TURBO
Continuous change



دفتر :

هندسة معوليّة

Reliability Engineering

الاسبوع (1-5)

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اللجنة الأكاديمية لقسم الهندسة الصناعية

2025

TurboIEG.Com

Turbo Team Youtube

What is Failure?



الشيء إلي بدي إياه بطل يقوم بشغله بالطريقة الموجود لأجلها

- Failure: an item unable to perform predefined functions
- System has inherent capacity to withstand the challenges. A failure may occur when challenges surpass the capacity of the system
- Include both internal and external challenges

- Failure is a random event!!!!

Random Predicted Pattern

lack of Knowledge

Failure

deterministic ← مش هيك بشكل مطلق وRandom ← بشكل مطلق عندي كل

لي ما عندي معلومات كاملة عن أتوقع إنه يخرب

لو كل المعلومات عندي بقدر أتنبأ

ومستحيل توصل ٪100

بتكون Function of time

رح يفل عن variability

distribution أو cycle ←

Objective of Reliability Engineering?

Reliability engineering attempts to study, characterize, measure, and analyze the failure in order to:

- Improve upon their operational use by (increasing their design life)
- Eliminating or reducing the likelihood of failures and safety risks ←
- And reducing downtime thereby increasing available operating time

Between them ← بدنا نعرف الفرق بينهم

أحمد الياس

→ Deterministic vs. Random Failures ←

- Traditional approach (نهج) to safety in engineering is to design into a product a high safety margin (هامش) or safety factor ←
  - a **deterministic** method in which a safety of factor (marjon) of perhaps 4 to 10 times the expected load or stress would be allowed for in the design (أضعاف)
- Safety factors often result in overdesign thus increasing costs

لازم أعرف بين ال Failures ←

Failures

بس بقدر أتوقعه ← Random ←

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Deterministic vs. Random Failures

- Approach taken in reliability is to treat (تعامل) failures as random or probabilistic occurrences (احداث احتمالية)
  - In theory, if we were able to comprehend the exact physics and chemistry of a failure process, many internal failures of a component could be predicted with certainty ←
  - With limited data on the physical state of a component (مكون), and an incomplete knowledge of the physical, chemical (and perhaps biological) processes which cause failures, failures will appear to occur at random over time ←
  - This random process may exhibit a pattern which can be modeled by some probability distribution

إذن لازم أفهم حتى أقدر أعمل Forecasting لـ Failure

هون أنا عندي معلومات محدودة ← كمان بصير Random Failure

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Random Phenomena



- Observed in practice when dealing with large numbers of components
- Statistically can predict the failure of these components
- Failures caused by events external to the component, such as environmental conditions like excessive heat or vibration, hurricanes or earthquakes, will appear to be random
- With sufficient understanding of the conditions resulting in the event as well as the effect such an event would have on the component, then we should also be able to predict these failures deterministically

Random Phenomena



- The uncertainty or incomplete information, about a failure process is a result of
  - Its complexity
  - Imprecise measurements of the relevant physical constants and variables
- The indeterminable nature of certain future events

Some Definitions

- **Reliability** is the probability that a component or system will perform a required function for a given period of time when used under stated operating conditions
- (It is the probability of non-failure)

احتمال عدم الفشل

R(t)

* مدة سلامية الجهاز قبل مايخرب

لم يتوقع عمر التلفون (راح يضل شغال لمدة خمس سنين بدون مايخرب ف (R(t)=5)

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Some Definitions

عند وقت معين يكون شغال

- **Availability** is the probability that a component or system is performing its required function at a given point in time when used under stated operating conditions

لم يكون أو النظام يعمل وظيفة بوقت معين ← شوفا (استخدمه بظروف التشغيلية)

A(t) .

$$A(t) = \frac{M + BF}{MTBF + MTTR}$$

$$A(t) = \frac{M + BF}{MTBF + MTTR}$$

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Some Definitions

قابلية الصيانة

- **Maintainability** is the probability that a failed component or system will be restored or repaired to a specified condition within a period of time when maintenance is performed in accordance with prescribed procedures

M(t)

← احتمال إني أصلح ال Failure الموجود خلال فترة زمنية معينة بعمل فيها الصيانة حسب الاجراءات المكتوبة

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Why Study Reliability?

