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دفتر :

# فيزياء عامة 2

Physics 2

للطالب : يوسف الخطيب

اللجنة الأكاديمية لقسم الهندسة الصناعية

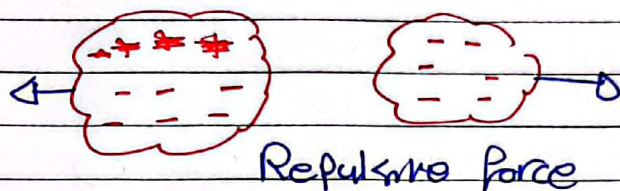
2023



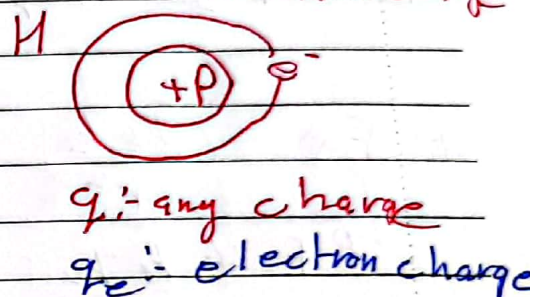
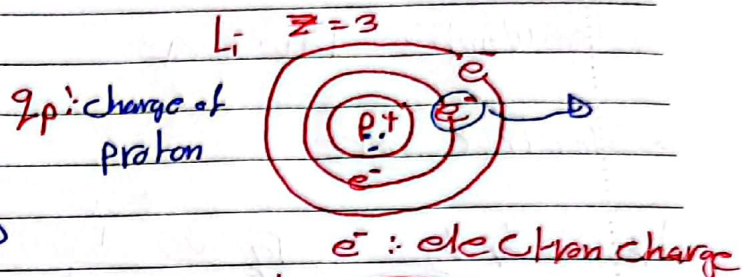
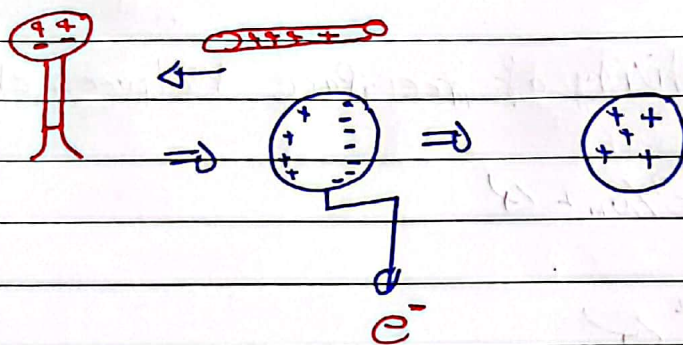
## Ch 23: Electric field:

~~→ E field~~

⇒ Electric charge:



⇒ induction:



⇒ Conservation of electric charge

⇒ electric charge can't be created or destroyed  
rather it can transfer from object to another

\* Quantization of electric charge

⇒ charge of any object equals an integer of elementary charge

$$\Rightarrow q = n \cdot q_e$$

$m_e = 9.11 \times 10^{-31} \text{ kg}$

$q_e = -1.6 \times 10^{-19} \text{ C}$



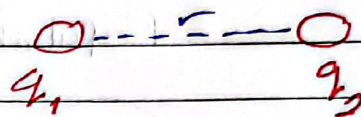
$$\Rightarrow q_p = 1.67 \times 10^{-19} \text{ C}, m_p = 1.6 \times 10^{-27} \text{ kg}$$

$\Rightarrow 1 \text{ mill: (mc)} = 10^{-3}$   
 $\Rightarrow 1 \text{ Micro: (mc)} = 10^{-6}$   
 $\Rightarrow 1 \text{ nano: (nc)} = 10^{-9}$   
 $\Rightarrow 1 \text{ pico: (pc)} = 10^{-12}$

$\Rightarrow$  Coulomb's law:

$\Rightarrow$  electric force

$$F_q \propto q_1 q_2, F_q \propto \frac{1}{r^2}$$



$$\Rightarrow F \propto \frac{q_1 q_2}{r^2}$$

$$\hookrightarrow F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

$1/4\pi\epsilon_0$ : Coulomb's constant

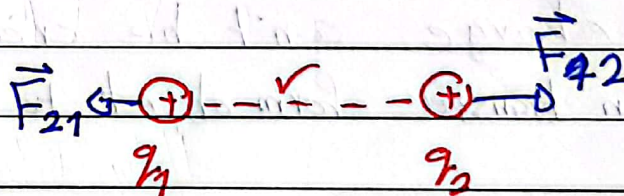
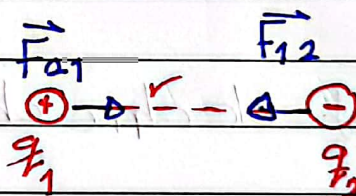
$\epsilon_0$ : permittivity of free space between charges

$$\epsilon_0 = 8.8 \times 10^{-12} \text{ C}^2/\text{Nm}^2$$

$$1/4\pi\epsilon_0 = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$$

$$\Rightarrow \vec{F}_{21} = -\vec{F}_{12}$$

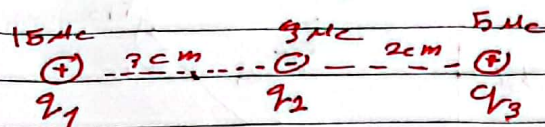
$$\Rightarrow |\vec{F}_{21}| = |\vec{F}_{12}|$$



$$\Rightarrow |\vec{F}_{12}| = |\vec{F}_{21}| = \frac{1}{4\pi\epsilon_0} \frac{|q_1| |q_2|}{r^2}$$

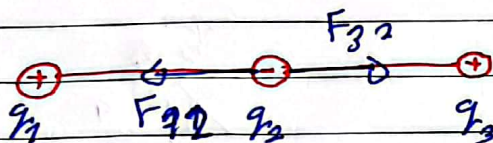


ex: (23.3) :- 3 point charges lie along axis as shown in the figure:



Find the magnitude and direction of resultant force on  $q_2$ ?

sol:-



$$F_{32} = \frac{k \cdot q_2 \cdot q_3}{r^2}$$

$$= \frac{(9 \times 10^9) (3 \times 10^{-6}) (5 \times 10^{-6})}{(2 \times 10^{-2})^2} = \frac{135 \times 10^{-3}}{4 \times 10^{-4}} = 33.75 \times 10^1 \text{ N}$$

$$F_{12} = \frac{k \cdot q_1 \cdot q_2}{r^2}$$

$$= \frac{(9 \times 10^9) (15 \times 10^{-6}) (3 \times 10^{-6})}{(3 \times 10^{-2})^2} = 450 \text{ N} \quad \text{--- (2)}$$

$$= 337.5 \text{ N} \quad \text{--- (1)}$$

$$F_{\text{net}} = 450 - 337.5 = 112.5 (-\hat{i}) \text{ N}$$

Resultant:

$$\vec{R} = \vec{F}_1 + \vec{F}_2$$

$$R = F_{\text{net}} = \vec{F}_1 + \vec{F}_2$$

$$|R| = |\vec{F}_1| + |\vec{F}_2|$$

$$\Rightarrow \frac{4}{F_{12}} = 180 \Rightarrow F_{21}$$

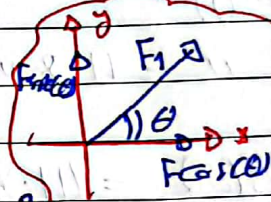
$$\vec{R} = -F_{12} + F_{21}$$

$$|R| = -|F_{12}| + |F_{21}|$$



$$|R| = \sqrt{F_1^2 + F_2^2}$$

$$\theta = \tan^{-1}\left(\frac{F_2}{F_1}\right)$$



$$F = F_x \hat{i} + F_y \hat{j}$$

$$= F \cos(\theta) \hat{i} + F \sin(\theta) \hat{j}$$

$$|F| = \sqrt{F_x^2 + F_y^2}$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

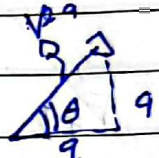
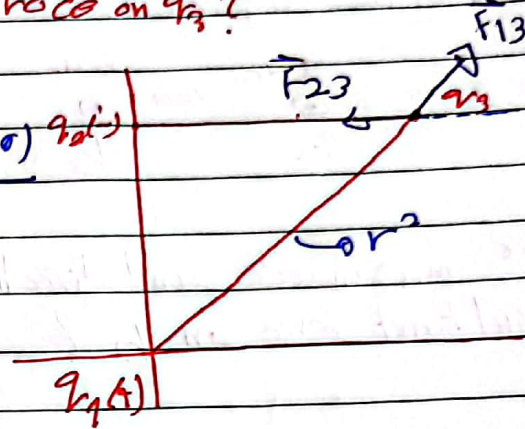


ex: (23.2): 3 point charges are located at the corners of a right angle triangle as shown in the figure,  $q_1 = q_2 = 5 \mu\text{C}$ ,  $q_3 = -2 \mu\text{C}$ ,  $a = 0.1 \text{ m}$ , find the net force on  $q_3$ ?

sol:

$$F_{23} = \frac{k q_2 q_3}{a^2} = \frac{(9 \times 10^9) (2 \times 10^{-6}) (5 \times 10^{-6})}{(0.1)^2}$$

$$= 9 \text{ N} \rightarrow$$



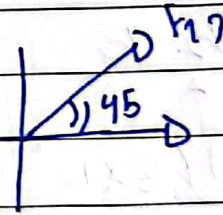
$$\vec{F}_{13} = \frac{k \cdot q_1 \cdot q_3}{r^2}$$

$$= \frac{(9 \times 10^9) (-5 \times 10^{-6}) (5 \times 10^{-6})}{(\sqrt{2} (0.1))^2}$$

$$= \frac{23.5}{2} \text{ N} = 11.8 \text{ N}$$

$$F_{13(x)} = F_{13} \cos(\theta) = (11.8) \cos(45) = 8.3 \text{ N}$$

$$F_{13(y)} = F_{13} \sin(\theta) = (11.8) \sin(45) = 8.3 \text{ N}$$



$$\Rightarrow \Sigma F(x) = 9 \cdot 8.3 - 9 = -0.7 \text{ N}$$

$$\Sigma F(y) = 8.3 \text{ N} \rightarrow$$

$$\Sigma F_{\text{net}} = -0.7 \text{ N} + 8.3 \text{ N}$$

$$|F| = \sqrt{(-0.7)^2 + (8.3)^2} = 8.32 \text{ N}$$

الناس للناس ما دام الوفاة



ex:- Two identical small charged spheres, each having a mass of  $3 \times 10^{-2} \text{ kg}$ , hang in equilibrium as shown in figure 23-9a. The length "L" of each string is 0.15m, and the angle  $\theta$  is  $5^\circ$ . find the magnitude of the charge on each sphere?

sol:-

$$\Rightarrow T \cos(\theta) = mg$$

$$T = \frac{mg}{\cos(\theta)} = \frac{29.4 \times 10^{-2}}{\cos(5^\circ)}$$

$$T = 29.5 \times 10^{-2}$$

$$T = 0.29 \text{ N} - \textcircled{1}$$

$$\Rightarrow T \sin(\theta) = \frac{k \cdot q^2}{r^2}$$

$$0.025 = \frac{(9 \times 10^9)(q^2)}{r^2}$$

$$\frac{(0.025)(0.025)^2}{9 \times 10^9} = q^2$$

$$q^2 = \frac{0.162 \times 10^{-4}}{9 \times 10^9}$$

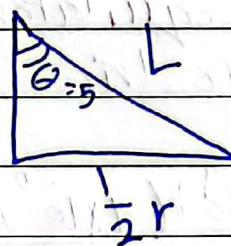
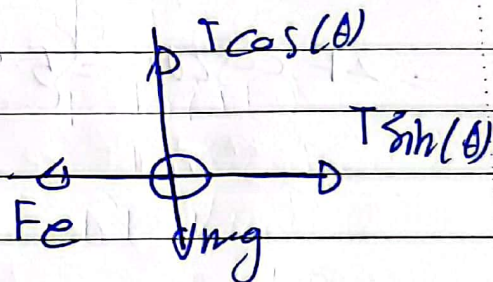
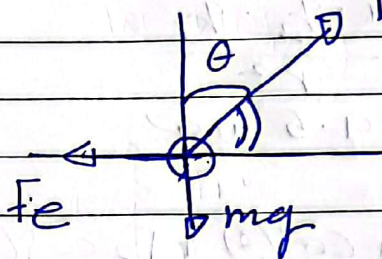
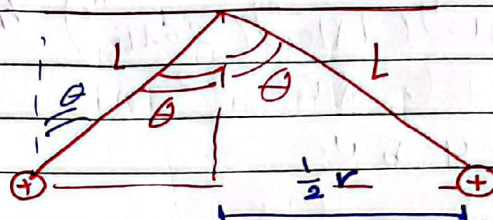
$$q^2 = \frac{(6.7) \times 10^{-4}(0.025)}{9 \times 10^9} = 0.162 \times 10^{-13}$$

$$= 0.018 \times 10^{-13}$$

$$q^2 = 0.18 \times 10^{-14}$$

$$q = \sqrt{0.18} \times 10^{-7}$$

$$q = 0.43 \times 10^{-7} \text{ C}$$



$$\sin(5^\circ) = \frac{\frac{1}{2} r}{L}$$

$$2L \sin(5^\circ) = r$$

$$r = 0.025$$



ex: Find force on  $q_3$ ?

$$F_1 = \frac{k q_1 q_3}{r^2}$$

$$= \frac{(9 \times 10^9)(1 \times 10^{-9})(25 \times 10^{-9})}{(5 \times 10^{-2})^2}$$

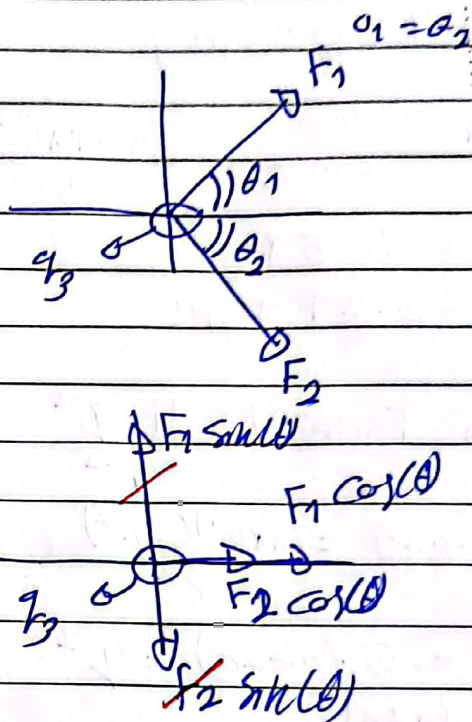
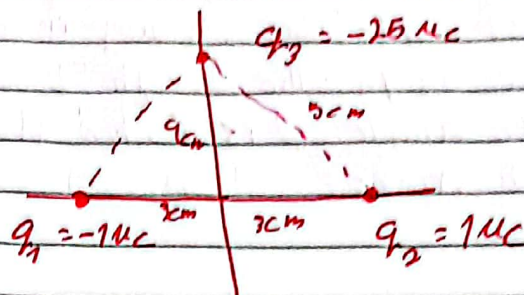
$$= 4.5 \text{ N}$$

$$F_2 = \frac{k q_2 q_3}{r^2}$$

$$F_2 = 4.5 \text{ N}$$

$$\Sigma F_y = F_1 \cos(\theta) + F_2 \cos(\theta)$$

$$= 2F \cos(\theta) = 5.4 \text{ N}$$



ex: Three charged particles are located at the corners of an equilateral triangle as shown in figure. Calculate the total electric force on the  $7 \mu\text{C}$  charge?

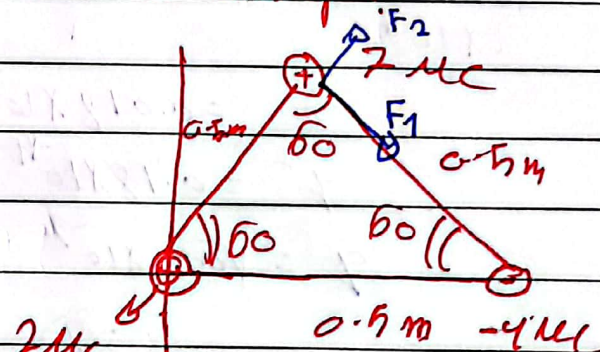
sol:

$$F_1 = \frac{(9 \times 10^9)(4 \times 10^{-9})(7 \times 10^{-9})}{(0.5)^2}$$

$$= 0.504 \text{ N} \quad (1)$$

$$F_2 = \frac{(9 \times 10^9)(7 \times 10^{-9})(2 \times 10^{-9})}{(0.5)^2}$$

$$= 1.008 \text{ N}$$





Ex

$$F_1(x) = F_1 \cos(60) = 0.252 \uparrow$$

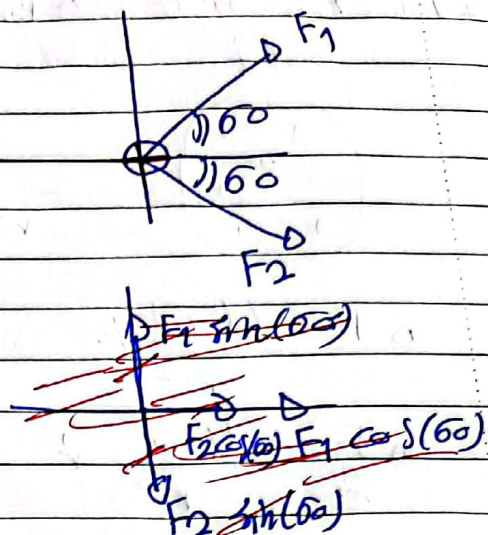
$$F_1(y) = F_1 \sin(60) = 0.436 \uparrow$$

$$F_2(x) = (1.008) \cos(60) = 0.504 \uparrow$$

$$F_2(y) = -0.872 \uparrow$$

$$\Sigma F = 0.756 \uparrow - 0.436 \uparrow$$

$$\theta = 330$$

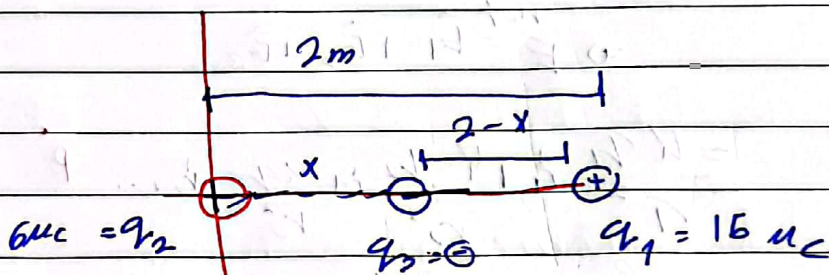


ex: 3 point charge are located along x-axis shown in the figure:

$$\Sigma F = 0$$

$$F_{23} + F_{12} = 0$$

$$\vec{F}_{23} = -\vec{F}_{12}$$



$$\frac{k \cdot q_2 q_3}{r^2} = + \frac{k \cdot q_1 q_2}{r^2}$$

$$\frac{q_2}{x^2} = \frac{q_1}{(2-x)^2} \Rightarrow q_2 (2-x)^2 = -q_1 x^2$$

$$\sqrt{q_2} (2-x) = \pm \sqrt{q_1} x$$

$$= \frac{2\sqrt{5}}{\sqrt{5} \pm \sqrt{5}}$$

$$x = \frac{2\sqrt{5}}{\sqrt{5} \pm \sqrt{5}} \text{ m}$$

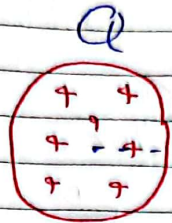
$$2\sqrt{q_2} - x\sqrt{q_2} = \pm \sqrt{q_1} x$$

$$2\sqrt{q_2} = x(\sqrt{q_2} \pm \sqrt{q_1})$$

$$x = \frac{2\sqrt{q_2}}{\sqrt{q_2} \pm \sqrt{q_1}} = \frac{2\sqrt{5} \times 10^{-6}}{\sqrt{5} \times 10^{-6} \pm \sqrt{16} \times 10^{-6}}$$

$$\sqrt{q_2} \pm \sqrt{q_1}$$

⇒ Electric field → Point charges



$q$

$q$ : test charge

قوة المجال  $q$

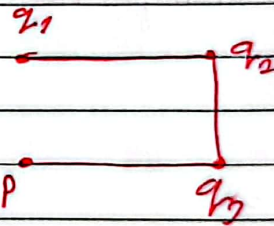
$$\Rightarrow F = \frac{k Q q}{r^2}$$

$$\Rightarrow \frac{F}{q} = \frac{k Q}{r^2} \Rightarrow \vec{E} = \frac{\vec{F}}{q} = \frac{k Q}{r^2}$$

$$\Rightarrow \vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots$$

$$\Rightarrow \vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

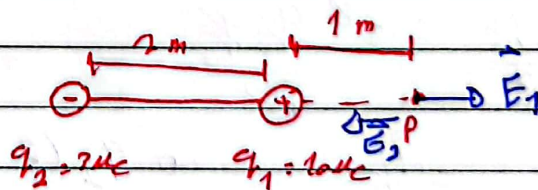
$$= \frac{k q_1}{r_1^2} \hat{r}_1 + \frac{k q_2}{r_2^2} \hat{r}_2 + \frac{k q_3}{r_3^2} \hat{r}_3 + \dots$$



ex: in the figure find the electric field at point "P" located at 1m from  $q_1$ .

$$\Rightarrow \vec{E}_1 = \frac{k \cdot q_1}{r^2} = \frac{(9 \times 10^9)(10 \times 10^{-6})}{1^2}$$

$$= 9 \times 10^3 = 9 \times 10^4 \text{ N/C} \rightarrow$$



$$\vec{E}_2 = \frac{k q_2}{r^2} = \frac{(9 \times 10^9)(2 \times 10^{-6})}{2^2} = 3 \times 10^3 = 0.3 \times 10^4$$

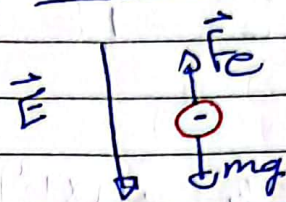


$$E_{\text{net}} = E_1 - E_2 = 90 \times 10^3 - 3 \times 10^3 = 87 \times 10^3 \text{ N/C} \uparrow$$

ex:- A small object of mass "3.8g" and charge " $-18 \mu\text{C}$ " is suspended motionless above the ground when immersed in a uniform electric field perpendicular to the ground what are the magnitude and direction of the electric field?

sol:-

$$\Rightarrow m = 3.8 \text{ g}, q = -18 \times 10^{-6}$$



$$F_e = mg$$

$$E = \frac{F_e}{q}$$

$$E \cdot q = mg$$

$$F_e = E \cdot q$$

$$E = \frac{mg}{q} = \frac{(3.8 \times 10^{-3})(9.8)}{(18 \times 10^{-6})} = 3.8 \times 10^4 \text{ N/C} \downarrow$$

ex:- A small 2g plastic ball is suspended by a 20 cm long string in a uniform electric field as shown in figure. If the ball is in equilibrium when the string makes a  $15^\circ$  angle with the vertical, what is the net charge on the ball?

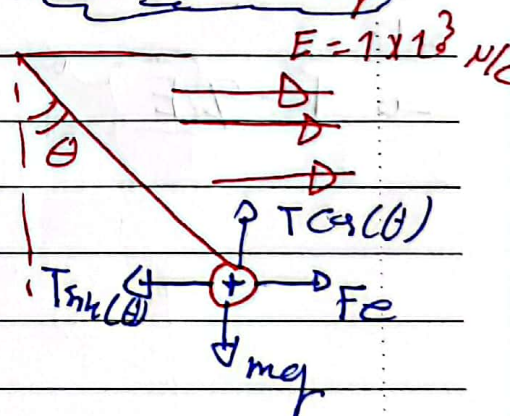
sol:-

$$\Sigma F = 0$$

$$F_e = T \sin(\theta)$$

$$mg = T \cos(\theta)$$

$$\frac{F_e}{mg} = \tan(\theta)$$



$$F_e = mg \tan(\theta) \Rightarrow E \cdot q = mg \tan(\theta)$$

$$= \frac{(2 \times 10^{-3})(9.8)(\tan(15))}{1 \times 10^{-3}} = 5.2 \times 10^{-3} \times 10^{-3} = 5.2 \times 10^{-6} \text{ C}$$

$$q = 5.2 \mu\text{C}$$

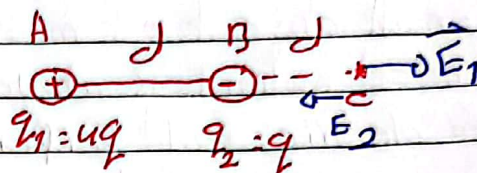


Ex: Find the electric field created by charges "A" and "B" at point C as shown in the figure.

Sol:

$$\vec{E}_1 = \frac{4 \cdot q}{r^2} = \frac{(9 \times 10^9)(4q)}{4d^2}$$

$$= \frac{44q}{d^2} = 4E \oplus$$



$$q = 5 \mu C, d = 2 \text{ cm}$$

→

$$\vec{E}_1 = \frac{4 \cdot q_1}{r^2} = \frac{(9 \times 10^9)(4(6 \times 10^{-6}))}{(4 \times 10^{-4})^2} = \frac{27 \times 10^7}{2} \text{ N/C}$$

$$\vec{E}_2 = \frac{(9 \times 10^9)(6 \times 10^{-6})}{4 \times 10^{-4}} = \frac{27 \times 10^7}{2} \text{ N/C}$$

$$\Sigma E = E_1 - E_2 = 0 \text{ N/C}$$

(2) Find the electric force on charge  $q = -2 \mu C$  placed at Point C?

Sol:

$$\Rightarrow \vec{F} = q(\vec{E}) = 0$$



ex: If the net electric field at point A in the figure is zero for  $q = 2 \text{ nC}$  and  $d = 2 \text{ cm}$ , find the charge of point D?

$$\Rightarrow E_D = E_B + E_C$$

$$E_D = E_A$$

$$\Rightarrow E_B \cos(60^\circ) = \frac{4 \cdot 2q}{(2 \times 10^{-2})^2} \cos(60^\circ)$$

$$= \frac{(9 \times 10^9)(2 \times 2 \times 10^{-9}) \cos(60^\circ)}{4 \times 10^{-4}}$$

$$\Rightarrow = 9 \times 10^4 \cos(60^\circ) \times 10^4$$

$$= \frac{9}{2} \times 10^4 \text{ (N/C)}$$

$$\Rightarrow E_C \cos(60^\circ) = E_B \cos(60^\circ) = \frac{9}{2} \times 10^4 \text{ N/C}$$

$$E_A = \frac{9}{2} \times 10^4 + \frac{9}{2} \times 10^4 = 9 \times 10^4 \text{ N/C}$$

$$E_D = 9 \times 10^4 \text{ N/C (N/C)}$$

$$9 \times 10^4 = \frac{9 \times 10^9 q_D}{(4 \times 10^{-2})^2} \Rightarrow q_D = \frac{(9 \times 10^4)(16 \times 10^{-4})}{9 \times 10^9}$$

$$= -4 \text{ nC}$$

سالو

لا تبالغ في حبك للناس لكي لا تنكسر

Yousef #



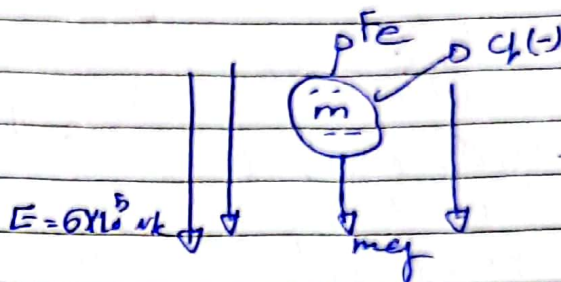
ex: a water droplet of mass " $1 \times 10^{-3} \text{ kg}$ " is located near the ground. an atmospheric electric field, of magnitude  $5 \times 10^5 \text{ N/C}$  points vertically downward near the droplet, if the droplet remains suspended at rest in air, what is the electric charge on the droplet?

sol:

$$\Sigma F = 0$$

$$F_e = mg$$

$$q \cdot E = mg$$



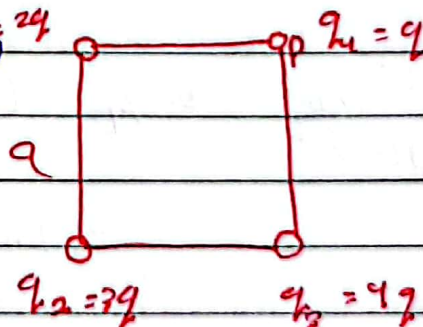
$$q = \frac{mg}{E} = \frac{(1 \times 10^{-3})(10)}{5 \times 10^5} = 2 \times 10^{-9} \text{ C} \Rightarrow q = -nC$$

ex: Four charged particles are placed at the corners of a square of side  $a = 1 \text{ m}$  as shown in the figure. Find the electric field at point "P" located at the position of charge  $q_4$ .

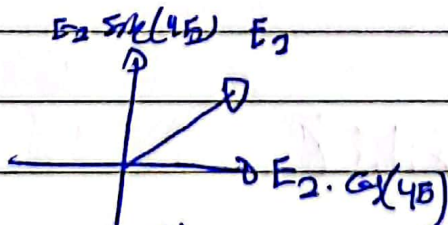
sol:

$$E_1 = \frac{(9 \times 10^9)(2)(1 \times 10^{-6})}{(1)^2} = 18 \times 10^3 \text{ N/C} \hat{i}$$

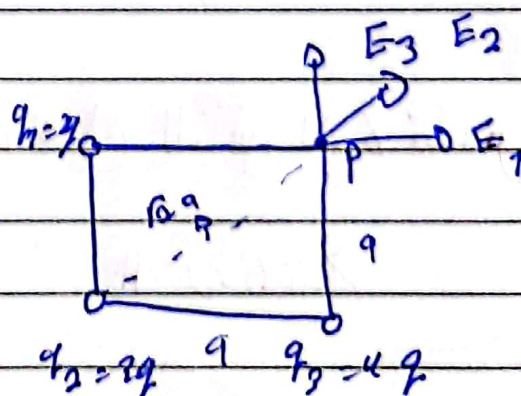
$$\Rightarrow 2q = 1 \mu\text{C}$$



$$E_3 = \frac{(9 \times 10^9)(4 \times 10^{-6})}{(1)^2} = 36 \times 10^3 \text{ N/C} \hat{j}$$



$$\Rightarrow E_2 = \frac{(9 \times 10^9)(3 \times 10^{-6})}{(1)^2} = 27 \times 10^3 \text{ N/C}$$



$$\Rightarrow E_{\text{net}} = E_1 + E_2$$

$$= 18 \times 10^3 + 27 \times 10^3 \left( \frac{1}{\sqrt{2}} \right) = (18 + \frac{13.5}{1.7}) \times 10^3 \text{ N/C} \hat{i} = 25 \times 10^3 \text{ N/C} \hat{i}$$



$$E_y = E_3 + E_2 \sin(45) = 36 \times 10^3 + \frac{27}{2} \cdot \frac{1}{\sqrt{2}} \times 10^3 = 47.4 \times 10^3 \text{ N/C}$$

$$\Sigma E = (25\hat{i} + 47.4\hat{j}) \times 10^3 \text{ N/C}$$

$$|\Sigma E| = \sqrt{(25 \times 10^3)^2 + (47.4 \times 10^3)^2}, \theta = \tan^{-1}\left(\frac{47.4}{25}\right)$$

(2) Find the force on a charge  $q = -4 \mu\text{C}$  placed at point "P":

$$\rightarrow F^{\text{net}} = qE^{\text{net}} = (-4 \times 10^{-6}) (25\hat{i} + 47.4\hat{j}) \times 10^3 \text{ N}$$

$$= (-100\hat{i} - 189.6\hat{j}) \times 10^{-3} \text{ N}$$

ex: A proton accelerates from rest in a uniform electric field of  $640 \text{ N/C}$ . At one later moment, its speed is  $1.2 \text{ Mm/s}$  (nonrelativistic because is much less than the speed of light).

