



TurbolEG.com



دفتر :

اساسيات الدوائر الكهربائية

Circuit

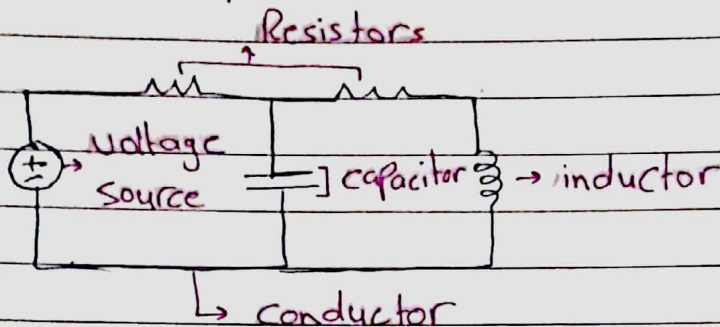
للمطالبة : راما خضير

اللجنة الأكاديمية لقسم الهندسة الصناعية

2023



Basic component and electrical circuit :-



Unit and scale :-

→ any measurable quantity can be describe by number and unit.

num $\leftarrow$ meter $\rightarrow$ unit

- | | | |
|------------------------------------|---|----------------------------------|
| * length $\rightarrow$ meter "m" | { | * temp $\rightarrow$ kelvin "K" |
| * mass $\rightarrow$ kilogram "kg" | | * work $\rightarrow$ Joule "J" |
| * time $\rightarrow$ second "s" | | * power $\rightarrow$ watt "W" |
| * current $\rightarrow$ Amper "A" | | * voltage $\rightarrow$ Volt "V" |

Prefixes :-

SI, L, W

- | | |
|--------------------------------------|--|
| * 10^{-18} $\rightarrow$ atto "a" | * 10^{-6} $\rightarrow$ micro " μ " |
| * 10^{-15} $\rightarrow$ femto "f" | * 10^{-3} $\rightarrow$ micro milli "m" |
| * 10^{-12} $\rightarrow$ pico "p" | * 10^{-2} $\rightarrow$ centi "c" |
| * 10^{-9} $\rightarrow$ nano "n" | |

- Ex :-
- ① $0.000006 \text{ m} = 6 \mu\text{m} = 6 \times 10^{-6} \text{ m}$
 - ② $6 \text{ mF} = 6 \times 10^{-3} \text{ F}$
 - ③ $7 \text{ pF} = 7 \times 10^{-12} \text{ F}$

* Prefixes :-

* 10^{-12} :- tera "T"

* 10^6 :- Mega "M"

* 10^9 :- Giga "G"

* 10^3 :- kilo "k"

Ex :- ① 10 kN $\rightarrow 10 \times 10^3$ N

② 5 M Ω $\rightarrow 5 \times 10^6$ Ω

*** Charge :-

* type of charge $\left\{ \begin{array}{l} + :- \text{Proton } +1.602 \times 10^{-19} \text{ C} \\ - :- \text{electron } -1.602 \times 10^{-19} \text{ C} \end{array} \right.$

* Unit of charge :- Coulomb "C"

* Charge $\rightarrow$ DC $\rightarrow$ Q (C+)
 $\rightarrow$ AC $\rightarrow$ q (C+)

*** Current :-

$\hookrightarrow$ time rate of flow of electrical charge through a conductor or circuit element

$$* i = \frac{dq}{dt} \rightarrow i = I = \text{current}$$

$$* \text{Unit} = \text{Amper} = A = \frac{C}{s}$$

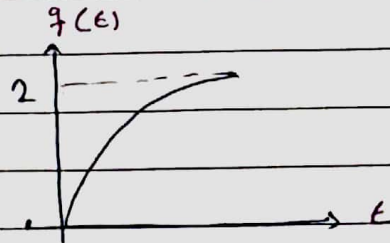
$$* \int_{t_0}^t i \cdot dt = \int_{t_0}^t dq$$

$$Q(t) - Q(t_0) = \int_{t_0}^t i(t) \cdot dt \quad **$$

Ex:- let $q(t) = \begin{cases} 0, & t < 0 \\ 2 - 2e^{-100t}, & t > 0 \end{cases}$

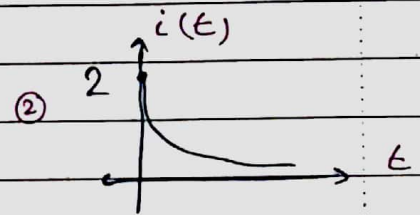
Plot $q(t), i(t)$

Sol:- graph $q(t) \rightarrow$

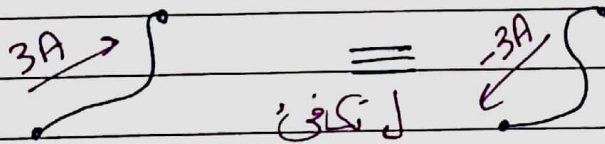


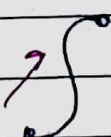
graph $i(t) \rightarrow$

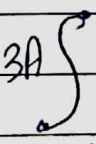
① $i(t) = \begin{cases} 0, & t < 0 \\ 200e^{-100t}, & t > 0 \end{cases}$



* Current is a vector quantity $\begin{cases} \text{magnitude} \\ \text{direction} \end{cases}$



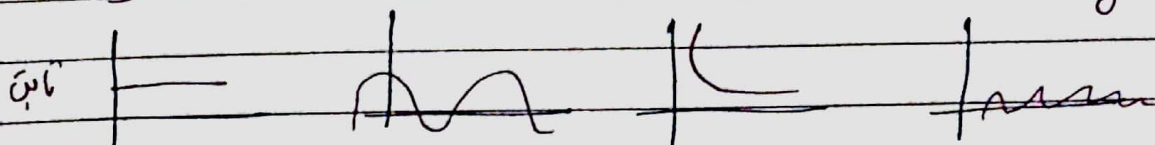
*  not current

*  not current

لا يكون
التيار في اتجاه
التيار

* type of current:

1) DC 2) AC 3) exp 4) damping



ooo Voltage :

is a measure of work required to move charge through element

* Unit = Volt = $V = \mathcal{V}$

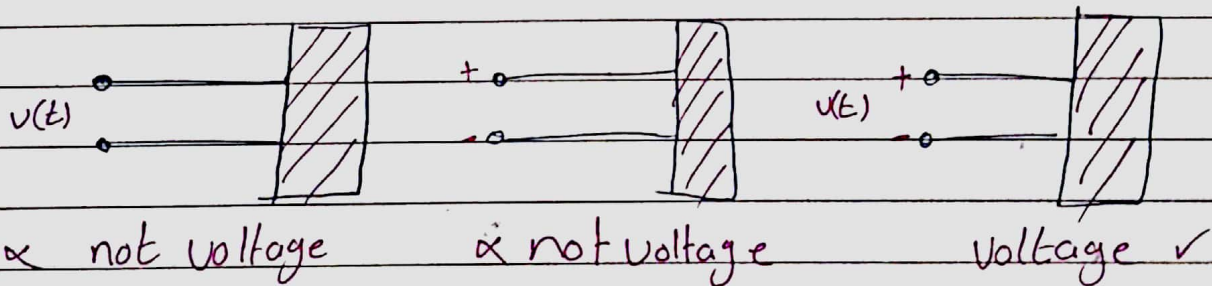
** Notes

Voltage can exist between pair of electrical terminal whether a current is flowing or not.

→ Voltage is a vector quantity $\begin{cases} \text{magnitude} \\ \text{Polarity } (+) \end{cases}$

$$\begin{array}{c} \oplus \\ \text{---} 5\mathcal{V} \text{---} \\ \ominus \end{array} = \begin{array}{c} \ominus \\ \text{---} -5\mathcal{V} \text{---} \\ \oplus \end{array}$$

* Voltage :



Power :-

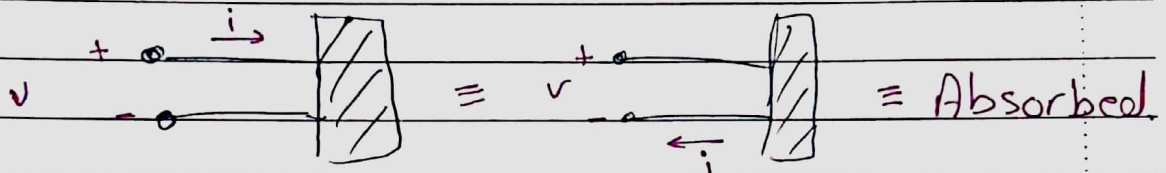
* Unit = Watts

* $P = VI$

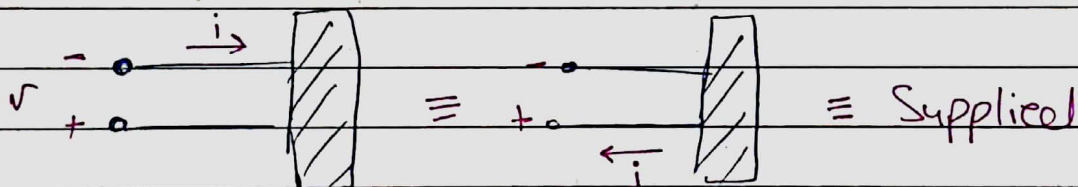
watt = Volt * Amper

** Passive sign convention :-

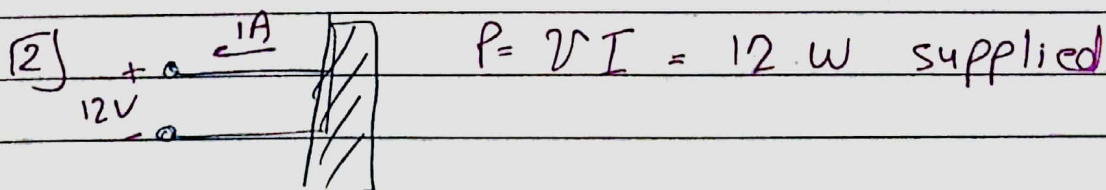
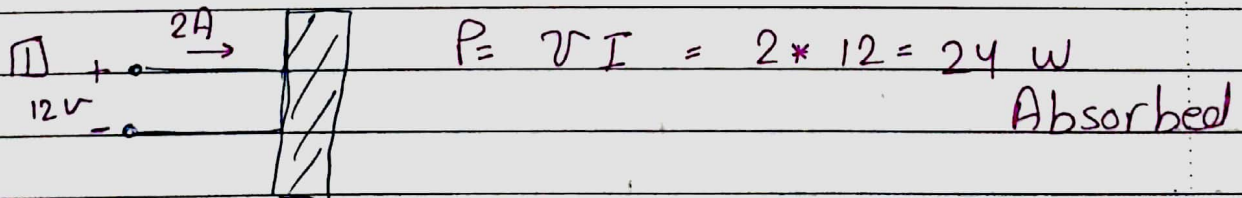
① If +ve current entering +ve terminal of element then power is Absorbed

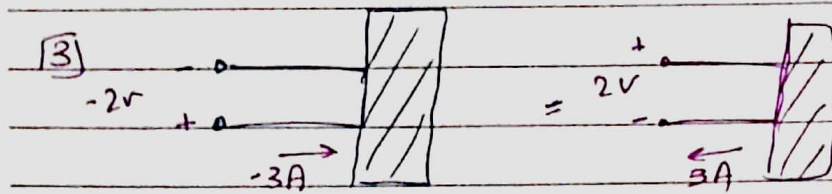


② If +ve current entering in -ve terminal of element then the power is Supplied



↳ Ex :- Find Power :-





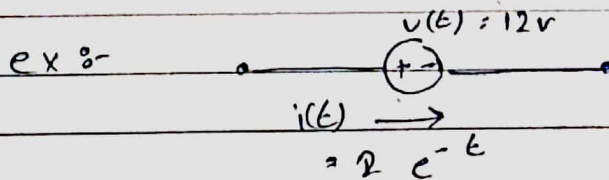
$$P = VI = 2 \times 3 = 6 \text{ W Absorbed}$$

* Note :- $P_{\text{absorbed}} = -P_{\text{supplied}}$

Energy :-

$$* W(t) = \int_{t_1}^{t_2} P(t) \cdot dt$$

$$P(t) = V(t) \times i(t)$$

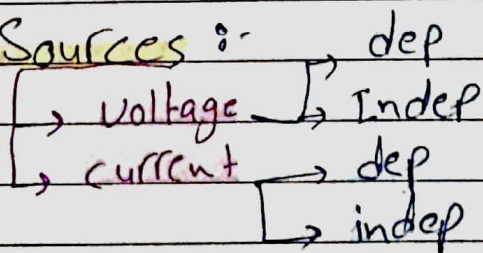


Find $w(t)$ between $0 \leq t \leq \infty$

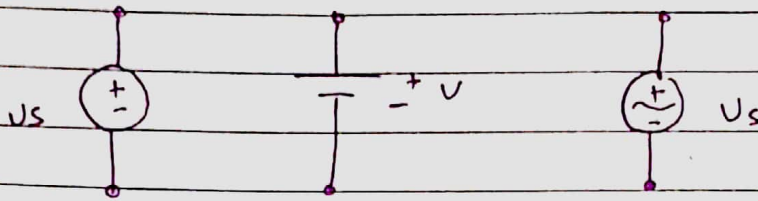
So :- $P(t) = V(t) \times I(t) = 24e^{-t} \text{ watt}$

$$W(t) = \int_0^{\infty} 24e^{-t} \cdot dt = 24 \text{ J}$$

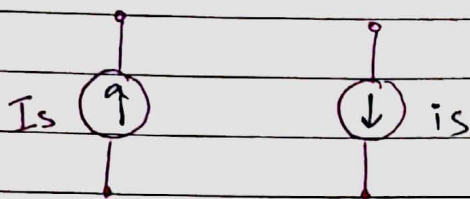
Sources :-



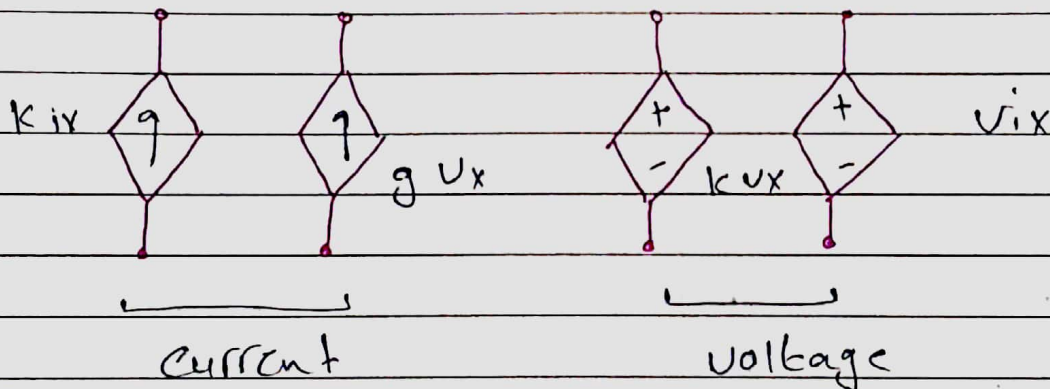
* Indep voltage source :-



* Indep. current source :-

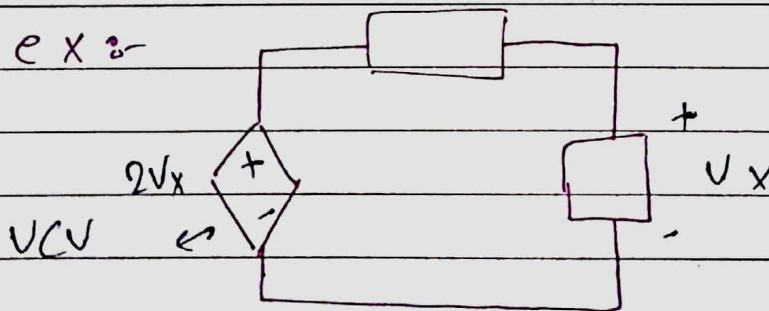


* dependent voltage and current sources :-



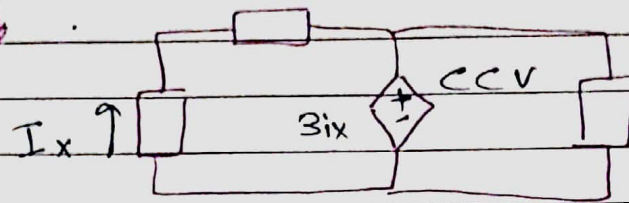
$\begin{matrix} \text{current} \\ \text{control} \end{matrix} \rightarrow \text{SCC}$
 $\begin{matrix} \text{current} \\ \text{control} \end{matrix} \rightarrow \text{VCC}$
 $\begin{matrix} \text{voltage} \\ \text{control} \end{matrix} \rightarrow \text{VCV}$
 $\begin{matrix} \text{voltage} \\ \text{control} \end{matrix} \rightarrow \text{CCV}$

ex :-



$\rightarrow u_x = 1, 2, 3$
 $\rightarrow 2u_x = 2, 4, 6$

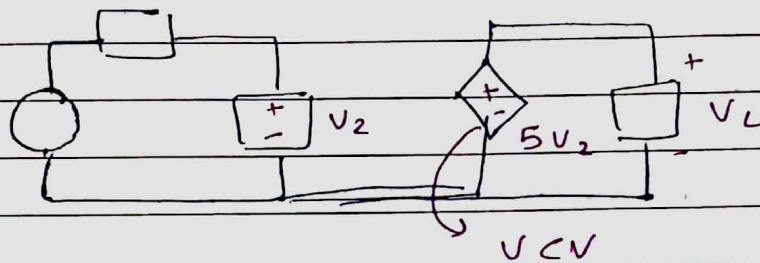
Ex :- ~~loop~~



$$i_x = 1, 2, 3$$

$$3i_x = 3, 6, 9$$

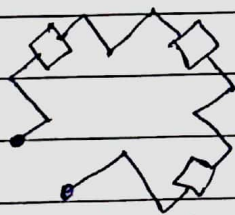
Ex :- Let $V_2 = 3V$ find V_L P



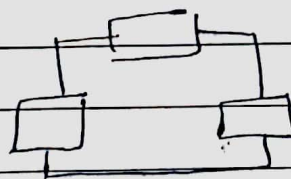
Sol :- $3 \times 5 = 15V_2$

$$V_2 = V_L \rightarrow V_L = 15$$

ooo Network + circuit :-

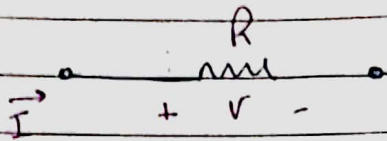


network.
open path.



circuit.
close path

ooo ohm Law :-



R:- resistor $\equiv \Omega \equiv \text{ohm}$

V:- voltage $\equiv V$

I:- current $\equiv A$

*** $\boxed{(+V = RI)}$



$\boxed{(-V = RI)}$

L $\rightarrow$ R, L, C $\rightarrow$ passive element

L $\rightarrow$ Sources $\rightarrow$ active element.