- The increased complexity and sophistication (تطورها) of systems → Random Behavior ↑
- Public awareness (الوعي العام) and insistence on product quality
- New laws and regulations (لوائح) concerning product liability (مسؤولية المنتج) → مسؤوليتي عن ال Product
- Government requirements to meet reliability and maintainability performance specifications
- Profit considerations resulting from the high cost of failures, their repairs, and warranty programs ← إلي بطرب وقت الكفالة

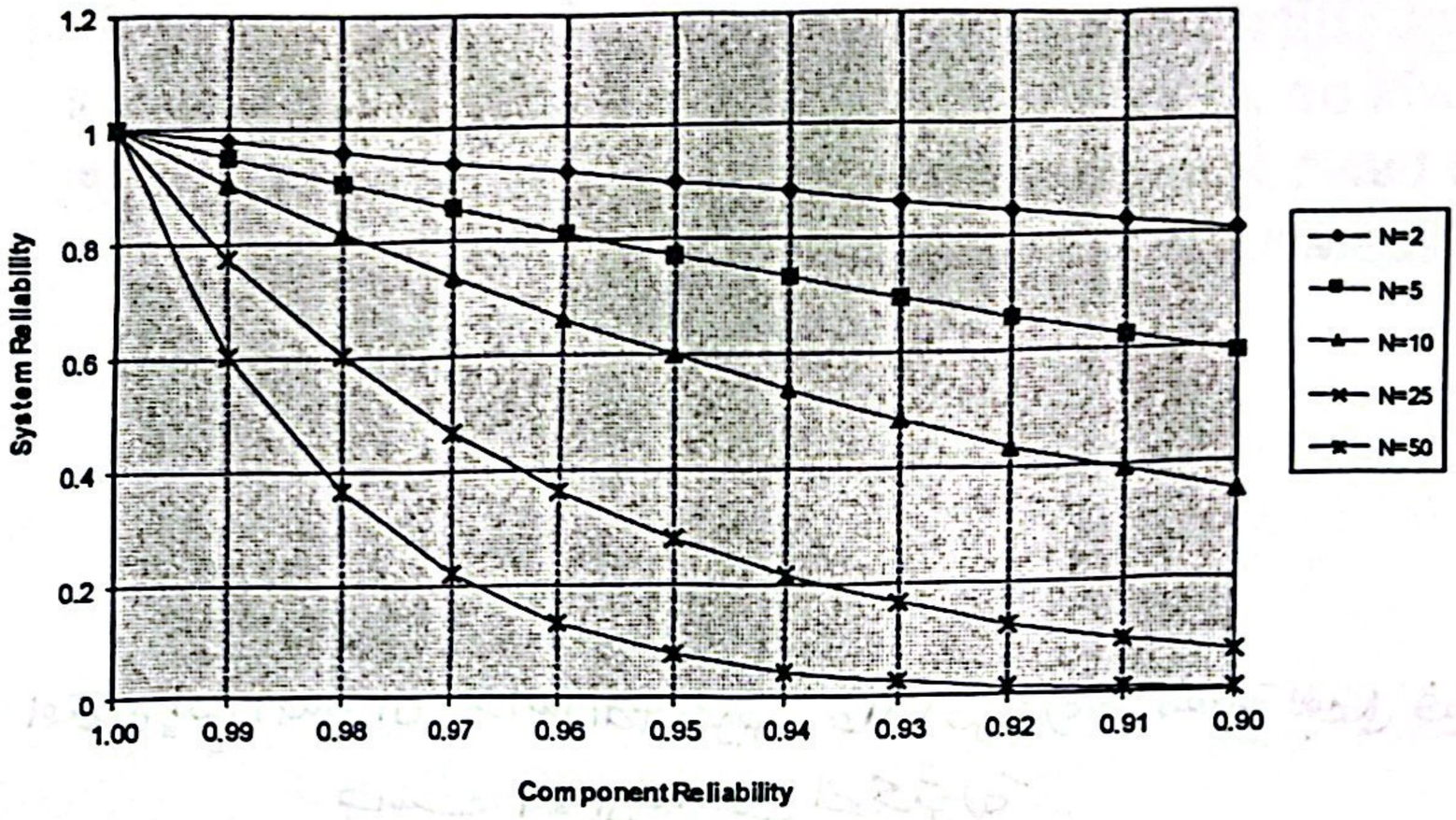
(اعتبارات)

cost ↓ مخان ↓

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Complexity and Reliability



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Government Regulations

معرفة فقط ←

موضوعنا ←

Food and Drug Act

Flammable Fabrics Act

Federal Hazardous Substance Act

National Traffic and Motor Vehicle Safety Act

Fire Research and safety Act

Child Protection and Toy Safety Act

Occupational Safety and Health Act

Federal Boat Safety Act

Consumer Product Safety Act



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Most Important Product Attributes (Survey)

Attribute	Average Score
Performance	9.5
Lasts long time (reliability)	9.0
Service	8.9
Easily Repaired (maintainability)	8.8
Warranty	8.4
Easy to Use	8.3
Appearance	7.7
Brand Name	6.3
Packaging/Display	5.8
Latest Model	5.4



Performance
Lasts long time (reliability)
Easily Repaired (Maintainability)