(a) Find the acceleration of the proton (b) over what time interval does the proton reach this speed? (c) How far does it move in this time interval? (d) what is its kinetic energy at the end of this interval?

$$\rightarrow q_p = 1.6 \times 10^{-19} \text{ C}$$

$$\rightarrow m_p = 1.67 \times 10^{-27} \text{ kg}$$

$$\rightarrow E = 640 \text{ N/C}$$

$$\rightarrow v_i = 0$$

Sol:-

$$F = ma$$

$$E \cdot q = ma \rightarrow a = \frac{q \cdot E}{m}$$

$$\cancel{F = ma}$$

$$a = \frac{(1.6 \times 10^{-19}) (640)}{1.67 \times 10^{-27}} = 6.1 \times 10^{-44} \text{ m/s}^2$$

$$\Rightarrow v_f = v_i + at \Rightarrow t = \frac{v_f}{a} \rightarrow t = \frac{12 \times 10^6}{6.1 \times 10^{-44}} = 1.96 \times 10^{50} \text{ s}$$

$$K = \frac{1}{2} m v^2 \rightarrow K = \frac{1}{2} (1.67 \times 10^{-27}) (12 \times 10^6)^2 = -$$

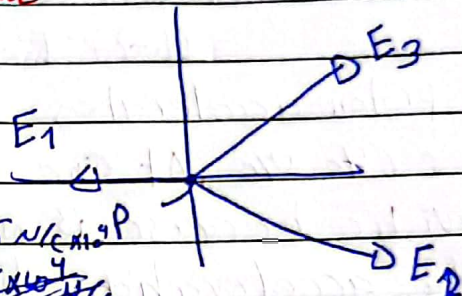
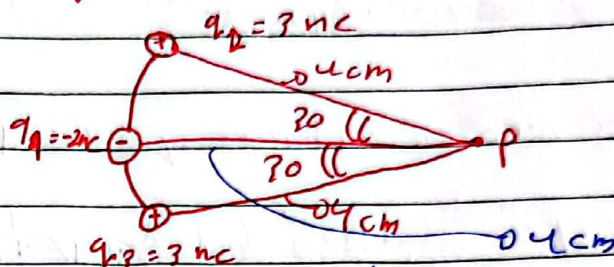


ex: Three point charges are located on a circular arc as shown in figure P23.31. (a) what is the total electric field at P, the center of the arc? (b) find the electric force that would be exerted on  $-5 \text{ nC}$  point charge placed at P?

sol:-

$$E_1 = \frac{(9 \times 10^9)(2 \times 10^{-9})}{(4 \times 10^{-2})^2}$$

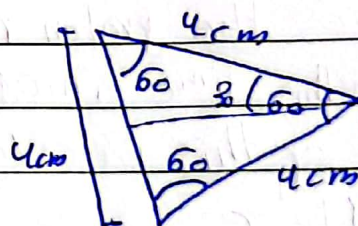
$$= \frac{18 \times 10^4}{16} = 1.125 \times 10^4 \text{ N/C } (-\hat{r})$$



$$E_2 = \frac{(9 \times 10^9)(3 \times 10^{-9})}{(4 \times 10^{-2})^2} = \frac{27 \times 10^4}{16} = 1.6875 \times 10^4 \text{ N/C}$$

$$E_{2(x)} = E_2 \cos(60) = 1.6875 \times 10^4 \times \frac{1}{2} = 8.4375 \times 10^3 \text{ N/C } (+\hat{x})$$

$$E_{2(y)} = E_2 \sin(60) = 1.6875 \times 10^4 \times \frac{\sqrt{3}}{2} = 1.46 \times 10^4 \text{ N/C } (-\hat{y})$$



$$E_3 = \frac{(9 \times 10^9)(3 \times 10^{-9})}{(4 \times 10^{-2})^2} = 1.6875 \times 10^4 \text{ N/C}$$

$$= 1.46 \times 10^4$$

$$E_{3(x)} = 8.4375 \times 10^3 \text{ N/C } (+\hat{x})$$

$$E_{3(y)} = 1.46 \times 10^4 \text{ N/C } (+\hat{y})$$

$$\Sigma E(x) = 1.125 + 1.7 \times 10^4 \text{ N/C } (+\hat{x})$$

$$\Sigma E(y) = 0$$

$$F = E \cdot q = (1.7 \times 10^4)(-5 \times 10^{-9}) = -8.5 \times 10^{-5} \text{ N}$$

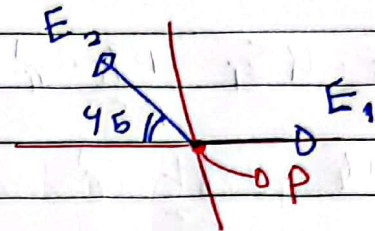
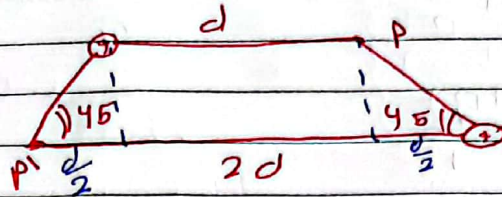


ex: Two equal positively charged particles are at opposite corners of a trapezoid as shown in figure P23.27. Find symbolic expressions for the total electric field at (a) the point "P" and (b) the point P'.

sol:-

$$\Rightarrow E_1 = \frac{k \cdot Q}{d^2} \quad \text{--- (1) (+)} \quad \hat{i}$$

$$\Rightarrow E_2 = \frac{k \cdot Q}{(d\sqrt{2})^2} \cdot 2 = 2E_1$$



$$\Rightarrow E_2(x) = E_1 \cdot \cos(45) = \frac{2E_1}{\sqrt{2}} = \sqrt{2} E_1 (-\hat{i})$$

$$E_2(y) = 2E_1 \sin(45) = \sqrt{2} E_1 (+\hat{j})$$

$$E(x) = E_1 + \sqrt{2} E_1 \quad E(y) = E_1 - \sqrt{2} E_1 = E_1(1 - \sqrt{2})$$

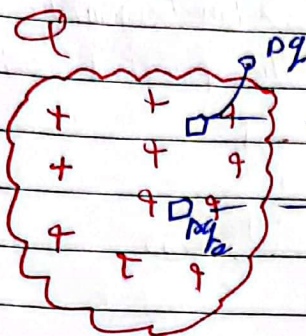
$$E(y) = \sqrt{2} E_1 (+\hat{j})$$

$$E_P = E_1(1 - \sqrt{2}) \hat{i} + \sqrt{2} E_1 \hat{j}$$

والله مال بعد الا وهتنام هو قتل نفسه بريئه بغير حق



⇒ continuous charge distribution and calculation of  $\vec{E}$ :



$$\Rightarrow dE_1 = \frac{1}{4\pi\epsilon_0} \frac{dq_1}{r_1^2} \hat{r}_1$$

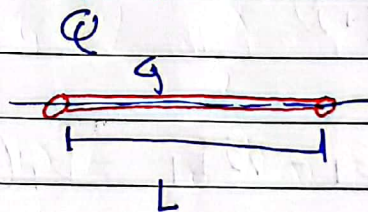
$$dE_2 = \frac{1}{4\pi\epsilon_0} \frac{dq_2}{r_2^2} \hat{r}_2$$

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0} \sum_i \frac{dq_i}{r_i^2} \hat{r}_i \Rightarrow \lim_{dq_i \rightarrow 0} \frac{1}{4\pi\epsilon_0} \sum_i \frac{dq_i}{r_i^2} \hat{r}_i$$

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{r}$$

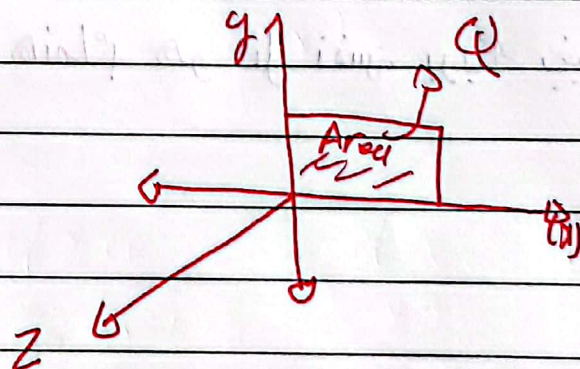
★ (1) linear charge distribution ((7)):

$$\Rightarrow \lambda = \frac{Q}{L} \text{ C/m}$$



★ (2) surface charge distribution ((6))

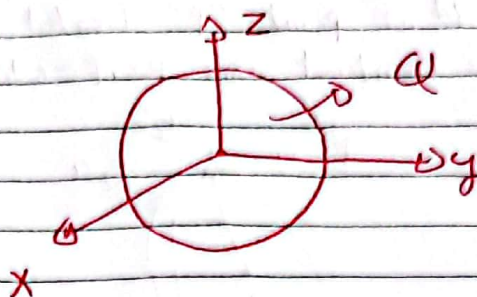
$$\Rightarrow \sigma = \frac{Q}{\text{Area}} \text{ C/m}^2$$





(3) Volume charge distribution ( $\rho$ ):

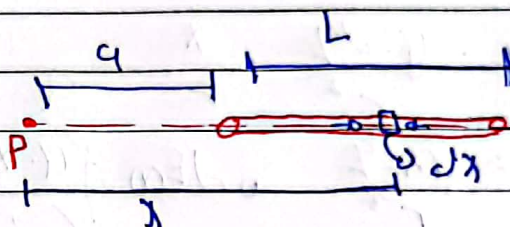
$$\Rightarrow \rho = \frac{Q}{V} \text{ C/m}^3$$



$$+ dq = \rho dV$$

ex:- A rod of length "L" has a uniform linear charge density " $\lambda$ " and a total charge point "P" located along the axis of the rod and a distance  $a$  from one end of the rod as shown in the figure:-

$$\lambda = \frac{Q}{L}$$



$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

$$\Rightarrow E_{rod} = \frac{1}{4\pi\epsilon_0} \frac{\lambda L}{a(a+L)}$$

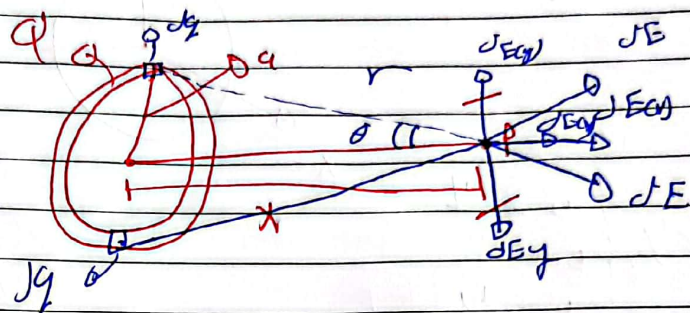
$$\Rightarrow dE = \frac{1}{4\pi\epsilon_0} \frac{\lambda dx}{x^2} \quad \Rightarrow \int_{a+L}^{L+a} dE = \int_{a+L}^{L+a} \frac{1}{4\pi\epsilon_0} \frac{\lambda dx}{x^2} = \frac{1}{4\pi\epsilon_0} \lambda \int_{a+L}^{L+a} \frac{dx}{x^2}$$

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0} \lambda \left| -\frac{1}{x} \right|_{a+L}^{L+a}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{\lambda L}{a(a+L)}$$



ex: A ring of radius  $a$  carries a positive charge  $Q$ , calculate the electric field of point "P" located at distance  $x$  from its center along the central axis perpendicular to the plane of the ring.



$$\Rightarrow dE_{\cos} = dE \cos(\theta)$$

$$\Rightarrow dE_{\cos} = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \cos(\theta)$$

$$dE_{\cos} = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \cdot \frac{x}{r}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{x dq}{r^3} \Rightarrow dE_{\cos} = \frac{1}{4\pi\epsilon_0} \frac{x dq}{(a^2 + x^2)^{3/2}}$$

$$Q = \int (2\pi a) \cdot \dots$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{x Q}{(a^2 + x^2)^{3/2}}$$

$$* x < a \Rightarrow E = 0$$

$$* x > a \Rightarrow E_{\cos} = \frac{1}{4\pi\epsilon_0} \frac{x Q}{(a^2 + x^2)^{3/2}} = \frac{1}{4\pi\epsilon_0} \frac{Q}{x^2}$$

إذا كان  $x$  أكبر بكثير من  $a$  فأنه  $E = \frac{1}{4\pi\epsilon_0} \frac{Q}{x^2}$

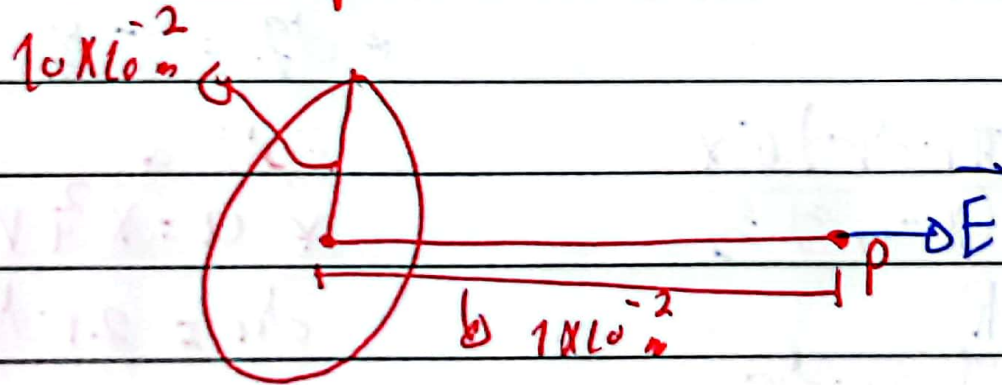


ex: A uniformly charged ring of radius 10 cm has a total charge of 75  $\mu\text{C}$ . Find the electric field on the axis of the ring at (a) 1 cm from the center of ring:

Sol:

$$Q = 75 \times 10^{-6} \text{ C}$$

$$E_p = \frac{4 \cdot Q \cdot x}{(x^2 + R^2)^{\frac{3}{2}}}$$



$$= \frac{(9 \times 10^9) (75 \times 10^{-6}) (1 \times 10^{-2})}{((1 \times 10^{-2})^2 + (10 \times 10^{-2})^2)^{\frac{3}{2}}} = \frac{675 \times 10^1}{(1 \times 10^{-4} + 100 \times 10^{-4})^{\frac{3}{2}}}$$

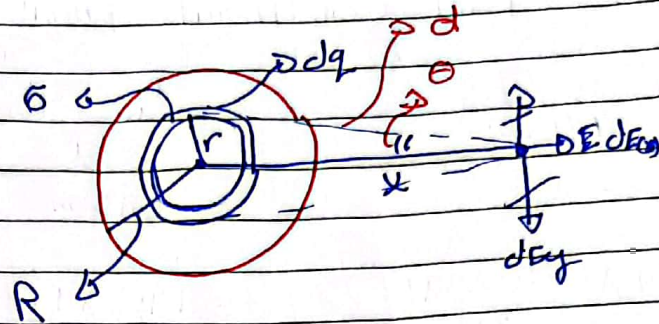
$$= \frac{675 \times 10^1}{(\sqrt{101 \times 10^{-4}})^3} = \frac{6750}{(10 \times 10^{-2})^3} = \frac{6750}{1000 \times 10^{-6}} = \frac{6.75 \times 10^1}{1 \times 10^{-3}} = 6.75 \times 10^4 \text{ N/C}$$



ex: a disk of radius  $R$ , has a uniform surface charge density  $\sigma$ , calculate the electric field at point "P" along the perpendicular central axis and a distance "x" from the center of the disk?

$$\Rightarrow Q = \sigma A \quad Q = \sigma \pi R^2$$

$$\Rightarrow dE(x) = dE \cos(\theta) = \frac{k dq \cdot \cos(\theta)}{d^2}$$



$$dE(x) = \frac{k \cdot dq \cdot x}{d^3}$$

$$\cos(\theta) = \frac{x}{d}$$

$$x \cdot d = (x^2 + r^2)^{1/2}$$

$$dE(x) = \frac{k \cdot dq \cdot x}{(x^2 + r^2)^{3/2}}$$

$$x \cdot d^3 = (x^2 + r^2)^{3/2}$$

$$dq = \sigma dA$$

$$\oint dE(x) = \int \frac{k (2\pi r dr) \sigma x}{(x^2 + r^2)^{3/2}}$$

substitution  $u = x^2 + r^2$   
 $du = 2r dr$   
 $r=0 \rightarrow u=x^2$   
 $r=R \rightarrow u=x^2 + R^2$

$$= k \cdot \sigma \cdot x \cdot \pi \int_0^R \frac{2r dr}{(x^2 + r^2)^{3/2}}$$

$$= k \sigma x \pi \int_{x^2}^{x^2 + R^2} \frac{du}{u^{3/2}} = k \sigma x \pi \int_{x^2}^{x^2 + R^2} u^{-3/2} du$$

$$= k \sigma x \pi \left( -\frac{1}{\sqrt{u}} \cdot 2 \right) \Big|_{x^2}^{x^2 + R^2} = -2k \sigma x \pi \left( \frac{1}{\sqrt{u}} \right) \Big|_{x^2}^{x^2 + R^2}$$

$$= -2k \sigma x \pi \left( \frac{1}{\sqrt{x^2 + R^2}} - \frac{1}{\sqrt{x^2}} \right) = 2k \sigma x \pi \left( 1 - \frac{x}{\sqrt{x^2 + R^2}} \right)$$

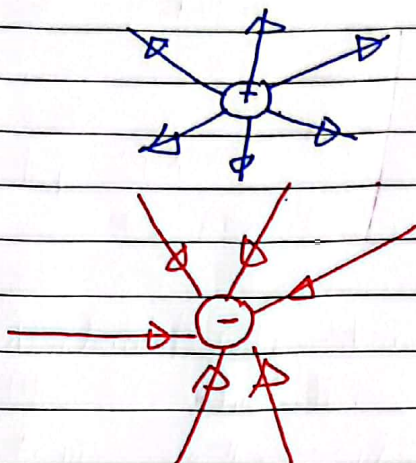
$$E(x) = 2k(6) \pi \left( 1 - \frac{x}{\sqrt{x^2 + R^2}} \right)$$

$$\sigma = \frac{Q}{A} = \frac{Q}{\pi R^2}$$

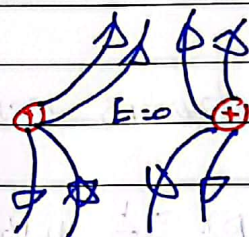
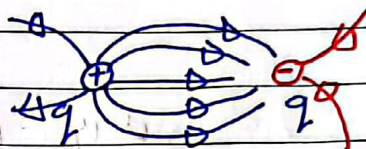


$$\Rightarrow R \rightarrow \infty \Rightarrow E = 2k\pi\sigma = 2\left(\frac{1}{4\pi\epsilon_0}\right)\sigma = \frac{\sigma}{2\epsilon_0}$$

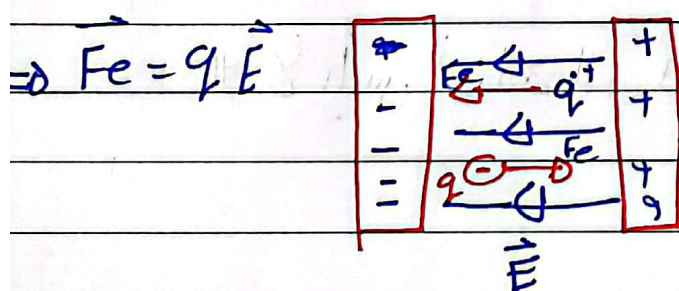
$\Rightarrow$  Electric Field lines:



\*  $\frac{N_2}{N_1} = \frac{Q_2}{Q_1}$   $\swarrow$  # of field line



$\Rightarrow$  uniform electric field



\*  $\Sigma F = F_e = qE = ma$

$$\vec{a} = \frac{q\vec{E}}{m}$$

$\Rightarrow$  motion under constant acceleration

$$\vec{v}_f = \vec{v}_i + \vec{a}t$$

$$v_f^2 = v_i^2 + 2ax$$

$$x = v_i t + \frac{1}{2} a t^2$$



ex: A uniform electric field directed along x-axis between parallel plates separated by a distance "d" as shown in the figure, a positive point charge of mass "m" is released from rest at point "A" next to the positive plate and acceleration to a point "B" next to the negative plate. Find the speed of the particle at point "B".

Sol:

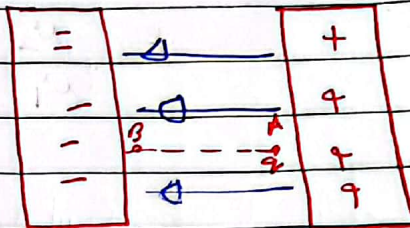
$$\Rightarrow v_B^2 = v_A^2 + 2ad$$

$$v_B^2 = 2ad$$

$$\Sigma F = F_e = qE = ma$$

$$a = \frac{qE}{m}$$

$$\Rightarrow v_B^2 = 2\left(\frac{qE}{m}\right)d \Rightarrow v_B = \sqrt{\frac{2qEd}{m}}$$



② Consider the system is nonisolated

$$\Rightarrow W = \Delta K \Rightarrow F_e d = \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2$$

$$v_B = \sqrt{\frac{2qEd}{m}}$$

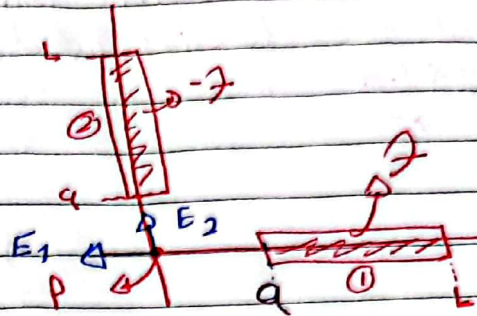
# لا بأس نحتاج للوقوف أميانياً كي نشر بروعة الوقوف



ex: calculat  $\vec{E}$  at P

$$\text{sol: } E_1 = \frac{4 \cdot 7 L}{q(a+L)} (-\hat{i})$$

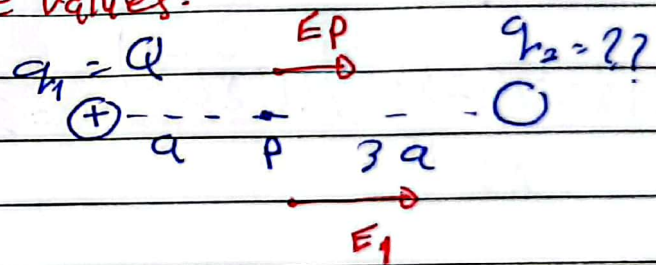
$$E_2 = \frac{4 \cdot 7 L}{q(a+L)} (+\hat{j})$$



$$\begin{aligned} E_P &= E_1 (-\hat{i}) + E_2 (+\hat{j}) = \frac{4 \cdot 7 L}{q(a+L)} (-\hat{i} + \hat{j}) \end{aligned}$$

ex: Two charged particles are located on the x-axis. The first is a charge "+Q at x=-a". The second is an unknown charge located at x=3a. The net electric field these charges produce at the origin has a magnitude of " $\frac{24Q}{q^2}$ ". Explain how many values are possible for the unknown charge and find the possible values.

$$\text{sol: } (1) E_P = \frac{24Q}{q^2}$$



$$E_1 = \frac{kQ}{a^2}$$

$$\Rightarrow E_P = E_1 + E_2$$

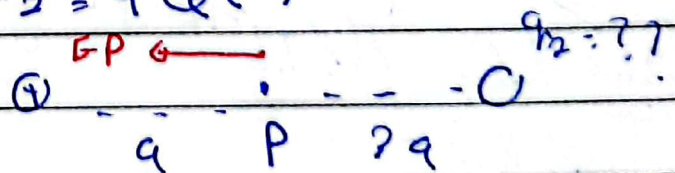
$E_P > E_1 \rightarrow$   $\frac{24Q}{q^2} > \frac{kQ}{a^2}$   $\Rightarrow \frac{24Q}{q^2} > \frac{kQ}{a^2}$

$$\frac{24Q}{q^2} = \frac{kQ}{a^2} + \frac{kq_2}{9a^2}$$

$$\frac{kQ}{a^2} = \frac{kq_2}{9a^2}$$

$$q_2 = 9Q (-)$$

$E_P < E_1 \rightarrow$   $\frac{24Q}{q^2} < \frac{kQ}{a^2}$   $\Rightarrow \frac{24Q}{q^2} < \frac{kQ}{a^2}$



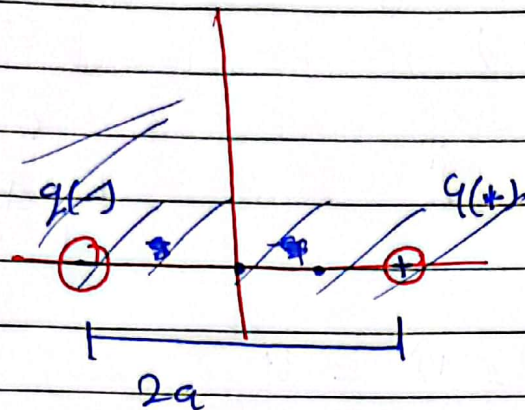
$$= E_2 - E_1$$

$$\frac{kQ}{a^2} - \frac{kq_2}{9a^2} = \frac{kQ}{a^2} \Rightarrow \frac{-q_2}{9a^2} = 0 \Rightarrow q_2 = 0$$

$$\Rightarrow q_2 = 27Q (+)$$

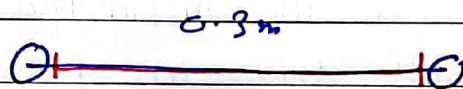


ex: Consider the electric dipole shown in figure p 23-36.  
 Show that the electric field at distant point on the x axis  
 is  $E_x = \frac{4kq \cdot a}{x^3}$



ex: Two identical conducting small spheres are placed with their centers 0.3 m apart. One is given a charge of 12 nC and the other charge of -18 nC. (a) Find the electric force exerted by one sphere on the other.

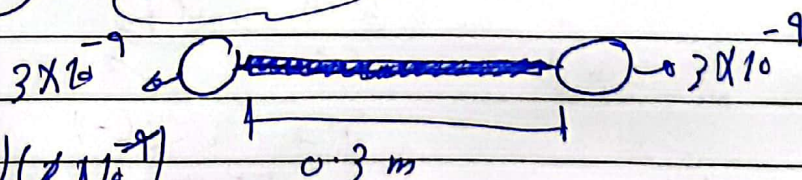
sol:  $F = \frac{k \cdot q_1 \cdot q_2}{r^2}$



$$= \frac{(9 \times 10^9)(12 \times 10^{-9})(18 \times 10^{-9})}{(0.3 \times 10^{-1})^2} = 216 \times 10^{-7} \text{ N}$$

(b) What if the spheres are connected by a conducting wire find the electric force each exerts on the other after they have come to equilibrium.

$Q = 6 \times 10^{-9}$



$$F = \frac{(9 \times 10^9)(3 \times 10^{-9})(3 \times 10^{-9})}{(0.3 \times 10^{-1})^2} = 9 \times 10^{-7} \text{ N}$$



ex: A Molecule of DNA is  $2.17 \mu\text{m}$  long. The ends of the molecule become singly ionized: negative on one end, positive on the other. The helical molecule acts like a spring and compresses  $1.00\%$  upon becoming charged. Determine the effective spring constant of the molecule;

$$\Rightarrow F_s = kx \quad \Leftrightarrow \quad F_s = kx \quad q_1 = q_2 = e^-$$

$$F = \frac{k q_1 \cdot q_2}{r^2} \Rightarrow F_s = F$$

$$x = 0.01 \mu \quad kx = \frac{(9 \times 10^9)(1.6 \times 10^{-19})^2}{(2.17 \times 10^{-6})^2}$$

$$k = \frac{(9 \times 10^9)(1.6 \times 10^{-19})^2}{(0.01)(2.17 \times 10^{-6})^3}$$

$$k = 2.25 \times 10^{-9} \text{ N/m}$$

ex: Consider the electric dipole shown in figure P23 shown that the electric field at distant point on the  $+x$  axis is  $E_x = \frac{4kq_1q_2}{x^3}$

$$E = \frac{kq}{r^2}$$

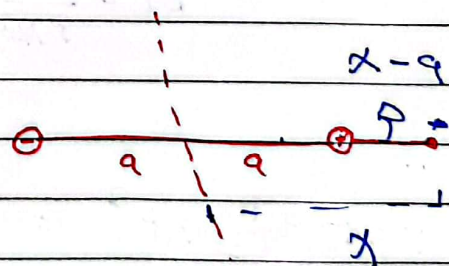
$$E = \frac{kq}{(x-a)^2} - \frac{kq}{(x+a)^2}$$

$$= \frac{kq(x+a)^2 - kq(x-a)^2}{(x^2-a^2)^2} = \frac{kq(x^2+2ax+a^2 - (x^2-2ax+a^2))}{(x^2-a^2)^2}$$

$$= \frac{kq(4ax)}{(x^2-a^2)^2} = \frac{4kqa}{(x^2-a^2)^2}$$

$$x \gg a$$

$$= \frac{4kqa}{x^3}$$





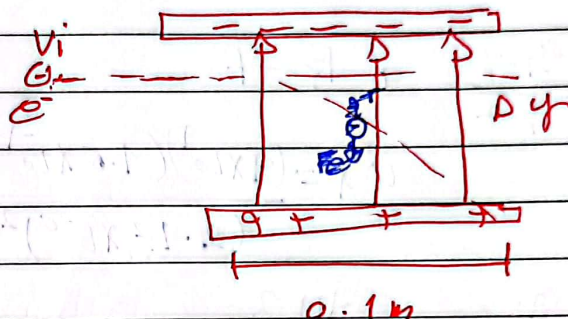
electron

ex: an electron enters a region of uniform electric field as shown in figure,  $v_i = 3 \times 10^6 \text{ m/s}$ ,  $E = 200 \text{ V/cm}$ , the horizontal length of the plates is  $0.1 \text{ m}$ , find (1) acceleration of electron (2) If the electron enters at time  $t=0$  find the time of which it leaves the electric field region. (3) what is the vertical position when it leaves the electric field?

Sol:

$$(1) q_e = -1.6 \times 10^{-19} \text{ C}$$

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$



$$\Rightarrow \Sigma F_y = ma_y$$

$$qE = ma_y$$

$$a_y = \frac{qE}{m} = \frac{(-1.6 \times 10^{-19})(200)}{9.11 \times 10^{-31}} = -3.51 \times 10^{10} \text{ m/s}^2$$

$$(2) \Delta x = v \Delta t \Rightarrow \Delta t = \frac{L}{v}$$

$$\Delta t = \frac{0.1}{3 \times 10^6} = 0.3 \times 10^{-7} \text{ s}$$

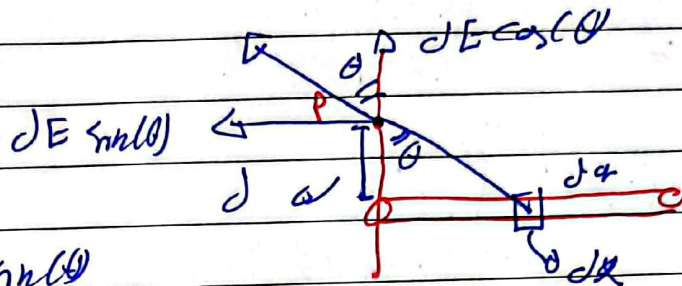
$$(3) \Delta y = v_{iy} t + \frac{1}{2} a_y t^2 = \frac{1}{2} (-3.51 \times 10^{10}) (0.3 \times 10^{-7})^2$$

$$= -1.95 \text{ cm}$$



Ex: a uniformly charged rod of length  $L$  and total charge  $Q$  lies along  $x$ -axis as shown in the figure. Find the  $x$  component of the electric field at point "P" on the  $y$ -axis at distance  $d$  from the origin;

⇒



$$\Rightarrow dE_x = -dE \sin(\theta)$$

$$= -\frac{1}{r^2} dq \cdot \sin(\theta)$$

$$\sin(\theta) = \frac{x}{r} \quad \Rightarrow dE_x = -\frac{1}{r^2} dq \cdot \frac{x}{r} \quad \text{where } r = \sqrt{x^2 + d^2}$$

$$dE(x) = -\frac{1}{(x^2 + d^2)^{3/2}} \cdot x \cdot dq$$

$$\Rightarrow dq = \lambda dx \quad \Rightarrow \int_0^L dE(x) = \int_0^L -\frac{1}{(x^2 + d^2)^{3/2}} \cdot x \cdot \lambda dx$$

$$= E(x) = -\lambda \int_0^L \frac{x dx}{(x^2 + d^2)^{3/2}}$$

$$u = x^2 + d^2$$

$$du = 2x dx$$

$$x=0 \rightarrow u = d^2$$

$$x=L \rightarrow u = d^2 + L^2$$

$$= E_x = -\lambda \int_{d^2}^{d^2 + L^2} \frac{\frac{du}{2}}{u^{3/2}}$$

$$= -\frac{\lambda}{2} \left[ \frac{u^{-1/2}}{-1/2} \right]_{d^2}^{d^2 + L^2} = \lambda \left[ \frac{1}{\sqrt{u}} \right]_{d^2}^{d^2 + L^2}$$

$$= \lambda \left[ \frac{1}{\sqrt{d^2 + L^2}} - \frac{1}{\sqrt{d^2}} \right]$$



$$\Rightarrow dE_y = dE \cos(\theta)$$

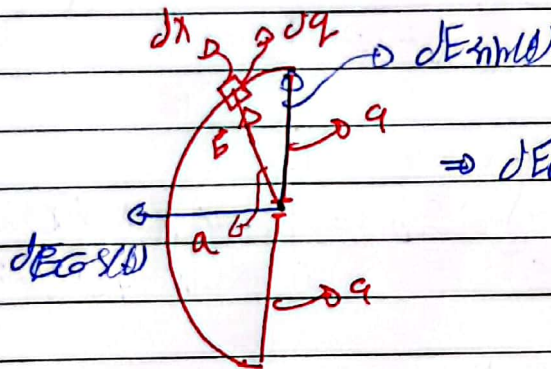
$$= \frac{4 dq}{r^2} \cos(\theta) \rightarrow \frac{d}{r}$$

$$= \frac{4 \cdot dq \cdot d}{r^3} = \frac{4 \cdot \lambda dx \cdot d}{(x^2 + d^2)^{3/2}}$$

$$E_y = 4 \cdot \lambda d \int_0^L \frac{dx}{(x^2 + d^2)^{3/2}}$$

ex: a semi circle is charged with a total charge of  $-8 \mu\text{C}$ , find magnitude and direction of  $E$  at center of semi circle?

$$\Rightarrow L = 14 \text{ cm}$$



$$\Rightarrow dE_x = -dE \cos(\theta)$$

$$= -\frac{4 dq}{r^2} \cos(\theta)$$

$$\Rightarrow dq = \lambda dx$$

$$x = a \theta$$

$$dx = a d\theta$$

$$\Rightarrow dq = \lambda a d\theta$$

$$E_x = \frac{-2k\lambda}{a} \int_0^{\pi/2} \cos(\theta) d\theta$$

$$= \frac{-2k\lambda}{a} [\sin(\theta) \Big|_0^{\pi/2}]$$

$$E_x = \frac{-2k\lambda}{a}$$

$$\int E_x = \int_0^{\pi/2} -\frac{4 \cdot \lambda a \cos(\theta)}{a^2} d\theta$$

$$E_x = \int_0^{\pi/2} -\frac{2k\lambda}{a} \cos(\theta) d\theta$$

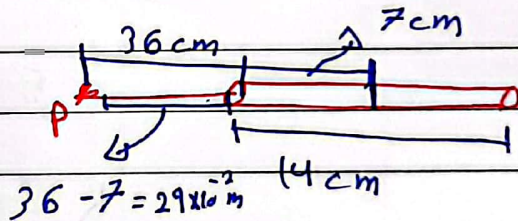


$$E_{CN} = \frac{-247}{9} \quad \lambda = \frac{Q}{L} \Rightarrow L = \frac{Q}{\lambda} \Rightarrow \lambda = \frac{L}{Q} = \frac{14 \times 10^{-2}}{3.14}$$

$$E(x) = \frac{(-2)(9 \times 10^9) \cdot \frac{14}{3.14} \times 10^{-4}}{14 \times 10^{-2}} = -2.31 \times 10^7 \text{ N/C}$$

$$\Rightarrow \int E_y = -\frac{4 \lambda q}{q^2} \ln(10)$$

ex:- A rod 14 cm long is uniformly charged and has a total charge of -22 μC. Determine (a) the magnitude and (b) the direction of the electric field along the axis of the rod at point 36 cm from its center  
sol:-



$$E_P = \frac{4Q}{D(D+L)}$$

$$= \frac{(9 \times 10^9)(22 \times 10^{-6})}{(29 \times 10^{-2})(29 + 14) \times 10^{-2}} = \frac{198 \times 10^3}{1247 \times 10^{-4}} = 0.158 \times 10^7 = 1.6 \times 10^6 \text{ N/C (to the left)}$$