⊛ Power $\equiv P = VI$ ①

$P = RI \times I = I^2 R$ ②

$P = VI$

$\frac{V \cdot V}{R} = \frac{V^2}{R}$ ③ (قوانين الطاقة)

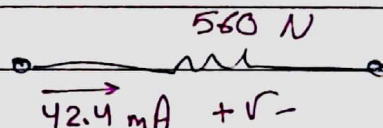
ooo Conductance :-

$\rightarrow G = 1/R$

$\rightarrow$ Unit $\rightarrow \text{mho}$

$\rightarrow P = VI = V^2 G = \frac{I^2}{G}$

ex: Find Power ?



sol:- $P = VI$

$V = RI = 560 \times 42.4 \times 10^{-3} = 23.7 \text{ V}$

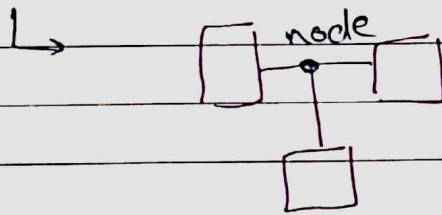
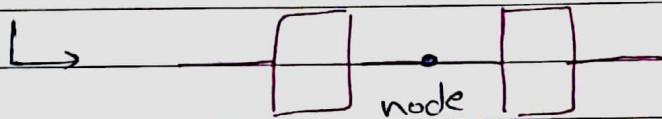
$\rightarrow P = 23.7 \times 42.4 \text{ m} = 1.005 \text{ W}$

method 2:- $P = I^2 R = (42.4 \times 10^{-3})^2 \times 560 = 1.005 \text{ W}$

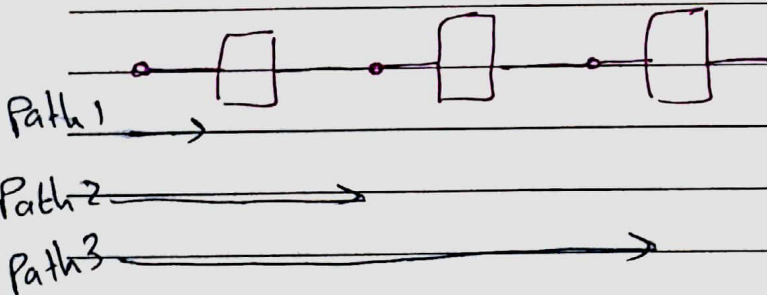
(absorbed) (lost) (Watt * 1)

Chapter 3 :- Voltage and current Laws:-

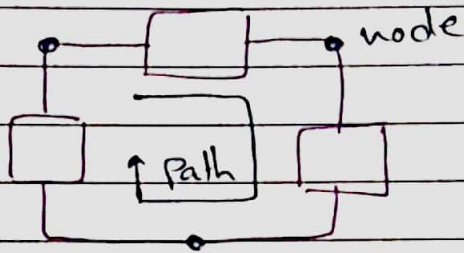
1] node :- a point at which two or more element have a common connection



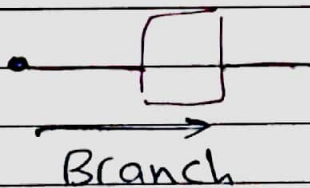
2] path :- a set of node and element that we pass through without passing through a node more than once



13 Loop :- if the node at which we started is the same as a node on which we end



14 Branch :- consist of one node and one element. «Path» في الدارة



000 kirchhoff's current law:- (KCL)

مقار + اتيان

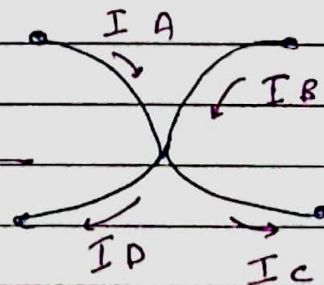
→ the algebraic sum of current entering any node is Zero

$$\sum_{\text{node}} I = 0$$

التيارات الداخلة = التيارات الخارجة

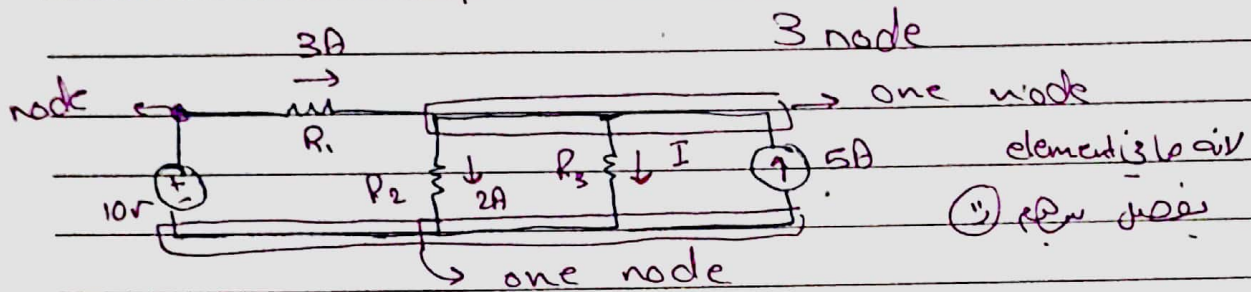
$$\textcircled{1} \checkmark I_A + I_B - I_C - I_D = 0$$

$$\textcircled{2} \checkmark -I_A - I_B + I_C + I_D = 0$$



دائماً افترض التيارات الداخلة = التيارات الخارجة

ex: Find I P

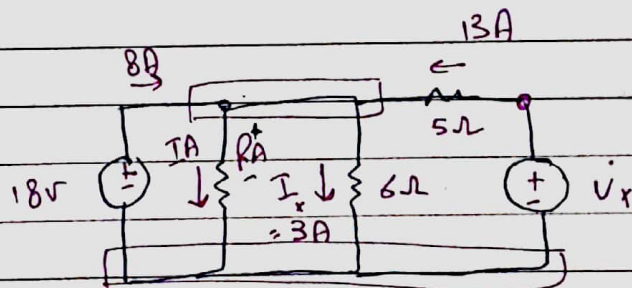


Using KCL

$$\rightarrow 3 + 5 = 2 + I \rightarrow I = 6A$$

or

ex: Find R P



* + VA RA IA

$$+ 18 = R_A \cdot I_A \rightarrow \text{using KCL: } 8 + 13 = I_A + 3$$

$$18 = R_A$$

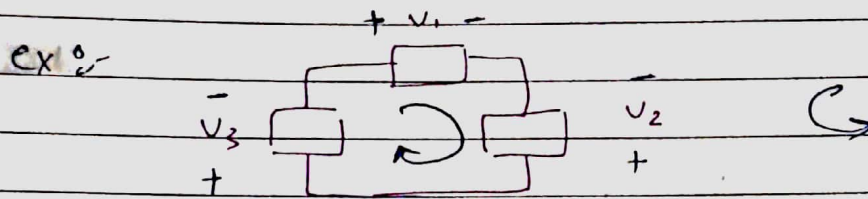
$$18$$

$$R_A = 1 \Omega$$

Kirchhoff's voltage Law :- ~~KVL~~ «KVL»

↳ the algebraic sum of the voltages around any loop is Zero

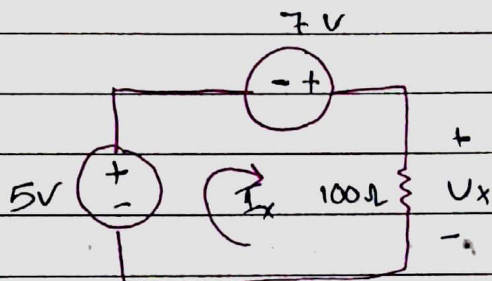
$$\sum_{\text{Loop}} V = 0$$



$$+V_1 - V_2 + V_3 = 0$$

$$\text{OR } -V_3 + V_2 - V_1 = 0$$

ex:- Find V_x , i_x ?



Sol:- ① Using the KVL

$$-5 - 7 + V_x = 0$$

$$V_x = 12V *$$

② using the ohm Law:-

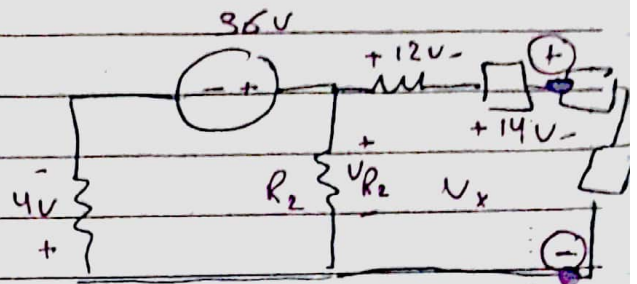
$$+ V_x = R I_x$$

$$12 = 100 I_x$$

$$I_x = 120 \text{ mA} *$$

Ex: Find V_{R_2} , V_x ?

Sol: ① using KVL :-



$$4 - 36 + V_{R_2} = 0$$

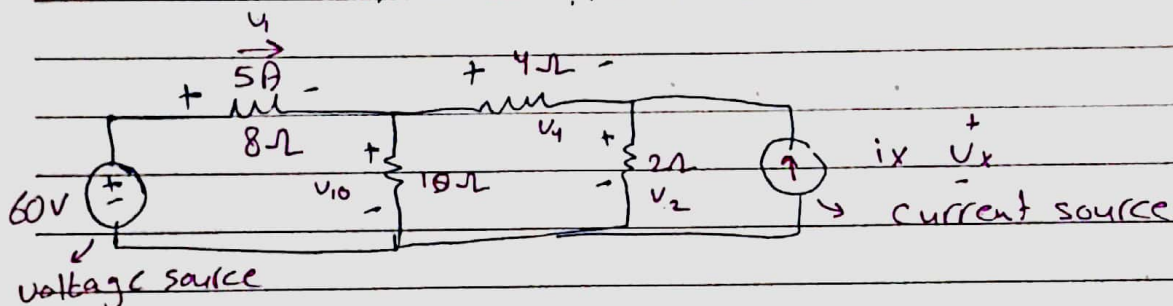
$$V_{R_2} = 32V$$

② using KVL :-

$$+4 - 36 + 12 + 14 + V_x = 0$$

$$V_x = 6V$$

Ex: Find V_x , I_x P!



$$\rightarrow ① V_1 = 8 \times 5 = 40V$$

② using the KVL on loop:-

$$-60 + 40 + V_{10} = 0$$

$$V_{10} = 20V$$

$$③ V_4 = 4 I_4 = 12V$$

④ using KVL :-

$$-V_{10} + V_4 + V_2 = 0$$

$$V_2 = 8V$$

$$V_x = 8V$$

$$\rightarrow ⑤ +V_{10} = 10 \times I_{10} \rightarrow I_{10} = 2A$$

KCL

⑥ using the KCL on loop:-

$$I_2 = I_4 + I_{10}$$

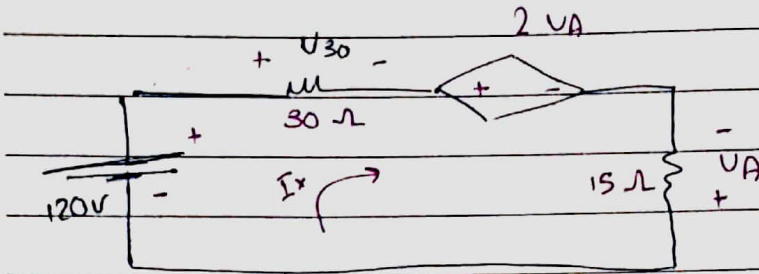
$$I_4 = 3A$$

$$⑦ I_2 = \frac{V_2}{2} = 4A$$

$$⑧ I_4 + I_x = I_2$$

$$I_x = 1A$$

Ex:- Find the power on each element:-



Using the KVL:-

$$* -120 + V_{30} + 2V_A - V_A = 0$$

$$-120 + 30 I_x + V_A = 0$$

$$-120 + 30 I_x - 15 I_x = 0 \rightarrow I_x = 8A$$

$$* -V_A = 15 I_x \rightarrow V_A = -15 I_x$$

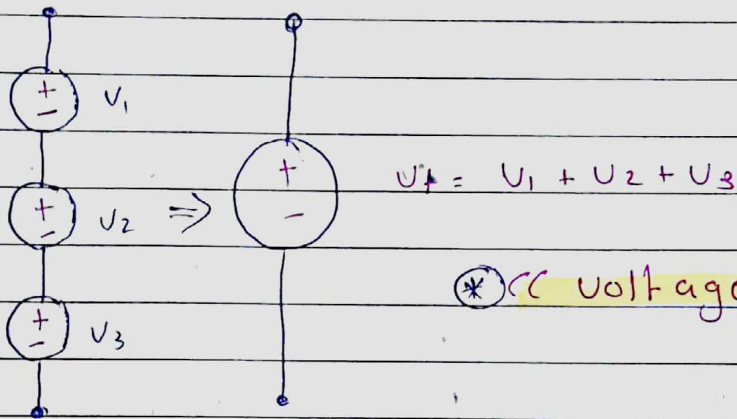
$$\textcircled{1} P_{120} = VI = 120 * 8 = 960W \text{ supplied} \\ = -960W \text{ absorbed}$$

$$\textcircled{2} P_{30} = I^2 R = 8^2 * 30 = 1920W \text{ absorbed}$$

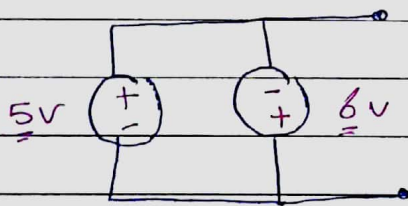
$$\textcircled{3} P_{2VA} = VI = 8 * 2 * -15 * 8 = -1920W \text{ absorbed}$$

$$\textcircled{4} P_{15} = I^2 R = 8^2 * 15 = 960W \text{ absorbed}$$

Series and Parallel connected sources :-



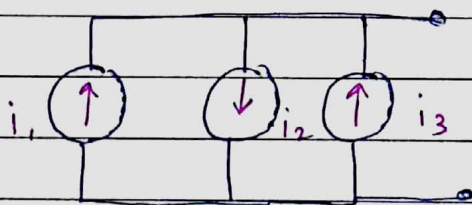
(*) « voltage in series »



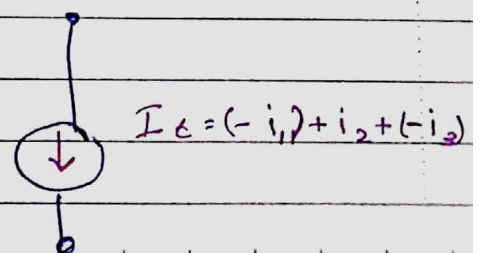
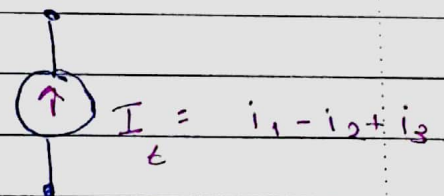
ما يقدر انهم لانه
الفولتية مختلفة

(*) « voltage in Parallel »

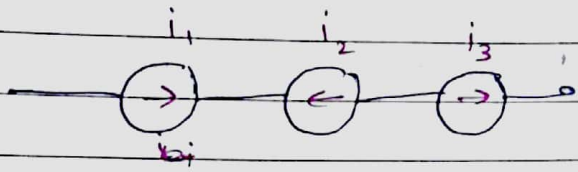
(*) current in parallel :-



$\Rightarrow$
or $\Rightarrow$



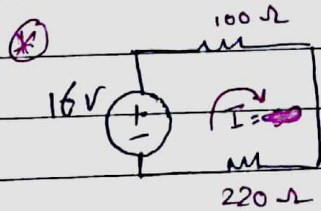
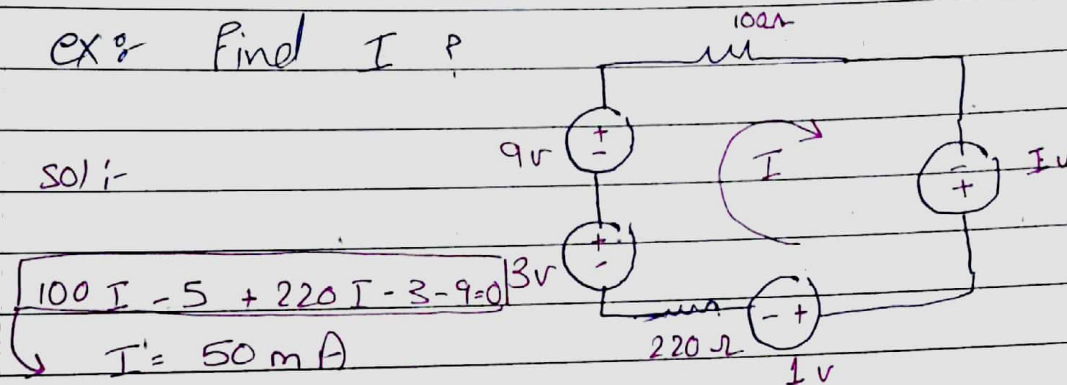
* Current in Series :-



ما يقدر الحظ
عنصر ال
لأنه البتة
(١) ملاحظة

ex: Find I

sol:-

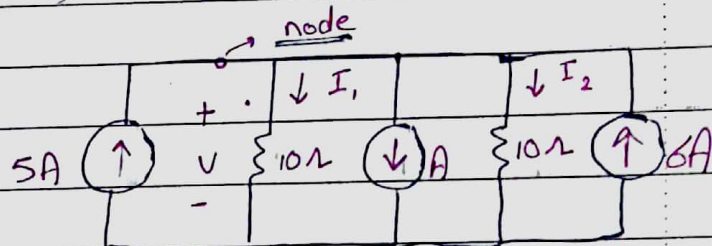


$$100I + 220I - 16 = 0$$

$$I = 50 \text{ mA}$$

ex :- Find V?

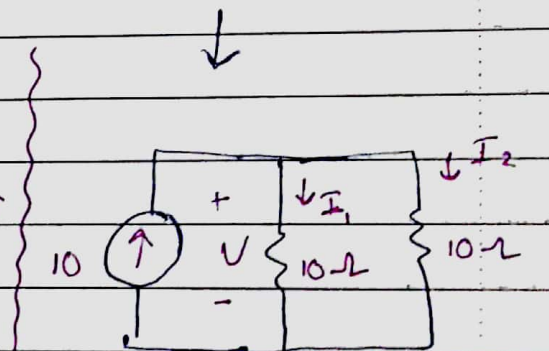
sol:- using KCL



$$5 + 6 = I_1 + I_2$$

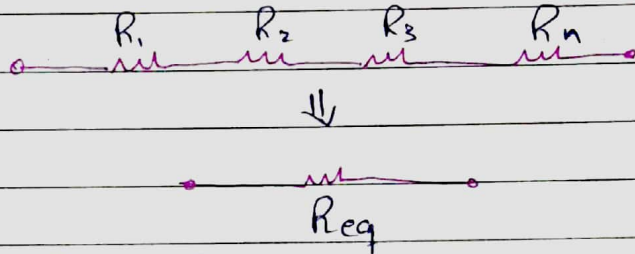
$$10 = I_1 + I_2$$

$$\frac{V}{10} + \frac{V}{10} \rightarrow V = 50 \text{ V}$$



$$10 = I_1 + I_2 \rightarrow V = 50 \text{ V}$$

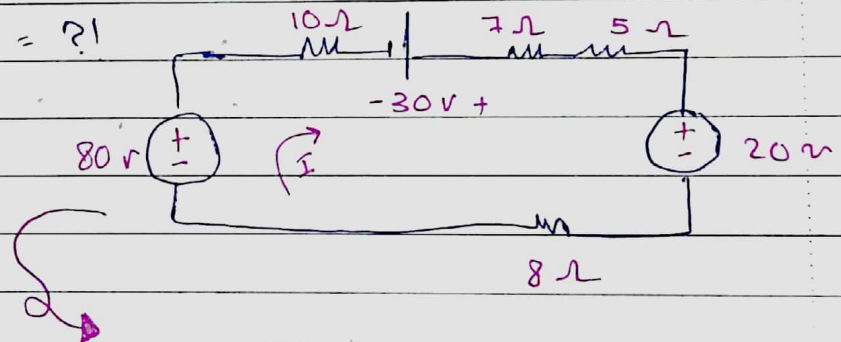
Resistor in series and parallel



$$R_{eq} = R_1 + R_2 + R_3 + \dots + R_n$$

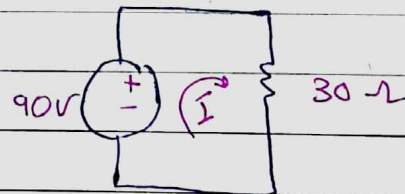
Resistor in series

ex:- Find $I = ?$

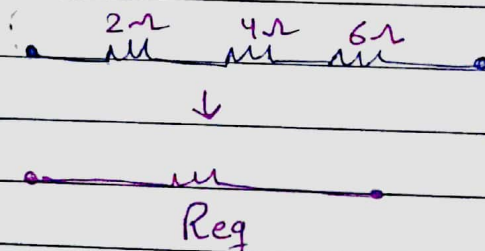


$$-90 + 30I = 0$$

$$I = 3A *$$

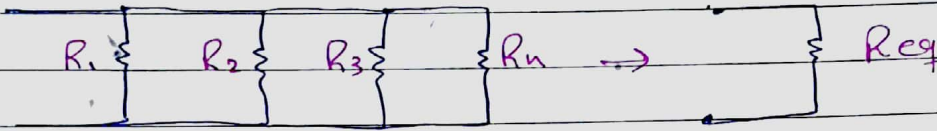


ex:- Find R_{eq} ?



