member}} = 1 \times 10^{-4} \text{ m}^2$$

$$\frac{1 \times 10^{-3}}{5} = \frac{SR/R}{4 + 2 SR/R}$$

$$\Rightarrow SR/R \approx 8 \times 10^{-4} (\Omega/\Omega)$$

$$\frac{\Delta R}{R} = \frac{\frac{4 \Delta E_0}{E_0}}{\left(1 - \frac{2 \Delta E_0}{E_i}\right)}$$

$$\checkmark 1. \text{ Strain} = \frac{8 \times 10^{-4}}{1.8} = 4.44 \times 10^{-4}$$

$$\checkmark 2. \text{ Stress} = E_m \cdot \epsilon$$

$$= (70 \times 10^9) (4.44 \times 10^{-4})$$

$$= 3.108 \times 10^7 \text{ N/m}^2$$

$$\checkmark 3. \text{ Force} = (3.108 \times 10^7) (1 \times 10^{-4})$$