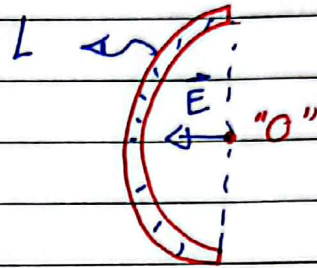
# كل شيء في الحياة يدور حول شيء واحد على التمثيل



ex: A uniformly charged insulating rod of length "14cm" is bent into the shape of a semicircle as shown in figure. The rod has a total charge of "-7.5 μC". find (a) the magnitude and (b) the direction of the electric field at "O", the center of the semicircle.

$$\Rightarrow Q = -7.5 \times 10^{-6} \text{ C}$$

$$\Rightarrow L = 14 \times 10^{-2}$$



$$E_0 = \frac{2k \cdot \lambda}{R}$$

$$\lambda = \frac{Q}{L}$$

$$E_0 = \frac{(2)(9 \times 10^9)(0.53 \times 10^{-4})}{4.7 \times 10^{-2}}$$

$$\lambda = \frac{7.5 \times 10^{-6}}{14 \times 10^{-2}} = 0.53 \times 10^{-4}$$

$$= 2.1 \times 10^7 \text{ N/C } \leftarrow$$

$$\Rightarrow Q = 2\pi R \lambda$$

$$7.5 \times 10^{-6} = 0.53 \times 10^{-4} (\pi) R$$

$$R = 4.7 \times 10^{-2} \leftarrow$$

# مقي الوراء بعد موتها لا يجيب أبت. #



ex: A uniformly charged disk of radius 35 cm carries charge with a density of  $7.9 \times 10^{-3} \text{ C/m}^2$ . Calculate the electric field on the axis of the disk at 5 cm

$$\Rightarrow E_P = 2\pi k \left[ 1 - \frac{x}{\sqrt{x^2 + R^2}} \right]$$

$$= 2\pi (9 \times 10^9) (7.9 \times 10^{-3}) \left[ 1 - \frac{5 \times 10^{-2}}{\sqrt{(5 \times 10^{-2})^2 + (35 \times 10^{-2})^2}} \right]$$

$$= 446.7 \times 10^6 \left[ 1 - \frac{5 \times 10^{-2}}{\sqrt{(1225 + 25) \times 10^{-4}}} \right]$$

$$= 446.7 \times 10^6 \left[ 1 - \frac{5 \times 10^{-2}}{\sqrt{1250}} \right]$$

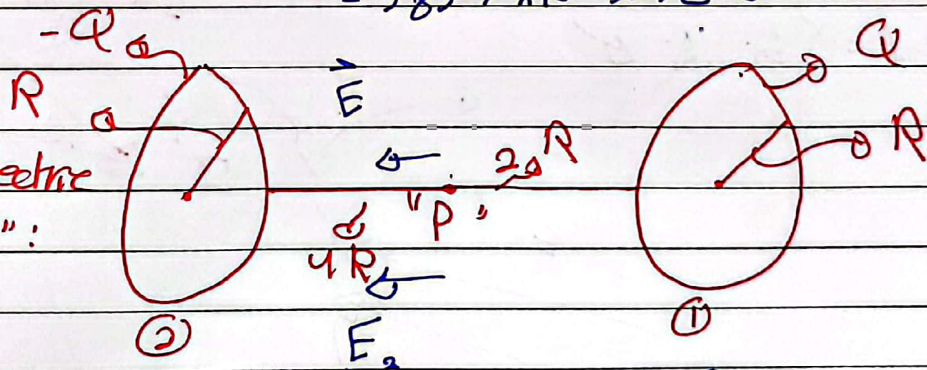
$$= 446.7 \times 10^6 \left[ 1 - \frac{5}{35.3} \right] = 446.7 \times 10^6 [1 - 0.141]$$

$$= 446.7 \times 10^6 [0.859]$$

$$= 383.7 \times 10^6 \text{ N/C}$$

ex:

calculate the electric field at point "P":



$$E_1 = \frac{k \times Q}{(x^2 + R^2)^{3/2}} = \frac{4(R)(Q)}{(\sqrt{R^2 + 4R^2})^3} = \frac{24 R Q}{(5 R^2)^{3/2}}$$

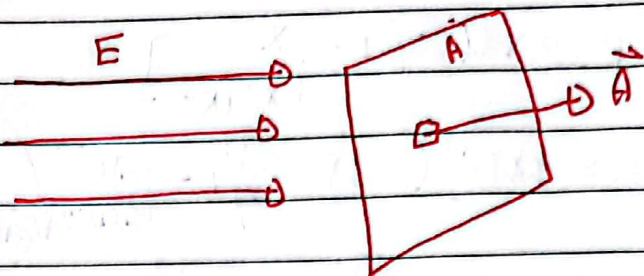
$$E_2 = \frac{4(4R)(Q)}{(R^2 + (6R)^2)^{3/2}} = \frac{44 R Q}{(17 R^2)^{3/2}} \Rightarrow E_P = \frac{24 R Q}{(5 R^2)^{3/2}} + \frac{44 R Q}{(17 R^2)^{3/2}}$$



⇒ Ch 24: Gauss's law

⇒ Electric Flux:

$$\Rightarrow \Phi = \vec{E} \cdot \vec{A}$$
$$= EA \cos(\theta) \text{ Nm}^2/\text{C}$$

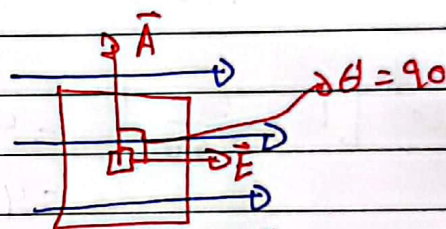


$\theta \Rightarrow \vec{E} \text{ and } \vec{A}$

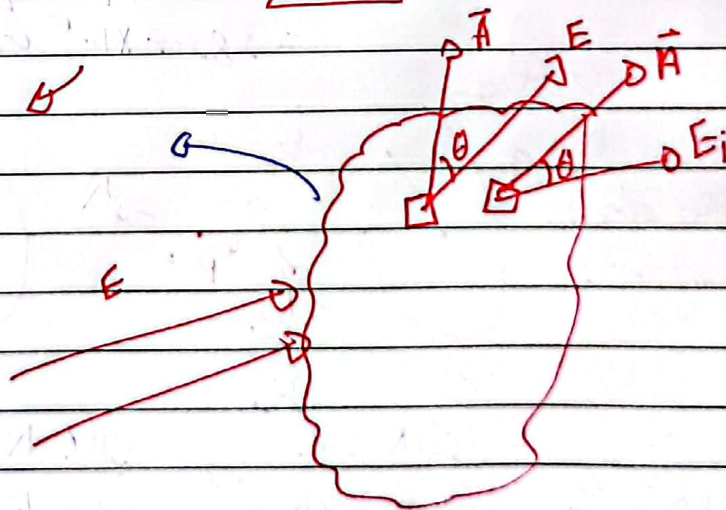
$$\Phi_{\vec{E} \parallel \vec{A}} \Rightarrow \theta = 0 \Rightarrow \cos(0) = 1$$

$$\Rightarrow \Phi_{\vec{E} \perp \vec{A}} = |\vec{E}| |\vec{A}| \cos(180) = -\vec{E} \cdot \vec{A}$$

$$\Rightarrow \Phi_{\vec{E}} = 0 \text{ at } \theta = \frac{\pi}{2}$$



$$\Rightarrow \Phi_{\text{total}} = \sum E_i dA_i$$



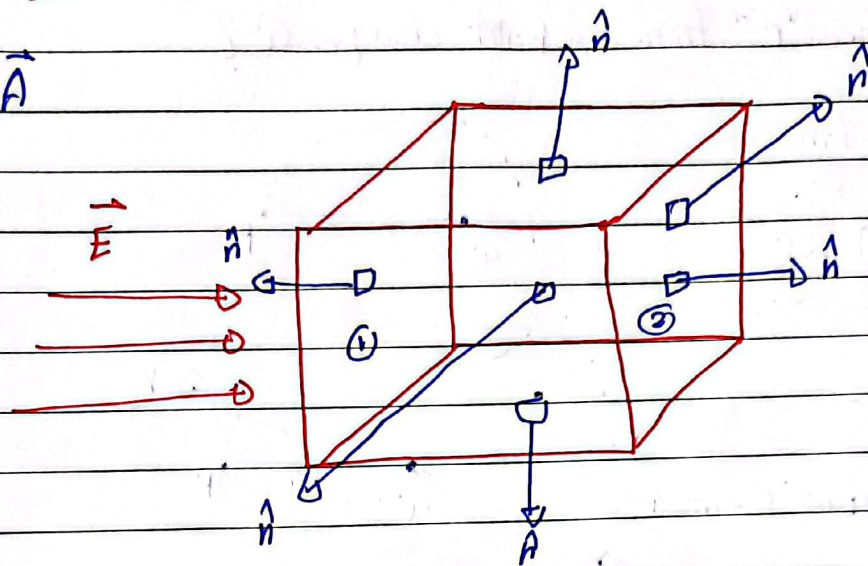


ex: find  $\Phi_{\text{total}}$ ?

$$\Phi_1 = -E \cdot \vec{A}$$

$$\Phi_2 = E \cdot \vec{A}$$

$$\Phi_{\text{total}} = 0$$

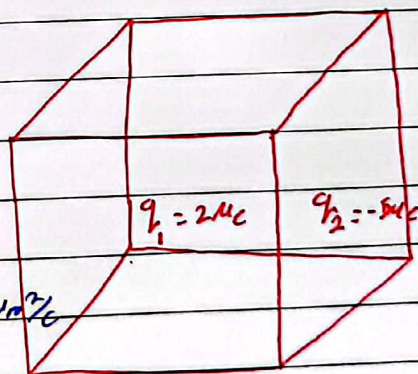


⇒ Gauss's law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$\Phi_{\text{net}} = ??$$

$$\Phi_1 = \frac{q_1}{\epsilon_0} = \frac{2 \times 10^{-6}}{8.8 \times 10^{-12}} = \frac{2}{8.8} \times 10^6 \text{ Nm}^2/\text{C}$$



$$\Phi_2 = \frac{q_2}{\epsilon_0} = \frac{-5 \times 10^{-6}}{8.8 \times 10^{-12}} = \frac{-5}{8.8} \times 10^6 \text{ Nm}^2/\text{C}$$

$$\Phi_{\text{net}} = \Phi_1 + \Phi_2 = \frac{2}{8.8} \times 10^6 - \frac{5}{8.8} \times 10^6 = \frac{-3}{8.8} \times 10^6 \text{ Nm}^2/\text{C}$$

⇒ net flux through only one surface:

$$= \frac{\Phi_{\text{net}}}{6} = \frac{-3}{(8.8)6} \times 10^6 \text{ Nm}^2/\text{C}$$



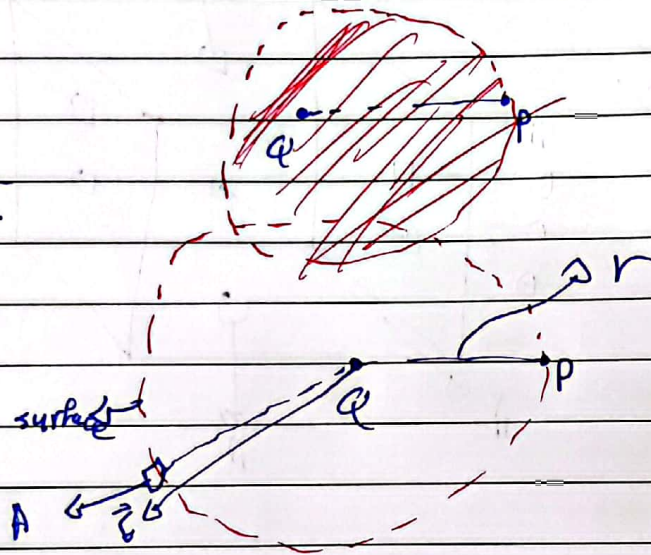
ex: using Gauss's law, find the electric field of point "p" located at a distance "r" from a point charge Q?

$$\Rightarrow \int E dA = \frac{q_{in}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$\Rightarrow E = \frac{Q}{4\pi\epsilon_0 r^2}$$

Gauss's surface



$$\Rightarrow E = \frac{k_e Q}{r^2}$$

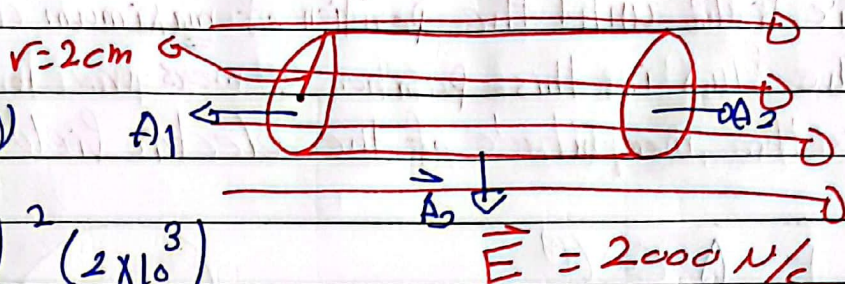


ex: Find the net electrical flux through the below cylinder:

$$\Phi_1 = \vec{A}_1 \cdot \vec{E} \cos(\theta)$$

$$= -(\pi)(2 \times 10^{-2})^2 (2 \times 10^3)$$

$$= -0.8 \pi \text{ Nm}^2/\text{C}$$



$$\Phi_2 = \vec{A}_2 \cdot \vec{E} \cos(\theta) = (\pi)(2 \times 10^{-2})^2 (2 \times 10^3) = 0.8 \pi \text{ Nm}^2/\text{C}$$

$$\Phi_3 = \vec{A}_3 \cdot \vec{E} \cos(\theta) \rightarrow \theta = \frac{\pi}{2} \Rightarrow 0$$

$$\Sigma \Phi = 0$$

ex: A flat surface of area "3.2 m<sup>2</sup>" is rotated in a uniform electric field of magnitude  $E = 6.2 \times 10^5 \text{ N/C}$ . Determine the electric flux through this area when the electric field is perpendicular to the surface:

perpendicular

$$\Phi = \vec{A} \cdot \vec{E} \cos(\theta)$$

$$= (3.2)(6.2) \times 10^5 \cos(0)$$

$$= 19.84 \times 10^5 \text{ Nm}^2/\text{C}$$

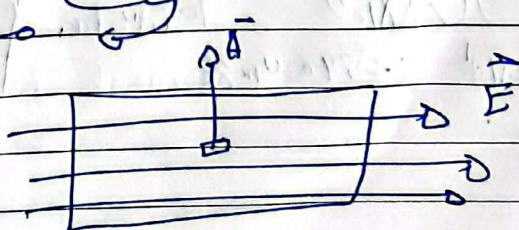


(B) when the electric field is parallel to the surface

parallel

$$\theta = \frac{\pi}{2}$$

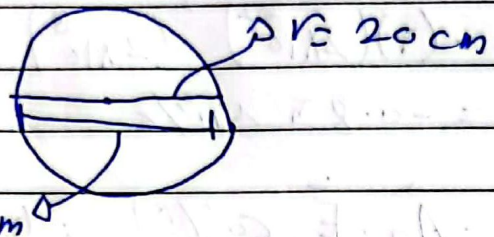
$$\Phi = 0$$





ex: A 40 cm diameter circular loop is rotated in a uniform electric field until the position of maximum electric flux is found. The flux in this position is measured to be  $5.2 \times 10^5 \text{ Nm}^2/\text{C}$ . What is the magnitude of the electric field?

$$\Rightarrow \Phi = \vec{E} \cdot \vec{A} \cos(\theta)$$

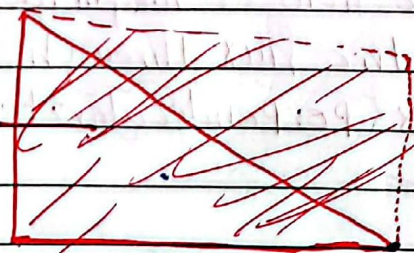
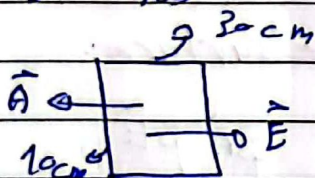


$$(5.2 \times 10^5) = E (\pi) (20 \times 10^{-2})^2$$

$$E = \frac{5.2 \times 10^5}{(\pi) (2 \times 10^{-1})^2} = 0.41 \times 10^7 \text{ N/C}$$

ex: Consider a closed triangular box resting within a horizontal electric field of magnitude  $E = 7.8 \times 10^4 \text{ N/C}$  as shown in figure p24.4. Calculate the electric flux through (a) the vertical rectangular surface:

$$\Rightarrow \Phi = \vec{A} \cdot \vec{E} \cos(\theta)$$



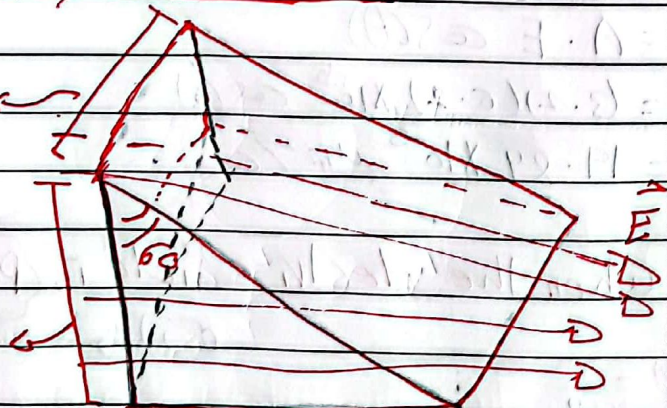
$$\Phi = -(30 \times 10^{-2} \times 10 \times 10^{-2}) (7.8 \times 10^4) \cos(90^\circ)$$

$$= -2340 \times 10^1$$

$$= -(300 \times 10^{-4}) (7.8 \times 10^3)$$

$$= -(3 \times 10^{-2}) (7.8 \times 10^3)$$

$$= -2340 \times 10^1 = -2340 \text{ Nm}^2/\text{C}$$



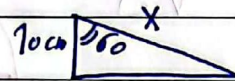
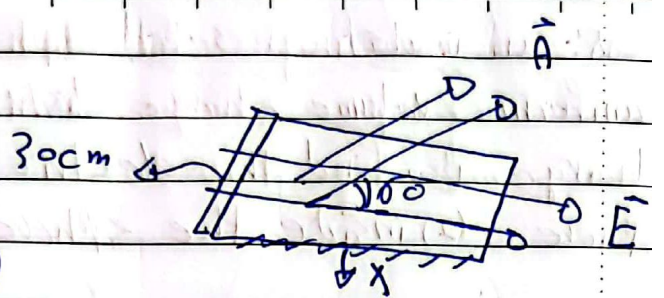


(B) the slanted surface

$$\Phi = \vec{E} \cdot \vec{A} \cdot \cos(\theta)$$

$$\Phi = (7.8 \times 10^4) \times (4.2 \times 0.31) \cos(60^\circ)$$

$$= 0.234 \times 10^4 = 2340$$



$$\Rightarrow \cos(\theta) = \frac{10 \times 10^{-2}}{x}$$

$$\frac{1}{2} = \frac{1 \times 10^{-1}}{x}$$

$$x = 2 \times 10^{-1} \text{ m}$$

(C) the entire surface of the box:

$$\Phi_{\text{total}} = 0$$

ex: Find the net electric flux through the spherical closed surface shown in figure. The two charges on the right inside the spherical surface

$$\Phi = \frac{\sum q_i}{\epsilon_0}$$

$$= \frac{1 \times 10^{-9} - 3 \times 10^{-9}}{8.85 \times 10^{-12}} = -226 \text{ Nm}^2/\text{C}$$

faces

$$q_2 = 2 \text{ nC}$$

$$q_1 = 1 \text{ nC}$$

$$q_2 = -3 \text{ nC}$$

ex: The following charges are located inside a submarine:

$q_1 = 5 \mu\text{C}$ ,  $q_2 = -9 \mu\text{C}$ ,  $q_3 = 27 \mu\text{C}$ ,  $q_4 = -84 \mu\text{C}$ . Calculate the net electric flux through the hull of the submarine.

$$\Rightarrow \Phi = \frac{\sum q_i}{\epsilon_0} = \frac{5 - 9 + 27 - 84}{8.85 \times 10^{-12}} = -6.89 \times 10^6 \text{ Nm}^2/\text{C}$$

$$= -6.89 \text{ MNm}^2/\text{C}$$

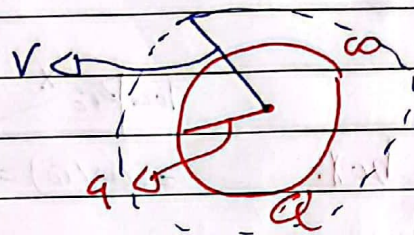
↳ Since the net electric flux is negative, more lines enter than leave the surface.

وهي الليت الفلدا يفتقد البدر



ex: an insulating solid sphere of radius  $a$  and has a uniform volume charge distribution  $\rho$ , and carries a total charge  $Q$ , find the electric field at point (1) Outside the sphere (2) inside the sphere:

(1)



$$\Rightarrow Q = \frac{Q}{V}$$

work done

$$\oint E dA = \frac{q_{in}}{\epsilon_0}$$

$$E (4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi r^2 \epsilon_0}$$

$$Q = \rho V$$

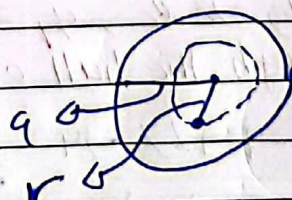
$$Q = \rho \left( \frac{4}{3} \pi a^3 \right)$$

$$E = \frac{\rho \left( \frac{4}{3} \pi a^3 \right)}{4\pi r^2 \epsilon_0} = \frac{\rho a^3}{3 \epsilon_0 r^2} \Rightarrow r > a$$

(2) inside the sphere

$$\oint E dA = \frac{q_{in}}{\epsilon_0}$$

$$E (4\pi r^2) = \frac{q_{in}}{\epsilon_0}$$



volume of gauss sphere

$$\Rightarrow q' = \rho V'$$

$$E (4\pi r^2) = \frac{q'}{\epsilon_0}$$

included charges  
inside gauss sphere



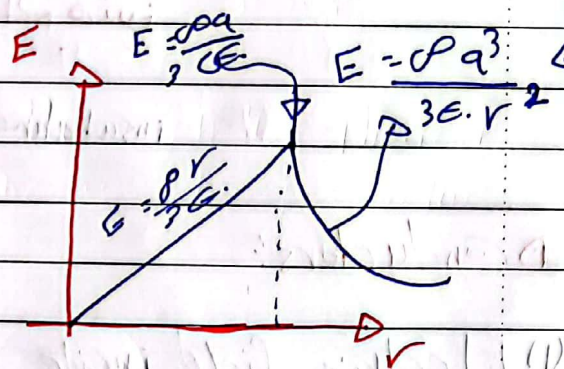
$$\Rightarrow q' = \infty \left( \frac{4}{3} \pi r^3 \right)$$

$$\Rightarrow E \left( 4 \pi r^2 \right) = \frac{\infty \left( \frac{4}{3} \pi r^3 \right)}{\epsilon_0}$$

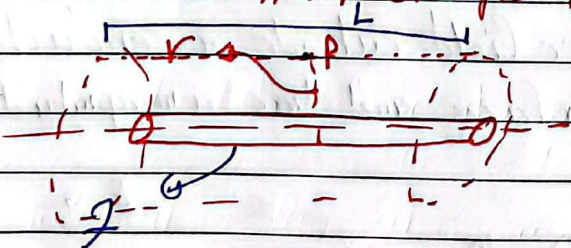
$$E = \frac{\infty r}{3 \epsilon_0} \Rightarrow r < a$$

$$\infty = \frac{Q}{\frac{4}{3} \pi a^3}$$

$$E = \frac{Q r}{4 \pi \epsilon_0 a^3}$$



ex: find the electric field at point "P" located at distance "r" of an infinite rod with a constant charge per unit length  $\lambda$ ?



$$\Rightarrow \oint E dA = \frac{q_{enc}}{\epsilon_0}$$

$$E \int dA = \frac{q_{enc}}{\epsilon_0}$$

$$E (2 \pi r L) = \frac{q_{enc}}{\epsilon_0} = \frac{\lambda L}{\epsilon_0}$$

$$\lambda = \frac{Q}{L}$$

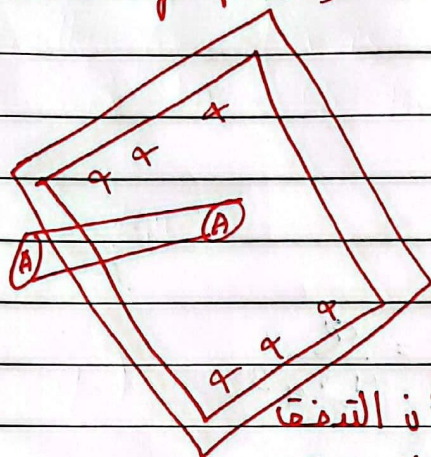
$$Q = \lambda L$$

$$E = \frac{\lambda}{2 \pi \epsilon_0 r} = \frac{\lambda}{2 \pi \epsilon_0 r} = \frac{\lambda}{2 \pi \epsilon_0 r}$$

# غداً تطير الصافير !



ex: calculate the Electric field of infinite plane of positive charge with uniform surface charge density  $\sigma$ ? (Non-conducting plane) :



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{en}}{\epsilon_0}$$

$$E \oint dA = \frac{\sigma A}{\epsilon_0}$$

$$2EA = \frac{\sigma A}{\epsilon_0}$$

من الشحنة  
من الجهتين

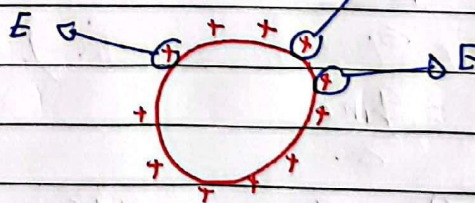
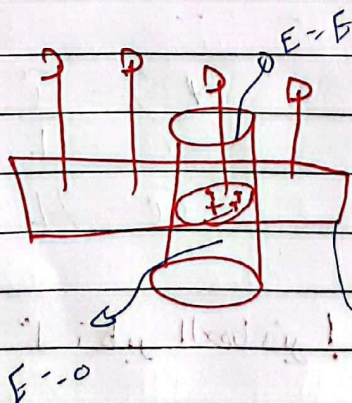
insulating

$$E = \frac{\sigma}{2\epsilon_0}$$

⇒ Conductors:

(1) Electric field inside a conductor = 0 for either solid conductor or hollow conductor.

(2) Electric field outside a conductor is perpendicular to the surface and has a magnitude of  $\frac{\sigma}{\epsilon_0}$ .



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{en}}{\epsilon_0}$$

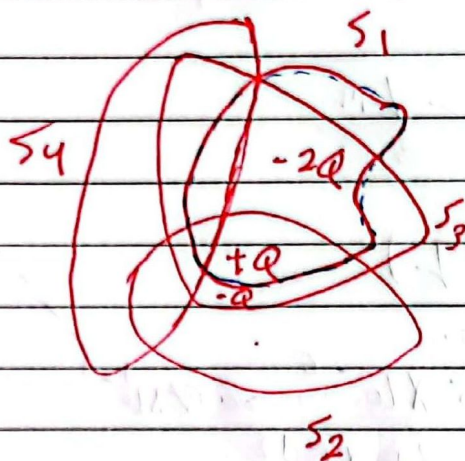
$$EA = \frac{\sigma A}{\epsilon_0} \Rightarrow E = \frac{\sigma}{\epsilon_0}$$



ex: Four closed surfaces,  $s_1$  through  $s_4$ , together with the charge  $-2Q$ ,  $+Q$ , and  $-Q$  are sketched in figure. (The colored lines are the intersections of the surfaces with the page). Find the electric flux through each surface.