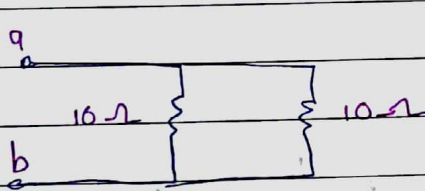
$$R_{eq} = 2 + 4 + 6 = 12\Omega$$

⊗ Resistor in Parallel :-



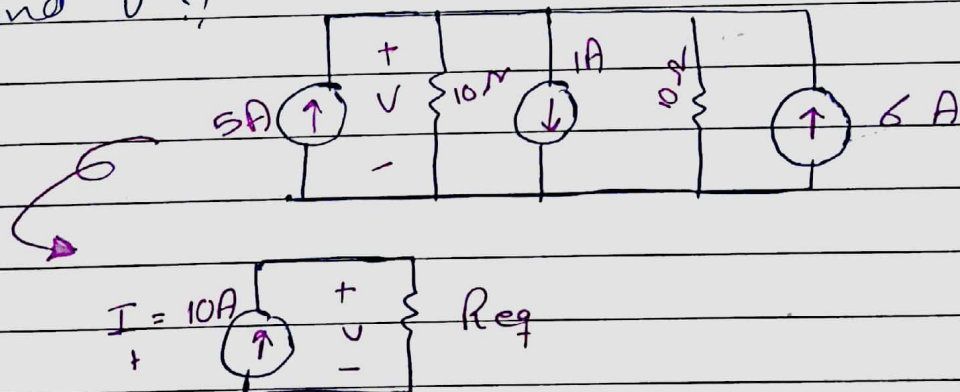
$$\therefore \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}$$

ex :- Find R_{eq} ?



Sol :- $\frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10}$

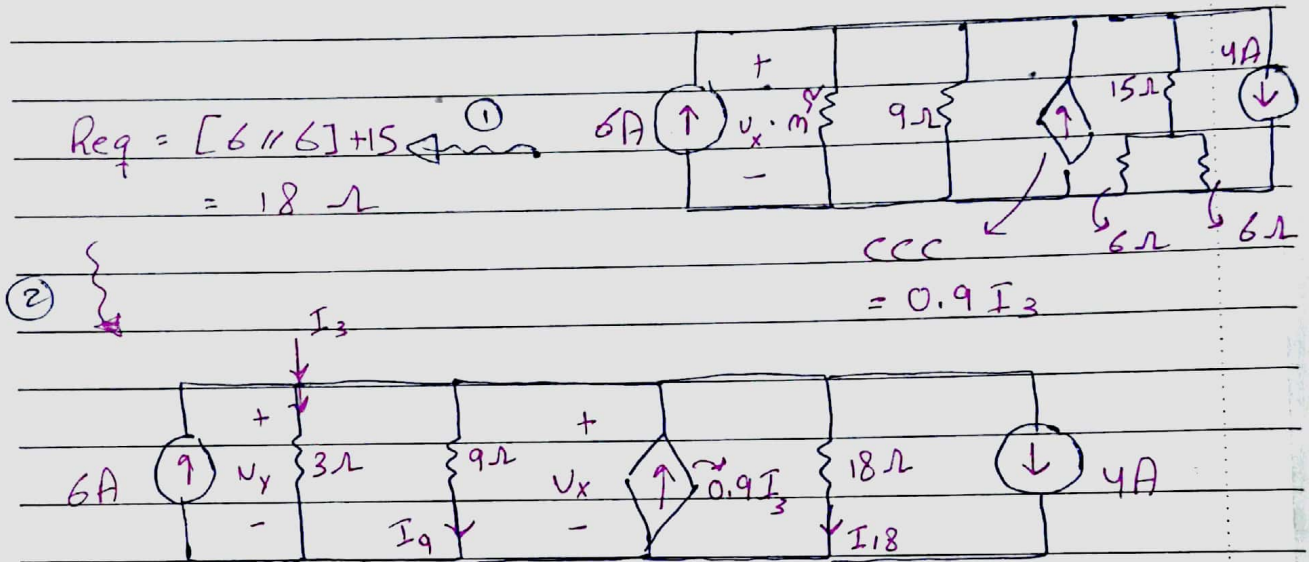
ex :- Find V ?



$$\therefore I_t = 5 - 1 + 6 = 10 \text{ A}$$

$$\therefore \frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10}$$

ex: Find the power in the dep. source:



$$P_{dep. source} = V I$$

$$= [V_x \times 0.9 I_3] \quad I_3 = \frac{V_x}{3}$$

$$P = 0.3 V_x^2$$

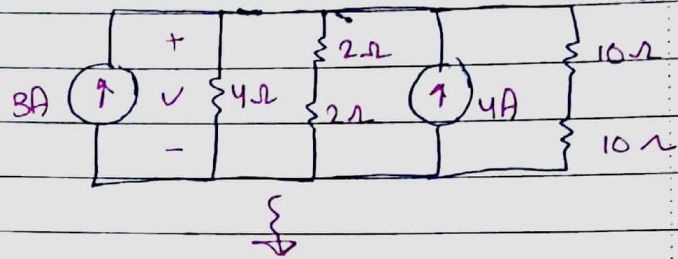
Using KCL:-

$$6 + 0.9 I_3 = I_3 + I_9 + I_{18} + 4$$

$$2 = 0.1 \left[\frac{V_x}{3} \right] + \left[\frac{V_x}{9} + \frac{V_x}{18} \right]$$

$$V_x = 10 V$$

$$P = 0.3 V_x^2 = 0.3 \times (10)^2 = 30 W \text{ supplied}$$

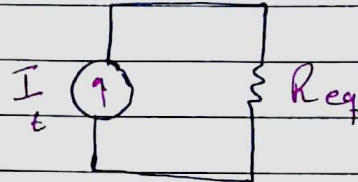
ex:- Find V ?

$$I_t = 3 + 4 = 7 \text{ A}$$

$$R_{eq} = (2+2) \parallel (10+10) \parallel 4$$

$$\left\{ \begin{array}{l} 4 \parallel 20 \parallel 4 \\ \frac{1}{R_{eq}} = \frac{1}{4} + \frac{1}{20} + \frac{1}{4} \end{array} \right.$$

$$R_{eq} = 1.818$$



$$\textcircled{*} V = RI = 1.818 * 7 \approx 14 \text{ Volt.}$$

* case 2:- dep + indep. current and voltage source in the complex network.

* $V_{th} = V_{oc}$

* $I_n = I_{sc}$

* $R_{th} = \frac{V_{oc}}{I_{sc}}$

→ ex:- find thevenin and Norton :-

→ sol :-

$V_{oc} = V_x$

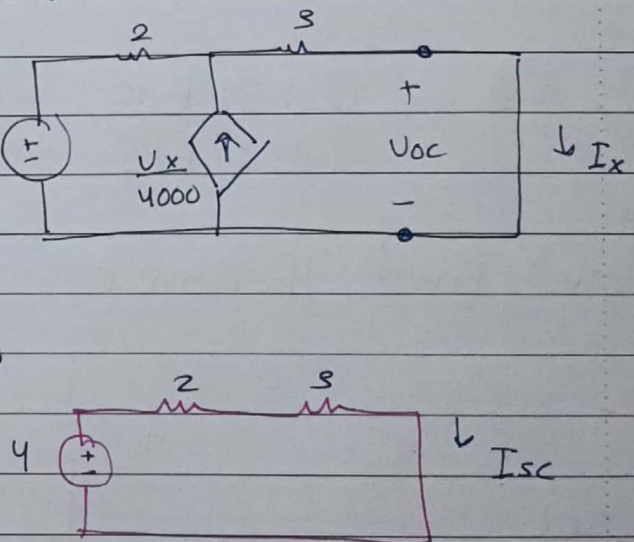
$= -4 + 2 \left[\frac{-V_x}{4000} \right] + V_x = 0$

$V_x = 8V$

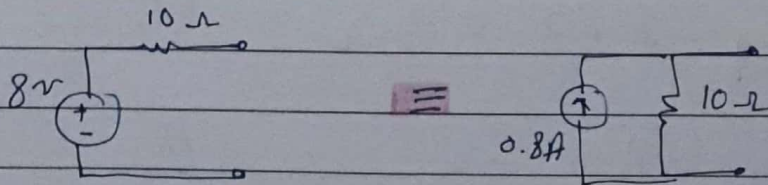
ce →

→ $-4 + 2 I_{sc} + 3 I_{sc} = 0$

$I_{sc} = 0.8mA$



→ $R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{8}{0.8} = 10 \Omega$



* Case 3 :-

only dep. source in the complex network.

→ Assume :- $I_{test} = 1A$ / $V_{test} = 1V$

→ Find :- $V_{test} = ?$ / $I_{test} = ?$

$$R_{Th} = \frac{V_{test}}{I_{test}}$$

$$R_{Th} = \frac{V_{test}}{I_{test}}$$

ex :- Find thevenin :-

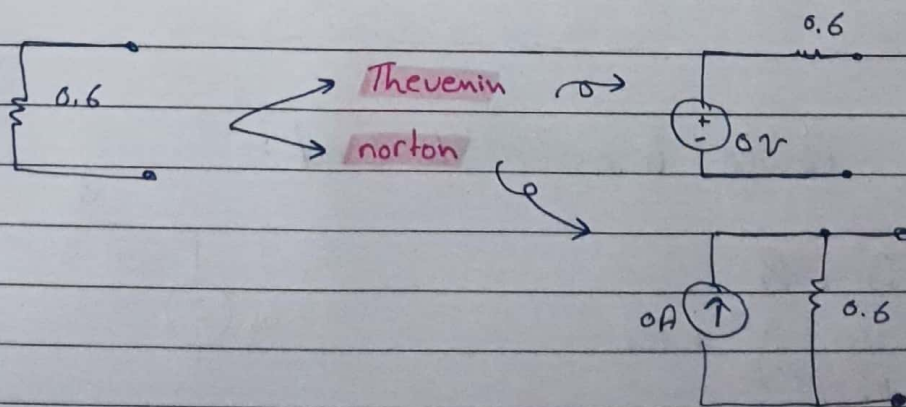
sol :-

$$R_{Th} = \frac{V_{test}}{I_{test}} = \frac{V_{test}}{1}$$

$$[I = -1] \#$$

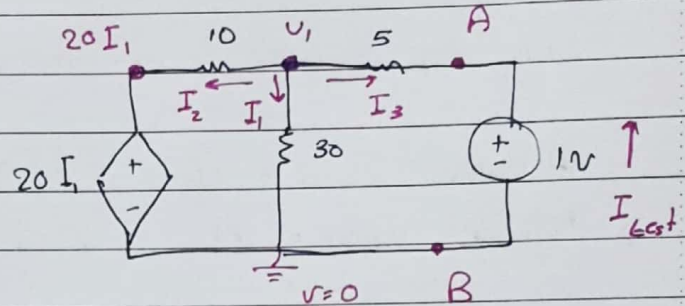
$$\rightarrow I_1 + I_2 + I_3 = 0$$

$$\frac{V_{test} - 1.5I}{3} + \frac{V_{test}}{2} - 1 = 0 \rightarrow V_{test} = 0.6V$$



ex: find thevenin :-

Sol'n



$$* R_{th} = \frac{V_{test}}{I_{test}} = \frac{1}{I_{test}}$$

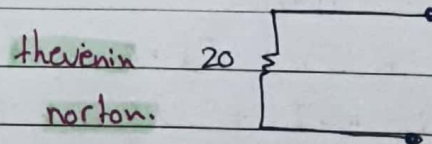
$$* -I_1 + I_2 + I_3 = 0$$

$$\frac{V_1 - 0}{30} + \frac{V_1 - 20I_1}{10} + \frac{V_1 - 1}{5} = 0 \rightsquigarrow V_1 = 30I_1$$

$$V_1 = 0.75V$$

$$* I_{test} = -I_3 = \left[\frac{V_1 - 1}{5} \right] = 50mA$$

$$\hookrightarrow R_{th} = \frac{1}{50} = 20\Omega$$



⊗ Max. Power transfer to Load :-

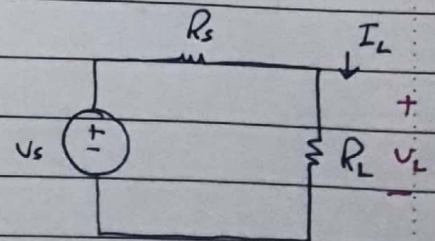
$$P = I_L^2 * R_L$$

$$\left(\frac{V_s}{R_L + R_s} \right)^2 * R_L$$

$$\rightsquigarrow \frac{dP}{dR_L} = 0 \rightsquigarrow \boxed{R_s = R_L} \#$$

max. Power

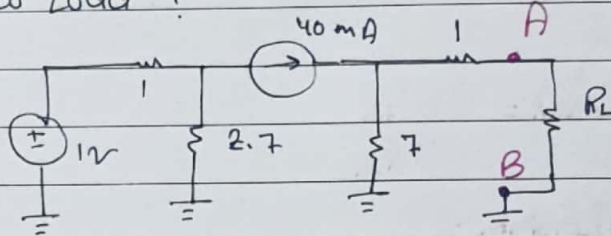
• 1993/10/10



$$P_{\max} = \frac{V_s^2}{4R_s} = \frac{V_{th}^2}{4R_{th}}$$

ex:- Find Power max to Load ?

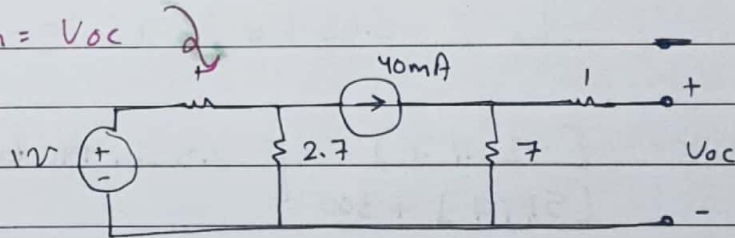
sol:- $P_{\max} = \frac{V_{th}^2}{4R_{th}}$



→ thevenin's theorem:-

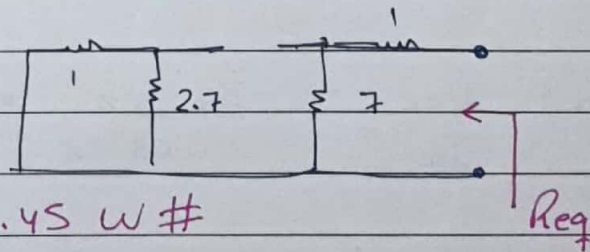
$$V_{th} = V_{oc}$$

$$V_{oc} = 7 \times 40 = 280 \text{ V}$$



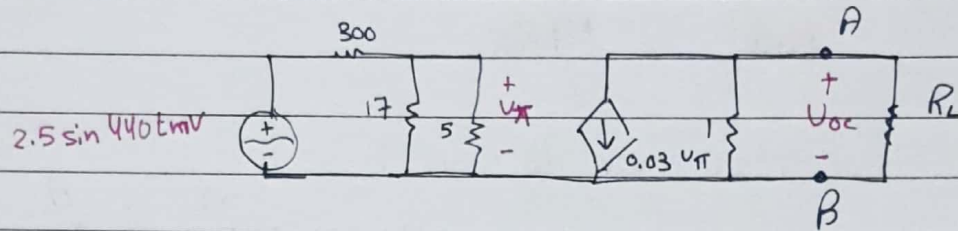
$R_{th} = R_{eq}$ after killing all source

$$R_{eq} = 1 + 7 = 8 \Omega$$



$$\therefore P_{\max} = \frac{(280)^2}{4(8)} = 2.45 \text{ W}$$

ex:- Find Max Power Transfer:-



$$P_{max} = \frac{V_{th}^2}{4R_{th}}$$

Thevenin:-

$$V_{th} = V_{oc}$$

$$\rightarrow [-0.03 v_{\pi}] [1] = -30 v_{\pi}$$

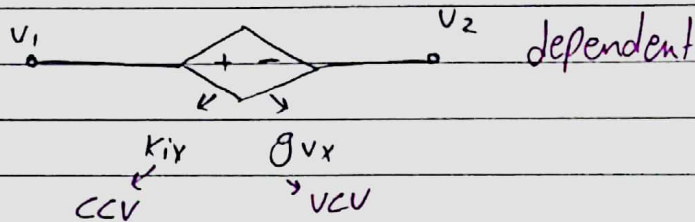
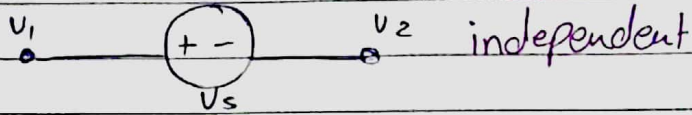
$$v_{\pi} = \frac{[5 \parallel 17]}{[5 \parallel 17] + 300} [2.5 \sin 440t \text{ mV}] = \text{---} v$$

$$I_{sc} = -0.03 v_{\pi}$$

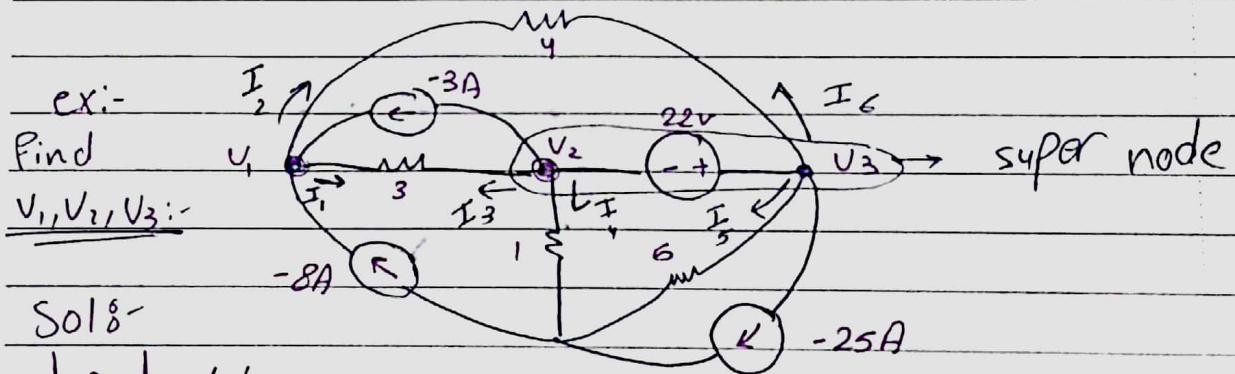
$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{-30 v_{\pi}}{-0.03 v_{\pi}} = 1 \Omega$$

$$P_{max} = \frac{V_{th}^2}{4R_{th}} = 1.211 \sin^2 440t \mu W.$$

* Super node :-



توضیح
 ① KCL ② super node
 voltage source



Sol:-

at node 1:-

$$-3 - 8 = I_1 + I_2$$

$$-11 = \frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{4} \quad \text{--- (1)}$$