$\Rightarrow$  Flux through  $s_1$ :

$$\begin{aligned}\Rightarrow \Phi_1 &= \frac{\sum q_{in s_1}}{\epsilon_0} \\ &= \frac{-2Q + Q}{\epsilon_0} = \frac{-Q}{\epsilon_0}\end{aligned}$$



$$\Rightarrow \Phi_2 = \frac{\sum q_{in s_2}}{\epsilon_0} = \frac{+Q - Q}{\epsilon_0} = 0$$

$$\Rightarrow \Phi_3 = \frac{\sum q_{in s_3}}{\epsilon_0} = \frac{-2Q + Q - Q}{\epsilon_0} = \frac{-2Q}{\epsilon_0}$$

$$\Rightarrow \Phi_4 = 0$$

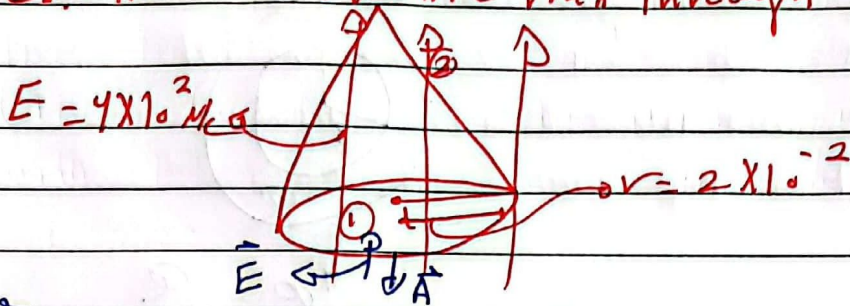
ex: A particle with charge of  $12 \mu C$  is placed at the center of spherical shell of radius  $22 \text{ cm}$ . What is the total electric flux through (a) the surface of the shell and (b) any hemispherical surface of the shell?

$$\begin{aligned}\Rightarrow \text{(A) } \Phi &= \frac{\sum q_{in}}{\epsilon_0} = \frac{12 \times 10^{-6}}{8.85 \times 10^{-12}} = 1.36 \times 10^6 \text{ Nm}^2/\text{C} \\ &= 1.36 \text{ MNm}^2/\text{C}\end{aligned}$$

$$\text{(B) } \Phi = \frac{\sum q_{in}}{2(\epsilon_0)} = \frac{1}{2} (1.36 \times 10^6) = 0.78 \times 10^6 \text{ Nm}^2/\text{C}$$



ex.: find the electric flux through each surface:



$$\Phi_1 = E \cdot A \cdot \cos(\theta) = (4 \times 10^2)(\pi)(2 \times 10^{-2})^2 \cos(180^\circ)$$

$$= -0.16 \pi \text{ Nm}^2/\text{C}$$

$$\Phi_{\text{net}} = 0 \quad \Phi_2 = 0.16 \pi \text{ Nm}^2/\text{C}$$

من بنى لنا في الروح مقاماً عشنا له في القلب منة

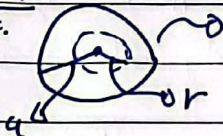
ex.: A solid insulating sphere of radius "a" carries a net positive charge Q uniformly distributed throughout its volume. A conducting spherical shell of inner radius "b" and outer radius "c" is concentric with the solid sphere and carries a net charge -2Q. Using Gauss's law, find the electric field in the regions labeled (1), (2), (3) and (4) in figure 24.19 and the charge distribution on the shell when the entire system is in electrostatic equilibrium:

⇒ region (1):  $r < a$

$$\oint E \cdot dA = \frac{q_{\text{en}}}{\epsilon_0}$$

$$\omega = \frac{Q}{\frac{4}{3}\pi a^3}$$

$$E(4\pi r^2) = \frac{q}{\epsilon_0}$$

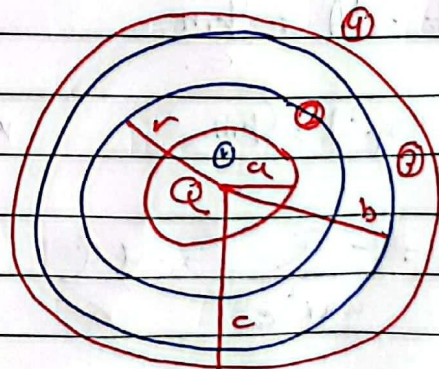


$$q = \omega \left( \frac{4}{3}\pi r^3 \right)$$

$$E(4\pi r^2) = \frac{\omega \left( \frac{4}{3}\pi r^3 \right)}{\epsilon_0}$$

??

$$E = \frac{\omega r}{3\epsilon_0} \Rightarrow E = \frac{r}{3\epsilon_0} \left( \frac{Q}{\frac{4}{3}\pi a^3} \right) \Rightarrow E = \frac{Qr}{4\pi \epsilon_0 a^3}$$



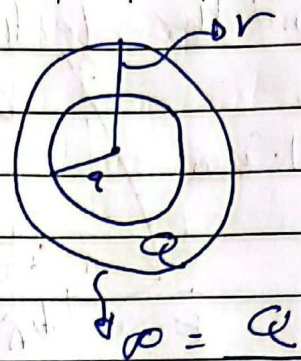


region (2)

$$\oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$$

$$E \cdot (4\pi r^2) = \frac{q_{in}}{\epsilon_0}$$

$$E = \frac{q}{4\pi\epsilon_0 r^2} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$



$$\frac{4}{3}\pi r^3$$

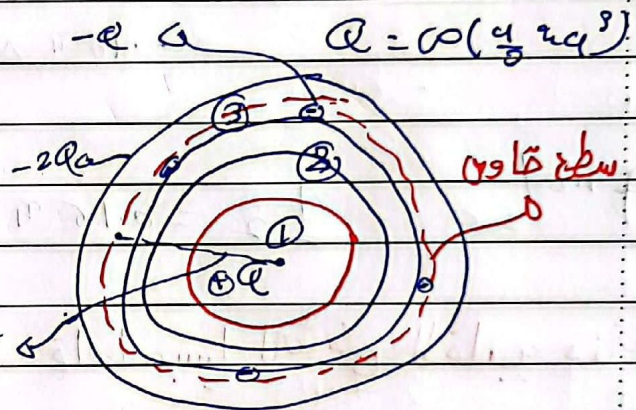
region (3)

$$\oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$$

$$E \cdot (4\pi r^2) = \frac{q_{in}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{Q + (-Q)}{\epsilon_0} = 0$$

$$E = 0$$



region (4):

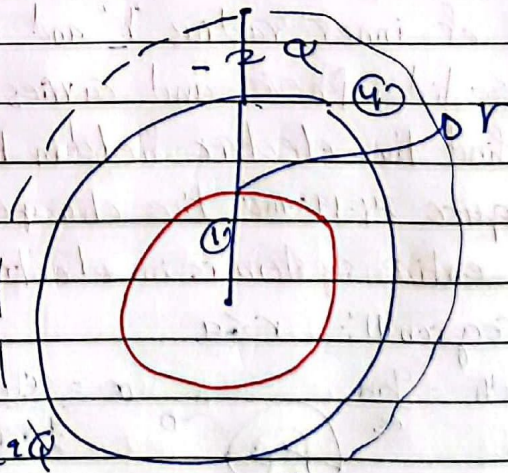
$$\oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{q_{in}}{\epsilon_0}$$

$$E = \frac{-Q}{4\pi r^2 \epsilon_0} = -\frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

$$q_{in} = -2Q$$

$$q_{in} = -Q$$



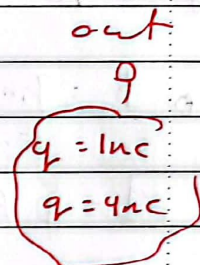
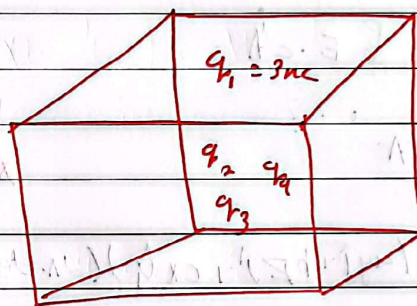


ex: charges of  $3\text{ nc}$ ,  $-2\text{ nc}$ ,  $-7\text{ nc}$ , and  $1\text{ nc}$  are contained inside a rectangular box with length  $1\text{ m}$ , width  $2\text{ m}$ , and height  $2.5\text{ m}$ . Outside the box are charges of  $1\text{ nc}$  and  $4\text{ nc}$ . what is the electric flux through the surface of the box?

$$\Rightarrow \Phi_{\text{out}} = 0$$

$$\Phi_{\text{in}} = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4$$

$$= \frac{q_1}{\epsilon_0} + \frac{q_2}{\epsilon_0} + \frac{q_3}{\epsilon_0} + \frac{q_4}{\epsilon_0}$$



$$= \frac{1}{\epsilon_0} (q_1 + q_2 + q_3 + q_4) = \frac{1}{8.85 \times 10^{-12}} (3 - 2 - 7 + 1) \times 10^{-9}$$

$$= \frac{-5 \times 10^{-9}}{8.85 \times 10^{-12}} \text{ } \frac{\text{Nm}^2}{\text{C}}$$

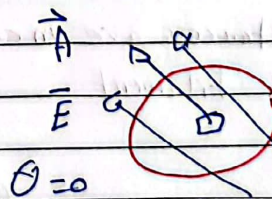
ex: A  $40\text{ cm}$  diameter circular loop is rotated in a uniform electric field until the position of maximum electric flux is found. The flux in this position is measured to be  $5.2 \times 10^5$ . what is the magnitude of the electric field?

$$\Rightarrow \Phi = 5.2 \times 10^5 \text{ } \frac{\text{Nm}^2}{\text{C}}$$

$$\Phi = E A$$

$$r = 20 \times 10^{-2} \text{ m}$$

$$R = 0.2 \text{ m}$$



$$5.2 \times 10^5 = E (\pi r^2)$$

$$E = \frac{5.2 \times 10^5}{(\pi)(0.2)^2} = \frac{5.2 \times 10^5}{0.125} = 41.5 \times 10^5 \text{ N/C}$$

# تغييرت قلوبهم أم عقولنا اضررت

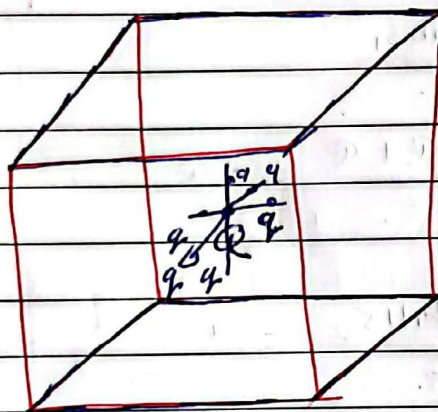


ex: A particle with charge  $Q = 5 \mu\text{C}$  is located at the center of cube of edge  $L = 0.1 \text{ m}$ . In addition, six other identical charged particles having  $q = -1 \mu\text{C}$  are positioned symmetrically around  $Q$  as shown in figure. Determine the electric flux through one face of the cube;

$$\Rightarrow \Phi_1 = \frac{\sum q_i}{\epsilon_0}$$

$$= \frac{6(q)}{\epsilon_0}$$

$$= \frac{-6 \times 10^{-6}}{8.85 \times 10^{-12}} = -0.67 \times 10^6$$



$$\Phi_2 = \frac{\sum q_i}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

$$= \frac{5 \times 10^{-6}}{8.85 \times 10^{-12}} = 0.56 \times 10^6 \text{ } \mu\text{m}^2/\text{C}$$

$$\Phi_{\text{net}} = (0.56 - 0.67) \times 10^6 = -0.11 \times 10^6 \text{ } \mu\text{m}^2/\text{C}$$

$$\Phi_{\text{one face}} = -0.018 \times 10^6 = -18 \times 10^3 \text{ } \mu\text{m}^2/\text{C} = -18 \text{ } \mu\text{N} \cdot \text{m}^2/\text{C}$$



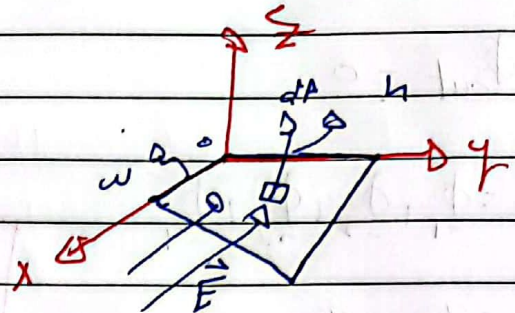
ex: A nonuniform electric field is given by the expression

$$\Rightarrow \vec{E} = ay\hat{i} + bz\hat{j} + cx\hat{k}, \text{ where } a, b, \text{ and } c \text{ are constants.}$$

Determine the electric flux through a rectangular surface in the  $xy$  plane, extending from  $x=0$  to  $x=w$  and from  $y=0$  to  $y=h$ .

$$\Rightarrow \Phi = \frac{q_{en}}{\epsilon_0} = \oint \vec{E} \cdot d\vec{A}$$

$$Q = \oint \vec{E} \cdot (d\vec{A}) \rightarrow ??$$



$$\Rightarrow \vec{E} \cdot d\vec{A} = (ay\hat{i} + bz\hat{j} + cx\hat{k}) \cdot (dx dy) \hat{k} \quad d\vec{A} = (dx dy) \hat{k}$$

$$\Rightarrow \vec{E} \cdot d\vec{A} = cx dx dy$$

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \int_0^w \int_0^h cx dx dy = \int_0^w cx dx \int_0^h dy$$

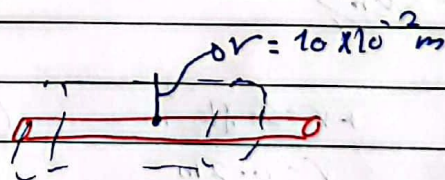
$$= cx \int_0^h dy \left[ \frac{x^2}{2} \right]_0^w = cx \frac{h}{2} \frac{w^2}{2}$$

$$\Phi = \frac{chw^2}{2} \epsilon_0$$

[24]

ex: The charge per unit length on a long straight filament is  $-10 \mu\text{C/m}$ . Find the electric field at  $10 \text{ cm}$  from the filament.

... where distances are measured perpendicular to the length of the filament.



$$\Rightarrow E(A) = \frac{\Sigma q_{en}}{\epsilon_0}$$

$$E(2\pi r h) = \frac{\Sigma q}{\epsilon_0}$$

$$E = \frac{\Sigma q}{2\pi r h \epsilon_0}$$

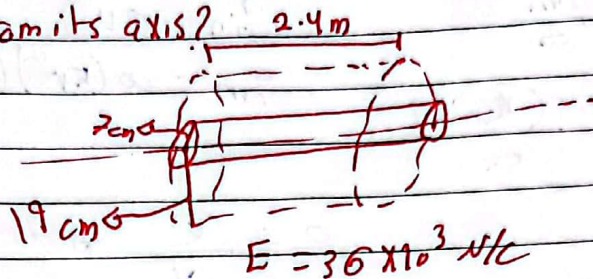
$$E = \frac{2(9 \times 10^9) (-10 \times 10^{-6})}{0.7} = -1.62 \times 10^5 \text{ N/C}$$

$$r = 10 \text{ cm}$$

$$0.7$$



ex: a cylindrical shell of radius 7cm and length of 2.4m has a uniformly distributed charge on its surface, the magnitude of Electric field of point 19cm radially out ward from its axis is  $36 \text{ kV/m}$ . ① Find the net charge  $Q$  on the shell. ② Find the field at point 4cm from its axis?



$$\Rightarrow \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon_0} \Rightarrow E(2\pi r L) = \frac{Q}{\epsilon_0}$$

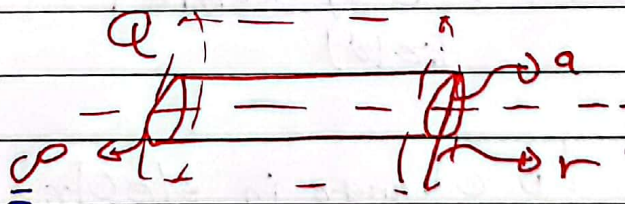
$$Q = E(2\pi r L) \epsilon_0$$

$$Q = (36 \times 10^3)(2)(2.4)(19 \times 10^{-2})(8.8 \times 10^{-12})$$

$$= 9.3 \times 10^{-7} \text{ C}$$

$$\Rightarrow \textcircled{2} E = 0$$

$\Rightarrow$  what if the cylinder is insulating



$$r > a$$

$$\Rightarrow E \cdot A = \frac{q_{\text{in}}}{\epsilon_0}$$

$$\Rightarrow \phi = \frac{Q}{\epsilon_0} = \frac{Q}{(\pi a^2)L}$$

$$E(2\pi r L) = \frac{Q}{\epsilon_0} \Rightarrow E(2\pi r L) = \frac{Q}{\epsilon_0}$$

$$Q = \phi(\pi a^2 L) \Rightarrow E = \frac{\phi(\pi a^2 L)}{2\pi r L \epsilon_0} = \frac{\phi a^2}{2 r \epsilon_0}$$



→ r < a

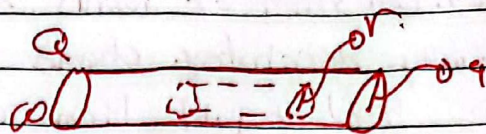
$$\Rightarrow E \cdot A = \frac{q_{in}}{\epsilon_0}$$

$$E(2\pi rL) = \frac{q_{in}}{\epsilon_0}$$

$$E(2\pi rL) = \frac{\omega(\pi r^2 L)}{\epsilon_0}$$

$$E = \frac{\omega r}{2\epsilon_0}$$

$$E = \left( \frac{Q}{2\pi a^2 L} \right) \left( \frac{r}{\epsilon_0} \right)$$



$$q_{in} = \omega V$$

$$q_{in} = \omega(\pi r^2)(L)$$

$$\omega = \frac{Q}{V} \Rightarrow \omega = \frac{Q}{2\pi a^2 L}$$

## Ch25: Electric Potential

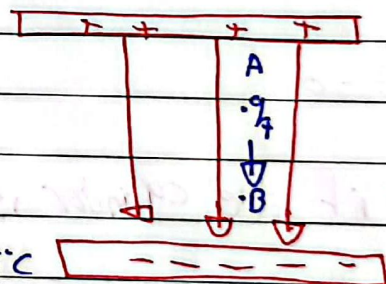
→ work:

$$W = \vec{F} \cdot \Delta \vec{x}$$

$$W = q \cdot \vec{E} \cdot \Delta \vec{x}$$

$$= q E \Delta x \cos(0)$$

$$\frac{W}{A \rightarrow B} = q E d \text{ (work done by electric field)}$$



→  $W = \Delta U$  change in electric potential energy

$$\Rightarrow \Delta U = U_B - U_A$$

$$\rightarrow W = q E d = -\Delta U$$

$$\Delta U = -q E d$$

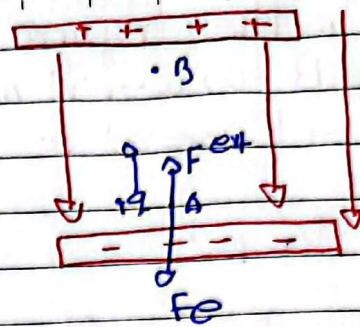
→ A positive charge move in the direction of the Electric field →  
 → work is done by the field (positive)  
 → electric potential energy decreases & ( $U_B < U_A$ )



$\Rightarrow W_{A \rightarrow B}$  (work done on the system)

$$\Rightarrow W_{A \rightarrow B} = +qEd = +DU$$

$$W_{A \rightarrow B} = DU \quad q$$



$\Rightarrow$  electric Potential

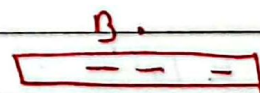
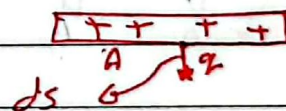
$$\Rightarrow V = \frac{U}{q} \Rightarrow DV = \frac{DU}{q}$$

$$V_B - V_A = \frac{U_B - U_A}{q}$$

$\Rightarrow$  electric potential of a point is referenced to  $\infty$   
( $V_{\infty} = U_{\infty} = 0$ )

$$DV = V_A - V_{\infty} = \frac{U_A - U_{\infty}}{q}$$

$$V_A = \frac{U_A}{q} \quad [J/C] = V$$



$$\Rightarrow [E] = N/C = V/m$$

$$\Rightarrow DV = V_B - V_A = - \int_A^B \vec{E} \cdot d\vec{s}$$

$$\Rightarrow D \cdot V = -E \cdot d$$

Relationship between DV and uniform electric field.

$$\Rightarrow DV = - \int_A^B \vec{E} \cdot d\vec{s} = - \int_A^B E \cdot ds \cos(0)$$

$$= -E \int_A^B ds = -E \int_A^B ds = -Ed$$



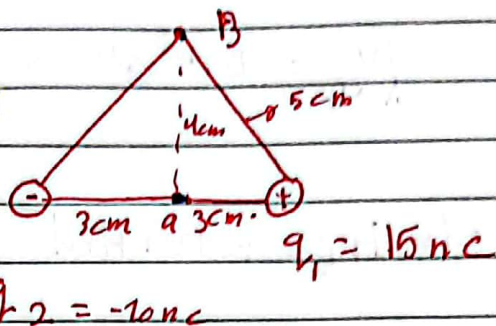
ex: In the figure. Find ① Potential difference between A and B. ② work needed to bring  $q = 2 \mu C$  from A to B. ③ work needed to bring  $q = 2 \mu C$  from A to  $\infty$ .

$$V_A = V_1 + V_2$$

$$= \frac{(9 \times 10^9)(15 \times 10^{-9})}{3 \times 10^{-2}} + \frac{(9 \times 10^9)(-10 \times 10^{-9})}{3 \times 10^{-2}}$$

$$= 9 \times 10^9 (15 - 10)$$

$$= \frac{45}{3} \times 10^2 = 15 \times 10^2 \text{ volt}$$



$$V_B = \frac{(9 \times 10^9)(15 \times 10^{-9})}{5 \times 10^{-2}} + \frac{(9 \times 10^9)(-10 \times 10^{-9})}{5 \times 10^{-2}}$$

$$V_B = 9 \times 10^2 \text{ volt}$$

$$\Rightarrow \frac{DU}{AD} = V_A - V_B = (15 - 9) \times 10^2 = 6 \times 10^2 \text{ volt}$$

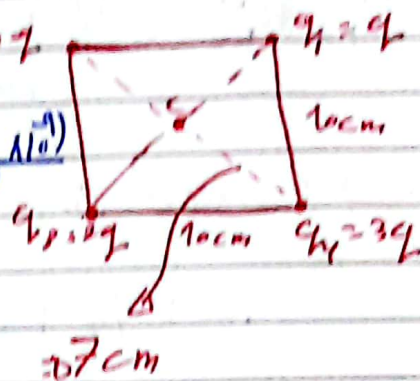
$$② \quad W_{A \rightarrow B} = \frac{DU}{A \rightarrow B} = q(V_B - V_A) = (2 \times 10^{-6})(-6 \times 10^2) = -12 \times 10^{-4} \text{ J}$$

$$③ \quad W_{A \rightarrow \infty} = \frac{DU}{A \rightarrow \infty} = q(V_{\infty} - V_A) = (2 \times 10^{-6})(-15 \times 10^2) = -30 \times 10^{-4} \text{ J}$$



ex: Find the electrical potential at 'c' if  $q = 7 \text{ nC}$

$$\begin{aligned} \Rightarrow V_c &= \frac{(9 \times 10^9)(2 \times 10^{-9})}{2 \times 10^{-2}} + \frac{(9 \times 10^9)(-2 \times 10^{-9})}{2 \times 10^{-2}} \\ &+ \frac{(9 \times 10^9)(14 \times 10^{-9})}{2 \times 10^{-2}} + \frac{(9 \times 10^9)(21 \times 10^{-9})}{(2 \times 10^{-2})} \\ &= 45 \times 10^2 \text{ volt} \end{aligned}$$



(b) If we want the work to bring an electron from  $\infty$  to 'c':

$$\Rightarrow W = \Delta U = q_e (V_c - V_{\infty}^0) = -72 \times 10^2 \text{ J}$$

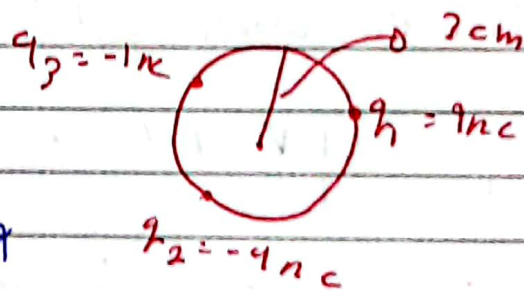
ex: what is the work needed to bring 'a' :  $q_a = 1.5 \times 10^{-19} \text{ C}$

$$\begin{aligned} W &= q_a [V_a - V_{\infty}^0] \\ &= 1.5 \times 10^{-19} V_a \end{aligned}$$

$$\Rightarrow V_a = \frac{(9 \times 10^9)(-1 + 9 - 4) \times 10^{-9}}{2 \times 10^{-2}}$$

$$V_a = 12 \times 10^2 \text{ volt}$$

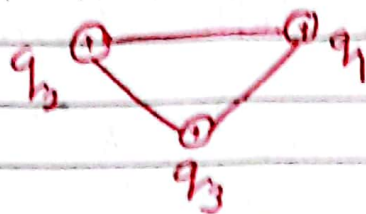
$$W = (1.5 \times 10^{-19})(12 \times 10^2) = 19.2 \times 10^{-17} \text{ J}$$





ex: Energy stored in  $q_1$ :

$$U_1 = U_{12} + U_{13} = \frac{k q_1 q_2}{r_{12}} + \frac{k q_1 q_3}{r_{13}}$$



\* energy stored in the system:

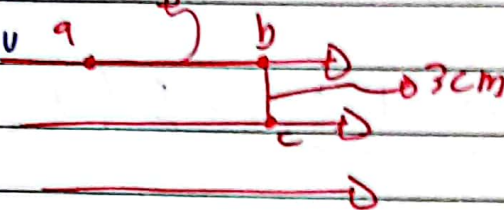
$$U_{\text{system}} = U_{12} + U_{13} + U_{23}$$

ex: Find potential difference between "a" and "b"

$$(1)(a) \quad \Delta V_{ab} = V_{b \rightarrow a} = -E d \cos \theta$$

$$E = 3 \times 10^3 \text{ N/C} \quad 10 \text{ cm}$$

$$\Delta V_{b \rightarrow a} = -(3 \times 10^3)(20 \times 10^{-2}) \cos(180) = 300 \text{ V}$$



$$\rightarrow \Delta V_{bc} = 0 \rightarrow V_b = V_c$$

$$\rightarrow \Delta V_{ac} = \Delta V_{ab} = 300 \text{ V}$$

(2) Find the work needed to move  $q = 2 \mu\text{C}$  from a to b:

$$\Rightarrow W = -q \Delta V = -\Delta U = -(2 \times 10^{-6})(300) = 600 \times 10^{-6} \text{ J}$$

(3) If  $q$  in Part (2) started from rest ( $m = 2 \text{ g}$ ), what is its final speed:

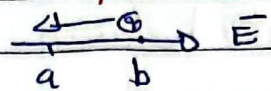
$$W = \Delta K$$

$$600 \times 10^{-6} = \frac{1}{2} m v_f^2$$

$$600 \times 10^{-6} = \frac{1}{2} (2 \times 10^{-3}) \cdot v_f^2 \rightarrow v_f = 0.77 \text{ m/s}$$

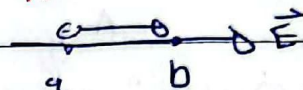


(4) Find the work needed to move  $q_1 = 5 \mu\text{C}$  from  $b$  to  $a$



$$W_{b \rightarrow a} = q_1 \Delta V = q_1 (V_a - V_b) = 5 \times 10^{-6} (300) = 1500 \times 10^{-6} \text{ J}$$

(5) Find the work needed to move  $q_1 = -1 \text{ nC}$  from  $a$  to  $b$



$$W_{a \rightarrow b} = q_1 \Delta V = q_1 (V_b - V_a) = (-1 \times 10^{-9}) (-200) = 200 \times 10^{-9} \text{ J}$$

(6) If  $V_a = 20 \text{ volt}$ , what is  $V_b$

$$\Rightarrow V_a - V_b = 300$$

$$20 - V_b = 300$$

$$V_b = -280 \text{ V}$$

(7) what is the work needed to move  $q_1$  from  $a \rightarrow b \rightarrow c \rightarrow a$  along the path  $a \rightarrow b \rightarrow c \rightarrow a$  :-

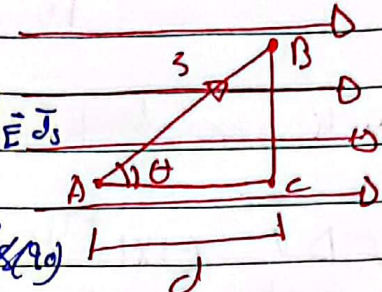
$$W = 0$$

# نرحل ويبقى الأثر



Ex:- Find the electric potential difference through Path (1) and Path (2) between A and B point?

⇒ Path (1)  $A \rightarrow C \rightarrow B$

$$\begin{aligned}
 DV &= V_B - V_A = - \int_A^C \vec{E} \cdot d\vec{s} - \int_C^B \vec{E} \cdot d\vec{s} \\
 &= - \int_A^C E ds \cos(0) - \int_C^B E ds \cos(90^\circ)
 \end{aligned}$$


$$\Rightarrow DV = V_B - V_A = -E \int_A^C ds = -Ed$$

⇒ Path (2)  $A \rightarrow B$

$$DV = V_B - V_A = - \int_A^B E ds \cos(0) = -E \int_A^B ds \left( \frac{d}{s} \right)$$

$$= -E s \cos(0) = -E s \frac{d}{s} = -Ed$$

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 لا تؤثر على الشحار  
 الشحار عبر مسار  
 ②  
 مختلف

$$\Rightarrow W = -DU = qEd$$

$$W = DV = q(DV)$$

⇒ If a charge ( $q$ ) equals the magnitude of electron charge " $e$ " and the potential is "1 V" then the charge in the energy  $DV = qEd = eV$

$$\Rightarrow 1 eV = 1.6 \times 10^{-19} \text{ J}$$



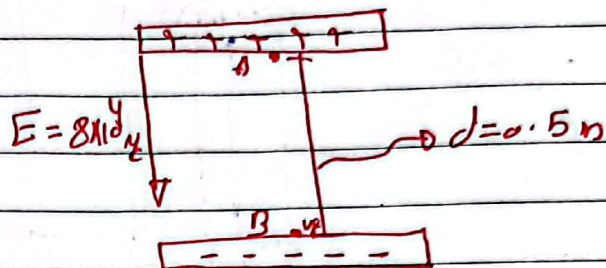
ex: A proton is released from rest at point "A" in uniform electric field of magnitude  $8 \times 10^4 \text{ V/m}$ , the proton undergoes a displacement of  $d = 0.5 \text{ m}$  to point B in the direction of electric field, find the speed of the proton at point B? ( $q_p = 1.6 \times 10^{-19} \text{ C}$ ,  $m_p = 1.67 \times 10^{-27} \text{ kg}$ )

$$\Rightarrow \Delta U = -\Delta V$$

$$\Delta U = -q \Delta V$$

$$\Delta U = -q(-Ed)$$

$$\Delta U = qEd$$



$$\frac{1}{2} m v_B^2 = qEd$$

$$v_B = \sqrt{\frac{2qEd}{m}} = \sqrt{\frac{(2)(1.6 \times 10^{-19})(8 \times 10^4)(0.5)}{1.67 \times 10^{-27}}} = 2.8 \times 10^6 \text{ m/s}$$

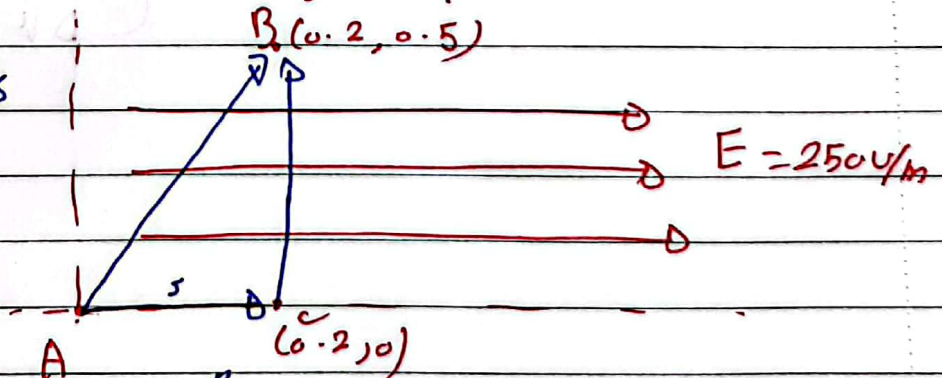
ex: a uniform electric field of magnitude  $250 \text{ V/m}$  is directed in the positive x-axis a  $12 \mu\text{C}$  charge moves from the origin to point  $(20 \text{ cm}, 50 \text{ cm})$ . (1) what is the potential difference between origin and point  $(20 \text{ cm}, 50 \text{ cm})$  (2) what is the change in the potential energy of the charge-field system?

$$\Delta V = V_B - V_A = -\int_A^B E \cdot ds$$

$$\Delta V = -E \int_0^{0.2} ds$$

$$= -E(0.2)$$

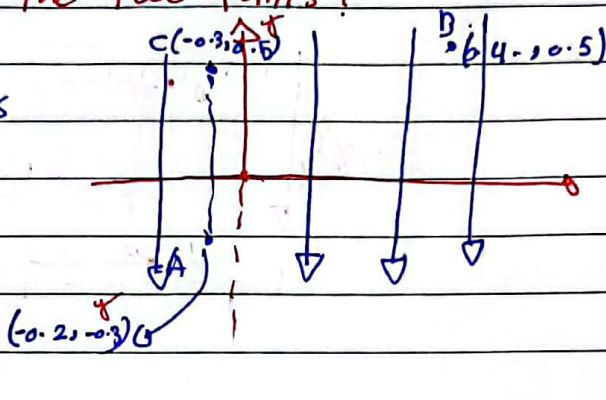
$$= -(250)\left(\frac{2}{10}\right) = -50 \text{ V}$$





$$\textcircled{2} \Delta V = q \cdot \Delta V = (12 \times 10^{-6}) (-50) = -6 \times 10^{-4} \text{ J}$$

(5) ex: A uniform electric field of magnitude  $325 \text{ V/m}$  is directed in the negative  $y$ -direction if the coordinates point (A) is  $(0.2, -0.3)$  and point "B"  $(0.4, 0.5)$  find  $\Delta V$  between the two points?

$$\begin{aligned} \Rightarrow \Delta V &= - \int_A^B E \, ds \\ &= -E \int_A^B ds \\ &= -E \int_{-0.3}^{0.5} ds \end{aligned}$$


$$= -E (0.5 + 0.3) \cos(180)$$

$$\Delta V = \oplus E (0.8) = \rightarrow (325)(0.8) = 260 \text{ V}$$

$$W = -\Delta U$$

$$\Delta U = q \cdot \Delta V \cos(\theta)$$

$$W = \Delta U = -\Delta U$$

$$W = \Delta U = -q \Delta V$$

$$\Delta V = -E \Delta y \cos(\theta)$$



مثال

ex: How much work is done (by a battery, generator, or some other source of potential difference) in moving Avogadro's number of electrons from an initial point is "9 V" to point where the electric potential is "-5 V"?

$$\Delta V = -5 - 9 = -14 \text{ V}$$

$$W = q \Delta V$$

$$q = N e = (6.02 \times 10^{23})(-1.6 \times 10^{-19}) = -9.63 \times 10^4 \text{ C}$$

$$W = (-9.63 \times 10^4)(-14) = 1.35 \times 10^3 \text{ J}$$

ex: An electron moving parallel to the x-axis has an initial speed of  $3.7 \times 10^6 \text{ m/s}$  at the origin. Its speed is reduced to  $1.4 \times 10^5 \text{ m/s}$  at point  $x = 2 \times 10^{-2} \text{ m}$ . (a) calculate the electric potential difference between the origin and that point. (b) which point is at the higher potential?

$$\Delta K = \frac{1}{2} m (v_f^2 - v_i^2) = -q \Delta V$$

$$\Delta V = -\frac{m (v_f^2 - v_i^2)}{q} = -\frac{(9.11 \times 10^{-31})((1.4 \times 10^5)^2 - (3.7 \times 10^6)^2)}{(-1.6 \times 10^{-19})}$$

$$= \frac{(9.11 \times 10^{-31})((1.4 \times 10^5)^2 - (3.7 \times 10^6)^2)}{1.6 \times 10^{-19}} = -38.9 \text{ V}$$

« لا تستسلم وأنت الذي إعتاد الجميع أن يراك قوياً »



ex: (A) Find the electric potential difference  $\Delta V_e$  required to stop an electron moving with an initial speed of  $(2.85 \times 10^7) \text{ m/s}$ .  
 (b) would a proton traveling at the same speed require a greater or lesser magnitude of electric potential difference? (c) Find a symbolic expression for the ratio of the proton stopping potential and the electron stopping potential,  $\Delta V_p / \Delta V_e$ .