~~at node 2/3~~

at node "supernode" :- (node 2 + node 3)

$$-3 + I_3 + I_4 + I_5 + I_6 + (-25) = 0$$

$$-3 + \frac{V_2 - V_1}{3} + \frac{V_2 - 0}{1} + \frac{V_3 - 0}{5} + \frac{V_3 - V_1}{4} - 25 = 0 \quad \text{--- (2)}$$

$$-V_2 + V_3 = 22 \quad \text{--- (3)}$$

$$\# V_1 = 1.071V$$

$$V_2 = \boxed{} V$$

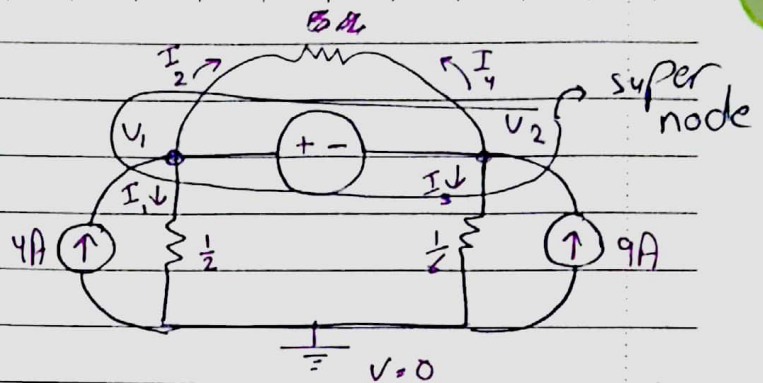
$$V_3 = \boxed{} V$$

20/3

ex:- Find V_1, V_2 ?

sol:-

→ at super node:-



$$4 + 9 = I_1 + I_2 + I_3 + I_4$$

$$13 = \frac{V_1 - 0}{1/2} + \frac{V_1 - V_2}{1/3} + \frac{V_2 - 0}{1/6} + \frac{V_2 - V_1}{1/3} \quad \text{--- (1)}$$

$$\rightarrow + V_1 - V_2 = 5 \quad \text{--- (2)}$$

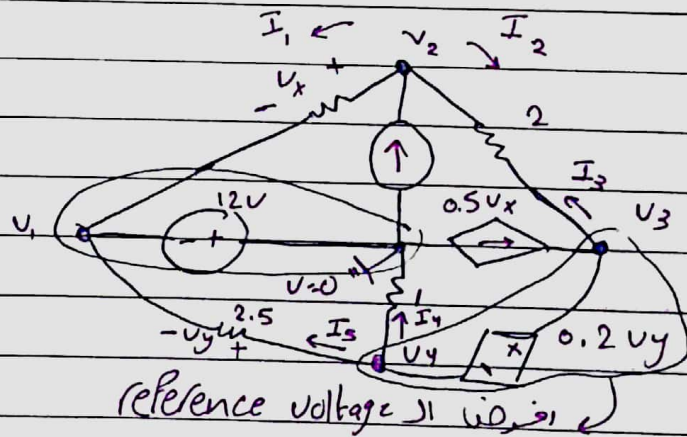
$$\rightarrow \# V_1 = 5.375 \text{ V}, V_2 = 375 \text{ mV}$$

ex:-

at super node:-

$$-V_1 + 0 = 12$$

$$V_1 = -12 \text{ V} \quad \text{--- (1)}$$



at node 2:-

$$I_1 + I_2 = 14$$

$$\frac{V_2 - V_1}{0.5} + \frac{V_2 - V_3}{2} = 14 \quad \text{--- (2)}$$

at super node:-

$$0.5 V_x = I_3 + I_4 + I_5$$

$$0.5 V_x = \frac{V_3 - V_2}{1} + \frac{V_4 - 0}{2.5} + \frac{V_4 - V_1}{2} \quad \text{--- (3)}$$

$$\rightarrow -\frac{V_1}{2} + \frac{V_3}{2} + \frac{V_4}{2.5} = \frac{14}{0.5} \quad \text{--- (3)}$$

next

Five Apple

$$+V_2 - V_1 = V_x$$

$$+V_4 - V_1 = V_y$$

$$-V_4 + V_3 = 0.2V_y$$

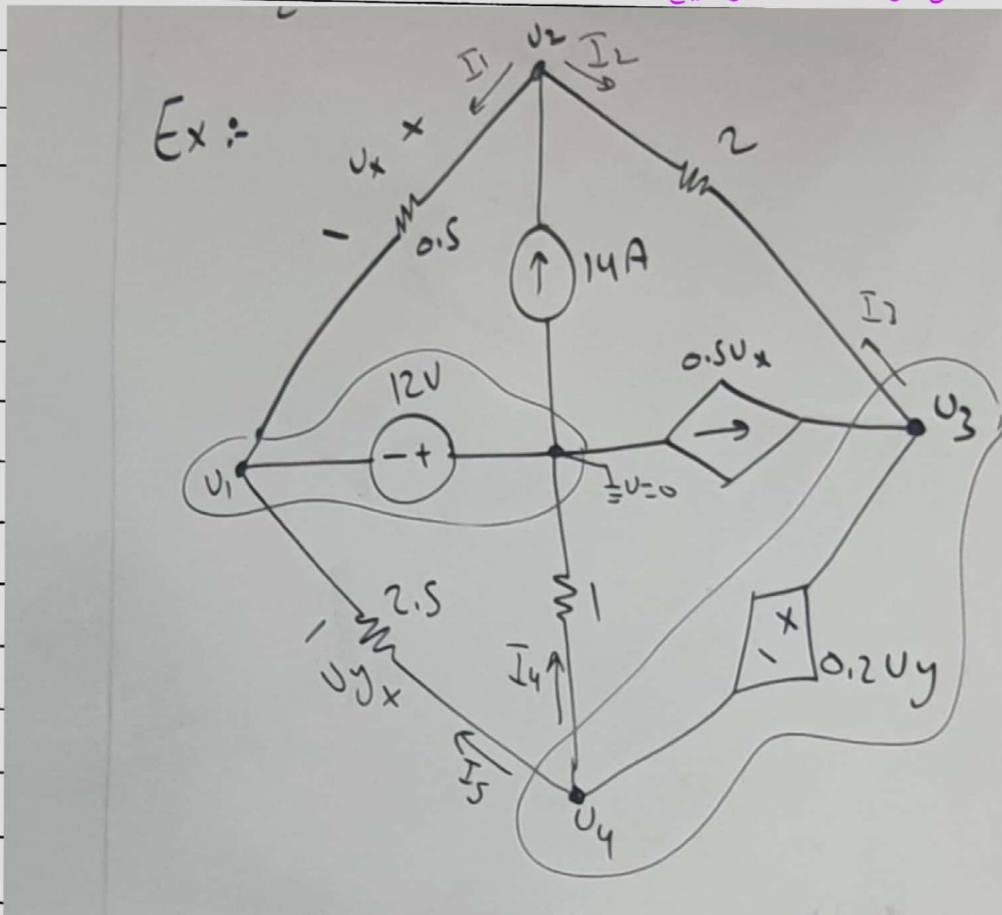
$$\rightarrow V_1 = -12V, V_2 = -4V, V_3 = 0V, V_4 = -2V$$

$$V_x = V_2 - V_1 = -4 + 12 = 8V$$

$$V_y = V_4 - V_1 = -2 + 12 = 10V$$

reference voltage * بالعادة افترضه دقة بس لو كانت
الدارة معقدة جدًا افترضه
كذا

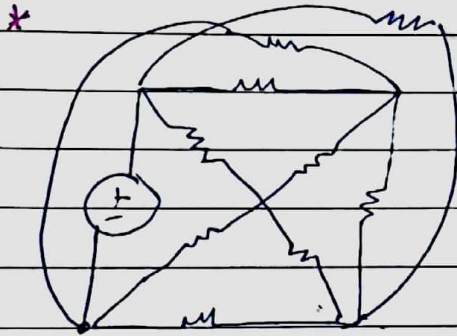
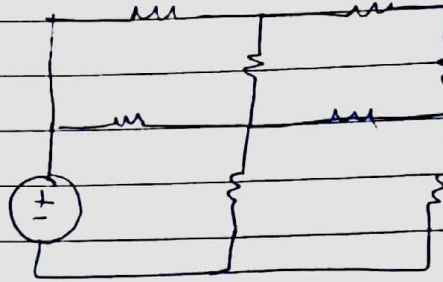
نفس الرسمة فقط للتوضيح



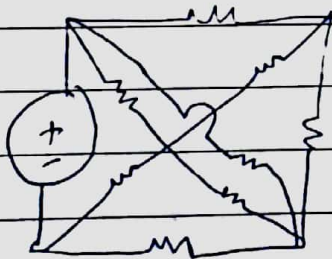
* mesh - current analysis :-

→ mesh analysis is applicable only to planer circuit

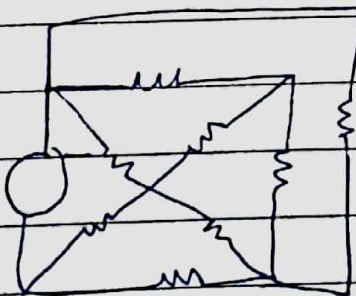
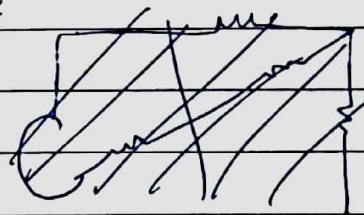
* planer circuit *



non planer circuit *



planer



Planer

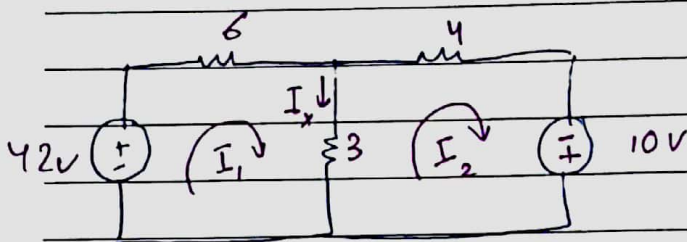
بالا سبقتوں کے
پیشے میں

Planer (٩)

* mesh analysis :-

- ① assume current on each loop.
- ② apply ~~KVL~~ KVL

* $\rightarrow$ Find I_x, I_2 ?



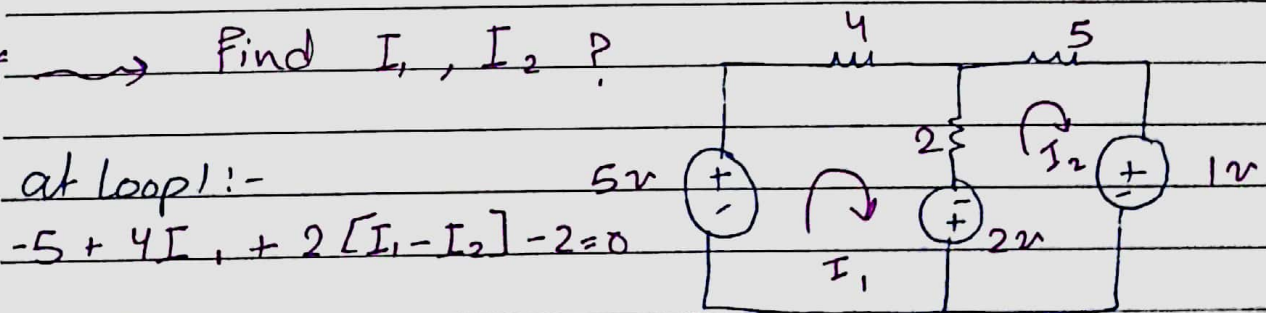
$$\rightarrow I_x = I_1 - I_2$$

at loop 1:- $-42 + 6I_1 + 3I_x = 0$ --- ①

at loop 2:- $-3I_x + 4I_2 - 10 = 0$ --- ②

$$\rightarrow I_1 = 6A \quad I_2 = 4A$$

* $\rightarrow$ Find I_1, I_2 ?



at loop 1:-

$$-5 + 4I_1 + 2[I_1 - I_2] - 2 = 0$$

at loop 2:- $2 + 2[I_2 - I_1] + 5I_2 + 1 = 0$

$$\rightarrow I_1 = 1.132A \quad I_2 = -0.1053A$$

$$* P_{2\Omega} = I_x^2 (2)$$

$$(I_1 - I_2)^2 \times 2 = 2.474 \text{ w absorbed}$$

* ex Find I_1, I_2, I_3 ?

at loop 1:-

$$-7 + 1[I_1 - I_2] + 6 + 2[I_1 - I_2] = 0$$

at loop 2:-

$$[I_2 - I_1] + 2I_2 + 3[I_2 - I_3] = 0$$

at loop 3:-

$$2[I_3 - I_1] - 6 + 3[I_3 - I_2] + I_3 = 0.$$

$$\sim I_1 = 3A \quad I_2 = 2A \quad I_3 = 3A \quad \#$$

* ex Find I_1, I_2, I_3 ?

at loop 1:-

$$-10 + 4[I_1 - I_2] + 3 = 0$$

at loop 2:-

$$4[I_2 - I_1] + 5I_2 + 9I_2$$

$$+ 10[I_2 - I_3] = 0$$

at loop 3:-

$$-3 + 10[I_3 - I_2] + I_3 + 7I_3 = 0$$

$$\sim I_1 = 2.22A, \quad I_2 = 470mA, \quad I_3 = \underline{\quad} A$$

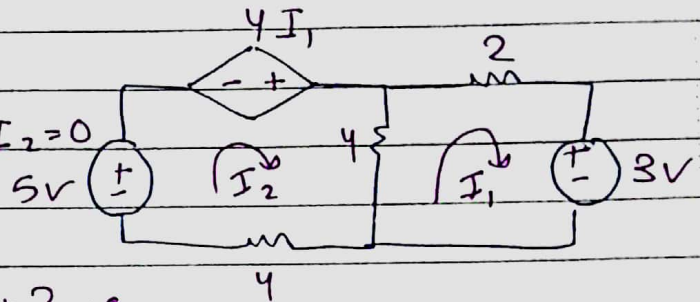
Find I_1, I_2 ?

at loop 1 :-

$$-5 + (-4I_1) + 4[I_2 - I_1] + 4I_2 = 0$$

at loop 2 :-

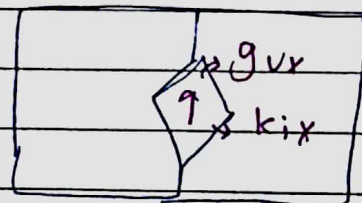
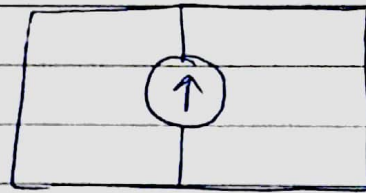
$$4[I_1 - I_2] + 2I_1 + 3 = 0$$



$$I_1 = -250 \text{ mA} \quad I_2 = 375 \text{ mA}$$

Super Mesh :-

لا نستخدم
KVL
في
الفجوة
التي
تحتوي
على
مصدر
تيار



ex:- Find current :-

at super mesh :-

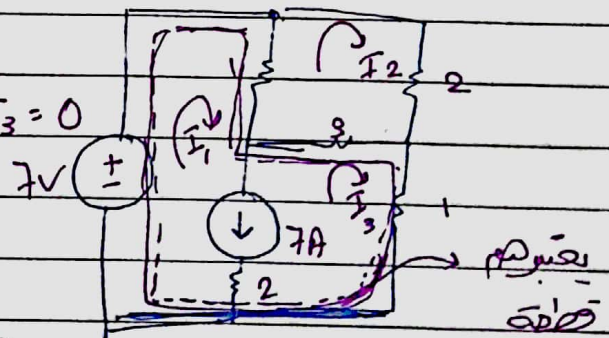
$$① -7 + [I_1 - I_2] + 3[I_3 - I_2] + I_3 = 0$$

$$② I_1 - I_3 = 7$$

at loop 2 :-

$$[I_2 - I_1] + 2I_2 + 3[I_2 - I_3] = 0$$

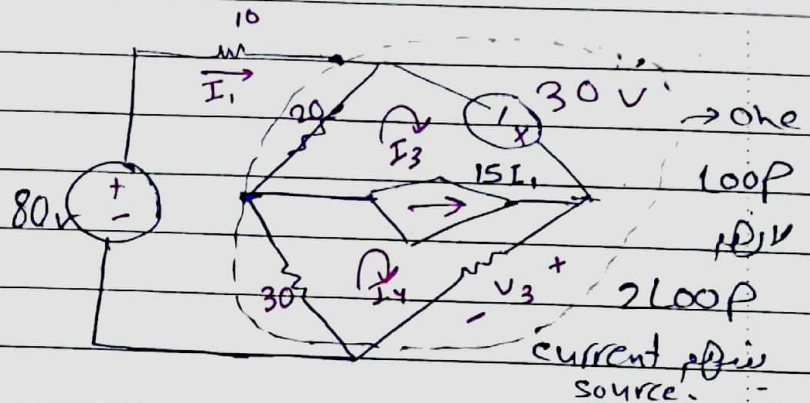
$$I_1 = 9 \text{ A} \quad I_2 = 2.5 \text{ A} \quad I_3 = 2 \text{ A}$$



Ex:- Find $V_3 = ?$

$$+V_3 = 40I_4$$

$$V_3 = 40I_4$$



at loop 1:-

$$-80 + 10I_1 + 20[I_1 - I_3] + 30[I_1 - I_4] = 0 \quad \text{--- (1)}$$

at super mesh:-

$$\rightarrow 20[I_3 - I_1] - 30 + 40I_4 + 30[I_4 - I_1] = 0 \quad \text{--- (2)}$$

$$\rightarrow I_4 - I_3 = 15I_1 \quad \text{--- (3)}$$

Final ans:- $I_1 = 0.584 \text{ A}$

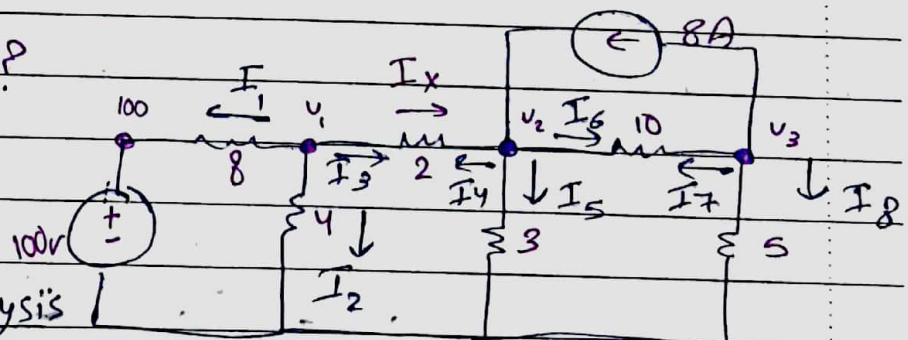
$$I_3 = -6.15 \text{ A}$$

$$I_4 = 2.605 \text{ A}$$

$$V_3 = 40I_4 = 104.20 \text{ V}$$

ex:- find I_x ?