(A)  $\Delta K = -q_e \Delta V$

$$\frac{1}{2} m_e v_f^2 - \frac{1}{2} m_e v_i^2 = +q_e \Delta V$$

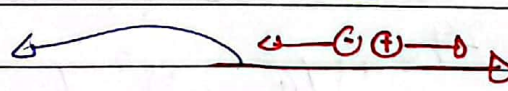
$$\Delta V = \frac{m_e v_i^2}{2 q_e} = \frac{(9.11 \times 10^{-31}) (2.85 \times 10^7)^2}{2 (-1.6 \times 10^{-19})}$$

$$\Delta V = -2.31 \times 10^3 \text{ V} = -2.31 \text{ kV}$$

(c)  $\Delta K = -q_p \Delta V$

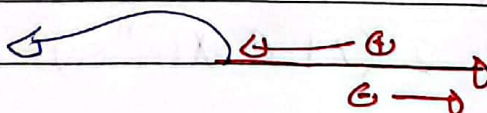
$$\frac{1}{2} m_p v_f^2 - \frac{1}{2} m_p v_i^2 = -q_p \Delta V \rightarrow \Delta V_p = \frac{m_p v_i^2}{2 q_p}$$

$$\frac{\Delta V_p}{\Delta V_e} = \frac{m_p v_i^2 / 2 q_p}{-m_e v_i^2 / 2 q_e} \rightarrow \Delta V_p / \Delta V_e = -\frac{m_p}{m_e} \frac{q_e}{q_p}$$

$$W = -\Delta U = -q_e (\Delta V) = \Delta K$$


$$W = \Delta U = q_e (\Delta V)$$

$$\Delta K = 0$$



$$+ \Delta V = -E d \cos(\theta)$$



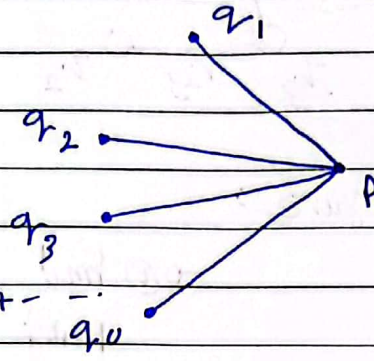
⇒ Electric Potential due point charges:

$$V_p = \frac{U \cdot q}{r}$$

① - - - - - P

$$\Rightarrow V_p = U \sum_i \frac{q_i}{r_i}$$

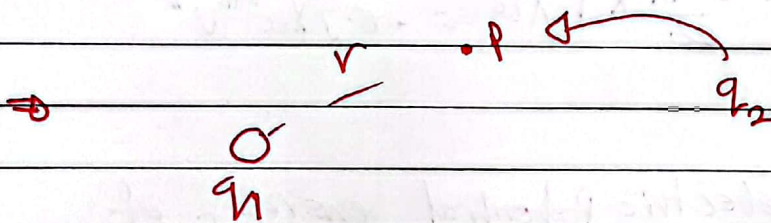
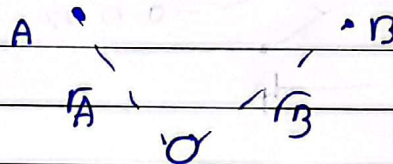
$$= U \frac{q_1}{r_1} + U \frac{q_2}{r_2} + \dots$$



$$\Rightarrow \Delta V = V_B - V_A$$

$$= \frac{Uq}{r_B} - \frac{Uq}{r_A}$$

$$= Uq \left( \frac{1}{r_B} - \frac{1}{r_A} \right)$$



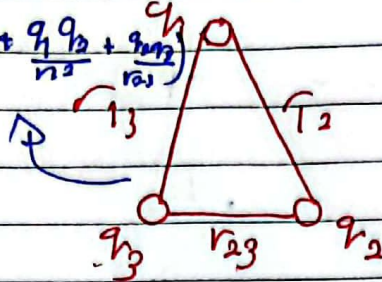
$$V_p = \frac{Uq_1}{r}$$

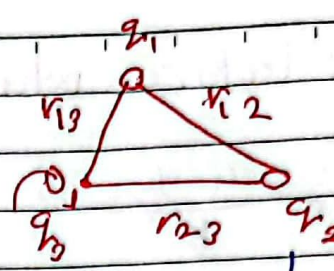
$$\Rightarrow \Delta V = q_2 \Delta V = q_2 (V_p - V_\infty)$$

$$= q_2 V_p$$

$$= \frac{U \cdot q_1 \cdot q_2}{r}$$

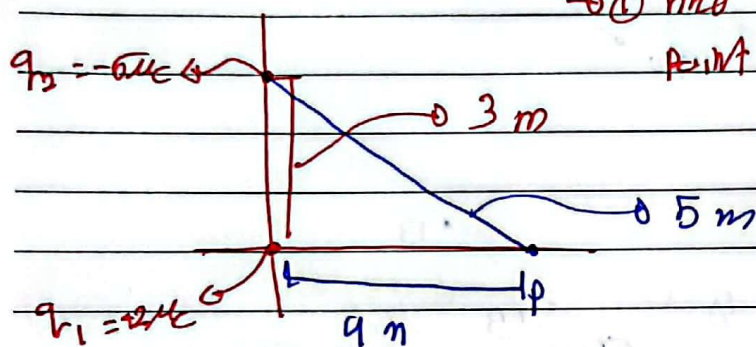


$$\Rightarrow U = k \left( \frac{q_1 q_3}{r_{13}} + \frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} \right)$$




$$\Rightarrow \Delta U = q_3 \Delta V_P$$

ex: in the figure:



① Find electric potential at point "P" which has coordinates (4, 0)

$$V_P = \frac{k q_1}{r_1} + \frac{k q_2}{r_2} = (9 \times 10^9) \left( \frac{2 \times 10^{-6}}{4} - \frac{6 \times 10^{-6}}{5} \right)$$

$$= (9 \times 10^3) \left( \frac{4}{4} - \frac{6}{5} \right) = -\frac{18}{5} \times 9 \times 10^3 = -6.3 \times 10^3 \text{ V}$$

② Find the change in electric potential energy of both charges plus a third charge  $q_3 = 3 \mu\text{C}$  placed at point P?

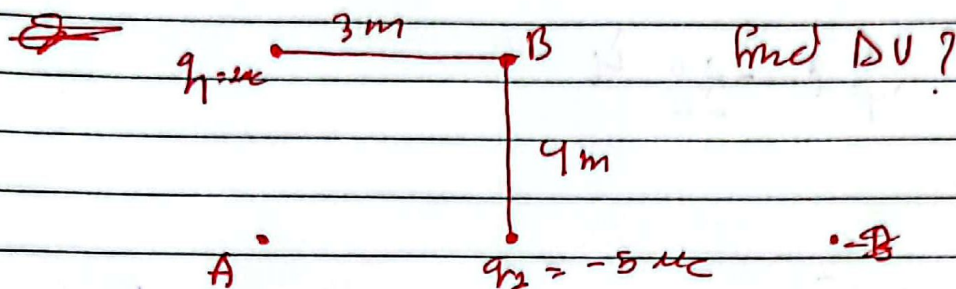
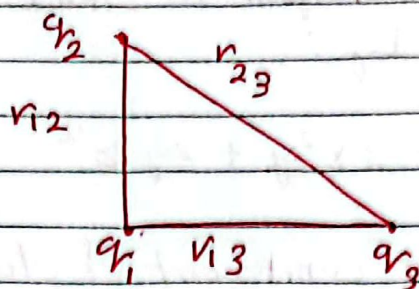
$$\Rightarrow \Delta U = q_3 \Delta V = q_3 (V_P - V_\infty) = q_3 \cdot V_P$$

$$= (3 \times 10^{-6}) (-6.3 \times 10^3) = -18 \times 10^{-3} \text{ J}$$



or

$$\Delta U = k \frac{q_1 \cdot q_2}{r_{12}} + k \frac{q_2 \cdot q_3}{r_{23}} + k \frac{q_1 \cdot q_3}{r_{13}}$$



$$\Delta U = U_B - U_A = k \left( \frac{q_1}{r_B} + \frac{q_2}{r_B} \right) - k \left( \frac{q_1}{r_A} + \frac{q_2}{r_A} \right)$$

$$= (9 \times 10^9) \left( \frac{2 \times 10^{-6}}{3} + \frac{5 \times 10^{-6}}{4} \right) - (9 \times 10^9) \left( \frac{2 \times 10^{-6}}{4} + \frac{5 \times 10^{-6}}{3} \right)$$

$$\Rightarrow V = - \int E \cdot ds \quad \left\{ \begin{array}{l} \text{If } V \rightarrow V(x, y, z) \text{ } \vec{E} ? \end{array} \right.$$

$$dV = - \vec{E} \cdot d\vec{s}$$

$$\vec{E} = - \nabla V = - \left( \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) V$$

$$E_x = - \frac{\partial V}{\partial x} \hat{i}$$

$$= \underbrace{- \frac{\partial V}{\partial x}}_{E_x} \hat{i} + \underbrace{- \frac{\partial V}{\partial y}}_{E_y} \hat{j} + \underbrace{- \frac{\partial V}{\partial z}}_{E_z} \hat{k}$$

$$E_y = - \frac{\partial V}{\partial y} \hat{j}$$

$$E_z = - \frac{\partial V}{\partial z} \hat{k}$$



ex(39): in a certain region in space the electric potential is given by:

$$V(x, y, z) = 5x - 3x^2y + 2yz^2$$

- ① find the  $x, y, z$  component of electric field:  
② " " magnitude of electric field at point "p" which has a coordinates of  $(1, 0, -2)$  m?

$$\textcircled{1} E = -\frac{\partial V}{\partial x} \hat{i} - \frac{\partial V}{\partial y} \hat{j} - \frac{\partial V}{\partial z} \hat{k}$$

$$\frac{\partial V}{\partial x} = 5 - 6xy$$

$$\frac{\partial V}{\partial y} = -3x^2 + 2z^2$$

$$\frac{\partial V}{\partial z} = 4yz$$

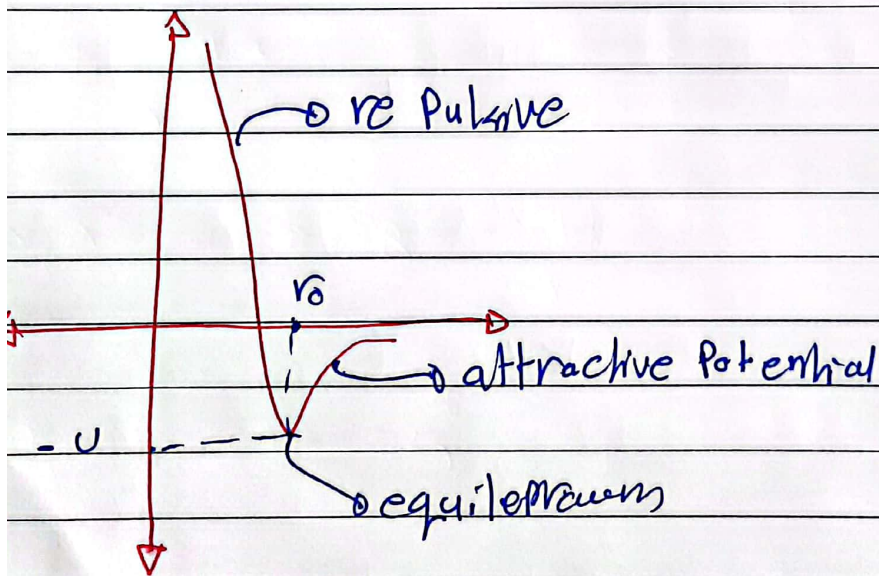
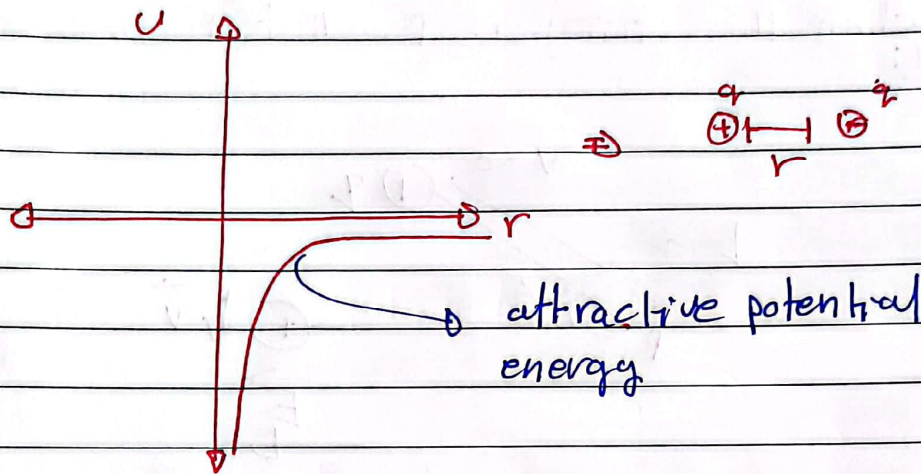
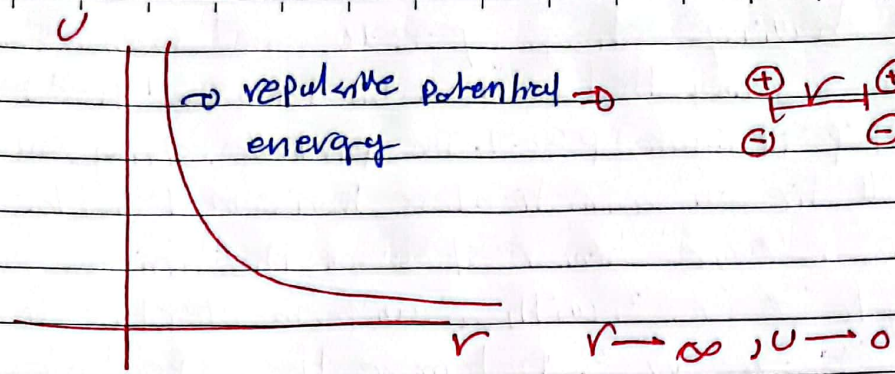
$$\textcircled{1} E = -(5 - 6xy)\hat{i} - (-3x^2 + 2z^2)\hat{j} - 4yz\hat{k}$$

② at point  $(1, 0, -2)$

$$E = -5\hat{i} - 5\hat{j}$$

$$|E| = \sqrt{(-5)^2 + (-5)^2} = \sqrt{50} \text{ N/C or V/m}$$





✽ اَعَانَ اللّٰهُ الصّٰلِحِيْنَ عَلَىٰ مَنَاجِجِ قُلُوْبِهِمْ ✽

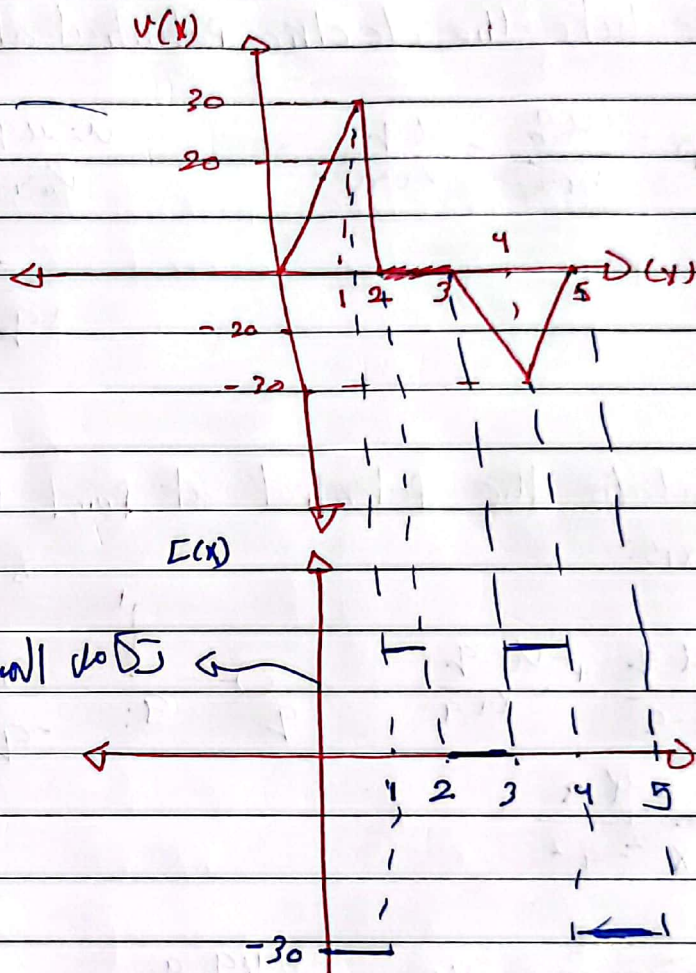


ex(18): An electric field in a region in a space is parallel to X-axis, the electric potential as a function of "x" is shown in the figure, graph the X-component of the electric field?

$$V(r) = \frac{4q}{r}$$

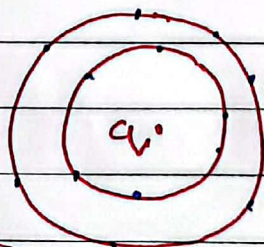
$$E = -\frac{dV}{dr} = -\left(-\frac{4q}{r^2}\right)$$

$$E = \frac{4q}{r^2}$$

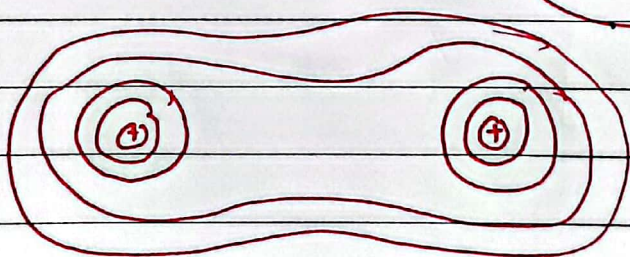


Potential up to 50 kV/m

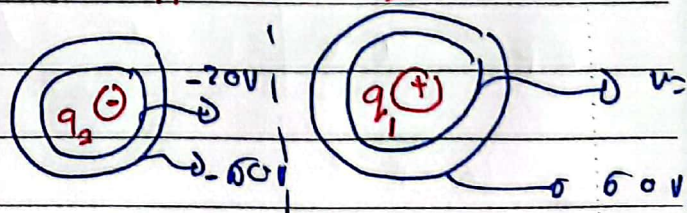
⇒ equipotential surface :-



equipotential surface



if  $q_1 = q_2$



$V=0$

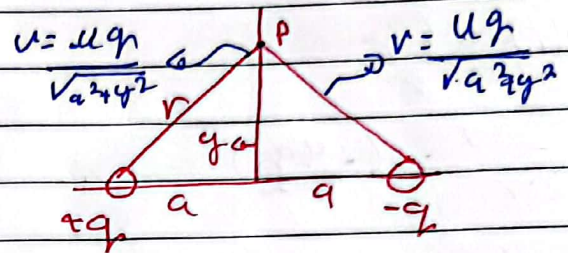


25.4

ex: An electric dipole consists of two charges of equal magnitude and opposite sign and separated by a distance  $2a$  as shown in the figure:

① Calculate the electric potential at point P on y-axis?

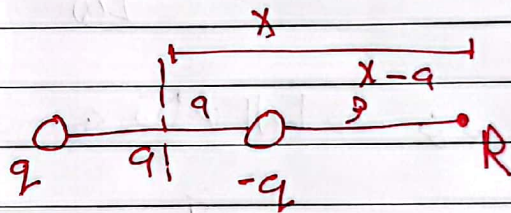
$$\Rightarrow V_P = \frac{Uq}{\sqrt{a^2+y^2}} - \frac{Uq}{\sqrt{a^2+y^2}} = 0$$



② Calculate the potential at point R on the positive x-axis:

$$V_R = \frac{-Uq}{x-a} + \frac{Uq}{x+a}$$

$$= \frac{-2Uqa}{x^2 - a^2}$$



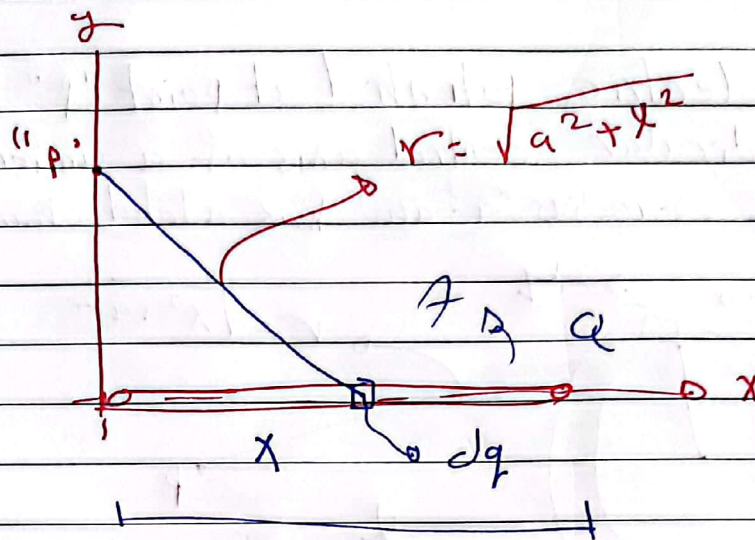
$$\Rightarrow x \gg a \Rightarrow V_R = \frac{-2Uqa}{x^2}$$

$$E \text{ ? } \Rightarrow E = -\frac{dV}{dx}$$



⇒ Potential due continuous charge distribution:-

ex: A rod of length "L" located along the x-axis has a total charge Q and a uniform linear charge density  $\lambda$ , find the electric potential of point "P" located along y-axis at distance a from origin:



$$\Rightarrow \int dV_p = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{r} = \int_0^L \frac{\lambda dx}{\sqrt{a^2 + x^2}}$$

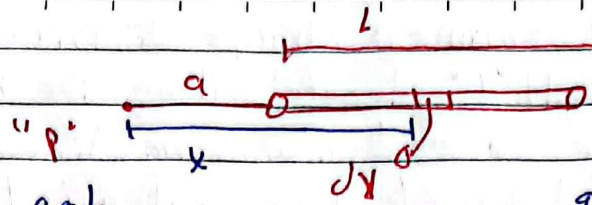
$$\Rightarrow V_p = \frac{1}{4\pi\epsilon_0} \lambda \int_0^L \frac{dx}{\sqrt{a^2 + x^2}} = \frac{1}{4\pi\epsilon_0} \lambda \ln(x + \sqrt{x^2 + a^2}) \Big|_0^L$$

$$= \frac{1}{4\pi\epsilon_0} \lambda (\ln(L + \sqrt{L^2 + a^2}) - \ln(a))$$

$$V_p = \frac{1}{4\pi\epsilon_0} \lambda \ln \frac{L + \sqrt{L^2 + a^2}}{a}$$

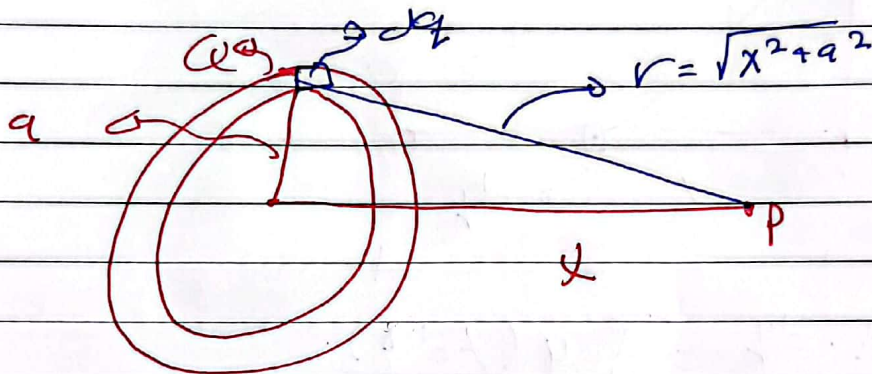
في الحياة إن لم نطلب من الضيق الأولي، فأنت تستحق الثاني





$$V_p = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{r} = \int_a^{a+L} \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda dx}{x} = \frac{\lambda}{4\pi\epsilon_0} \ln x \Big|_a^{a+L} = \frac{\lambda}{4\pi\epsilon_0} \ln\left(\frac{a+L}{a}\right)$$

ex:- Find the electric potential at point "p" located on the perpendicular central axis of a uniformly charged ring of radius "a" and has a total charge.



$$\Rightarrow dV_p = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{r} = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{\sqrt{x^2 + a^2}}$$

$$\Rightarrow \int dV_p = \int \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{\sqrt{x^2 + a^2}} \Rightarrow V_p = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{\sqrt{x^2 + a^2}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{\sqrt{x^2 + a^2}}$$

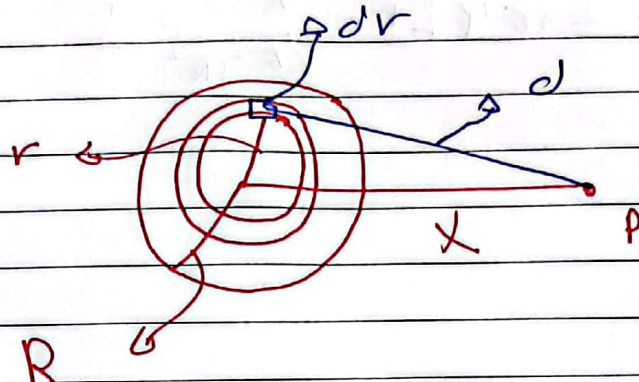
$$\Rightarrow V_p = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{\sqrt{x^2 + a^2}}, \quad E_p = -\frac{dV}{dx} = -\frac{1}{4\pi\epsilon_0} \cdot \left[ \frac{Q}{(\sqrt{x^2 + a^2})^2} \right]$$

$$E_p = -\frac{1}{4\pi\epsilon_0} \cdot \left[ \frac{Q}{2} \cdot \frac{2x}{(x^2 + a^2)^{3/2}} \right]$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{Qx}{(x^2 + a^2)^{3/2}}$$



ex: A uniformly charged disk has a radius "R" and a uniform surface charge density  $\sigma$ , find the electric potential at point 'p' along the perpendicular central axis of the disk??



$$\Rightarrow dV_p = \frac{q \cdot dq}{d} = \frac{q \cdot dq}{(r^2 + x^2)^{\frac{1}{2}}}$$

$$\Rightarrow dq = \sigma \cdot dA = \sigma (2\pi r dr)$$

$$\Rightarrow dV_p = \frac{q \cdot \sigma [2\pi r dr]}{(r^2 + x^2)^{\frac{1}{2}}}$$

$$\int dV_p = 4 \cdot \sigma (2\pi) \int_0^R \frac{2r dr}{(r^2 + x^2)^{\frac{1}{2}}}$$

$$\Rightarrow V_p = 4\sigma\pi \int_{x^2}^{x^2 + R^2} \frac{du}{u^{\frac{1}{2}}} = 4\sigma\pi \int_{x^2}^{x^2 + R^2} u^{-\frac{1}{2}} du$$

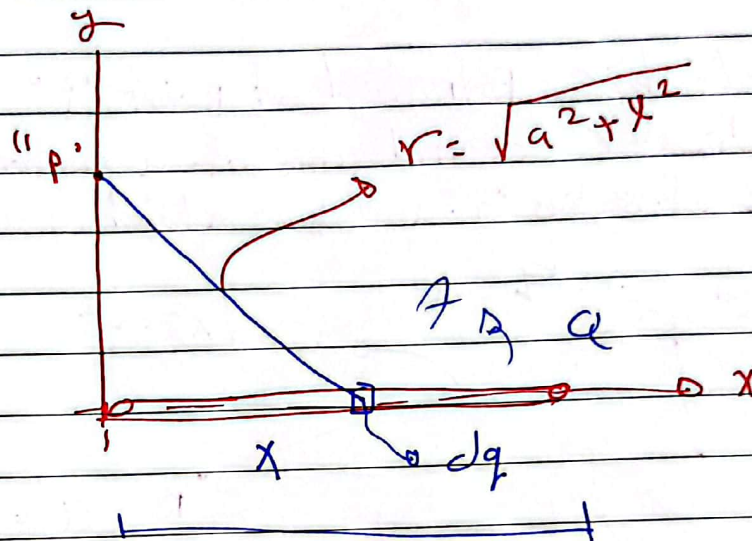
$$V_p = 24\sigma\pi u^{\frac{1}{2}} \Big|_{x^2}^{x^2 + R^2}$$

$$= 2\pi 4\sigma (\sqrt{x^2 + R^2} - x)$$

$$\begin{aligned} \Rightarrow u &= r^2 + x^2 \Rightarrow du = 2r dr \\ r=0 &\Rightarrow u = x^2 \\ r=R &\Rightarrow u = x^2 + R^2 \end{aligned}$$



⇒ Potential due continuous charge distribution:-  
 ex: A rod of length "L" located along the x-axis has a total charge Q and a uniform linear charge density  $\lambda$ , find the electric potential of point "P" located along y-axis at distance a from origin.



$$\Rightarrow \int dV_p = \frac{1}{r} \cdot dq = \int \frac{\lambda dx}{\sqrt{a^2 + x^2}}$$

$$\Rightarrow V_p = \lambda \int_0^L \frac{dx}{\sqrt{a^2 + x^2}} = \lambda \cdot \ln(x + \sqrt{x^2 + a^2}) \Big|_0^L$$

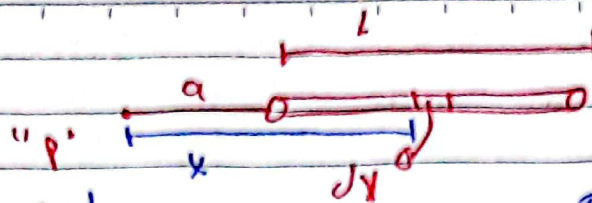
$$= \lambda \left( \ln(L + \sqrt{L^2 + a^2}) - \ln(a) \right)$$

$$V_p = \lambda \cdot \ln \frac{L + \sqrt{L^2 + a^2}}{a}$$

في الحياة إن لم نتعلم من التجربة الأولى، فأنت تستحق الثانية

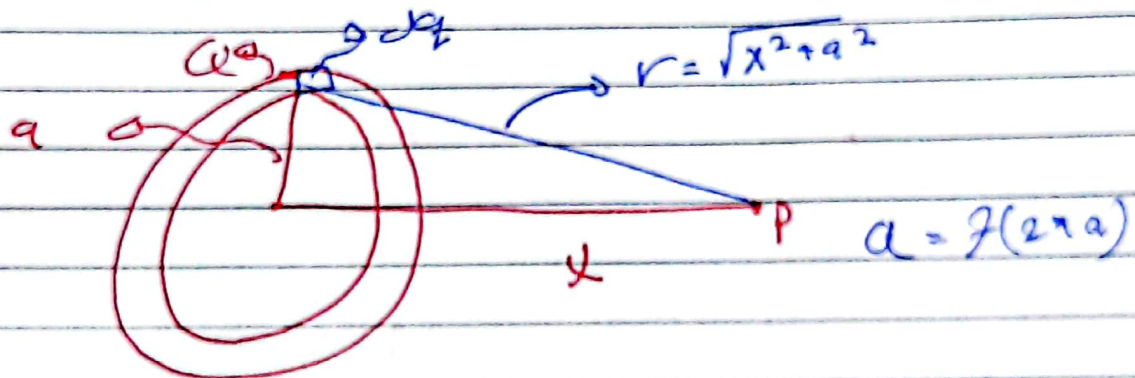
✍





$$V_P = k \cdot \frac{dq}{x} = \int_a^{a+L} \frac{k \cdot \lambda dx}{x} = k \lambda \ln x \Big|_a^{a+L} = k \lambda \ln \left( \frac{a+L}{a} \right) \lambda$$

ex: Find the electric potential at point "P" located on the perpendicular central axis of a uniformly charged ring of radius "a" and has a total charge.



$$\Rightarrow dV_P = \frac{k \cdot dq}{r} = \frac{k dq}{\sqrt{x^2 + a^2}}$$

$$\Rightarrow \int dV_P = \int \frac{k \cdot dq}{\sqrt{x^2 + a^2}} \Rightarrow V_P = \frac{kQ}{\sqrt{x^2 + a^2}} \int_0^Q dq = \frac{kQ}{\sqrt{x^2 + a^2}}$$

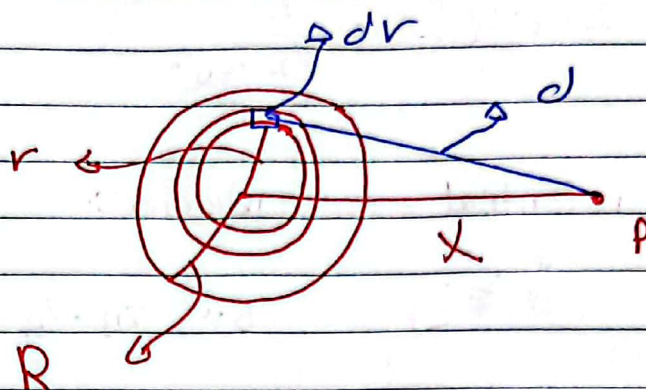
$$\Rightarrow V_P = \frac{kQ}{\sqrt{x^2 + a^2}}, \quad E_P = -\frac{dV}{dx} = -kQ \left[ \frac{-1}{(x^2 + a^2)^{3/2}} \cdot 2x \right]$$

$$E_P = kQ \left[ \frac{1 - (x^2 + a^2)^{-1/2} \cdot 2x}{x^2 + a^2} \right]$$

$$= \frac{kQx}{(x^2 + a^2)^{3/2}}$$



ex: A uniformly charged disk has a radius "R" and a uniform surface charge density  $\sigma$ , find the electric potential at point "p" along the perpendicular central axis of the disk??



$$\Rightarrow dV_p = \frac{q \cdot dq}{d} = \frac{q \cdot dq}{(r^2 + x^2)^{\frac{1}{2}}}$$

$$\Rightarrow dq = \sigma \cdot dA = \sigma (2\pi r dr)$$

$$\Rightarrow dV_p = \frac{q \cdot \sigma (2\pi r dr)}{(r^2 + x^2)^{\frac{1}{2}}}$$

$$\int dV_p = \frac{q \cdot \sigma (2\pi)}{(r^2 + x^2)^{\frac{1}{2}}} \int_0^R \frac{r dr}{(r^2 + x^2)^{\frac{1}{2}}}$$

$$\Rightarrow V_p = 4\sigma\pi \int_{x^2}^{x^2 + R^2} \frac{du}{u^{\frac{1}{2}}} = 4\sigma\pi \int_{x^2}^{x^2 + R^2} u^{-\frac{1}{2}} du$$

$$V_p = 24\sigma\pi u^{\frac{1}{2}} \Big|_{x^2}^{x^2 + R^2}$$

$$Q = \sigma (\pi R^2)$$

$$= 2\pi 4\sigma (\sqrt{x^2 + R^2} - x)$$

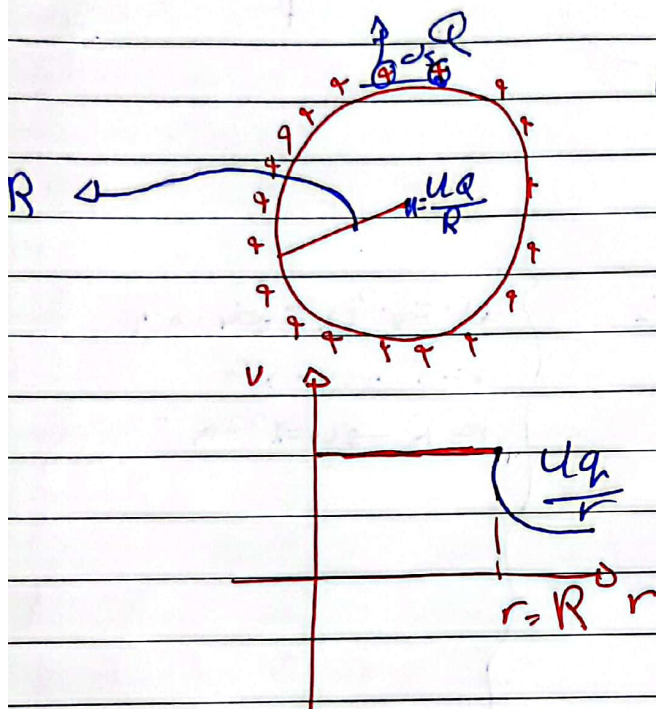


$$\Rightarrow E_x = -\frac{dv}{dx} = -2\pi\epsilon_0 \frac{d}{dx} (\sqrt{x^2 + R^2} - x)$$

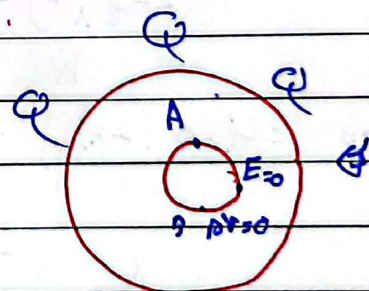
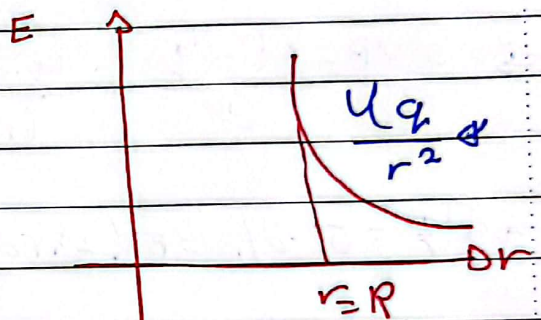
$$= -2\pi\epsilon_0 \left( \frac{1}{2}(x^2 + R^2)^{-\frac{1}{2}}(2x) - 1 \right)$$

$$= 2\pi\epsilon_0 \left[ 1 - \frac{x}{\sqrt{x^2 + R^2}} \right]$$

$\Rightarrow$  Electric Potential of a conductor:



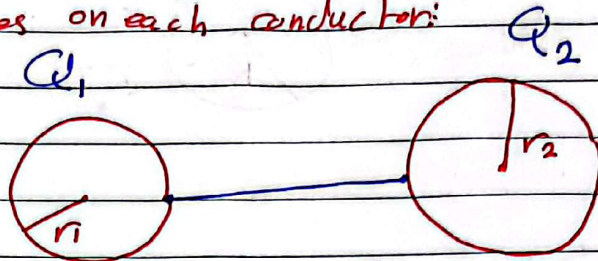
$$dv = v_B - v_A = - \int_A^B E_{\text{rad}} ds = 0$$



الكمية  
في الدائرة الكهربائية داخل الموصل يساوي صفر  
الدائرة على سطحه.



ex: (28.8): two spherical conductors of radii  $r_1$  and  $r_2$ , separated by a distance longer than  $r_1$  and  $r_2$ . the two spheres are connected by a conducting wire as shown in the figures at equilibrium, the charges on each conductor:



→ Find the ratios of the electric field at the surface of the sphere?

at equilibrium:  $V_1 = V_2$

$$\Rightarrow \frac{kq_1}{r_1} = \frac{kq_2}{r_2} \Rightarrow \frac{q_1}{q_2} = \frac{r_1}{r_2}$$

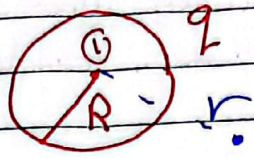
$$\Rightarrow E_1 = \frac{kq_1}{r_1^2}, \quad E_2 = \frac{kq_2}{r_2^2}$$

$$\Rightarrow \frac{E_1}{E_2} = \frac{\frac{kq_1}{r_1^2}}{\frac{kq_2}{r_2^2}} = \frac{q_1}{q_2} \cdot \frac{r_2^2}{r_1^2} = \frac{r_1}{r_2} \cdot \frac{r_2^2}{r_1^2} = \frac{r_2}{r_1}$$

⇒ حلوين بس الد هه ضرب ..... ملا ههنا



## ① Conducting Sphere

①  $V_{in} = \frac{kq}{R} = V_{surface}$  ②   $\Rightarrow q = 5(4\pi R^2)$

②  $r > R$  outside

$$V_{out} = \frac{kq}{r}$$

## ① Insulating Sphere:



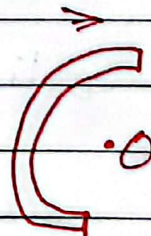
$$q = \rho \left( \frac{4}{3} \pi R^3 \right)$$

①  $r < R \Rightarrow V_{in} = \frac{kq}{2R} \left( 3 - \frac{r^2}{R^2} \right)$

②  $r > R \Rightarrow V_{out} = \frac{kq}{r}$

ex: (44): A uniformly charged insulating rod of length  $14 \times 10^{-2} \text{ m}$  is bent into the ~~sphere~~ shape of a semicircle as shown in figure 'p.25.44'. The rod has a total charge of  $q = -7.5 \times 10^{-6} \text{ C}$ . Find the electric potential at 'o' the center of the semicircle:

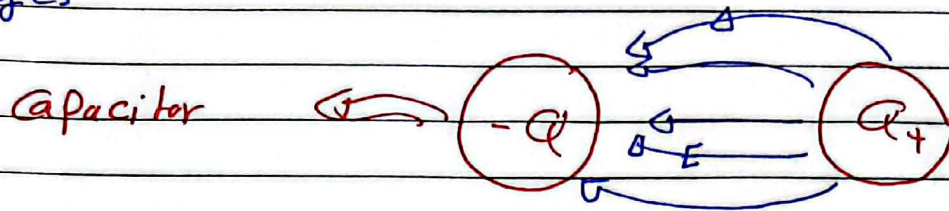
$$\begin{aligned} \Rightarrow V_o &= k \int \frac{dq}{r} \quad r = \frac{L}{\pi} \\ &= (9 \times 10^9) \left( \frac{-7.5 \times 10^{-6}}{14 \times 10^{-2}} \right) (3.14) \\ &= 15.1 \times 10^5 \text{ volt} \end{aligned}$$





## Ch(26): Capacitance and Dielectrics

⇒ Capacitor: two conductors of equal and opposite charges



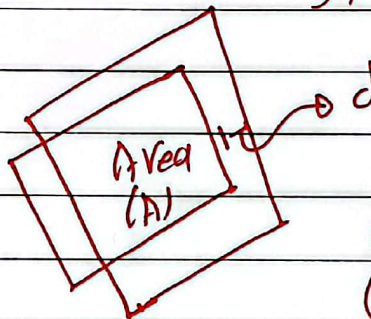
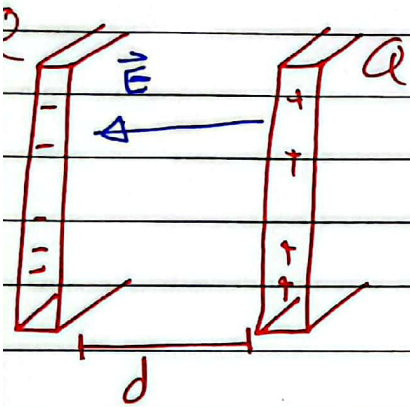
⇒ Capacitance (C):

$$\Rightarrow C = \frac{|Q|}{|DV|} \quad (C/V)$$

$$\Rightarrow \frac{C}{V} = |Farad(F)|$$

A: area of the plate  
d: separation distance between two plates.

① Parallel-Plate capacitor:



$\epsilon$ : dielectric constant

C depends on geometry of capacitor.

$$\Rightarrow E = \frac{\sigma}{\epsilon} = \frac{Q}{A\epsilon}$$

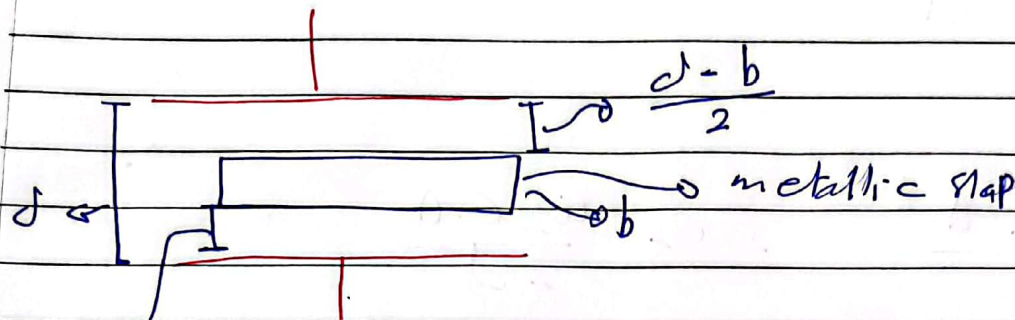
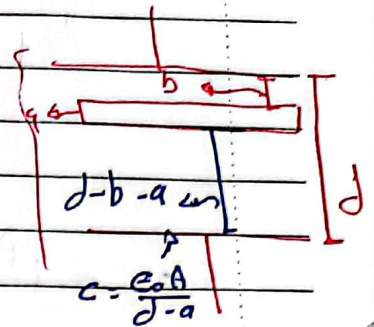
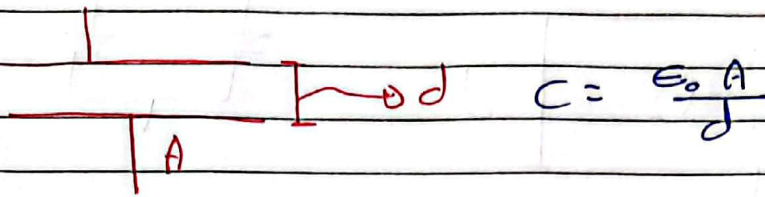
$$C = \frac{Q}{DV} = \frac{A\sigma}{d}$$

$$DV = Ed \quad \Rightarrow DV = \frac{Qd}{A\epsilon}$$

∴ Permittivity of free space,  $\epsilon_0 = 8.85 \times 10^{-12} \text{ Nm}^2/\text{C}^2$

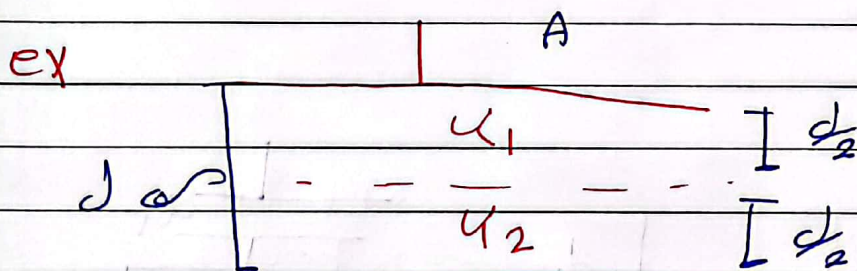


→ "Metallic Slap inside C" (بارة معدنية)

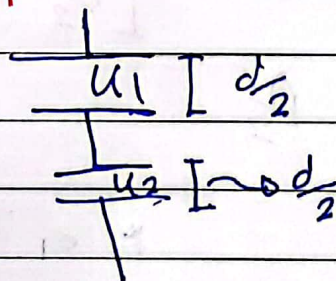


$$C = \frac{\epsilon A}{(d-b)}$$

~~ex~~



$$C_0 = \frac{\epsilon A}{d}$$



$$C_1 = \frac{\kappa_1 \epsilon A}{d/2}$$

$$C_2 = \frac{\kappa_2 \epsilon A}{d/2}$$

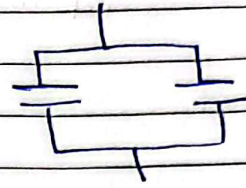
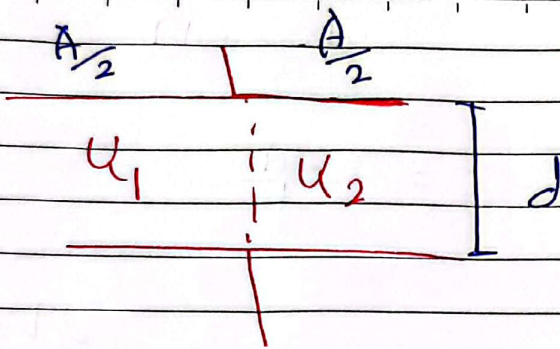
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C} = \frac{1}{2\kappa_1 \epsilon A} + \frac{1}{2\kappa_2 \epsilon A}$$

$$\frac{1}{C} = \frac{d}{2\kappa_1 \epsilon A} + \frac{d}{2\kappa_2 \epsilon A} \quad \frac{1}{C} = \frac{d(\kappa_2 + \kappa_1)}{2\kappa_1 \kappa_2 \epsilon A}$$



ex:-



$$C_0 = \frac{\epsilon_0 A}{d}$$

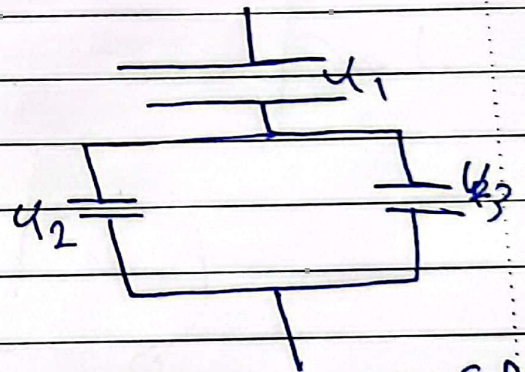
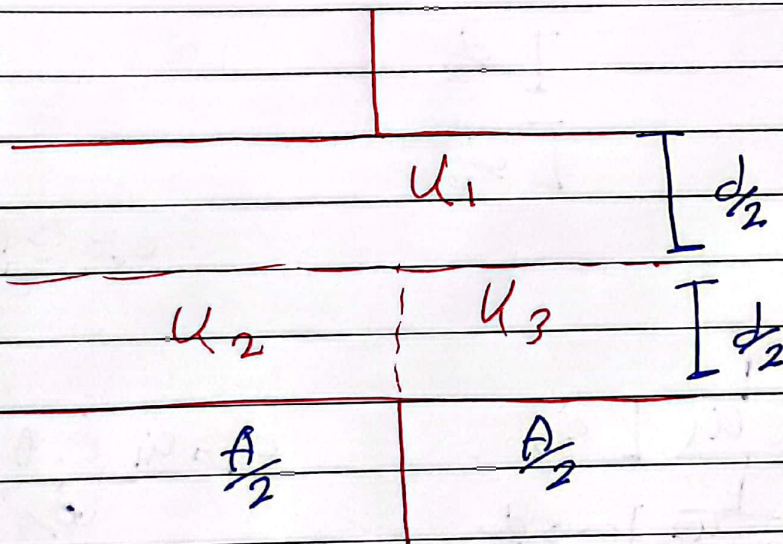
$$C_1 = \frac{U_1 \epsilon_0 A}{2d}$$

$$C_2 = \frac{U_2 \epsilon_0 A}{2d}$$

$$C_{eq} = C_1 + C_2 = \frac{U_1 \epsilon_0 A}{2d} + \frac{U_2 \epsilon_0 A}{2d}$$

$$= \left( \frac{U_1 + U_2}{2} \right) \frac{\epsilon_0 A}{d} = \left( \frac{U_1 + U_2}{2} \right) C_0$$

ex:-



$$C_0 = \frac{\epsilon_0 A}{d}$$

$$C_1 = \frac{2U_1 \epsilon_0 A}{d} = C_0$$

$$C_2 + C_3 = C_4$$

$$C_2 = \frac{U_2 \epsilon_0 A/2}{d/2} = \frac{U_2 \epsilon_0 A}{d}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_4}$$

$$C_3 = \frac{U_3 \epsilon_0 A}{d} = C_0$$



ch(27): current and resistance:

$$\frac{I}{av} = \frac{\Delta q}{\Delta t} \leadsto \Delta q = I \Delta t \leadsto \text{Ampere}$$

$$I = \frac{dq}{dt} \leadsto \Delta q = \int_{t_1}^{t_2} I dt$$

in instantaneous

ex: If  $q = 4t^2 - 5t + 1$ , find the current at  $t = 2s$ :-

$$\leadsto q = 4t^2 - 5t + 1$$

$$I = \frac{dq}{dt} = 8t - 5 = 16 - 5 = 11 \text{ A}$$

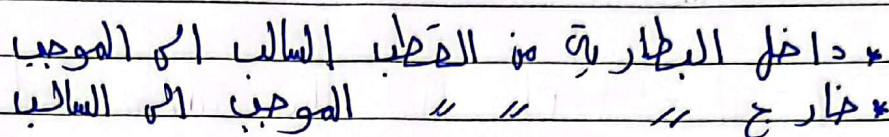
$$\textcircled{2} I_{av} = \frac{\Delta q}{\Delta t} = \frac{q(2) - q(0)}{t=2 - t=0} = \frac{7 - 1}{2} = \frac{6}{2} = 3 \text{ A.}$$

ex:  $I = 6t - 2$  find charge from  $t=0$  to  $t=1$

$$q = \int_0^1 (6t - 2) dt = \left[ \frac{6t^2}{2} - 2t \right]_0^1 = 3t^2 - 2t \Big|_0^1 = 1 \text{ C}$$

~~-1 C~~





٤. خندق الجهد بين طرفي المقاومة يساوي الجهد بين طرفي البطارية

$$\rightarrow T = \frac{q}{f} \rightarrow q = hf$$

عبد الإكبر دنانة التي  
تعتبر الموصلة

$$I = n' q v_d A$$

$n'$  → charge per particle  
 $q$  → cross-sectional area  
 $v_d$  → drift speed  
 $A$  → cross-sectional area

$$* BV = E \cup$$

$$F = q E$$

concentration of moving charge particles

$$n \text{ of } V_G = \frac{L_{\text{eff}}}{f}$$

$$J = \frac{I}{A} = n' e v_d \quad \text{or} \quad J = \sigma E$$

قَسَمَ الْقَائِدُ

↳ current density

$$\sigma = \frac{n^2 q^2 \tau}{m \gamma}$$

$$P_A = IV = I^2 R = \frac{V^2}{R}$$

$$U = p \times t$$



$q$  = charge inside the resistance

$I$  = current,  $t$  = time of current moving

$\Delta V$  = potential (voltage) difference across the R.

$n$  = number of electrons pass through the wire

$n'$  = electron's density ( $e/m^3$ )

$V_d$  = drift velocity

$A$  = Area of the wire ( $m^2$ )

$L$  = length of the wire

$J$  = current density ( $A/m^2$ )

$R$  = Resistance ( $\Omega$ )

$\rho$  = resistivity ( $\Omega \cdot m$ )

$\sigma$  = conductivity ( $(\Omega \cdot m)^{-1}$ )

$E$  = electric field,  $P$  = Power (energy rate) ~ watt

$U$  = energy in R.,  $\Delta t$  = av. time interval between two

ex: ~~Resistance of long successive collisions~~

ex: Resistance of length "10m" and radius of ( $\frac{2}{\sqrt{\pi}}$  cm), has a conductivity of  $4 \times 10^6 (\Omega \cdot m)^{-1}$ , "connected" across a p.d. diff of "20-v" for "10s". Find:

resistivity ( $\rho$ )

$$L = 10m, \quad r = \frac{2}{\sqrt{\pi}} \text{ cm}, \quad \sigma = 4 \times 10^6 (\Omega \cdot m)^{-1}$$

$$A = \pi r^2$$

$$= 4 \times 10^{-4} m^2$$

$$t = 10s$$

$$\Delta V = 20V$$

$$\rho = \frac{1}{\sigma} = \frac{1}{4 \times 10^6} = 0.25 \times 10^{-6} (\Omega \cdot m)$$

② Resistance:-

$$R = \frac{\rho L}{A} = \frac{(0.25)(10) \times 10^{-6}}{4 \times 10^{-4}} = \frac{2.5 \times 10^{-2}}{4} = 0.625 \times 10^{-2} \Omega$$



③ current:

$$R = \frac{V}{I} \quad I = \frac{V}{R} = \frac{20 \times 10^2}{0.625} = 32 \times 10^2 \text{ A}$$

④ charge:

$$I = \frac{Q}{t} \quad Q = It = (32 \times 10^2)(10) = 32 \times 10^3 \text{ C}$$

⑤ numbers of "e" passes through the wire (n):

$$Q = ne \Rightarrow n = \frac{Q}{e} = \frac{32 \times 10^3}{1.6 \times 10^{-19}}$$

⑥ drift velocity:

$$V_d = \frac{L}{t} = \frac{10}{10} = 1 \text{ m/s}$$

⑦ numbers of "e" per unit volume ( $n'$ ):

$$I = n' e V_d A$$

$$n' = \frac{I}{e V_d A} = \frac{32 \times 10^2}{(1.6 \times 10^{-19})(0.54 \times 10^{-4})} = \frac{32 \times 10^2}{126.4 \times 10^{-23}} = 0.253 \times 10^{25} \text{ (e/m}^3\text{)}$$

⑧ current density ( $J$ ):

$$J = \frac{I}{A} = \frac{32 \times 10^2}{4 \times 10^{-4}} = 8 \times 10^6 \text{ A/m}^2$$

⑨ electric field:

$$\Delta V = E \cdot L \Rightarrow E = \frac{V}{L} = \frac{20}{10} = 2 \text{ V/m}$$



(10) Power:  $P = I \cdot V = (32 \times 10^2)(20) = 64 \times 10^3 \text{ watt}$

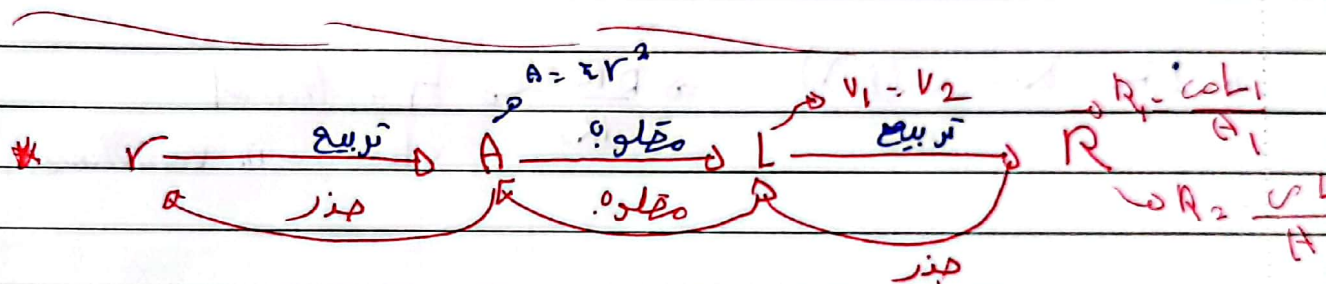
(11) energy:  $U = P \cdot t = (64 \times 10^3) \times 10 = 64 \times 10^4 \text{ J}$

(12) Force on e.

$F = e \cdot E = (1.6 \times 10^{-19})(2) = 3.2 \times 10^{-19} \text{ N}$

(13) time between 2 successive collisions ( $\tau$ ):

$\Rightarrow \sigma = \frac{n e^2 \tau}{m e} \Rightarrow \frac{\sigma \cdot m e}{n \cdot e^2} = \tau$

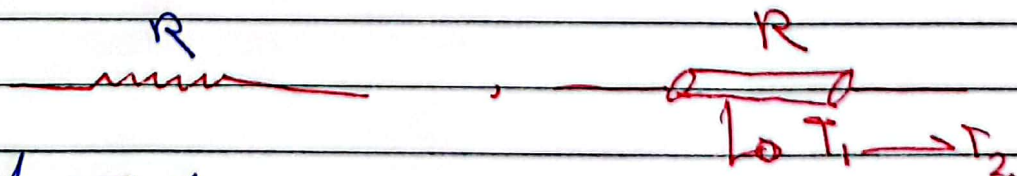


$R_2 = 2 R_1 \rightarrow A_2 = \frac{1}{4} A_1 \rightarrow L_2 = \frac{1}{4} L_1 \rightarrow R_2 = \frac{1}{16} R_1$

$A_2 = 2 A_1 \rightarrow L_2 = \frac{1}{2} L_1 \rightarrow R_2 = \frac{1}{4} R_1$

$R_2 = 3 R_1 \rightarrow L_2 = \sqrt{3} L_1 \rightarrow A_2 = \frac{1}{\sqrt{3}} A_1 \rightarrow r_2 = \frac{1}{\sqrt{3}} r_1$

$\Rightarrow$  Resistance:



$R_0 \equiv$  original resistance

$R \equiv$  new

$\alpha \equiv$  Temperature coefficient of resistance ( $\frac{1}{T}$ )



$$\Rightarrow R = R_0 (1 + \alpha (T_f - T_i))$$

$$\Rightarrow \infty = \infty (1 + \alpha (T_f - T_i))$$

ex: If  $\alpha = 2 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$ ,  $T_i = 50^\circ\text{C}$ ,  $R_0 = 10\Omega$   
 $R = 20\Omega$

$$20 = 10 (1 + 2 \times 10^{-5} (T_f - 50))$$

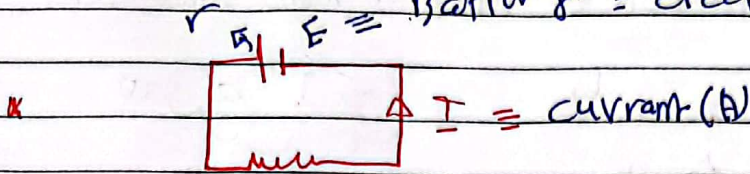
$$T_f = 500050^\circ\text{C}$$

\*  $\Delta R = R_0 \cdot \alpha (\Delta T) \Rightarrow \frac{\Delta R}{R_0} \sim$  Fractional change in resistance



# ⇒ Electromotive force and circuits "ch 28"

$\mathcal{E} \equiv$  Battery  $\equiv$  electromotive force (V)



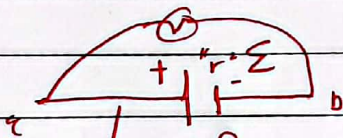
$R \equiv$  external resistance ( $\Omega$ )

⇒  $\frac{\mathcal{E}}{I} = R + r \Rightarrow V_{ab} = IR$

$PR = I \cdot V_{ab}$

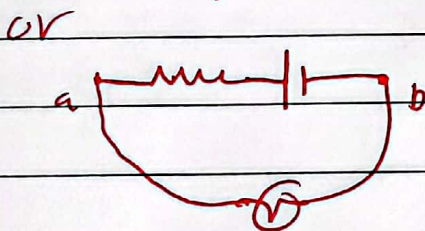
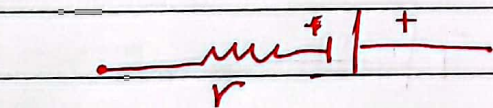
$U = P \cdot t$

⇒  $\frac{\mathcal{E}}{I} = R + r$ ,  $\frac{\mathcal{E}}{I} = R$



Let if  $r=0$  then  $V_{ab} = \mathcal{E}$

$V_{ab} = \mathcal{E}$  if  $r=0$   
 $I=0$



$\mathcal{E} \rightarrow I = \frac{\mathcal{E}}{R+r}$   
 $\Rightarrow V_{ab} = \mathcal{E} - Ir$  internal resistance

terminal voltage  
source with internal resistance

$P_{\text{out}} = V_{ab} \cdot I = (\mathcal{E} - Ir) \cdot I$

Ammeter

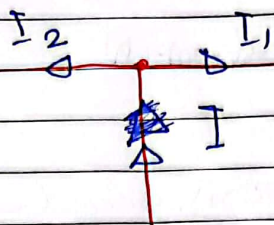
$P_{\text{in}} = \mathcal{E}I - I^2r$

voltmeter



⇒ Kirchhoff's junction rule

A Junction: is a point where three or more conductors meet.



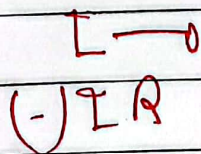
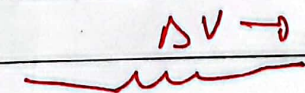
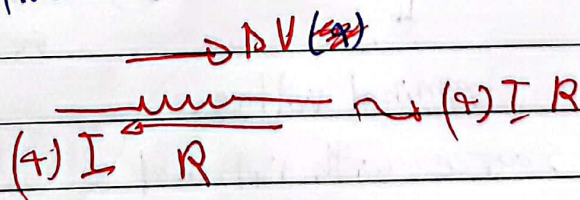
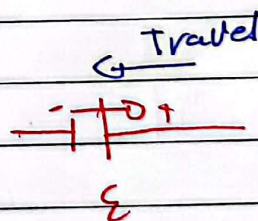
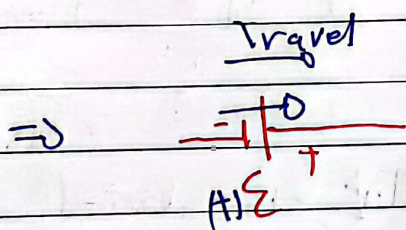
$$\sum I = 0 \quad \text{--- (1)}$$

⇒ Kirchhoff's loop rule

A loop: is any closed conducting path

$$\sum V = 0 \quad \text{--- (2)}$$

⇒ sign conventions for the loop rule:





(B) Calculate the power delivered to the load resistor, the power delivered to the internal resistance of battery and the power delivered by the battery:

$$P = IV = I^2 R$$

$$\Rightarrow P_R = (3.9)^2 (3) = 46.7 \text{ W}$$

$$\Rightarrow P_r = I^2 r = (3.9)^2 (0.05) = 0.772 \text{ W}$$

$$\Rightarrow P_{\text{battery}} = P_{\text{total}} = 46.34 + 0.772 = 47.1 \text{ W}$$

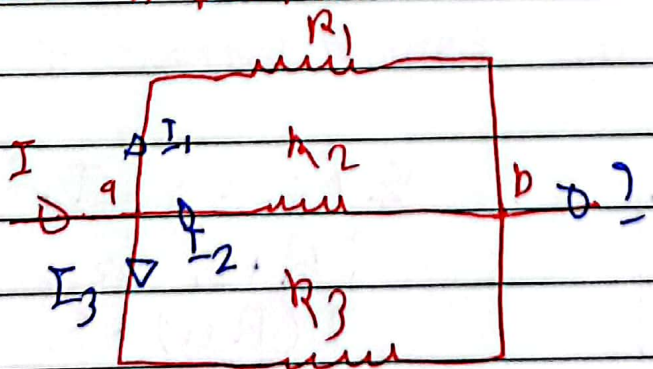
$\Rightarrow$  Resistors in series:



$$\Rightarrow R_{\text{eq}} = R_1 + R_2 + R_3 + \dots$$

$$V_{ab} = V_1 + V_2 + V_3 + \dots$$

$\Rightarrow$  Resistors in parallel:



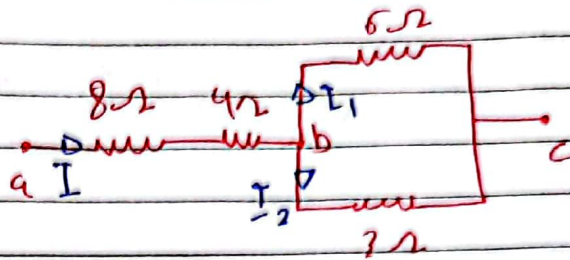
$$\Rightarrow \frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$



ex: (28.4): Four resistors are connected as shown in figure (28.14). (a) Find the equivalent resistance between points a and c.

$$R_{(8,4)} = 12 \Omega$$

$$\frac{1}{R_{(6,2)}} = \frac{1}{6} + \frac{1}{2} = \frac{2}{3} \Rightarrow R_{(6,2)} = \frac{3}{2} = 1.5 \Omega$$

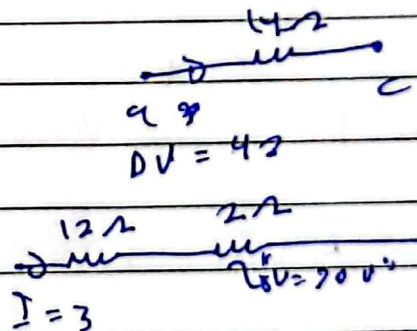


$$R_{(6,2)} = 1.5 \Omega \Rightarrow R_{eq} = 14 \Omega$$

(b) what is the current in each resistor if potential difference of "42V" is maintained between a and c?

$$\Rightarrow I = \frac{\Delta V}{R}$$

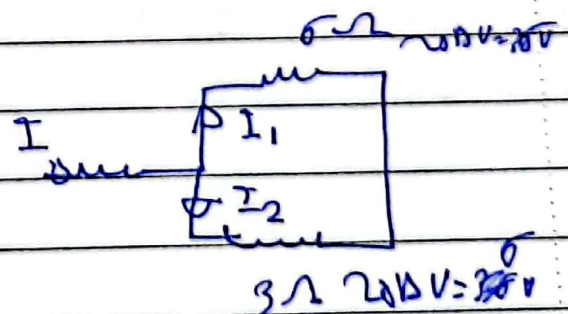
$$I = \frac{42}{14} = 3 \text{ A}$$



$$\Rightarrow R_{(2)} \Rightarrow \Delta V_{(2)} = (3)(2) = 6 \text{ V}$$

$$\Rightarrow I_2 = \frac{6}{3} = 2 \text{ A}$$

$$R_{(2)} \Rightarrow \Delta V_{(2)} = (3)(2) = 6 \text{ V}$$



$$\Rightarrow I_2 = \frac{6}{3} = 2 \text{ A}$$

$$I_1 = 1 \text{ A}$$



ex: (28.5): Three resistors are connected in parallel as shown in figure "B.11". A potential difference of "18V" is maintained between a and b.