sol:-



Using nodal analysis

at node 1:- $I_1 + I_2 + I_3 = 0$

$$\frac{V_1 - 100}{8} + \frac{V_1 - 0}{4} + \frac{V_1 - V_2}{2} = 0 \quad \text{--- (1)}$$

at node 2:-

$$8 = I_4 + I_5 + I_6$$

$$\rightarrow 8 = \frac{V_2 - V_1}{2} + \frac{V_2 - 0}{3} + \frac{V_2 - V_3}{10} \quad \text{--- (2)}$$

at node 3:-

$$8 + I_7 + I_8 = 0$$

$$8 + \frac{V_3 - V_2}{10} + \frac{V_3 - 0}{5} = 0 \quad \text{--- (3)}$$

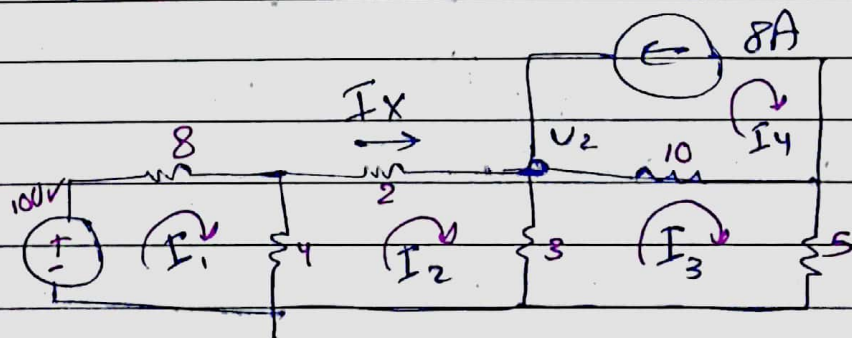
Final ans:-

$$V_1 = 25.89 \text{ V} / V_2 = 20.31 \text{ V} / V_3 = \boxed{} \text{ V}$$

$$I_x = \frac{V_1 - V_2}{2} = 2.79 \text{ A}$$

(التي - اليمين)

Method 2 :- Using mesh analysis.



at loop 1:- $-100 + 8I_1 + 4[I_1 - I_2] = 0 \quad \text{--- (1)}$

at loop 2:- $4[I_2 - I_1] + 2I_2 + 3[I_2 - I_3] = 0 \dots (2)$

at loop 3:- $3[I_3 - I_2] + 10[I_3 - I_4] + 5I_3 = 0 \dots (3)$

Final Ans is: $I_4 = -8 \text{ A} / I_1 = 2.79 \text{ A} / I_2 = 1 \text{ A}$

$I_3 = 2.79 \text{ A} / I_4 = 1 \text{ A}$

→ $\left[\begin{array}{l} \text{nodal potential} \leftarrow \text{نودال پوتنشل} \\ \text{mesh potential} \leftarrow \text{میش پوتنشل} \end{array} \right]$

First

Second (CH5 + CH6)

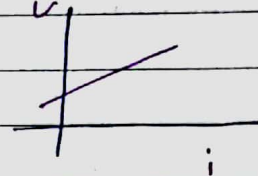
CH5
27/3

* CH5:

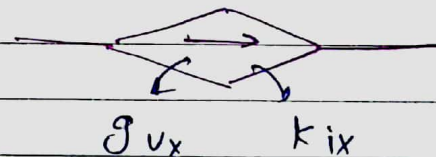
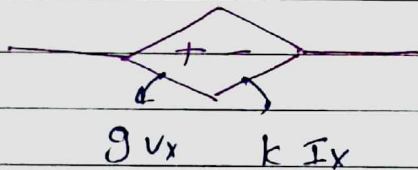
① Linear element and Linear circuit

① linear element:- element that has linear voltage - current relation v

$$v(t) = R i(t)$$



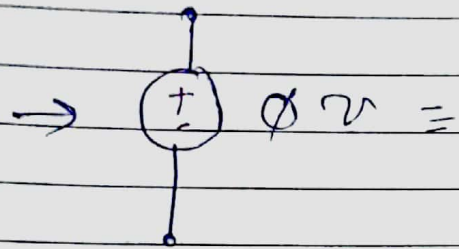
② linear dep. source:- is dep. voltage or current ~~voltage~~ source whose out put current or voltage is ~~proportional to~~ first power of current or voltage. $\hookrightarrow$ Proportional to



③ linear circuit:- the circuit that composed of : ① indep. source ② dep. linear source ③ linear element.

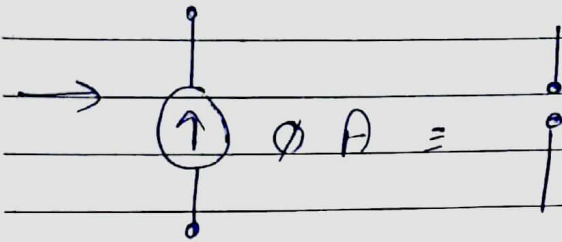
* Super position :- the response in a linear circuit having more than one indep. source can be obtained by adding the responses caused by the seperat indep source acting alone.

27/3



S.C

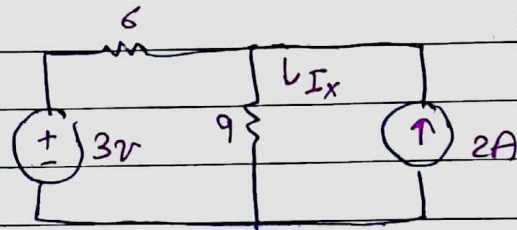
لوبى اقبل الفولتية
short circuit
ر. ز. يقصر



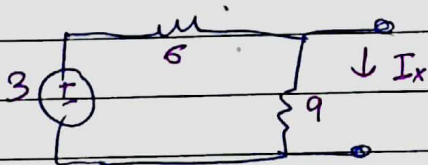
O.C "open circuit"

ex:- Find I_x using super position:-

$$I_x = I_x \downarrow_{3\Omega} + I_x \downarrow_{2A}$$



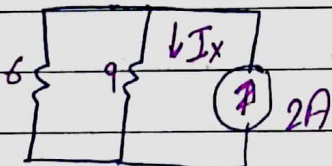
$$\rightarrow * I_x \downarrow_{3\Omega}$$



$$-3 + 6I_x + 9I_x = 0$$

$$I_x = 0.2A$$

$$\rightarrow I_x \downarrow_{2A}$$

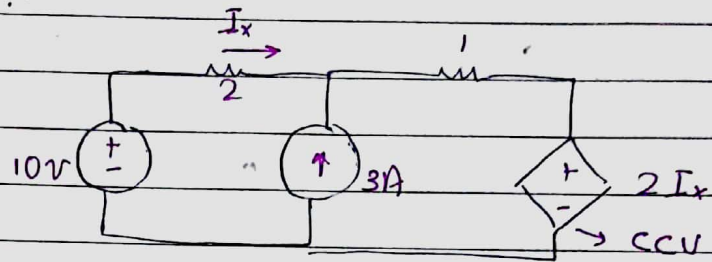


$$I_x = \frac{1}{\frac{1}{6} + \frac{1}{9}} * [2] = 0.8A$$

$$\rightarrow I_{x \text{ tot}} = 0.2 + 0.8 = 1A$$

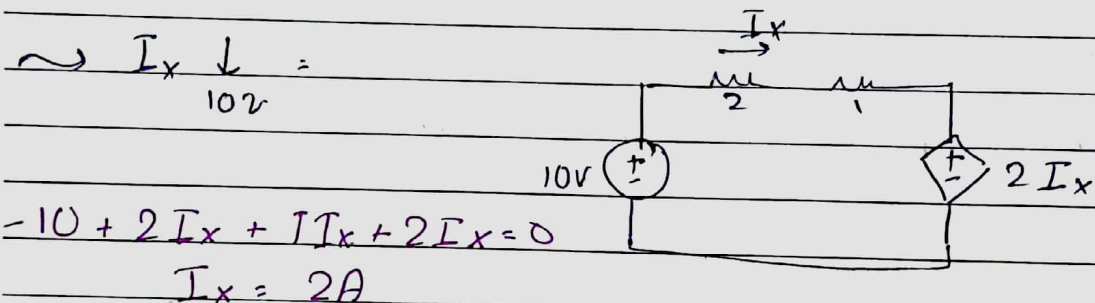
* Super position :-

ex $\rightarrow$ Find I_x ?



$$\text{Sol:- } I_{x \downarrow} = I_{x \downarrow} + I_{x \downarrow}$$

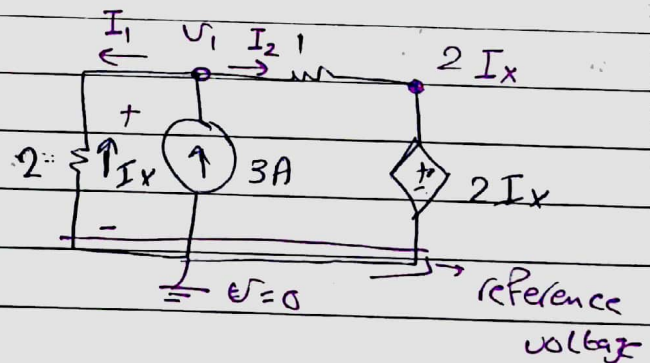
tot 10V 3A



$\sim I_{x \downarrow} =$

3A

nodal analysis



$$3 = I_1 + I_2$$

$$3 = \frac{V_1 - 0}{2} + \frac{V_1 - 2I_x}{1}$$

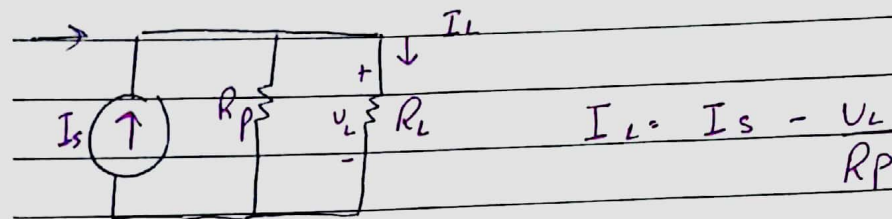
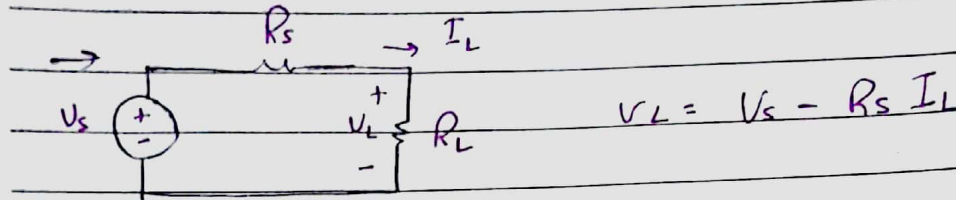
$$\sim -V_1 = 2I_x \text{ HA}$$

$\rightarrow$ final ans :- $I_x = -6.6A$ / $I_{x \downarrow} = 1.4A$

tot

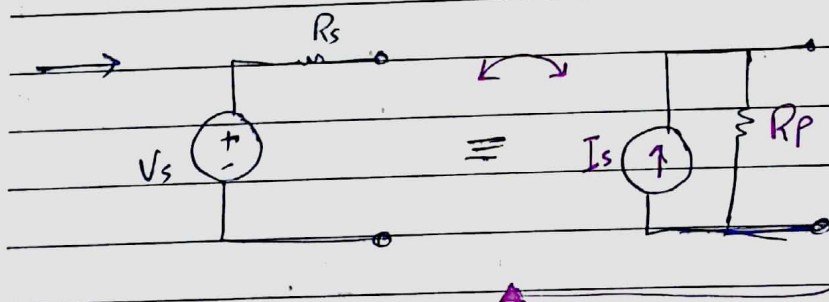
29/3

⊛ Source transformation:- (source $\leftrightarrow$ current voltage)



* $V_L = \frac{R_L}{R_L + R_s} [V_s]$

x $V_L = I_L R_L$
 $= \frac{1/R_L}{1/R_L + 1/R_p} [R] [I_s]$

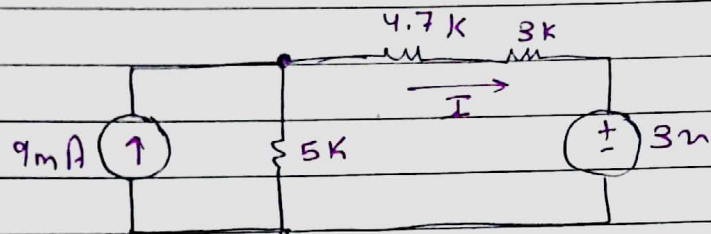


بقدر دخول
 نزي شكل به
 عن طريق قوانین

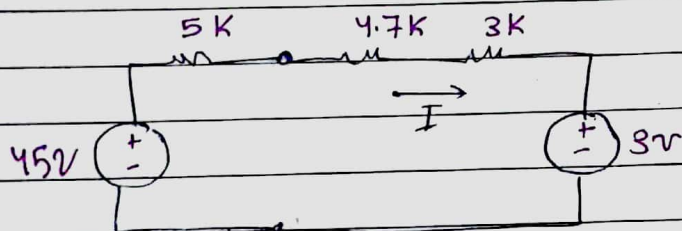
- ① $R_s = R_p$
- ② $V_s = I_s R_p = \text{ohm's Law.}$
- ③ $I_s = V_s / R_s = \text{ohm's Law}$

ex: Find I ?

$$k \rightarrow 10^3 / m \rightarrow 10^{-3}$$



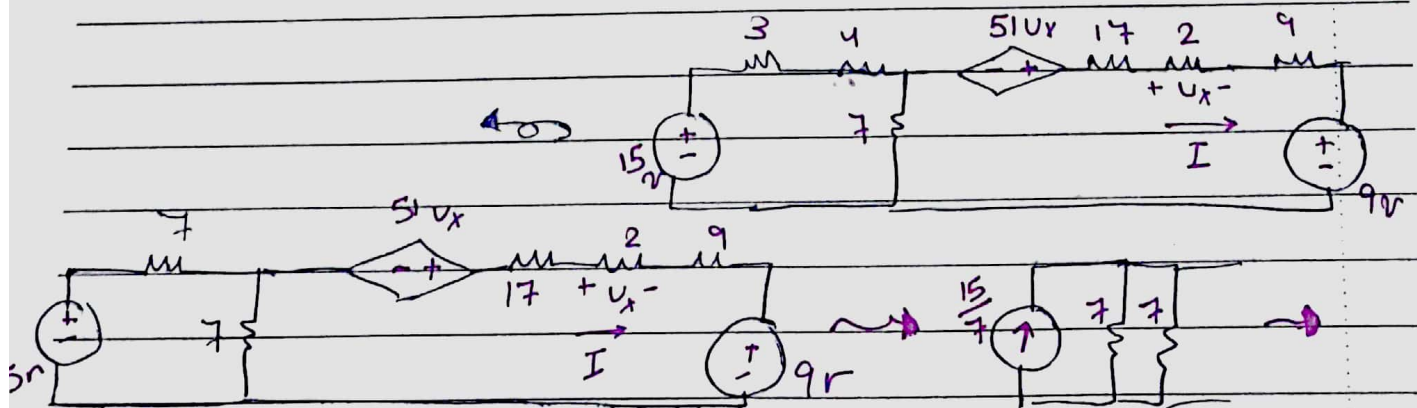
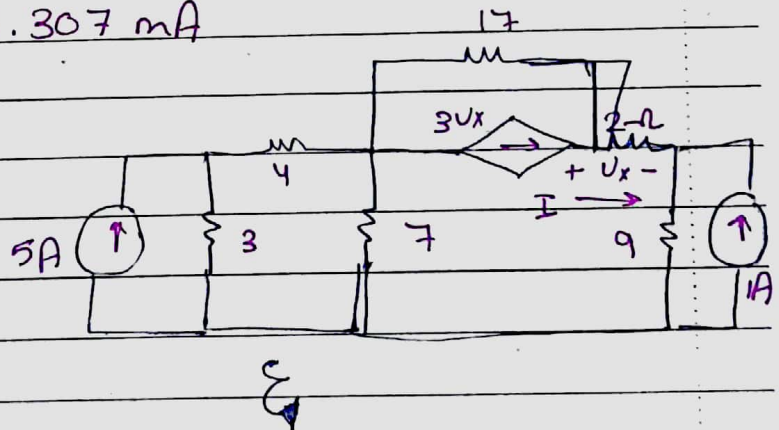
حلها الى
2 loop
عن طريق القوانين



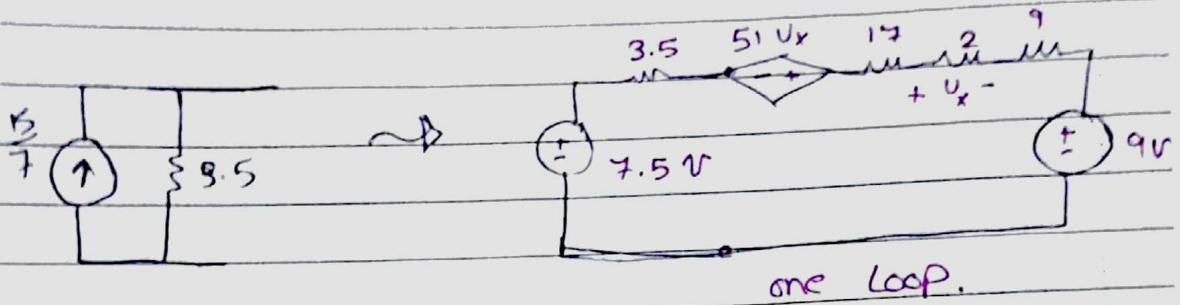
$$\textcircled{1} R_s = R_p = 5k$$

$$\textcircled{2} V_s = I_s R_p = 5k * 9m = 45V$$

Using KVL :- $-45 + 5kI + 4.7kI + 3kI + 3 = 0$
 $\rightarrow I = 3.307 \text{ mA}$

ex:- Find I, V_x ?

29/3



Using KVL:-

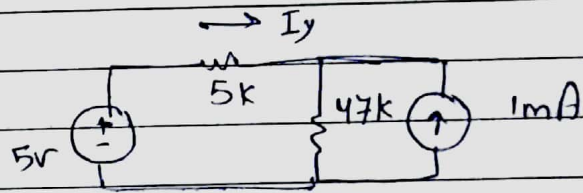
$$\rightarrow -7.5 + 3.5 \times I - 51V_x + 17I + 2I + 9I + 9 = 0$$

$$\sim V_x = 2I$$

$$\rightarrow \text{Final ans:- } I = 21.28 \text{ mA}$$

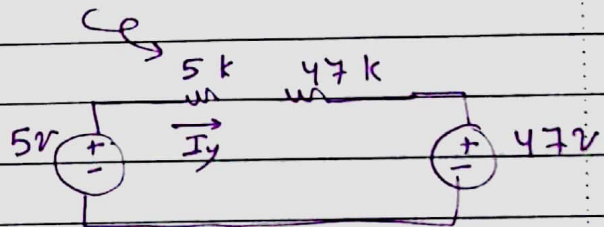
$$V_x = 42.2 \text{ mV}$$

ex:- Find I_y ?

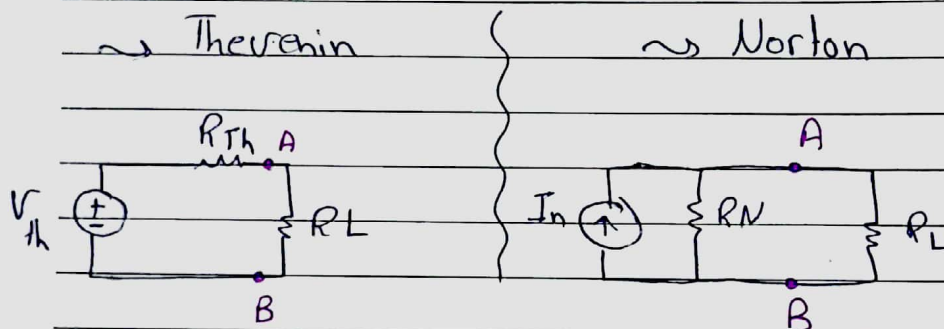
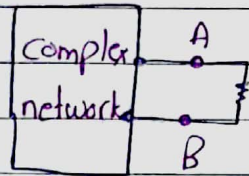


$$-5 + 5kI_y + 47kI_y + 47 = 0$$

$$I_y = -A$$



* Thevenin and Norton Equivalent:



⊗ Using Source transformation:

① $R_n = R_{th}$

② $I_n R_n = V_{th}$

③ $I_n = \frac{V_{th}}{R_{th}}$

نorton is the equivalent of thevenin (مساوي)

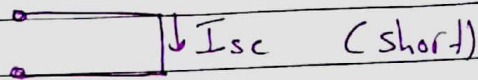
Case one:- Only independent current and voltage sources

→ $V_{th} = V_{oc}$

The diagram shows an open circuit between terminals 'a' and 'b'. The voltage across the open terminals is labeled V_{oc} (open).