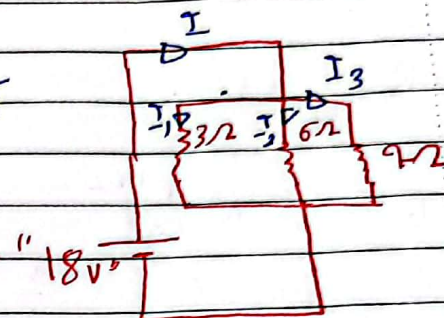
$$\frac{1}{R_{eq}} = \frac{1}{9} + \frac{1}{6} + \frac{1}{3} \times 2 = \frac{1}{9} + \frac{1}{2} = \frac{11}{18}$$

$$R_{eq} = \frac{18}{11} = 1.64 \Omega$$

$$\Rightarrow I = \frac{18}{1.64} = 11 \text{ A}$$

$$\Rightarrow I_1 = \frac{18}{3} = 6 \text{ A}, I_2 = \frac{18}{6} = 3 \text{ A}, I_3 = \frac{18}{9} = 2 \text{ A}$$

$$I_{tot} = I_1 + I_2 + I_3 = 11 \text{ A}$$



f) calculate the power delivered to each resistor and the total power delivered to the combination of resistors.

$$P_1 = I_1^2 R_1 = (6)(3) = 108 \text{ W}$$

$$P_2 = (3)(6) = 54 \text{ W}$$

$$P_3 = (2)(9) = 36 \text{ W}$$

$$P_{tot} = P_1 + P_2 + P_3 = 198$$

$$\text{or } P_{to} = \frac{(18)^2}{R_{eq}} = \frac{(18)^2}{1.64} = 198 \text{ W}$$



ex: (1): A battery has an emf "15 V". The terminal voltage of the battery is "11.6 V" when it is delivering 20 W of power to an external load resistor (R). (A) what is the value of "R"? (B) what is the internal resistance of the battery?

$$\Rightarrow R = \frac{DV}{I}$$

$$\begin{aligned}\Sigma &= 15 \text{ V} \\ DV &= 11.6 \\ P &= 20 \text{ W}\end{aligned}$$

$$\Rightarrow P = I \cdot V$$

$$\frac{20}{11.6} = I$$

$$I = 1.72 \text{ A}$$

$$\Rightarrow R = \frac{11.6}{1.72} = 6.74 \Omega$$

$$\Rightarrow DV = \Sigma - Ir$$

$$Ir = \Sigma - DV$$

$$r = \frac{\Sigma - DV}{I} = \frac{(15 - 11.6)}{1.72} = \frac{3.4}{1.72} = 1.97 \Omega$$

ex(2): An automobile battery has an emf of "12.6 V" and an internal resistance of  $0.08 \Omega$ . The headlights together are an equivalent resistance of "5  $\Omega$ ". what is the pV across on the headlight bulbs (a) when they are the only load on the battery :-

$$\begin{aligned}\Sigma &= 12.6 \text{ V} & r &= 0.08 & I &= \frac{\Sigma}{R+r} = \frac{12.6}{5.08} = 2.48 \text{ A} \\ R &= 5 \Omega\end{aligned}$$

$$DV = (I)(R) = (2.48)(5) = 12.4 \text{ V}$$



(B) When the starter motor is operated, with "35 A" of current in the motor:

$$\Rightarrow I_1 = I_2 + I_3$$

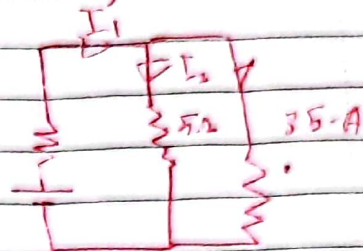
$$\Sigma -I_1 R - I_2 R = 0$$

$$\Sigma - (I_2 + 35)R - I_2 R = 0$$

$$12.0 = (I_2 + 35)(0.08) + I_2(5)$$

$$I_2 = 1.93 \text{ A}$$

$$V_{R2} = (1.93)(5) = 9.65 \text{ V}$$



ex: find currents  $I_1, I_2, I_3$  in the circuit shown in the figure:

$$I_3 = I_1 + I_2 \quad \text{--- (1)}$$

$$\Sigma V = 0$$

$$V_{bb} = 0 \Rightarrow -4I_2 - 14 + 6I_1 - 10$$

$$6I_1 - 4I_2 - 24 = 0 \quad \text{--- (2)}$$

$$V_{aa} = 0$$

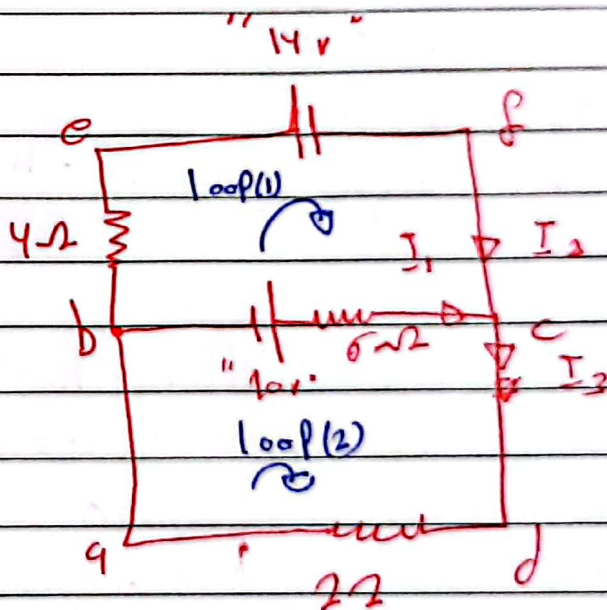
$$\Rightarrow 10 - 6I_1 - 2I_3 = 0$$

$$10 - 6I_1 - 2(I_1 + I_2) = 0$$

$$\Rightarrow 10 - 6I_1 - 2I_1 - 2I_2 = 0$$

$$-2 \times (10 - 8I_1 - 2I_2) = 0$$

$$-20 + 16I_1 + 4I_2 = 0$$





$$\Rightarrow -20 + 15 I_1 + 4 I_2 = 0$$

$$6 I_1 - 4 I_2 - 24 = 0$$

---

$$-44 + 22 I_1 = 0$$

$$22 I_1 = 44$$

$$I_1 = 2 \text{ A}$$

$$I_2 = -3 \text{ A}$$

$$I_3 = I_1 + I_2 = 5 \text{ A}$$



$$I_3 = I_1 + I_2 = 5A$$

ex(2):- Two "1.5V" batteries, with their positive terminal in the same directed are inserted in series into a flash light one battery has a resistance of  $0.255\Omega$  and the other is  $0.15\Omega$ , when the switch is closed, the bulb carries a current '600mA', (i) what is the bulb resistance (ii) what fraction of chemical energy transforms internal energy?

$$\textcircled{a} I = \frac{\sum \mathcal{E}}{\sum R}$$

$$\sum R = \frac{3}{0.6} = 5\Omega$$

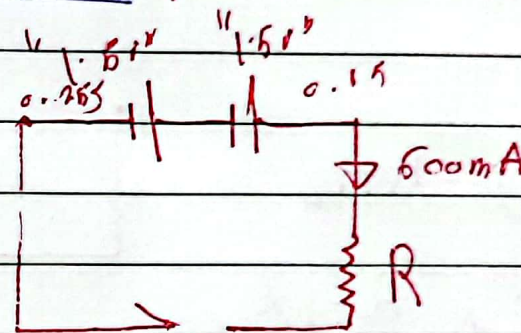
$$r_1 + r_2 + R = 5\Omega$$

$$R = 4.5\Omega$$

$$\textcircled{b} P = I \cdot V$$

$$= I^2 \cdot R = \text{to}$$

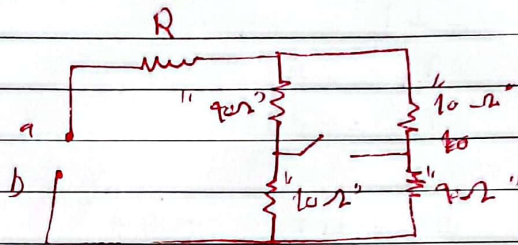
$$= (0.6)^2 (0.255 + 0.15) = 0.146W$$





ex: (14): (a) when the switch "S" in the circuit of Figure P28.14 is closed, will the equivalent resistance between points "a" and "b" increase or decrease? state your reasoning. (b) Assume the equivalent resistance drops by 50% when the switch is closed. Determine the value of "R".

before closing switch:



$$R_{b,10} = 100 \Omega$$

$$R_{b,10} = 10 \Omega$$

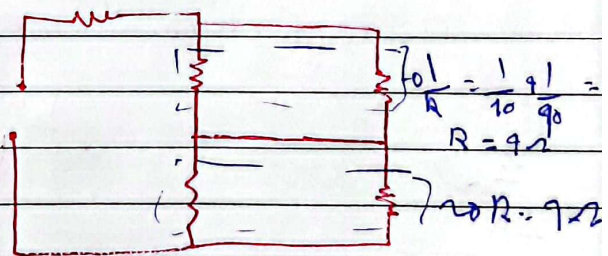
$$\frac{1}{R_{eq}} = \frac{1}{100} + \frac{1}{100}$$

$$R_{eq} = R +$$

$$R_{eq} = 50 \Omega$$

$$R_{eq} = 50 + R \quad (1)$$

after closing switch



$$(R_{eq}) \left( \frac{50}{100} \right) = R + 18$$

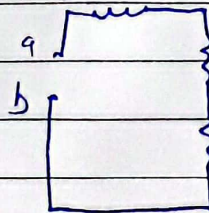
$$(R + 50) \left( \frac{50}{100} \right) = R + 18$$

$$\frac{R}{2} + 25 = R + 18$$

$$25 - 18 = \frac{R}{2}$$

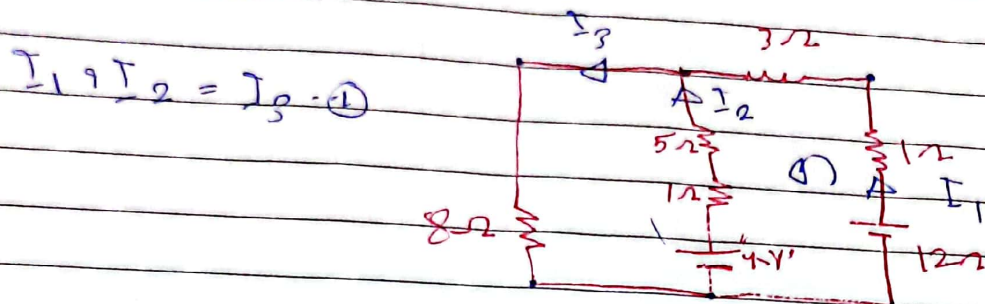
$$7 = \frac{R}{2}$$

$$R = 14 \Omega$$





ex: (22): Find Potential difference across each resistor:



$$I_1 + I_2 = I_3 \quad \text{--- (1)}$$

$$\Rightarrow 12 - 4I_1 + 6I_2 - 4 = 0 \quad \text{--- (2)}$$

$$8 - 4I_1 + 6I_2 = 0 \quad \text{--- (2)}$$

$$\Rightarrow 4 - 6I_2 - 8I_3 = 0 \quad \text{--- (3)}$$

$$\Rightarrow 4 - 6I_2 - 8I_1 - 8I_2 = 0$$

$$-16 + 8I_1 - 12I_2 = 0$$

$$-12 - 24I_2 = 0$$

$$I_2 = -0.5$$

$$I_1 = 1.25$$

$$I_3 = -0.5 + 1.25 = 0.75$$

$$\begin{aligned} * V_1 &= I_1 \cdot R \\ &= (1.25)(1) \end{aligned}$$

$$V_2 = (1.25)(3)$$

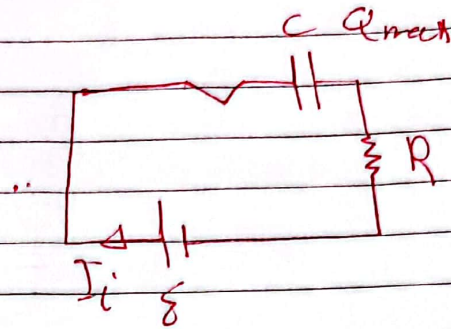
$$V_3 = (0.5)(5)$$



⇒ RC-circuit:

at  $t=0$ ,

$$I_i = \frac{\mathcal{E}}{R}$$



at  $t=\infty$

$$q = Q_{max}$$

$$I = 0$$

$$\mathcal{E} - IR - \frac{q}{C} = 0$$

$$IR = \frac{\mathcal{E}}{R} - \frac{q}{CR}$$

$$I = \frac{\mathcal{E}C - q}{RC}$$

$$I = -\left(\frac{q - \mathcal{E}C}{RC}\right)$$

$$I = \frac{dq}{dt} = -\left(\frac{q - \mathcal{E}C}{RC}\right)$$

$$\int \frac{dq}{q - \mathcal{E}C} = \int \frac{-dt}{RC}$$

$$\Rightarrow \ln(q - \mathcal{E}C) = \frac{-t}{RC}$$

$$\frac{q - \mathcal{E}C}{-\mathcal{E}C} = e^{\frac{-t}{RC}}$$

$$q - \mathcal{E}C = -\mathcal{E}C e^{\frac{-t}{RC}}$$

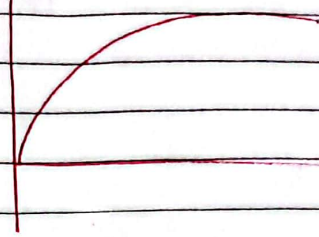
$$q = \mathcal{E}C - \mathcal{E}C e^{\frac{-t}{RC}} \Rightarrow q(t) = \mathcal{E}C(1 - e^{\frac{-t}{RC}})$$

$$\text{at } t=0 \quad q=0$$

$$t=\infty \quad q = \mathcal{E}C = Q_{max}$$

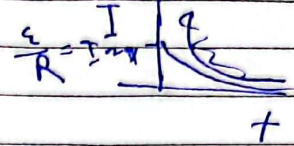


$q(t)$



\* Time constant

$$\tau = RC$$



ex(29): The ammeter shown in figure P28.29 reads "2A".  
Find (a)  $I_1$ ,  $I_2$  and (b)  $\mathcal{E}$ .

$$\Rightarrow 15 - 7I_1 - 2(5) = 0$$

$$7I_1 = 5$$

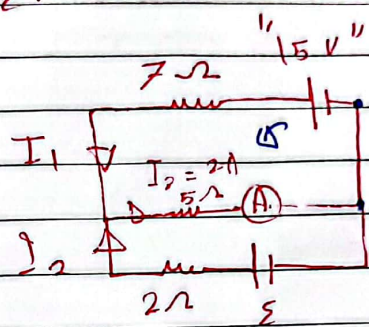
$$I_1 = \frac{5}{7} \text{ A}$$

$$I_2 = \frac{9}{7} \text{ A}$$

$$\Rightarrow \mathcal{E} - \left(\frac{9}{7}\right)(2) - (2)(5) = 0$$

$$\mathcal{E} = \left(2 + \frac{18}{7}\right) \text{ V}$$

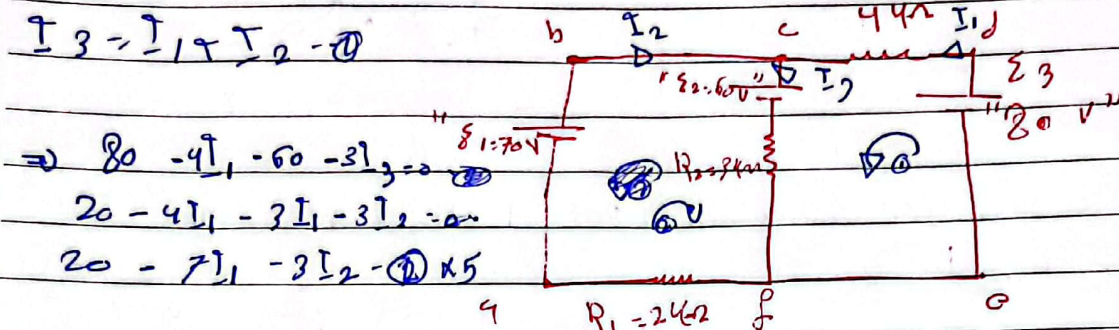
$$\mathcal{E} = 12.57 \text{ V}$$



$$I_3 = I_1 + I_2$$



ex: Using Kirchhoff's rules, (a) find the current in each resistor shown in figure p 28-31 and (b) find the potential difference between points c and f.



$$I_3 = I_1 + I_2 \quad \text{--- (1)}$$

$$\Rightarrow 80 - 4I_1 - 60 - 3I_3 = 0$$

$$20 - 4I_1 - 3I_1 - 3I_2 = 0$$

$$20 - 7I_1 - 3I_2 = 0 \quad \text{--- (2) } \times 5$$

$$\Rightarrow -60 - 3I_3 - 2I_2 + 70 = 0$$

$$10 - 3(I_1 + I_2) - 2I_2 = 0$$

$$-3 \times 10 - 3I_1 - 5I_2 = 0 \quad \text{--- (3)}$$

$$\Rightarrow -30 + 9I_1 + 15I_2 = 0$$

$$100 - 35I_1 - 15I_2 = 0$$

$$\cancel{130 - 26I_1 = 0}$$

$$\cancel{130 = 26I_1}$$

$$I_1 =$$

$$70 = 20I_1 \rightarrow$$

$$I_1 = \frac{70}{20}$$

$$\Rightarrow 100 - 35\left(\frac{70}{20}\right) - 15I_2 = 0$$

$$I_2 = \frac{5}{13} \text{ A}$$

$$I_3 = I_1 + I_2$$

$$= 3.2 \text{ A}$$

$$\Delta V = V_f - V_c$$

$$\Rightarrow \cancel{60} +$$

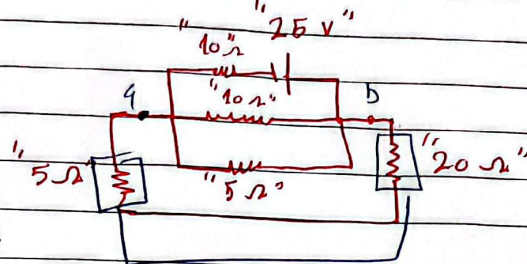
$$= -60 - 3I_3$$

$$= (-60) - (3)(3.2) \times 10^3 = -9660 \text{ V}$$



ex. Consider the circuit shown in figure P 28.9. Find (a) the current in the "20  $\Omega$ " resistor and (b) the potential difference between points a and b.

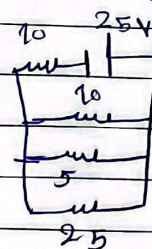
$\Rightarrow$



$$\frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{10} + \frac{1}{25}$$

$$R_{eq} = 2.94 \Omega$$

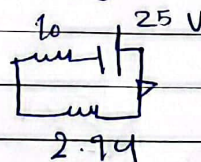
$$I = \frac{\Delta V}{R} = \frac{25}{2.94}$$



$$\Delta V_1 = \Delta V_2 = \Delta V_3$$

$$\Rightarrow -(2.94 + 10)I + 25 = 0$$

$$I = 1.93$$



$$\Rightarrow \Delta V = I R$$

$$= (1.93)(2.94) = 5.68 V$$

$$V(25) = 5.68$$

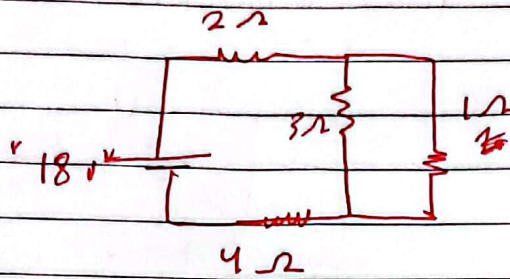
$$I = \frac{V(25)}{R} = \frac{5.68}{25} = 0.227 A$$

$$I(20) = 0.227 A \rightarrow \text{كس التيار}$$

$$I = 0.227 A \quad V_{ab} = 5.68 V$$

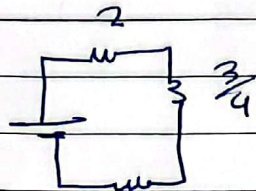


ex: (19): calculate the power delivered to each resistor in the circuit shown in Figure P2819.



$$\Rightarrow \frac{1}{R_{eq}} = \frac{1}{3} + 1$$

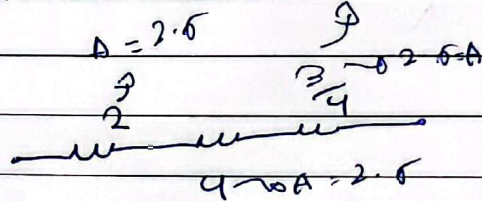
$$R_{eq} = \frac{3}{4} \Omega$$



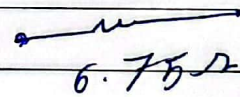
$$= 2.0 \left( \frac{3}{4} \right) = 1.5$$

$$PV = I \cdot R$$

$$R_{eq} = 2 + 4 + \frac{3}{4} = \frac{27}{4} = 6.75 \Omega$$



$$I = \frac{18V}{R} = \frac{18}{6.75} = \boxed{2.6A}$$



$$\Rightarrow P_{(2)} = I^2 \cdot R = (2.5)^2 (2) = 12.5 W$$

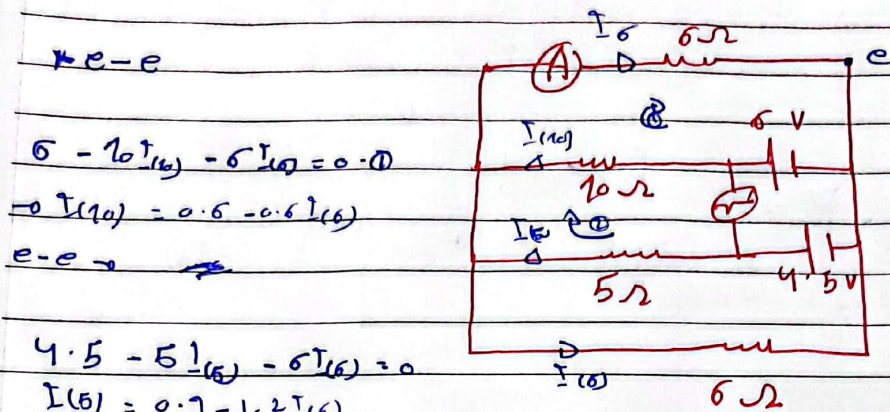
$$P_{(4)} = (2.5)^2 (4) = 25 W$$

$$P_3 = \frac{V^2}{R} = \frac{(1.5)^2}{3} = 0.75 W$$

$$P_1 = \frac{V^2}{R} = (1.5)^2 = 2.25 W$$

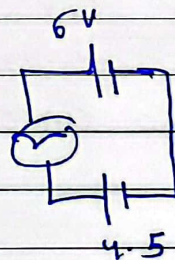


ex:- what are the expected readings of (a) the ideal ammeter and (b) the ideal voltmeter in Figure P28.25?



$$\Rightarrow 6 + PV - 4.5 = 0$$

$$PV = 1.5 \text{ V}$$





ex: In the circuit of figure p28.32, the current  $I_1 = 3A$  and the values of  $\mathcal{E}$  for the ideal battery and  $R$  are unknown. what are the currents (a)  $I_2$  and (b)  $I_3$ ? (c) can you find the values of  $\mathcal{E}$  and  $R$ ? If so, find their values. If not, explain.

$$\Rightarrow 24 - 6(3) + 3I_2 = 0$$

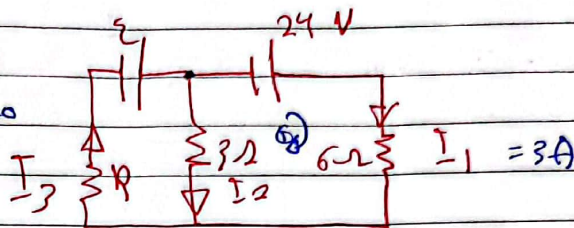
$$24 - 18 + 3I_2 = 0$$

$$6 = -3I_2$$

$$I_2 = -2A$$

$$I_2 = -2A$$

$$I_3 = 1A$$



$$I_1 + I_2 = I_3$$

$$3 - 2 = I_3$$

$$I_3 = 1A$$

ex: Can the circuit shown in figure p28.36 be reduced to a single resistor connected to a battery? explain.

calculate the currents (b)  $I_1$ , (c)  $I_2$  and (d)  $I_3$ .

$$I_3 = I_1 + I_2$$

$$I_1 = I_2 \quad \text{and} \quad I_1 - I_2 = I_3$$

$$\Rightarrow 24 - 6I_1 - 3I_2 = 0$$

$$12 - 6I_2 + 3I_3 = 0$$

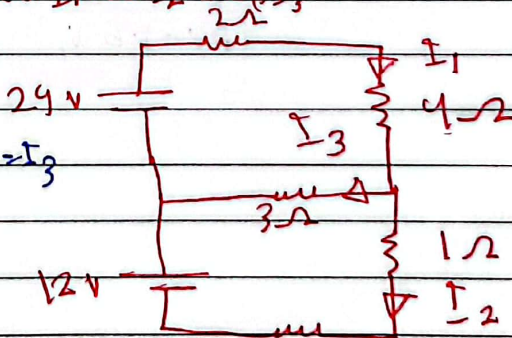
$$\Rightarrow 24 - 6I_1 - 3(I_1 - I_2) = 0$$

$$24 - 6I_1 - 3I_1 + 3I_2 = 0$$

$$\Rightarrow 12 - 6I_2 + 3I_1 - 3I_2 = 0$$

$$I_1 = 3.5A$$

$$I_3 = 1A$$



$$24 - 6I_1 + 3I_2 = 0$$

$$3(12 - 9I_2 + 3I_1) = 0$$

$$36 - 27I_2 + 9I_1 = 0$$

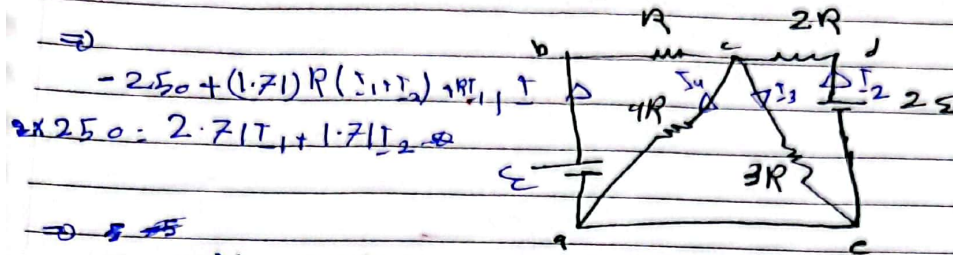
$$24 - 9I_1 + 3I_2 = 0$$

$$60 = 24I_2$$

$$I_2 = 2.5A$$



ex. (25): Taking  $R = 1.4 \Omega$  and  $\mathcal{E} = 250 \text{ V}$  in Figure P28.27. Determine the direction and magnitude of the current  $I$  in the horizontal wire between  $a$  and  $c$ .



$$\Rightarrow -250 + (1.71)R(I_1 + I_2) + 1.71I_1 = 0$$

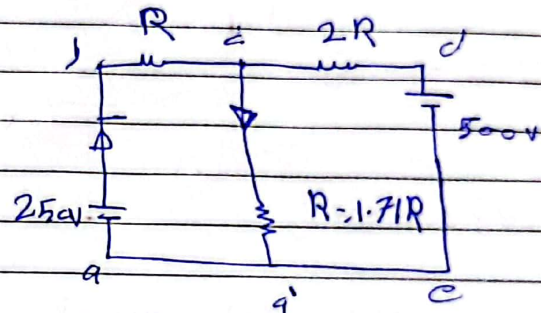
$$2 \times 250 = 2.71I_1 + 1.71I_2$$

$$\Rightarrow -500 + 2R I_2 + 1.71(I_1 + I_2) = 0$$

$$500 = 2I_2 + 1.71I_1 + 1.71I_2$$

$$500 = 3.71I_2 + 1.71I_1$$

$$500 = 5.42I_2 + 1.71I_1$$



$$\Rightarrow (250 = 2.71I_1 + 1.71I_2) \times 1.7$$

$$\Rightarrow 4.95I_1 + 2.89I_2 = 425 \quad (1)$$

$$\Rightarrow (1.7I_1 + 3.71I_2 = 500) \times (-2.7)$$

$$-4.95I_1 - 9.9I_2 = -1350$$

$$4.95I_1 + 2.89I_2 = 425$$

$$\frac{-10.9I_2}{10.9} = \frac{-925}{10.9}$$

$$I_2 = 130.1 \text{ mA}$$

$$I_2 = 10.1 \text{ mA}$$

$$V_{ca} = (I_1 + I_2)(1.7R)$$

$$= 240 \text{ V}$$

$$I_4 = \frac{240}{4R}$$

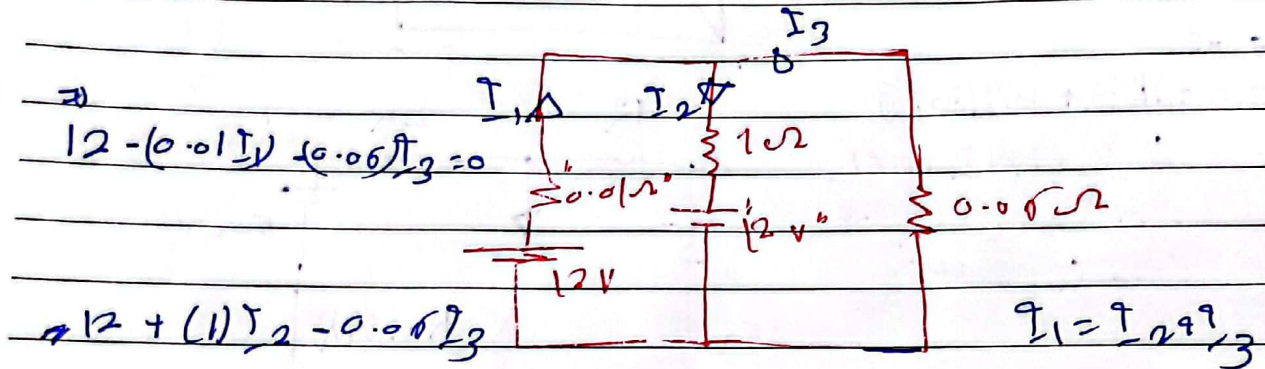
$$= 60 \text{ mA}$$

$$I_3 = \frac{240}{3R}$$

$$I_3 = 0.08 \text{ mA}$$



ex: Jumper cables are connected from a fresh battery in one car to charge a dead battery in another car. Figure shows the circuit diagram for this situation. While the cables are connected, the ignition switch of the car with dead battery is closed and the starter is activated to start the engine. Determine the current in (a) the starter and (b) the dead battery.



$$\Rightarrow 12 - 0.01(I_2 + I_3) - 0.06I_3 = 0$$

$$12 - 0.01I_2 - 0.07I_3 = 0$$

$$0.01 \times 12 + I_2 - 0.06I_3 = 0$$

$$0.12 + I_2 - 0.06I_3 = 0 \quad \times$$

$$12 \cdot 12 = 0.0706I_3$$

$$I_3 = 1.72 \text{ A}$$

$$I_2 = -1.7 \text{ A} = 1.7 \text{ A} \sim \uparrow$$



ex(37): An uncharged capacitor and a resistor are connected in series a source of emf. If  $\mathcal{E} = 9\text{V}$ ,  $C = 20\text{ }\mu\text{F}$ ,  $R = 100\text{ }\Omega$ , find (a) the time constant of circuit, (b) the maximum charge on the capacitor and (c) the charge on the capacitor at a time equal to one time constant after the battery is connected.

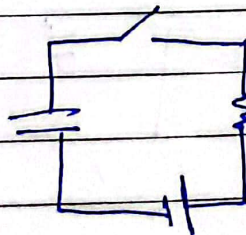
$$\Rightarrow \tau = R \cdot C = (100)(20\text{ }\mu\text{F}) = 2\text{ ms}$$

time constant

$$\Rightarrow Q_{\text{max}} = \mathcal{E} \cdot C = (9\text{V})(20\text{ }\mu\text{F}) = 1.8 \times 10^{-4}\text{ C}$$

$$\begin{aligned} \Rightarrow q(t) &= \mathcal{E} \cdot C \left(1 - e^{-\frac{t}{R \cdot C}}\right) \\ &= 1.8 \times 10^{-4} \left(1 - e^{-\frac{R \cdot C}{R \cdot C}}\right) \\ &= 1.8 \times 10^{-4} \left(1 - \frac{1}{e}\right) = 1.14 \times 10^{-4}\text{ C} \end{aligned}$$

ex: consider a series RC circuit as in figure p 2838 for which  $R = 1\text{ M}\Omega$ ,  $C = 5\text{ }\mu\text{F}$  and  $\mathcal{E} = 30\text{ V}$ . Find (a) the time constant of the circuit and (b) the maximum charge on the capacitor after the switch is thrown closed. (c) Find the current in the resistor  $10\text{ s}$  after the switch is closed.



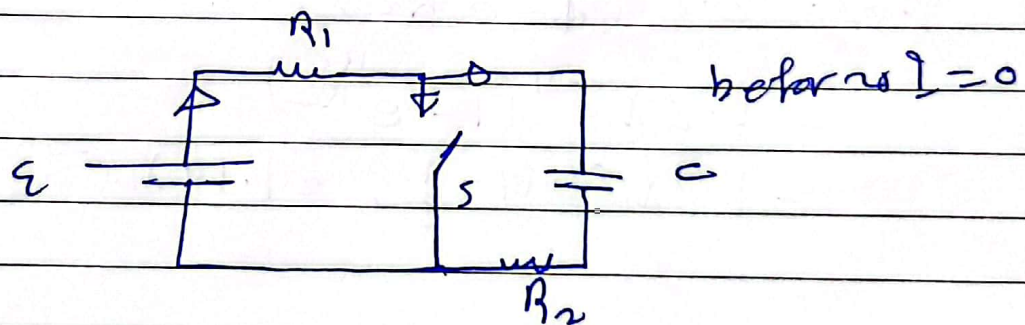


$$\Rightarrow \tau = C \cdot R = (1 \times 10^{-6}) (5 \times 10^{-6}) = 5 \text{ s}$$

$$\Rightarrow Q_{\max} = \xi \cdot C = (5 \times 10^{-6}) (30) = 150 \text{ C}$$

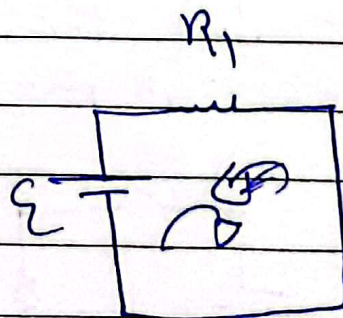
$\Rightarrow$

ex: In the circuit of figure p28.41, the switch "S" has been open for a long time. It is then suddenly closed. Take  $\xi = 10 \text{ V}$ ,  $R_1 = 50 \text{ k}\Omega$ ,  $R_2 = 100 \text{ k}\Omega$ ,  $C = 10 \mu\text{F}$ . Determine the time constant (a) before the switch is closed and (b) after the switch is closed. (c) Let the switch be closed at  $t=0$ . Determine the current in the switch as a function of time.



before  $\tau = (R_1 + R_2) C = (1.5 \times 10^5) (10 \times 10^{-6}) = 1.5 \text{ s}$

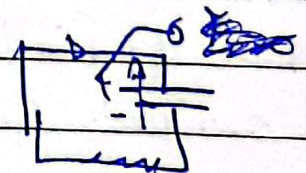
after  $\tau = R_2 C = (1 \times 10^5) (10 \times 10^{-6}) = 1 \text{ s}$



$$\xi - R_1 I_1 = 0$$

$$\xi = R_1 I_1$$

$$I_1 = \frac{\xi}{R_1} = \frac{10}{50 \times 10^3} = 200 \text{ A}$$



$$\Rightarrow \frac{\xi}{C} - I_2 R_2 = 0$$

$$\frac{\xi}{C} = I_2 R_2$$

$$I_2 = \frac{\xi}{C \cdot R_2}$$



$$I_2 = \frac{q}{R_2 C} = \frac{\epsilon \cdot \epsilon_0}{R_2 C}$$

$$= \frac{\epsilon}{R_2}$$

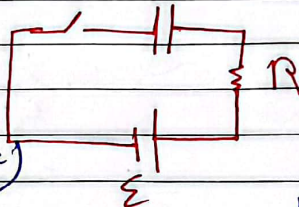
$$= \frac{10}{1000} = 10^{-4} \text{ A}$$

$$I \rightarrow 0 = \frac{20}{200000} = 10^{-4} \text{ A}$$

$$Q_{\max} = \epsilon \cdot C$$

$\Rightarrow$  RC-circuit:

charging



$$q(t) = Q_{\max}(1 - e^{-t/RC})$$

$$\epsilon \cdot C$$

$$I(t) = I_0 e^{-t/RC}$$

$$I_{\max}$$

$$Q_{\max} = \epsilon \cdot C$$

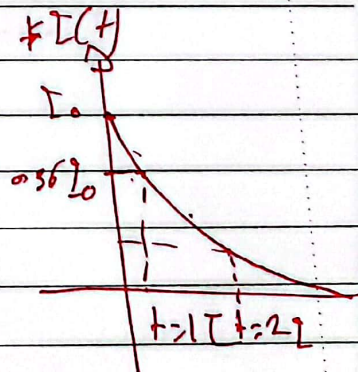
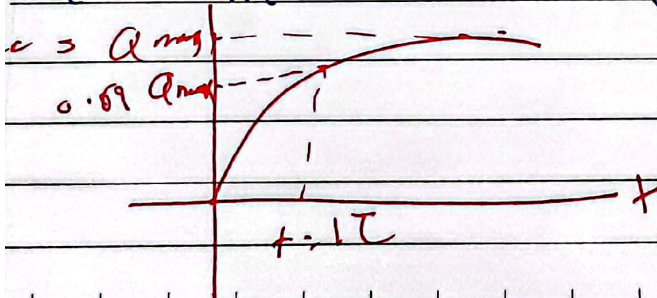
$$I_{\max(0)} = \frac{\epsilon}{R}$$

$R \cdot C = \tau$  Time constant or circuit constant

\* at  $t = \tau = RC$

$$I = I_0 e^{-t/RC} = I_0 e^{-RC/RC} = \frac{I_0}{e}$$

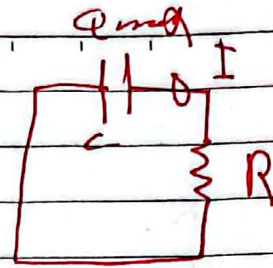
$$Q = Q_{\max}(1 - e^{-1}) = Q_{\max}(1 - \frac{1}{e})$$





Discharging

$$I = -\frac{q}{R \cdot C}$$



$$\frac{dq}{dt} = -\frac{q}{R \cdot C}$$

$$\int \frac{dq}{q} = \int \frac{dt}{R \cdot C}$$

$$q_i(\text{max})$$

$$\ln \frac{q}{q_i} = -\frac{t}{R \cdot C} \quad \leadsto \quad \ln \frac{q}{q_i} = -\frac{t}{R \cdot C}$$

$$\frac{q}{q_i} = e^{-\frac{t}{R \cdot C}}$$

$$q(t) = q_i e^{-\frac{t}{R \cdot C}}$$

$$I(t) = -\frac{q_i}{R \cdot C} e^{-\frac{t}{R \cdot C}}$$

$$I(t) = I_{\text{max}} e^{-\frac{t}{R \cdot C}}$$

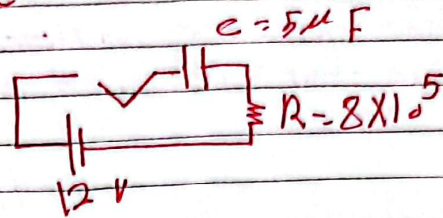
$$I = \frac{Q_i}{\text{Time } (\tau = R \cdot C)}$$



ex: an uncharged capacitor and a resistor are connected as shown in the figure

① time constant:-

$$\tau = RC = (8 \times 10^5)(5 \times 10^{-6}) = 4 \text{ s}$$



② Maximum current in the circuit:-

$$I_0 = \frac{\mathcal{E}}{R} = \frac{12}{8 \times 10^5} = \frac{12}{8} \times 10^{-5} \text{ A}$$

③ Maximum charge on capacitor:-

$$Q_{\text{max}} = \mathcal{E} \cdot C = (12)(5 \times 10^{-6}) = 60 \mu\text{C}$$

④ charge and current after two <sup>time</sup> constants?

$$\begin{aligned} Q(t) &= Q_{\text{max}} \left( 1 - e^{-\frac{t}{RC}} \right) \\ &= Q_{\text{max}} \left( 1 - e^{-\frac{2RC}{RC}} \right) \\ &= (60 \mu\text{C}) \left( 1 - \frac{1}{e^2} \right) \end{aligned}$$

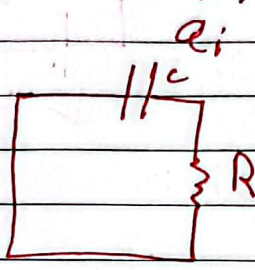
$$\begin{aligned} I &= I_0 e^{-\frac{t}{RC}} \\ &= \frac{12}{8} \times 10^{-5} \left( \frac{1}{e^2} \right) \end{aligned}$$



ex: Consider a capacitor of capacitance  $C = 10 \mu F$  that being discharged through a resistor of  $R = 6 \times 10^5 \Omega$  :-

① how many time constants is the charge on capacitor is one-fourth its initial charge?

$$q(t) = Q_{max} e^{-t/Rc}$$

$$\frac{Q_c}{4} = Q_i e^{-t/Rc}$$


$$q(t) = \frac{Q_c}{4}$$

$$\ln \frac{1}{4} = -\frac{t}{Rc}$$

$$R \cdot c \ln 4 = t$$

$$\rightarrow \ln 4 = \frac{t}{Rc}$$

$$t = (1.39)(6 \times 10^5)(10 \times 10^{-6})$$

$$= (60)(1.39) \text{ s} \approx 84 \text{ s}$$

② After how many time constants the energy stored in the capacitor is one-fourth its initial value?

$$U_i = \frac{Q_i^2}{2C}$$

$$U_f = \frac{1}{4} U_i$$

$$U_f = \frac{q_f^2}{2C}$$

$$= \frac{(Q_i e^{-t/Rc})^2}{2C}$$

$$\frac{Q_i^2 e^{-2t/Rc}}{2C} = \frac{1}{4} \frac{Q_i^2}{2C}$$

$$e^{-2t/Rc} = \frac{1}{4}$$

$$\ln e^{-2t/Rc} = \ln \frac{1}{4}$$

$$U_f = \frac{Q_i^2 \cdot e^{-2t/Rc}}{2C}$$

$$+2 \frac{t}{R \cdot c} = \ln 4$$

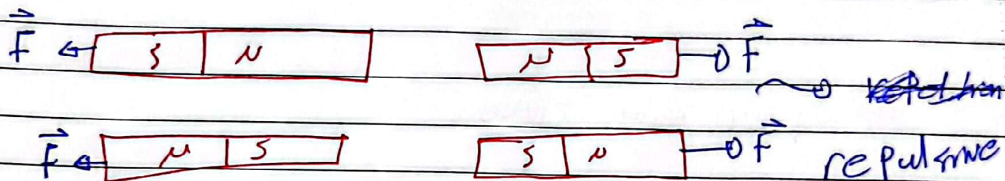
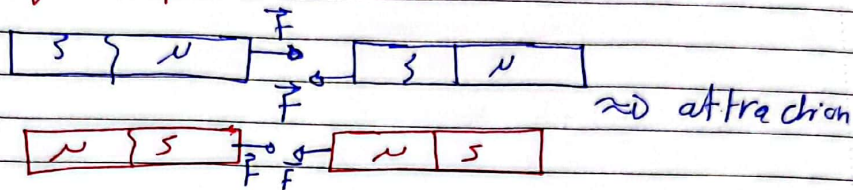
$$t = \frac{Rc}{2} \ln 4$$

$$= \frac{1.39}{2} (6 \times 10^5)(10 \times 10^{-6}) \approx 42 \text{ s}$$



## Ch(29): Magnetic Fields

### ~ Magnetic poles:

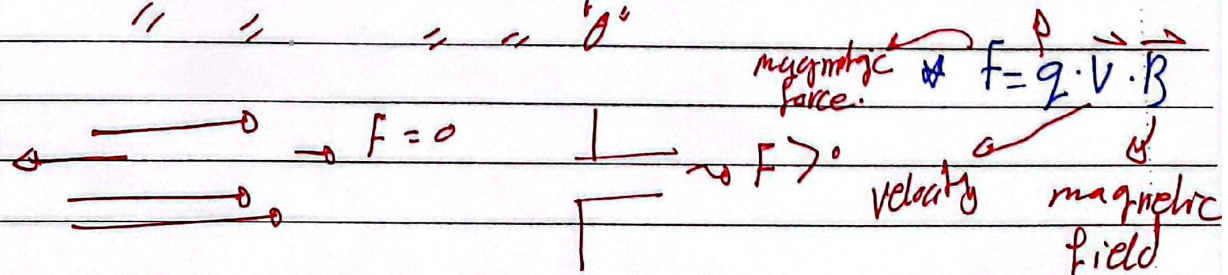


### \* Similarities between magnetic and electric force

- ① The magnetic force is proportional to the charge  $q$  of the particle.
- ② The "m.f" on a negative charge is directed opposite to the force on positive charge moving in the same direction.
- ③ The "m.f" is 'pro' to magnitude of magnetic field vector  $B$ .

~ The "m.f" depends on speed of the particle particle charge

" " " " " " " "





$$\Rightarrow \vec{F}_B = q \cdot \vec{v} \times \vec{B}$$

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$* \vec{A} \times \vec{B} = \vec{C} \sim \vec{C} = |\vec{A}| |\vec{B}| \sin(\theta)$$

$$|\vec{A}| = \sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2}$$

$$|\vec{B}| = \sqrt{(B_x)^2 + (B_y)^2 + (B_z)^2}$$

$$\vec{C} = \vec{A} \times \vec{B}$$

$$\vec{C} = \hat{i} (A_y B_z - A_z B_y)$$

$$- \hat{j} (A_x B_z - B_x A_z)$$

$$+ \hat{k} (A_x B_y - A_y B_x)$$

|             |       | $\hat{k}$ |
|-------------|-------|-----------|
| $( \quad )$ | $A_y$ | $A_z$     |
| $B_x$       | $B_y$ | $B_z$     |



$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 3 \\ 1 & 3 & -2 \end{vmatrix}$$

$$\vec{A} = 2\hat{i} + 3\hat{k}$$

$$\vec{B} = \hat{i} + 3\hat{j} - 2\hat{k}$$

$$\vec{C} = \hat{i}(0 - 9) - \hat{j}(-4 - 3) + \hat{k}(6)$$

$$= -9\hat{i} + 7\hat{j} + 6\hat{k}$$

$$\Rightarrow \hat{i} \cdot \hat{j} = \hat{k}$$

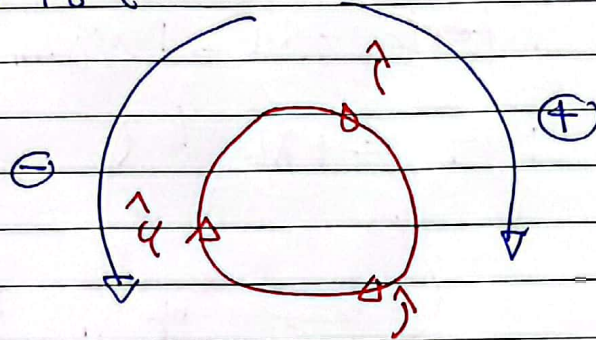
$$\hat{j} \cdot \hat{k} = \hat{i}$$

$$\hat{k} \cdot \hat{i} = \hat{j}$$

$$\Rightarrow \hat{k} \cdot \hat{j} = -\hat{i}$$

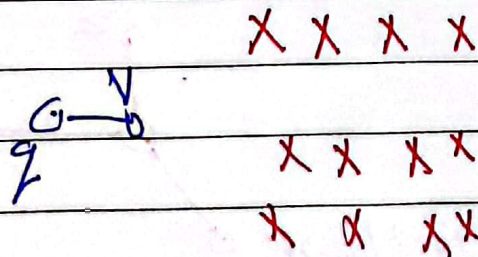
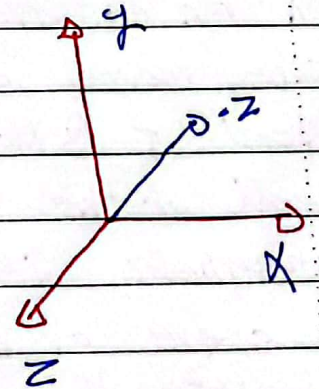
$$\hat{i} \cdot \hat{k} = -\hat{j}$$

$$\hat{j} \cdot \hat{i} = -\hat{k}$$



$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

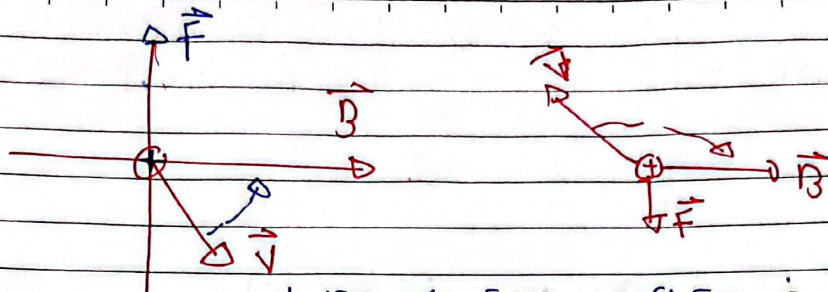
\* (x) → into the page  
 ○ → out of the page



if  $q(+)$   $\vec{F} \uparrow$   
 if  $q(-)$   $\vec{F} \downarrow$

$$\vec{F} = q \vec{v} \times \vec{B}$$

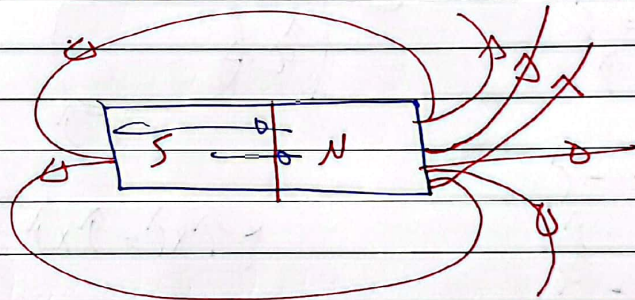




لقد تم تعريف القوة المغناطيسية على أنها القوة التي تؤثر على الجسيم المشحون المتحرك في المجال المغناطيسي

$$1 \text{ tesla} = 1 \text{ T} = 1 \text{ N/A} \cdot \text{m}$$

$$1 \text{ G} = 1 \times 10^{-4} \text{ T}$$



ex: An electron in an old-style television picture tube moves toward the front of the tube with a speed of  $8 \times 10^5 \text{ m/s}$  along the x-axis. Surrounding the neck of the tube are coils of wire that create a magnetic field of magnitude  $0.025 \text{ T}$  directed at an angle of  $60^\circ$  to the x-axis and lying in the xy-plane. Calculate the magnetic force on the electron.

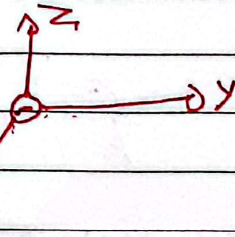
$$v = 8 \times 10^5, q = 1.6 \times 10^{-19}$$

$$B = 0.025 \text{ T}$$

$$\theta = 60^\circ$$

$$\vec{F}(B) = (1.6 \times 10^{-19})(8 \times 10^5)(0.025) \sin(60^\circ)$$

$$= 2.3 \times 10^{-14} \text{ N} (-y)$$





ex: A proton moves with a velocity of  $\vec{v} = 2\hat{i} - 4\hat{j} + \hat{k}$  in a region where the magnetic field is  $\vec{B} = (\hat{i} + 2\hat{j} - \hat{k})$  T. What is the magnitude of the magnetic force this particle experiences?

$$q = 1.6 \times 10^{-19}, \quad \vec{v} = 2\hat{i} - 4\hat{j} + \hat{k}, \quad \vec{B} = (\hat{i} + 2\hat{j} - \hat{k})$$

$$\vec{C} = 4\hat{i} + 2\hat{j} + 4\hat{k} + 4\hat{i} + \hat{j} - 2\hat{k}$$

$$\vec{C} = 8\hat{i} + 3\hat{j} + 2\hat{k}$$

$$|\vec{v}| = \sqrt{2^2 + (-4)^2 + 1^2}$$

$$|\vec{B}| = \sqrt{1^2 + 2^2 + (-1)^2}$$

$$\Rightarrow |\vec{F}| = |q| |\vec{v}| |\vec{B}| \sin(\theta)$$

$$\vec{F}_B = (1.6 \times 10^{-19})(2\hat{i} + 3\hat{j} + 8\hat{k})$$

$$\vec{F}_B = (8.2\hat{i} + 4.8\hat{j} + 12.8\hat{k}) \times 10^{-19} \text{ N}$$

$$|\vec{F}| = \sqrt{(8.2)^2 + (4.8)^2 + (12.8)^2} \times 10^{-19}$$

ex: A proton travels with a speed of  $5.02 \times 10^5$  m/s in a direction that makes an angle of  $60^\circ$  with the direction of magnetic field of magnitude  $0.18$  T in the positive x direction. What are the magnitudes of (a) the magnetic force on the proton and (b) the proton's acceleration?

$$v = 5.02 \times 10^5, \quad B = 0.18, \quad \theta = 60^\circ$$

$$F = q \cdot v \cdot B \cdot \sin(\theta) = (1.6 \times 10^{-19})(5.02 \times 10^5)(0.18)(\sin(60^\circ))$$

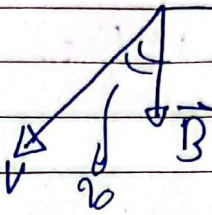
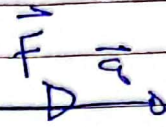


ex: A proton moves perpendicular to a uniform magnetic field  $B$  at a speed of  $1 \times 10^7 \text{ m/s}$  and experiences an acceleration of  $2 \times 10^{13} \text{ m/s}^2$  in the positive "x" direction when its velocity is in the positive "z" direction. Determine the magnitude and direction of the field.

$$v = 1 \times 10^7$$

$$a = 2 \times 10^{13}$$

$$q = 1.6 \times 10^{-19}$$

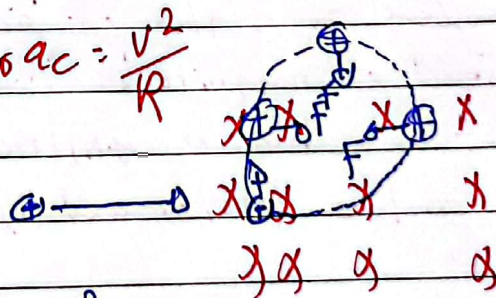


$$F = ma = q \cdot v \cdot B \sin(\theta)$$

$$B = \frac{ma}{q \cdot v \sin(\theta)} = \frac{(1.67 \times 10^{-27})(2 \times 10^{13})}{(1.6 \times 10^{-19})(1 \times 10^7)} = 2 \times 10^{-2} \text{ T} \quad (-\hat{y})$$

⇒ Motion of charged particles in a magnetic field

$$F_{(B)} = q \cdot v \cdot B = \frac{mv^2}{R}$$



$$\Rightarrow q \cdot v \cdot B \sin(90^\circ) = \frac{m \cdot v^2}{R}$$

$$q \cdot v \cdot B = \frac{m \cdot v^2}{R}$$

$$\text{radius } R = \frac{mv}{q \cdot B}$$



$$\Rightarrow \omega = \frac{v}{R} = \frac{qB}{m} \rightarrow \text{cyclotron frequency}$$

$$T_{\text{period}} = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

ex:- A proton is moving in a circular orbit of radius 14 cm in a uniform 0.35 T magnetic field perpendicular to the velocity of the proton. Find the speed of the proton.

$$R = 14 \times 10^{-2} \text{ m}, \quad B = 0.35 \text{ T}, \quad q = 1.6 \times 10^{-19} \text{ C}, \quad m = 1.67 \times 10^{-27} \text{ kg}$$

$$\begin{aligned} \frac{v}{R} &= \frac{qB}{m} \Rightarrow v = \frac{qB \cdot R}{m} \\ &= \frac{(1.6 \times 10^{-19})(0.35 \text{ T})(0.14)}{1.67 \times 10^{-27} \text{ kg}} \\ &= 4.7 \times 10^6 \text{ m/s} \end{aligned}$$



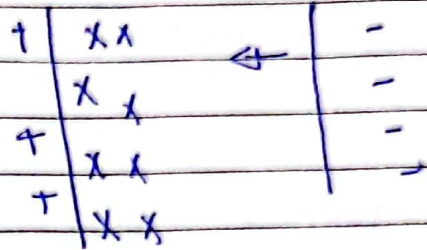
ex (29.3):  $V_i = 0$ ,  $\Delta V = 350 \text{ V}$ ,  $v_f = ??$ ,  $R = 7.5 \times 10^{-2} \text{ m}$

(A) what is the magnitude of the magnetic field?

$$q = -1.6 \times 10^{-19}$$

$$m = 9.11 \times 10^{-31} \text{ kg}$$

$$B = \frac{m v}{q R}$$



$$\Rightarrow \Delta K + \Delta U = 0$$

$$\frac{1}{2} m (v_f^2 - v_i^2) + q \Delta V = 0$$

$$v_f^2 = \frac{-2q \Delta V}{m} \Rightarrow v_f = \sqrt{\frac{-2q \Delta V}{m}}$$

$$v_f = \sqrt{\frac{(-2)(-1.6 \times 10^{-19})(350 \text{ V})}{9.11 \times 10^{-31}}} = 1.11 \times 10^7 \text{ m/s}$$

$$B = \frac{(9.11 \times 10^{-31})(1.11 \times 10^7)}{(1.6 \times 10^{-19})(0.075)} = 8.4 \times 10^{-4} \text{ T}$$

(B) what is the angular speed of the electrons?

$$\omega = \frac{v}{R} = \frac{1.11 \times 10^7}{0.075} = 1.5 \times 10^8 \text{ rad/s (Hz)}$$



# Physics (102)

Ex: (13): An electron moves in a circular path perpendicular to a uniform magnetic field with a magnitude of 2 mT. If the speed of the electron is  $1.5 \times 10^7 \text{ m/s}$ , determine (a) the radius of the circular path and (b) the time interval required to complete one revolution:

$$\rightarrow v = 1.5 \times 10^7 \text{ m/s}, B = 2 \times 10^{-3} \text{ T}, q = 1.6 \times 10^{-19} \text{ C}$$

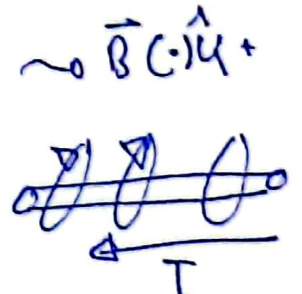
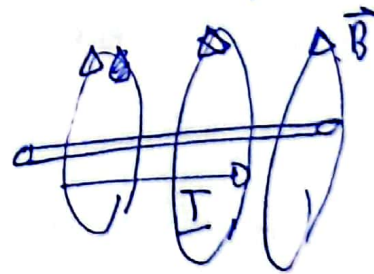
$$m = 9.1 \times 10^{-31} \text{ kg}$$

$$\rightarrow R = \frac{mv}{qB} = \frac{(9.1 \times 10^{-31})(1.5 \times 10^7)}{(2 \times 10^{-3})(1.6 \times 10^{-19})} = \frac{13.65 \times 10^{-2}}{3.2} \text{ m}$$

$$= 4.3 \times 10^{-2} \text{ m}$$

$$T = \frac{2\pi R}{v} = \frac{2(\pi)(4.3 \times 10^{-2})}{1.5 \times 10^7} = \frac{27 \times 10^{-9}}{1.5} = 18 \times 10^{-9} \text{ s}$$

$\Rightarrow$  Magnetic field of a straight current-carrying wire:



$\Rightarrow$  Force on a current-carrying conductor

$$F = (nAL)(qvB) = (nqvA)(LB)$$

$v \equiv$  drift velocity

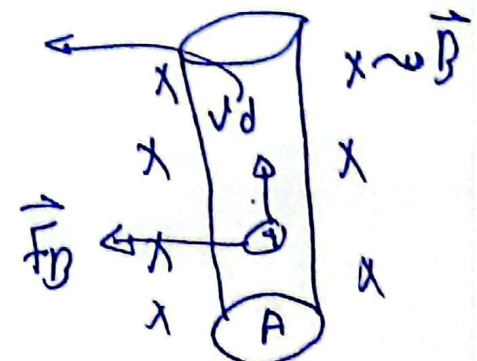
$$F = I \vec{L} \times \vec{B}$$

$$= ILB \sin(\theta)$$

$$n = \frac{q}{v}$$

$$q = nvA$$

$$= nAL$$





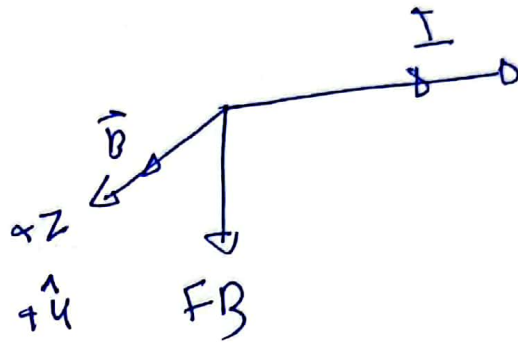
ex(33):  $I = 15 \text{ A}$   $\hat{x}$  ,  $\frac{F}{L} = 0.12 \text{ N/m}$  ( $-y$ )

(a) the magnitude and (b) the direction of the magnetic field in the region through which the current passes.

(a)  $\frac{F}{L} = I B \sin(90)$

$B = \left(\frac{F}{L}\right) \cdot \frac{1}{I}$

$= (15)(0.12) = 8 \times 10^{-3} \text{ T}$   $\hat{y}$



ex(34): " $L = 2.8 \text{ m}$ " " $I = 5 \text{ A}$ " ,  $\vec{B} = 0.39 \text{ T}$ . calculate the magnitude of the magnetic force on the wire assuming the angle between the magnetic field and the current is (a)  $60^\circ$  (b)  $90^\circ$  and (c)  $120^\circ$

$F = I \vec{L} \times \vec{B} = I L B \sin(\theta)$

$I L B = 5.46$

$F_1 = 5.46 \sin(60) = 4.7 \text{ N}$   $\hat{y}$

(b)  $F_2 = 5.46 \sin(90) = 5.46 \text{ N}$   $\hat{y}$

(c)  $F_3 = 5.46 \sin(120) = 4.7 \text{ N}$   $\hat{y}$



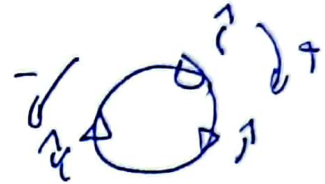
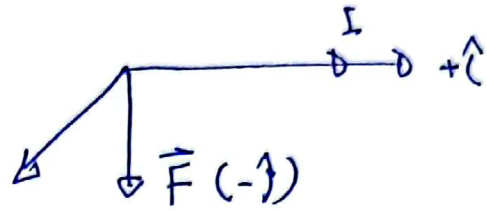
ex(35):  $I = 2.4 \text{ A}$   $L = 0.75 \text{ m}$   $\vec{B} = 1.6 \hat{y} \text{ T}$

If the current is in the " $+\hat{x}$ ", what is the magnetic force on the section of wire?

$$\vec{F} = I \vec{L} \times \vec{B} = I L(\hat{x}) \times B \hat{y}$$

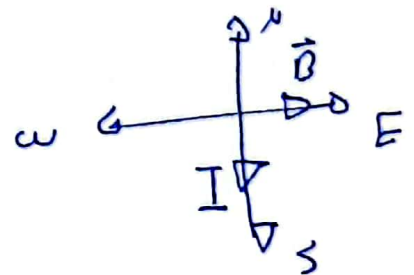
$$= (2.4)(0.75)(1.6) \hat{x} \times \hat{y}$$

$$= -2.9 \hat{z} \text{ N}$$



ex(39):  $\frac{m}{L} = 0.5 \text{ g/cm}$ ,  $I = 2 \text{ A}$  south what are (a) the direction and (b) the magnitude of the minimum magnetic field needed to lift this wire vertically upward?

$$\frac{m}{L} = \frac{0.5 \times 10^{-3} \text{ kg}}{1 \times 10^{-2}} = 0.5 \times 10^{-1} \text{ kg/m} = 0.05 \text{ kg/m}$$



$$\vec{F} = I \vec{L} \times \vec{B}$$

$$\sum F = 0$$

$$I L B \sin(90) = mg$$

$$B = \frac{mg}{IL} = \frac{(0.05 \times 10^{-1})(10)}{2} = 2.5 \times 10^{-1} \text{ T} = 0.25 \text{ T}$$

east direction



ex: (37):  $m = 0.72 \text{ kg}$   $R = 6 \times 10^{-2} \text{ m}$

$d = 12 \times 10^{-2} \text{ m}$ ,  $L = 45 \times 10^{-2}$ ,  $I = 48 \text{ A}$ ,  $\vec{B} = 0.24 \text{ T}$

$v_i = 0$  find  $v_f$

$\Rightarrow v_f^2 = v_i^2 + 2a\Delta x$

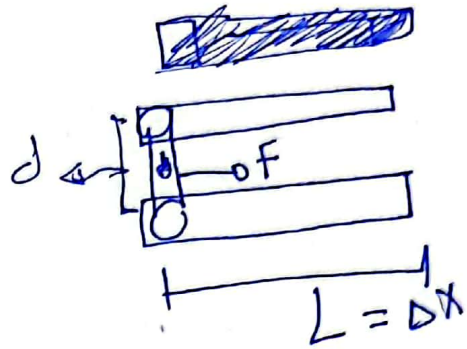
$v_f = \sqrt{2a\Delta x}$   
??

$F = I\vec{L} \times \vec{B}$

$\Sigma F = ma = ILB$

$a = \frac{ILB}{m} = \frac{(48)(0.24)(12 \times 10^{-2})}{0.72} = 192 \text{ m/s}^2$

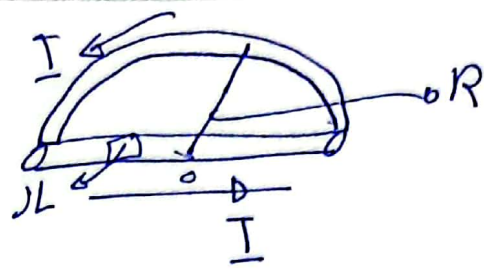
$v_f = \sqrt{(2)(192)(0.45)} = 13.14 \text{ m/s}$





ex: (29. 4):

find the magnitude and direction of the magnetic force acting on the straight portion of the wire and on the curved portion



~ straight portion:

$$\begin{aligned}\vec{F} &= I \, d\vec{L} \times \vec{B} = I \, dL \, B \sin(90^\circ) \\ &= I B \int_{-R}^R dL = I B L \Big|_{-R}^R \\ &= I B L (2R)\end{aligned}$$

~ curved portion

$$\begin{aligned}dF &= I \, d\vec{L} \times \vec{B} = I \, dL \, B \sin(\theta) \\ &= I R \, d\theta \cdot B \sin(\theta) \\ &= I R B \int_0^\pi \sin(\theta) \cdot d\theta \\ &= I R B (-\cos(\theta)) \Big|_0^\pi\end{aligned}$$

$$\begin{aligned}L &= R\theta \\ dL &= R \, d\theta\end{aligned}$$

$$= -I R B (\cos(\pi) - \cos(0)) = 2 I R B \hat{y}$$

لم تنته الحكاية... غذا تطير العصفير