→ $R_{th} = R_{eq}$ after killing all sources

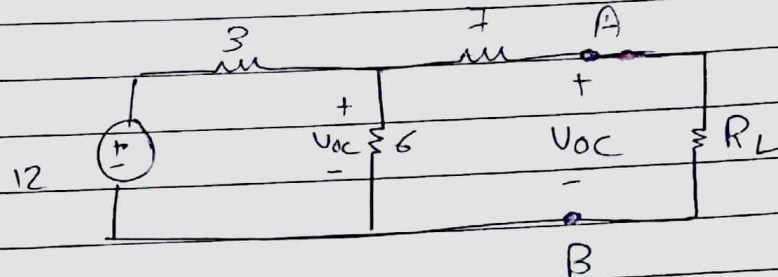
$$\rightarrow I_N = I_{sc}$$



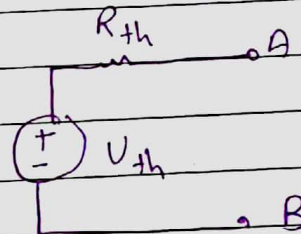
$$\rightarrow R_{th} = R_N$$

ex:-

Find thevenin?

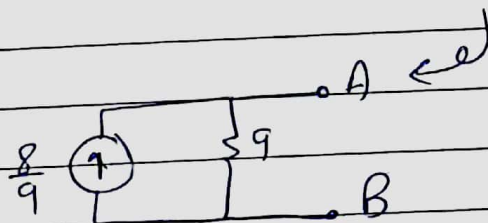
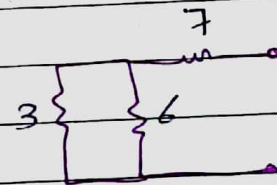


$$\begin{aligned} \rightarrow \text{sol:- } V_{th} &= V_{oc} \\ &= \left[\frac{6}{3+6} \right] [12] \\ &= 8V \end{aligned}$$

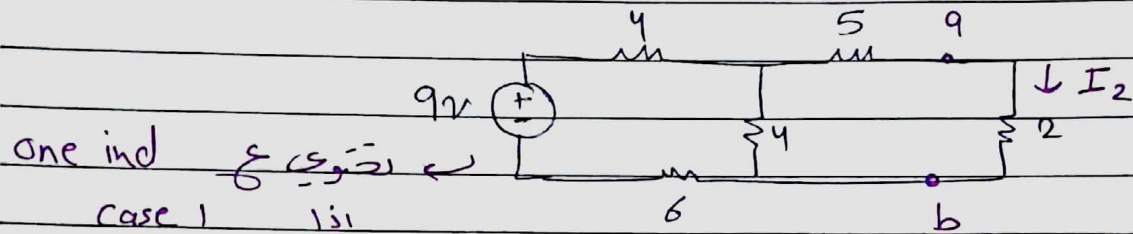


$$\therefore R_{eq} = [3 \parallel 6] + 7 = 9 \Omega$$

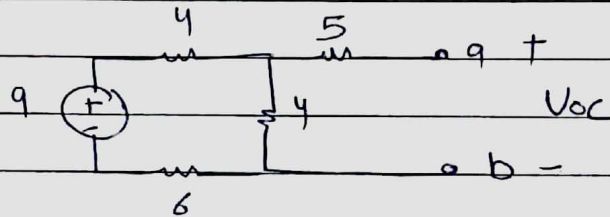
$$\therefore I_N = I_{sc} = \frac{V_{th}}{R_{th}} = \frac{8}{9} A$$



ex:- Find thevenin and $I_2 = ?$



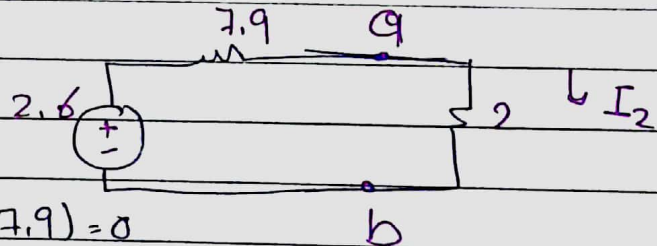
Sol:-



$$\rightarrow V_{th} = V_{oc}$$

$$= \frac{4}{4+4+8} \times 9 = 2.6V$$

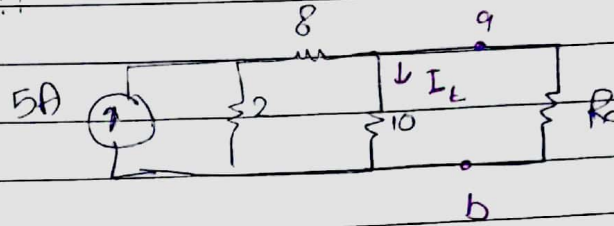
$$\rightarrow R_{th} = R_{eq} = \left(\frac{10 \times 4}{10+4} \right) + 5 = 7.9$$



$$-2.6 + 7.9(I_2) + 2(7.9) = 0$$

$$I_2 = 0.26A$$

Find thevenin ?



sol :-

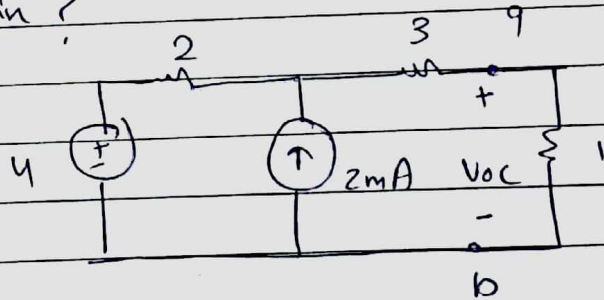
$$V_{th} = V_{oc}$$

$$= 10 I_L$$

$$= 10 \left[\left(\frac{1/2}{1/2 + 1/2} \right) * 5 \right] = 5V$$

$$R_{th} = \frac{(2+8) * 10}{(2+8)+10} = 5\Omega$$

ex:- Find thevenin ?



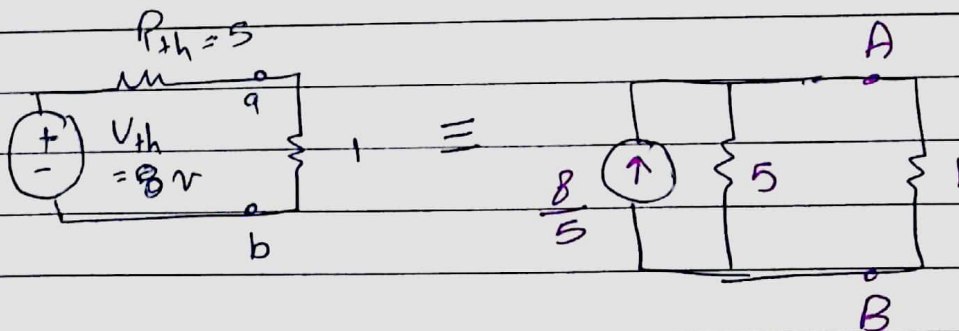
sol:-

$$V_{th} = V_{oc}$$

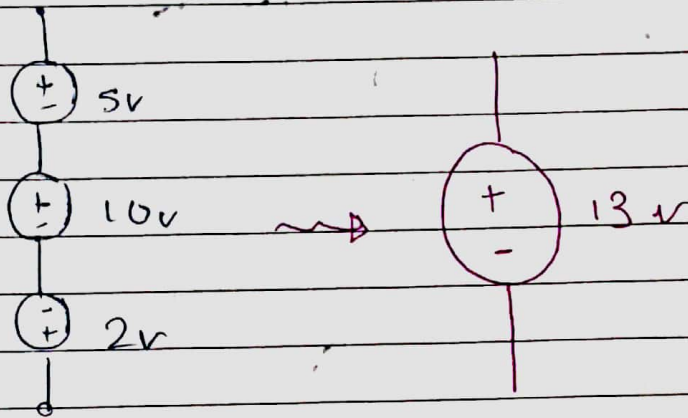
$$= 4 + (2 * -2) + 3 * 0 + V_{oc}$$

$$V_{oc} = 8V$$

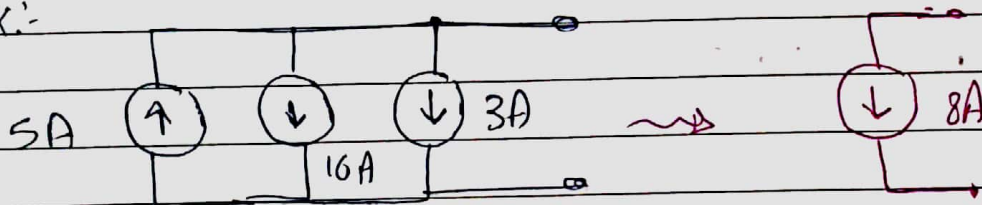
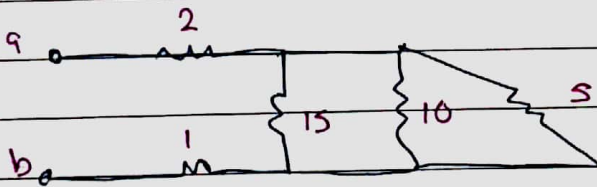
$$R_{th} \rightarrow R_{eq} = 2 + 3 = 5\Omega$$



ex:-



ex:-

ex:- Find R_{eq} between a, b.?

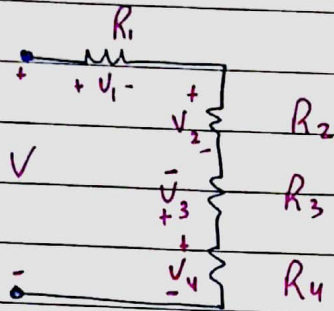
$$R_{eq} = (5 \parallel 10 \parallel 15) + 2 + 1$$

$$= \left[\frac{1}{5} + \frac{1}{10} + \frac{1}{15} \right]^{-1} + 2 + 1$$

$$= 5 \Omega$$

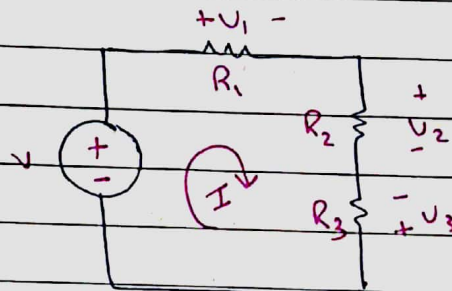
ooo Voltage and current division :-

↳ Voltage division :-



$$V_n = \left[\frac{R_n}{R_1 + R_2 + \dots + R_n} \right] \times V$$

ex:-



من كذا
التي
 $V_3 = -R_3 I$

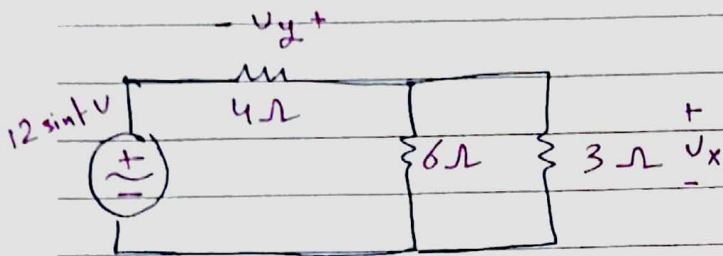
$$\begin{aligned} -V + V_1 + V_2 - V_3 &= 0 \\ -V + R_1 I + R_2 I - [-R_3 I] &= 0 \end{aligned}$$

$$I = \frac{V}{R_1 + R_2 + R_3}$$

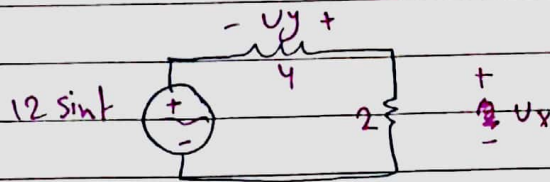
$$V_1 = R_1 I = R_1 \left[\frac{V}{R_1 + R_2 + R_3} \right]$$

$$V_3 = -R_3 I = -R_3 \left[\frac{V}{R_1 + R_2 + R_3} \right]$$

ex:- Find V_x , V_y ?



sol :- $R_{eq} \ 3 \parallel 6 = 2 \Omega$

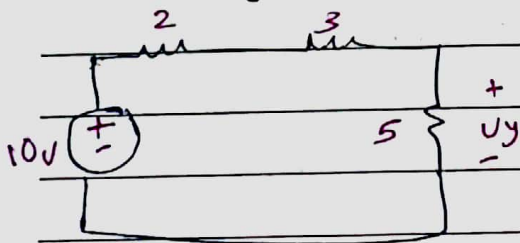
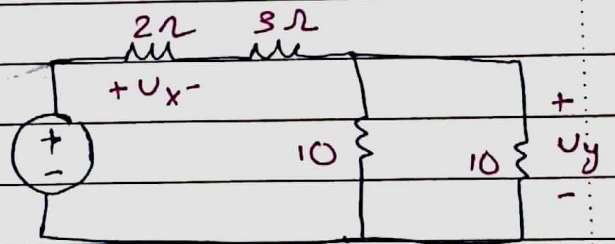


$$V_x = \frac{2}{4+2} * 12 \sin t = 4 \sin t \text{ V}$$

$$V_y = \frac{4}{4+2} * -12 \sin t = -8 \sin t \text{ V}$$

ex:- Find V_x , V_y ?

$R_{eq} = 10 \parallel 10 = 5 \Omega$ sol



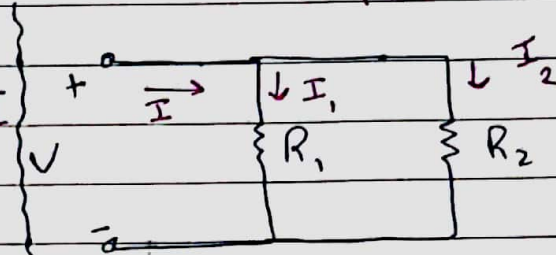
$$V_y = \frac{5}{5+2+3} * 10 \text{ V} = 5 \text{ V}$$

$$V_x = \frac{2}{10} * 10 = 2 \text{ V}$$

→ Current division :-

→ "R in Parallel"

$$I_n = \left[\frac{1/R_n}{1/R_1 + 1/R_2 + \dots} \right] * I$$

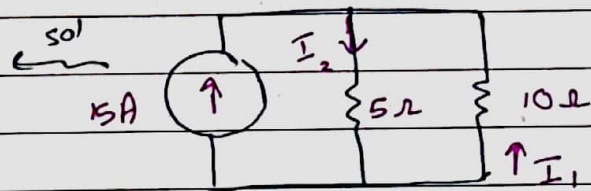


* -1 ← Ujip (Self, li) *

$$I_1 = \left[\frac{1/R_1}{1/R_1 + 1/R_2} \right] * I ; I_2 = \left[\frac{1/R_2}{1/R_1 + 1/R_2} \right] * I$$

ex:- Find I_1 ; $I_2 = ?$

$$I_1 = \left[\frac{1/10}{1/10 + 1/5} \right] * (-15)$$



$$I_2 = \left[\frac{1/5}{1/5 + 1/10} \right] * 15$$

→ method 2 :-

Ujip!!

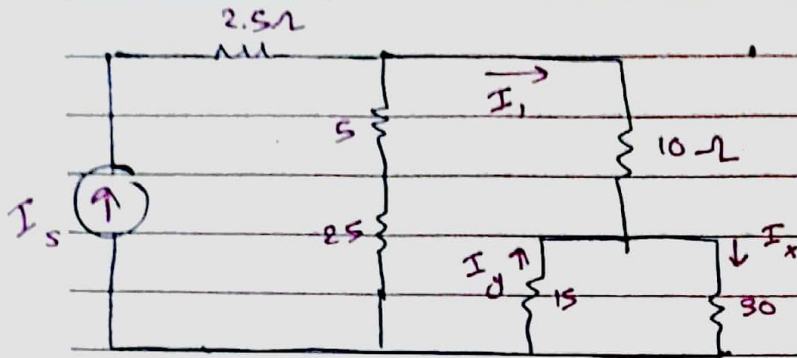
$$R_{eq} = 5 // 10$$

$$V = 15 * (5 // 10)$$

$$I_1 = \frac{(15)(5 // 10)}{5}$$

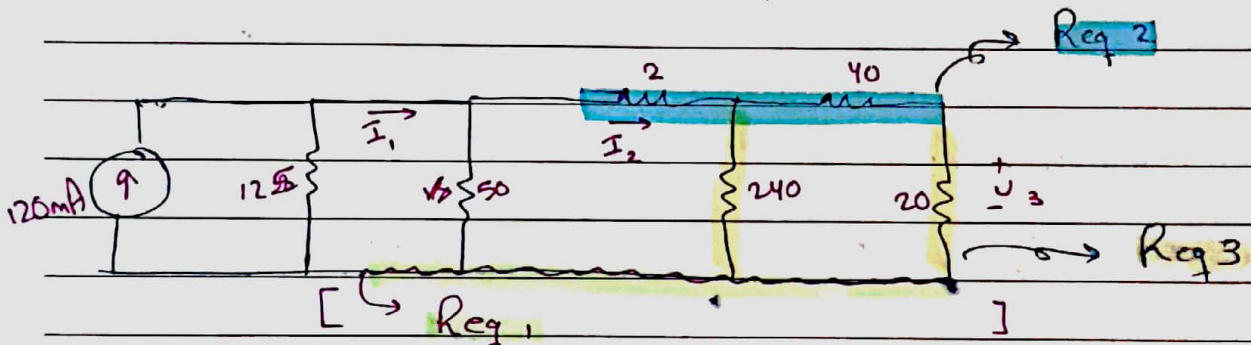
$$I_2 = \frac{-15 * (5 // 10)}{10}$$

13/3

ex: Find I_x , I_y , & $I_1 = 100 \text{ mA}$ 

sol $\rightarrow I_x = \left[\frac{1/30}{1/30 + 1/15} \right] * 100 \text{ mA} = 33.33 \text{ mA}$

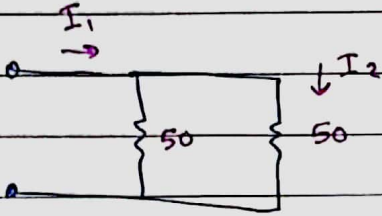
$I_y = \left[\frac{1/15}{1/15 + 1/30} \right] * -0.1 = -72.77 \text{ mA}$
 (current flowing into the node)

ex:- Find I_1 , I_2 , & V_3 & P 

$\rightarrow Req_1 = \left[\left[(20 + 40) \parallel 240 \right] + 2 \right] \parallel 50 = 25 \Omega$

$I_1 = \left[\frac{1/25}{1/25 + 1/12} \right] * 120 \text{ mA} = 100 \text{ mA}$

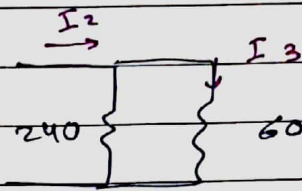
$$R_{eq2} = [(20 + 40) \parallel 240] + 2 = 50 \Omega$$



$$I_2 = \left[\frac{1/50}{1/50 + 1/50} \right] \times 100 \text{ mA}$$

$$= 50 \text{ mA}$$

$$R_{eq3} = 20 + 40 = 60$$



$$I_3 = \left[\frac{1/60}{1/60 + 1/240} \right] \times 50 \text{ mA}$$

$$= 40 \text{ mA}$$

$$V_3 = 20 \times 40 \text{ mA} = 0.8 \text{ volt}$$

* Solving equations:-

$$A) a_{11} X_1 = b_1$$

$$B) a_{11} X_1 + a_{12} X_2 = b_1$$

$$a_{21} X_1 + a_{22} X_2 = b_2$$

$$C) a_{11} X_1 + a_{12} X_2 + a_{13} X_3 = b_1$$

$$a_{21} X_1 + a_{22} X_2 + a_{23} X_3 = b_2$$

$$a_{31} X_1 + a_{32} X_2 + a_{33} X_3 = b_3$$

CH 4:- Node and Mesh analysis:-

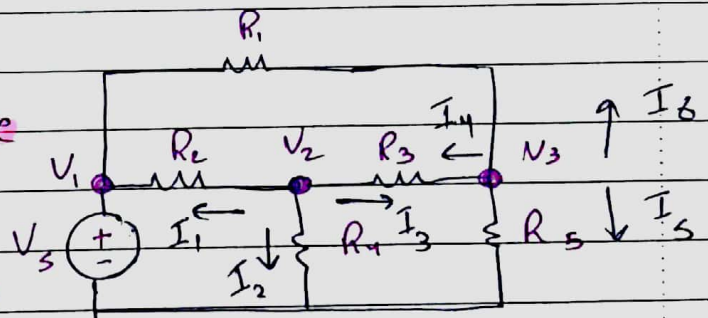
* node analysis

→ steps:-

- ① assume ~~reference~~ voltage
- ② assume voltage on each node

- ③ assume current entering each Resistor

- ④ apply KCL



→ at node 1:- $-0 + V_1 = V_s$
 $\rightarrow V_1 = V_s \quad \text{--- (1)}$

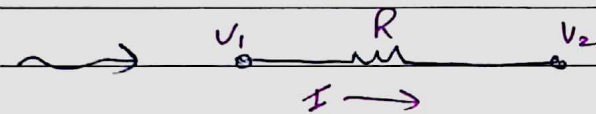
→ at node 2:- $I_1 + I_2 + I_3 = 0$
 $\left[\frac{V_2 - V_1}{R_2} + \frac{V_2 - 0}{R_4} + \frac{V_2 - V_3}{R_3} = 0 \right] \quad \text{--- (2)}$

cont:-

at node 3 :-

$$I_4 + I_5 + I_6 = 0$$

$$\frac{V_3 - V_2}{R_3} + \frac{V_3 - 0}{R_5} + \frac{V_3 - V_1}{R_1} = 0 \quad \text{--- (3)}$$



$$\therefore I = \frac{V_1 - V_2}{R}$$



$$\therefore I = \frac{V_2 - V_1}{R}$$

ex:- Find V_1, V_2 ?

→ at node 1 :-

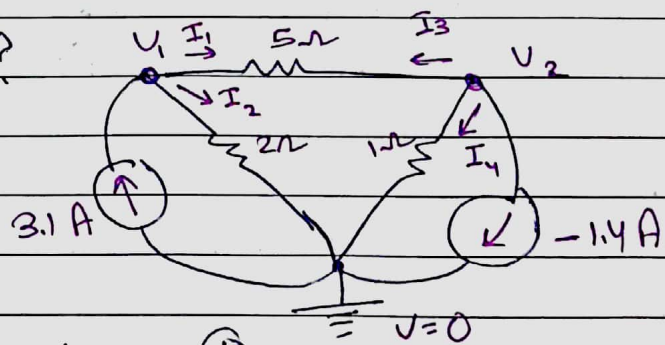
$$3.1 = I_1 + I_2$$

$$3.1 = \frac{V_1 - V_2}{5} + \frac{V_1 - 0}{2} \quad \text{--- (1)}$$

→ at node 2 :-

$$-1.4 + I_3 + I_4 = 0$$

$$1.4 = \frac{V_2 - V_1}{5} + \frac{V_2 - 0}{1} \quad \text{--- (2)}$$



$$V_1 = 5V$$

$$V_2 = 2V$$

$$I_1 = \frac{V_1 - V_2}{5} = \frac{3}{5} A$$

Ex:-

at node 1:-

$$-3 - 8 = I_1 + I_2$$

$$-11 = \frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{4} \quad (1) \quad V=0$$

at node 2:-

$$-3 + I_3 + I_4 + I_5 = 0$$

$$3 = \frac{V_2 - V_1}{3} + \frac{V_2 - 0}{1} + \frac{V_2 - V_3}{7} \quad (2)$$

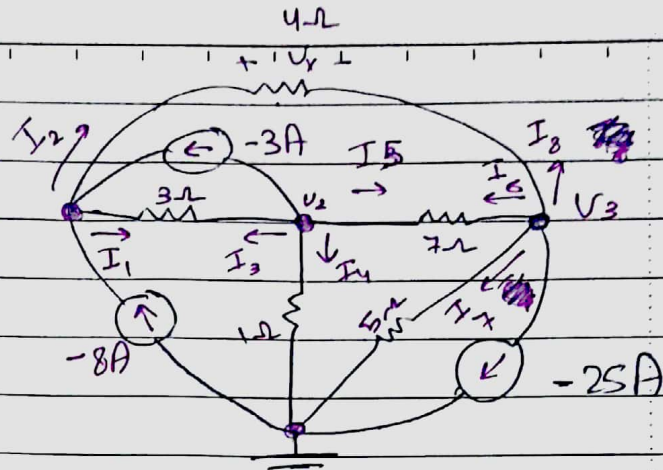
at node 3:-

$$-25 + I_6 + I_7 + I_2 = 0$$

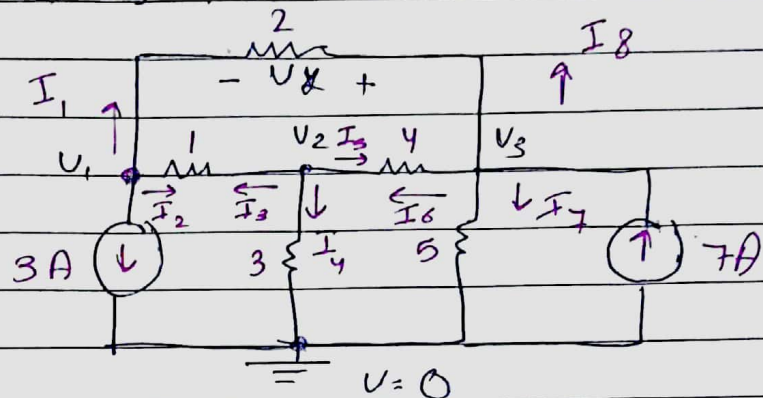
$$25 = \frac{V_3 - V_2}{7} + \frac{V_3 - 0}{5} + \frac{V_3 - V_1}{4} \quad (3)$$

$$\begin{aligned} V_1 &= 5.412 \text{ V} \\ V_2 &= 7.738 \text{ V} \\ V_3 &= 46.32 \text{ V} \end{aligned}$$

$$V_x = V_1 - V_3 \quad \#$$



ex:- Find V_1, V_2, V_3 P



at node ①:-

$$I_1 + I_2 + I_3 = 0$$

$$3 + \frac{V_1 - V_2}{1} + \frac{V_1 - V_3}{2} = 0 \quad \text{--- (1)}$$

at node ②:-

$$I_3 + I_4 + I_5 = 0$$

$$\frac{V_2 - V_1}{1} + \frac{V_2 - 0}{3} + \frac{V_2 - V_3}{4} = 0 \quad \text{--- (2)}$$

at node ③:-

$$7 = I_6 + I_7 + I_8$$

$$7 = \frac{V_3 - V_2}{4} + \frac{V_3 - 0}{5} + \frac{V_3 - V_1}{2} \quad \text{--- (3)}$$

$$\rightarrow V_1 = 5.235 \text{ V} / V_2 = 5.12 \text{ V} / V_3 = 11.217 \text{ V}$$

$$V_y = -V_1 + V_3 \text{ \#}$$

10/4

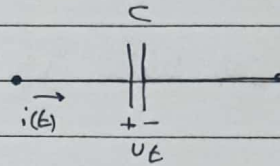
CH 7

* Capacitor and Inductors:-

↳ Both the inductors and capacitor are passive element that are capable of storing and delivering finite amount of energy.

* Capacitor

unit = Farad.



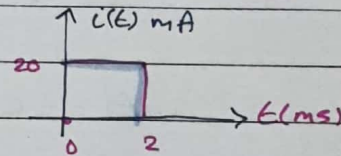
$i(t) dt =$

$$\rightarrow i(t) = C \frac{dv(t)}{dt}$$

$$i(t) dt = C dv(t) \rightarrow \int_{t_0}^t dv(t) = \frac{1}{C} \int_{t_0}^t i(t) \cdot dt$$

$$* v(t) - v(t_0) = \frac{1}{C} \int_{t_0}^t i(t) \cdot dt$$

ex:- Find $v(t)$ if $C = 5 \mu F$



$$\rightarrow i(t) = \begin{cases} 0, & -\infty \leq t \leq 0 \\ 20 \times 10^{-3}, & 0 \leq t \leq 2 \\ 0, & 2 \leq t \leq \infty \end{cases}$$

Let $v(t \rightarrow \infty) = 0$

$$\textcircled{1} v(t) - v(-\infty) = \frac{1}{5 \mu} \int_{-\infty}^t 0 \cdot dt \rightarrow v(t) = 0 \text{ #}$$

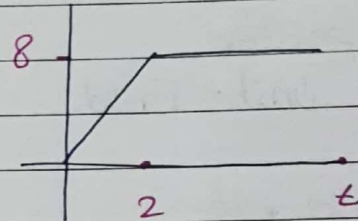
$$\textcircled{2} v(t) - v(0) = \frac{1}{5} \int_0^t 20 \times 10^{-3} dt \rightarrow v(t) = 4000t \text{ #}$$

$$\textcircled{3} v(t) - v(2) = \frac{1}{5} \int_2^t 0 \cdot dt \rightarrow v(t) = 8 \text{ #}$$

$$v(t) = \begin{cases} 0 & , -\infty \leq t \leq 0 \\ 4000t & , 0 \leq t \leq 2 \\ 8 & , 2 \leq t < \infty \end{cases} \quad \#$$

~~***m***~~

oğel börs dör zü dör ←



⊛ Power :-

$$P = VI \\ = v \left[c \frac{dv(t)}{dt} \right]$$

⊛ Energy :-

$$w(t) = \int_{t_0}^t P(t) \cdot dt$$

$$w(t) = c \int_{t_0}^t v(t) \cdot dv(t)$$

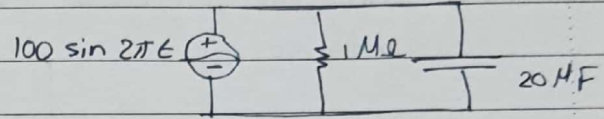
$$= c \left[\frac{v^2(t)}{2} \right]_{t_0}^t$$

$$\frac{1}{2} c [v^2(t) - v^2(t_0)] \rightarrow \text{let } v(t_0) = 0$$

$$\rightarrow w_c(t) = \frac{1}{2} c v^2(t)$$

ex :- Find $w_c(t)$, $t = \frac{1}{4} \text{ s}$, $w_R(t)$ $0 \leq t \leq 0.5$

$$\textcircled{1} w_c(t) = \frac{1}{2} C V^2$$



$$= \frac{1}{2} [20 \times 10^{-6}] [100 \sin 2\pi \left(\frac{1}{4}\right)]^2$$

$$= 100 \text{ mJ}$$

$$\textcircled{2} w_R(t) = \int_0^{0.5} p(t) \cdot dt = \int_0^{0.5} \frac{[100 \sin 2\pi t]^2}{1 \times 10^6} dt$$

$$= 2.5 \text{ mJ}$$

* Characteristic of Ideal capacitor :

1] there is no current through capacitor if a voltage across it is not changing with time capacitor is open circuit

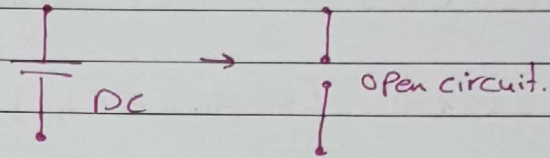
$$i(t) = C \frac{dV(t)}{dt}$$

2] Finite amount of energy can be stored in a capacitor even if the circuit is zero such that voltage is constant $w_c(t) = \frac{1}{2} C V^2(t)$

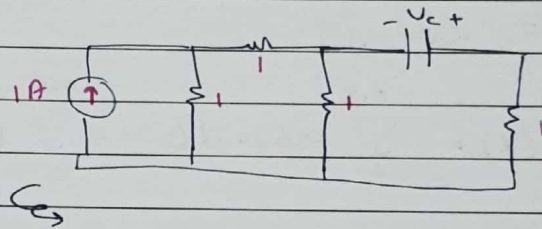
3] It's impossible to change the voltage across capacitor in finite time

$$V_c(0^+) = V_c(0^-)$$

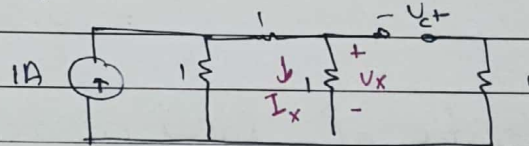
iv) Capacitor never dissipates energy but only store it, although this is true in mathematical model it is not true for physical model due to finite amount of resistance



ex:- Find $V_c(t)$?



$$-V_x - V_c = 0$$



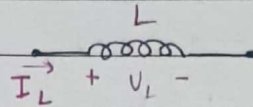
$$\Rightarrow V_x = 1 I_x$$

current

$$1 \left[\frac{1/2}{1/2 + 1/2} \right] [1] = \frac{1}{3} \text{ v.}$$

$$\Rightarrow V_c = -V_x = -\frac{1}{3} \text{ v}$$

→ Inductors :-



Unit :- henries.

$$V(t) = L \frac{dI(t)}{dt}$$

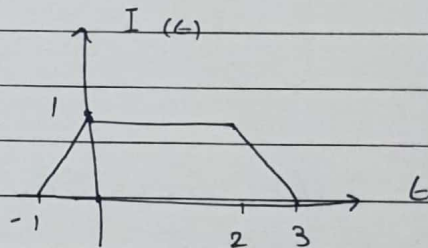
$$\int_{t_0}^t V(t) dt = L \int_{t_0}^t \frac{dI(t)}{dt} dt = L [I(t) - I(t_0)]$$

$$* I(t) - I(t_0) = \frac{1}{L} \int_{t_0}^t V(t) dt$$

ex:- Find $V(t)$ for $L = 3H$

sol:-

$$I(t) = \begin{cases} 0, & -\infty \leq t \leq -1 \\ 1, & 0 \leq t \leq 2 \\ 0, & 3 \leq t \leq \infty \end{cases}$$



$$* y - y_1 = m(x - x_1)$$

$$I - I_0 = m(t - t_0)$$

$$\Rightarrow (-1, 0) (0, 1)$$

$$I - 0 = 1(t + 1)$$

$$I = t + 1$$

$$\rightarrow (2, 1) (3, 0)$$

$$I - 0 = (-1)(t - 3) \Rightarrow I = -t + 3$$

معادلة الخط المستقيم

لاوجد الفترات بين $[-1 \rightarrow 1]$ $[2 \rightarrow 3]$

$$I(t) = \begin{cases} 0, & -\infty \leq t \leq -1 \\ t+1, & -1 \leq t \leq 0 \\ 1, & 0 \leq t \leq 2 \\ -t+3, & 2 \leq t \leq 3 \\ 0, & 3 \leq t \leq \infty \end{cases}$$

$$\rightarrow V(t) = L \frac{dI(t)}{dt} = 3 \frac{dI(t)}{dt}$$

$$\frac{dI(t)}{dt} = \begin{cases} 0 & -\infty \leq t \leq -1 \\ 1 & -1 \leq t \leq 0 \\ 0 & 0 \leq t \leq 2 \\ -1 & 2 \leq t \leq 3 \\ 0 & 3 \leq t \leq \infty \end{cases}$$

$$\frac{dI}{dt} = \begin{cases} 0 & -\infty \leq t < -1 \\ 1 & -1 \leq t \leq 0 \\ 0 & 0 \leq t \leq 2 \\ -1 & 2 \leq t \leq 3 \\ 0 & 3 \leq t \leq \infty \end{cases}$$

$$v(t) = \begin{cases} 0 & \\ 3 & \\ 0 & \\ -3 & \\ 0 & \end{cases}$$

ex:- $v(t) = 6 \cos 5t$, $L = 2H$, $I(-\frac{\pi}{2}) = 1A$
Find $I(t)$?

$$\rightarrow I(t) - I(-\frac{\pi}{2}) = \frac{1}{2} \int_{-\frac{\pi}{2}}^t 6 \cos 5t \cdot dt$$

$$I(t) = 0.6 \sin 5t + 1.6$$

* Power:- $VI \sim I \left[L \frac{dI(t)}{dt} \right]$

* Energy:- $w_e(t) = \int_{t_0}^t P_e(t) dt = \int_{t_0}^t I \left[L \frac{dI(t)}{dt} \right] dt$

$$= L \int_{t_0}^t I dI(t) = L \left. \frac{I^2(t)}{2} \right|_{t_0}^t$$

$$= \frac{1}{2} L [I(t)^2 - I(t_0)^2]$$

Let $I(t_0) = \text{zero} \rightarrow W(t) = \frac{1}{2} L I^2$ #

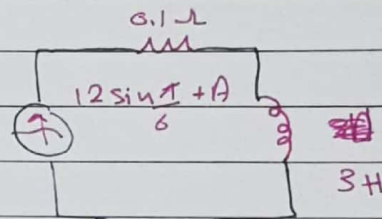
ex:- Find $W_L(t)$ at $t=3s$, $W_R(t)$ at $0 \leq t \leq 6$

Sol:- ① $W_L(t) = \frac{1}{2} L I(t)^2$

$= \frac{1}{2} [3] \left[\frac{12 \sin \pi t}{6} \right]^2$

at $t=3$

$W = 216 \text{ J}$



② $W_R(t) = \int_0^6 [12 \sin \frac{\pi t}{6}]^2 [0.1] dt = 43.2 \text{ J}$

* Characteristic of Ideal inductor :-

- ① There is no voltage across inductor if the current through it isn't changing with time inductor short circuit to DC

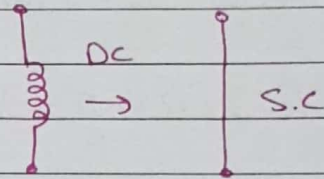
$\rightarrow V(t) = L \frac{dI(t)}{dt}$

- ② Finite amount of energy can be stored in inductor even if the voltage is zero such that current is constant

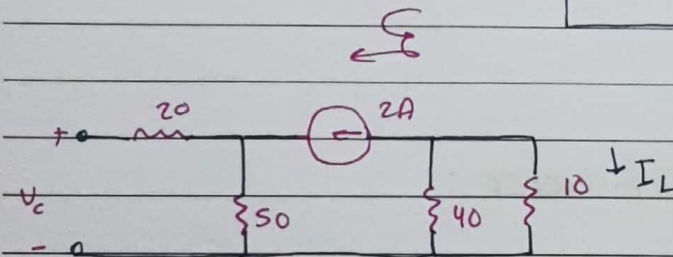
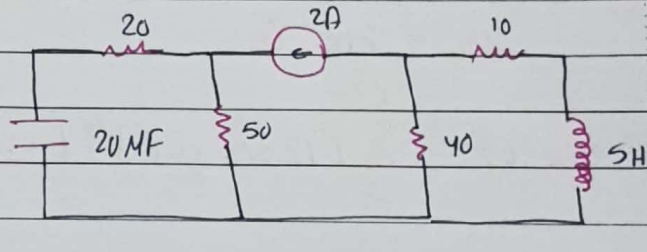
$\rightarrow W_L(t) = \frac{1}{2} L I^2(t)$

$$\textcircled{3} I_L(0^+) = I_L(0^-)$$

④ the inductor never dissipate energy but only store it although this is true for math. model it is not true for physical Model due to internal resistance



ex:- Find $W_L(t) = W_C(t)$



$$\textcircled{1} W_C(t) = \frac{1}{2} C V^2$$

$$V_C = 50(2) = 100$$

$$W_C(t) = \frac{1}{2} [20 \times 10^{-6}] [100]^2$$

$$= 0.1 \text{ J}$$

$$\textcircled{2} W_L(t) = \frac{1}{2} L I_L^2(t)$$

$$I_L = \left[\frac{1/10}{1/10 + 1/40} \right] [-2] = -1.6 \text{ A}$$

$$W_L(t) = \frac{1}{2} [5] [-1.6]^2 = 6.4 \text{ J}$$

* General ohm Law :-

$$V = Z I \rightarrow \begin{matrix} \text{impedance (ohm)} \\ \text{current (A)} \\ \text{voltage (V)} \end{matrix}$$

$$* Z_R = R, \quad * Z_L = j\omega L, \quad * Z_C = \frac{1}{j\omega L}$$

→ KVL :-

$$\sum_{\text{Loop}} V(t) = 0 \rightarrow \sum_{\text{Loop}} V = \text{zero}$$

8

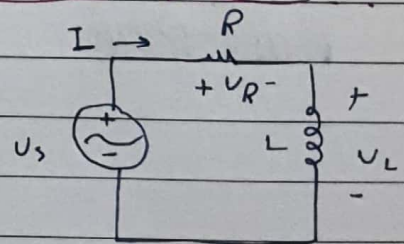
→ KCL

$$\sum_{\text{node}} i(t) = 0 \rightarrow \sum_{\text{node}} I = 0$$

$$-U_s + U_R + U_L = \text{Zero}$$

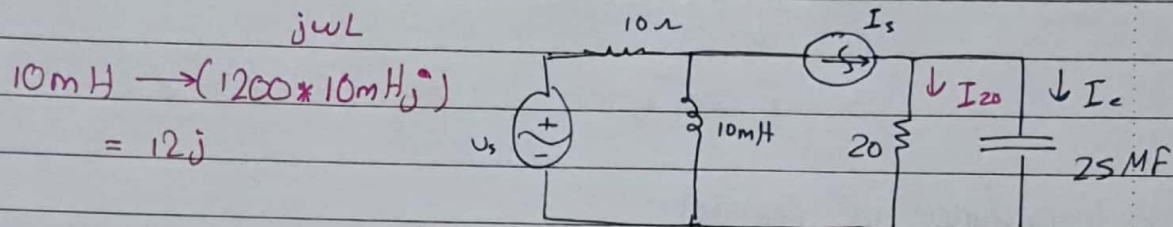
$$-U_s + RI + (j\omega L)I = 0$$

$$I = \frac{U_s}{R + j\omega L} \quad \#$$



ex:- Let $\omega = 1200 \text{ rad/s}$, $I_c = 1.2 \angle 28^\circ \text{ A}$

$I_L = 3 \angle 33^\circ \text{ A}$ Find I_s , V_s , $i_R(t)$



$$25 \text{ mF} \rightarrow \frac{1}{j\omega C} = \frac{1}{j(1200)(25 \text{ mF})} = -33.33j$$

$$V_c = (-33.33j)(1.2 \angle 28^\circ) = 40 \angle -62^\circ \text{ V}$$

$$I_{20} = \frac{40 \angle -62^\circ}{20} = 2 \angle -62^\circ \text{ A}$$

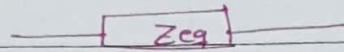
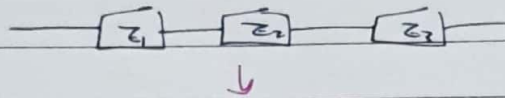
$$I_s = I_{20} + I_c = 2.332 \angle -31.04^\circ \text{ A}$$

$$I_R = I_s + I_L = 3.986 \angle 17.42^\circ$$

$$i_R(t) = 3.986 \cos(1200t + 17.42^\circ) \text{ A}$$

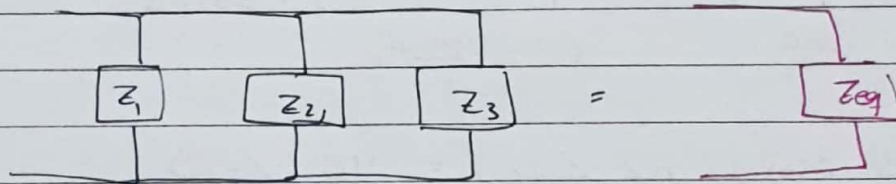
$$\rightarrow -V_s + 10 I_R + 12j [3 \angle 33^\circ] = 34.86 \angle 74.55^\circ$$

* Impedance in Series:-



$$Z_{eq} = Z_1 + Z_2 + Z_3 \dots$$

* Impedance in parallel:-



$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \dots$$

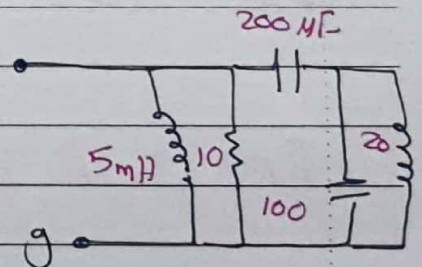
Ex:- Find $Z_{eq} = ?$, $\omega = 1000 \text{ rad/s}$

$$5 \text{ mH} \rightarrow j\omega L = 5j$$

$$20 \text{ mH} \rightarrow j\omega L = 20j$$

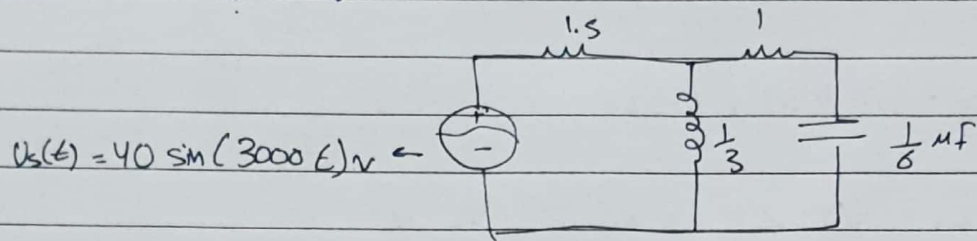
$$200 \text{ nF} \rightarrow 1/j\omega C = -5j$$

$$100 \text{ nF} \rightarrow 1/j\omega C = -10j$$



$$Z_{eq} = [[20j \parallel -10j] + (-5j)] \parallel 10 \parallel 5j$$

ex:- Find $i(t)$ p.u.



$$\rightarrow v_s(t) = 40 \sin(3000t)$$

$$\omega = 3000$$

$$V_m = 40$$

$$= 40 \cos(3000t - 90)$$

$$\rightarrow v_s = 40 \angle -90^\circ \text{ V}$$

$$1/3 \rightarrow j\omega L \rightarrow 1 \text{ k}\Omega$$

$$1/6 \rightarrow 1/j\omega L \rightarrow -2 \text{ k}\Omega$$

$$Z_{eq} = [(-2 \text{ k}\Omega) + 1 \text{ k}\Omega] \parallel 1 \text{ k}\Omega + 1.5 \text{ k}\Omega$$

$$= 2.5 \angle 36.87^\circ \text{ k}\Omega$$

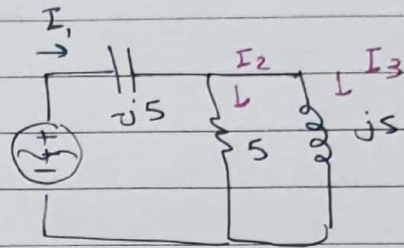
$$I = \frac{v_s}{Z_{eq}} = \frac{40 \angle -90^\circ}{2.5 \angle 36.87^\circ} = 16 \angle -126.9^\circ \text{ mA}$$

$$i(t) = 16 \cos(3000t - 126.9^\circ) \text{ mA}$$

ex :- Find I_1, I_2, I_3

$$Z_{eq} = (j5 \parallel 5) + (-j5)$$

$$I_1 = \frac{100 \angle 0}{Z_{eq}} = 28.28 \angle 45^\circ \text{ A}$$



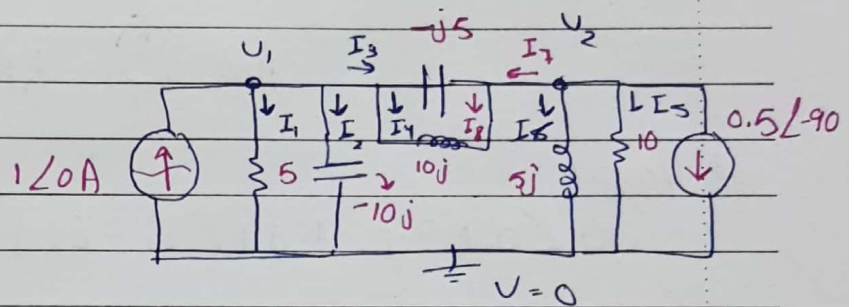
$$I_2 = \frac{1/5}{1/5 + 1/j5} [28.28 \angle 45] = 20 \angle 90^\circ \text{ A}$$

$$I_3 = \frac{1/j5}{1/5 + 1/j5} [28.28 \angle 45] = 20 \angle 0^\circ \text{ A}$$

* nodal analysis:-

- ① assume reference voltage
- ② assume voltage on each node
- ③ assume current entering each Z
- ④ apply KCL

ex:-



at node 1:-

$$1\angle 0 = I_1 + I_2 + I_3 + I_4$$

$$1\angle 0 = \frac{V_1}{5} + \frac{V_1}{-j10} + \frac{V_1 - V_2}{-j5} + \frac{V_1 - V_2}{j10} \quad \text{--- (1)}$$

at node 2:-

$$0.5\angle -90 + \frac{V_2}{10} + \frac{V_2}{2j} + \frac{V_2 - V_1}{-j5} + \frac{V_2 - V_1}{j10} = 0 \quad \text{--- (2)}$$

$$\sim \rightarrow V_1 = 2.24\angle -63.4 \quad / \quad V_2 = 4.47\angle 116.6^\circ$$

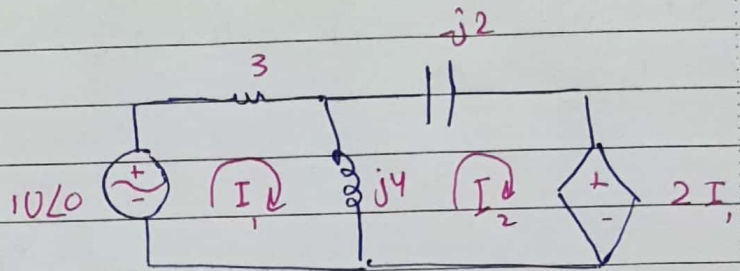
$$V_1(t) = 2.24 \cos(\omega t - 63.4) \quad / \quad V_2(t) = 4.47 \cos(\omega t + 116.6^\circ)$$

⊗ mesh analysis

① assume current on each loop

② apply KVL.

ex:- $\omega = 10^3$



at loop 1 :-

$$-10 \angle 0 + 3I_1 + j4[I_1 - I_2] = 0$$

at loop 2 :-

$$j4[I_2 - I_1] - j2I_2 + 2I_1 = 0$$

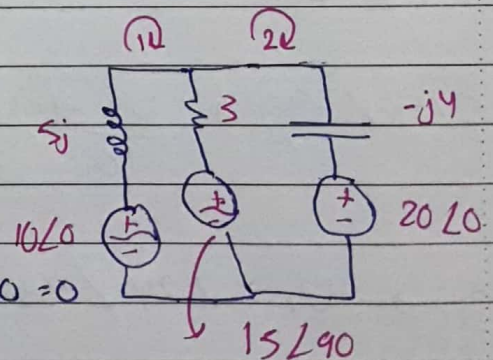
$$\rightarrow I_1 = 1.24 \angle 24.7^\circ \text{ A} \sim i(t) = 1.24 \cos(10^3 t + 24.7^\circ)$$

$$\rightarrow I_2 = 2.77 \angle 56.3^\circ \text{ A} \sim i(t) = 2.77 \cos(10^3 t + 56.3^\circ)$$

α:- Find $I_1, I_2 = ?$

at loop 1:-

$$-10 \angle 0 + 5j I_1 + 3[I_1 - I_2] + 15 \angle 90 = 0$$



at loop 2:-

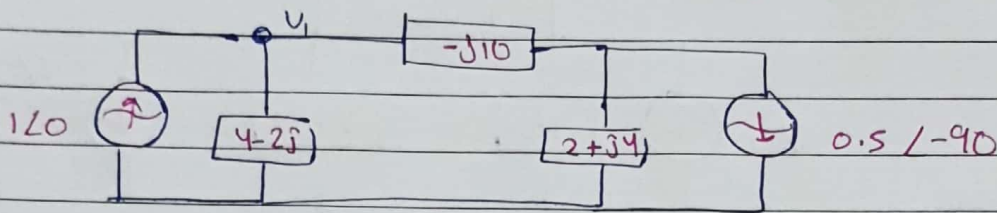
$$-15 \angle 90 + 3(I_2 - I_1) - j4 I_2 + 20 \angle 0 = 0$$

$$\rightarrow I_1 = 4.87 \angle -164.6^\circ \text{ A}$$

$$I_2 = 7.17 \angle -144.9^\circ \text{ A}$$

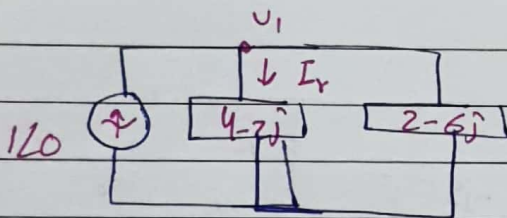
ex:- Find v_i ?

(superposition)



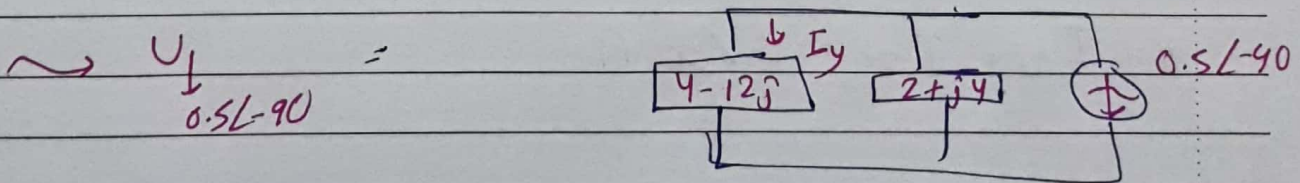
Using superposition :-

$$v_i = v_{i1} + v_{i2} = \boxed{1-j2} + \boxed{-1}$$

 ~~$v_i(4-2j)$~~

$$\begin{aligned} v_i &= (4-2j) I_x \\ &= 4-2j \times \frac{1 \angle 0}{(1/4-2j) + (1/2-j6)} \end{aligned}$$

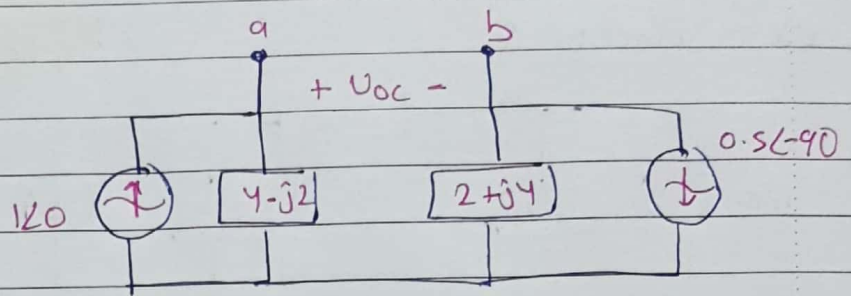
$$= \boxed{2-j2} \text{ V}$$



$$v_i = (4-2j) I_y$$

$$v_i = 4-2j \left[\frac{1/(4-j12)}{1/(4-j12) + (1/(2+j4))} \right] [0.5 \angle -90] = \boxed{-1}$$

* Find Thevenin



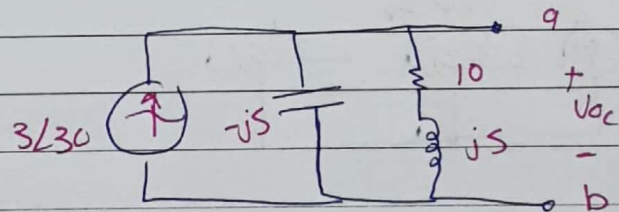
$$U_{th} = V_{oc}$$

$$V_{oc} = (4-j2)(120) + (2+j4)(0.5\angle-90) = 6-j3 \text{ V}$$

$$Z_{th} = (4-j2) + (2+j4) = 6+j2$$

* Find Thevenin

$$* U_{th} = V_{oc}$$



$$V_{oc} = [(10+j5) \parallel -j5] \cdot 3\angle30$$

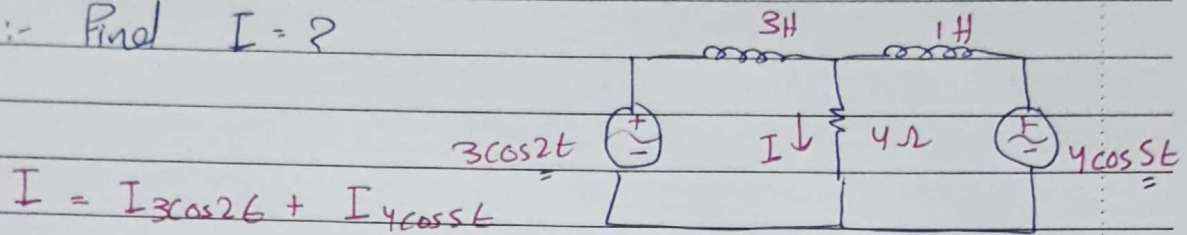
$$= 16.77 \angle -33.43 \text{ V}$$

* $Z_{th} = Z_{eq}$ after killing all sources

$$(10+j5) \parallel -j5 = 2.5-j5 \text{ } \Omega$$

$$* I_N = I_{sc} = 3\angle30 \text{ A}$$

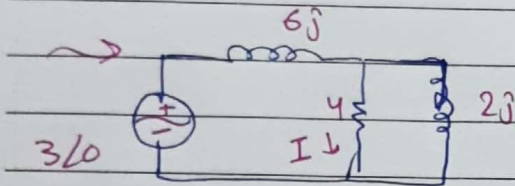
ex:- Find $I = ?$



$$I = I_{3\cos 2t} + I_{4\cos 5t}$$

super position
position

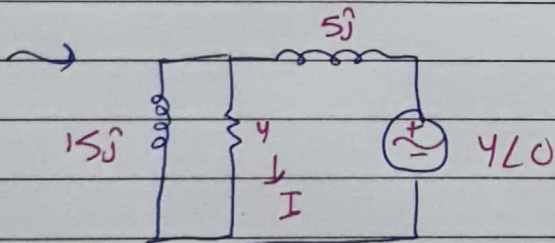
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$$I = \frac{V}{Y} = \frac{1}{4} \left[\frac{2j \parallel 4}{2j \parallel 4 + 6j} \right] [3\angle 0]$$

$$= 175.6 \angle -20.56 \text{ mA}$$

$$i(t) = 175.6 \cos(2t - 20.56) \text{ mA}$$



$$I = \frac{V}{Y} = \frac{1}{4} \left[\frac{4 \parallel 15j}{4 \parallel 15j + 5j} \right] [4\angle 0]$$

$$= 547.1 \angle -43.15 \text{ mA}$$

$$i(t) = 547.1 \cos(5t - 43.15) \text{ mA}$$

$$\rightarrow I_{\text{total}} = 175.6 \cos(2t - 20.56) + 547.1 \cos(5t - 43.15) \text{ mA}$$