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Reliability vs Quality



- Quality is the amount by which a product satisfies the users' (customers') requirements/ Product quality is in part a function of design and conformance to design specifications during manufacture
- Reliability is concerned with how long the product continues to function once it becomes operational/Therefore reliability can be viewed as the quality of the product's operational performance over time, and as such it **extends quality into the time domain**

quality overtime

Reliability → جودة الأداء التشغيلي للمنتج مع الوقت

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Reasons for Evaluating Reliability

تقييم

- Assessing characteristics of materials → Knowledge (تقييم)
- Predict product reliability in design stage → lifecycle for Product (توقع)
- Assessing the effect of a proposed design change → لو غيرت ال design حتى اذا يغير على ال cost ..
- Comparing two or more different venders (بائع)
- Assess product reliability in field → ممكن يكونوا قريب لل Random
- Predict product warranty costs (توقع التكلفة (الضمان))

لما اكمارح إلى / 2 أدفعها كالكفالة

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Example 1.1

- Company manufactures small motors for use in household appliances
- designed a new motor which has experienced an abnormally high failure rate with 43 failures reported from among the first 1000 motors produced
- Possible causes of these failures included faulty design, defective material, or a manufacturing (tolerance) problem
- The company initiated an aggressive accelerated life testing program where they observed that those motors produced near the end of a production run were failing at a higher rate than those at the start of the run

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Example 1.1

- Table 1.1 summarizes the results of testing program
- The failure rate is computed by dividing the number of failures by the total number of hours on test
- It was assumed that the production process was going "out of control" and design tolerances were not being met
- Additional emphasis placed on quality control

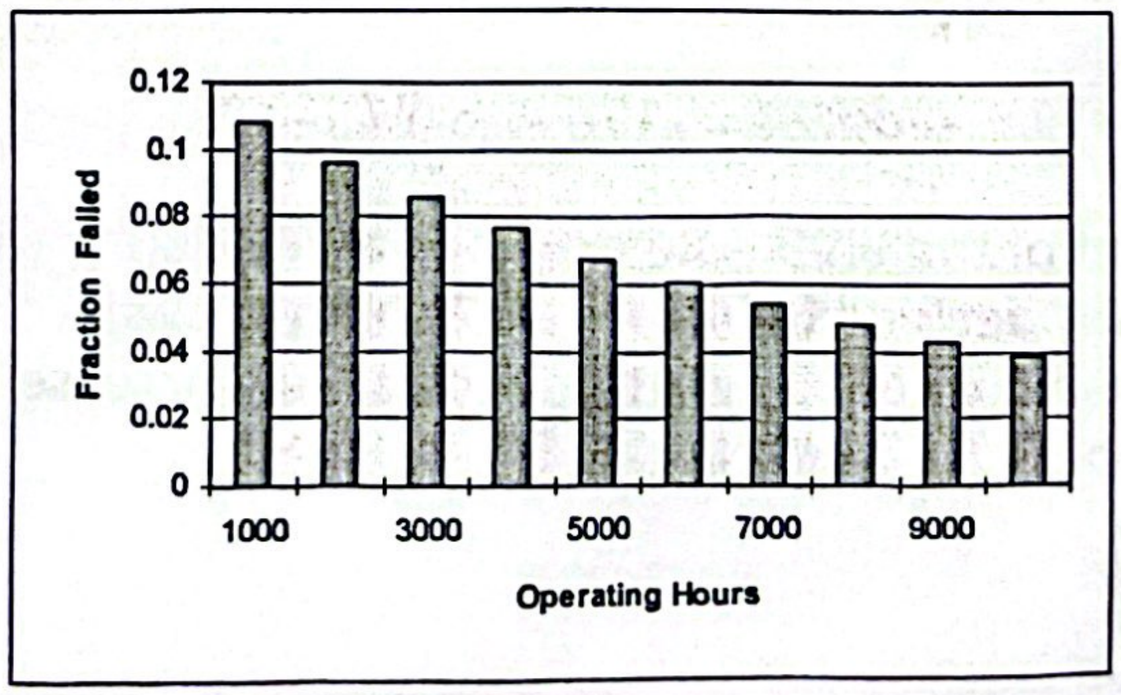
motor	# 1-100	# 101-200	# 201-300	# 301-400	# 401-500	Total
number tested	12	11	12	12	15	62
hours on test	2540	2714	2291	1890	2438	11873
number failed	1	0	1	5	7	14

2438 / 7 → Failure rate

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Example 1.2

- For a new VCR unit produced by the XYZ Company, the following distribution of the time to failure was obtained from a reliability testing program



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Example 1.2

video cassette recorder

- From this data, F(t) was derived where F(t) is the probability of a VCR failure occurring by time t (in operating hours):

$$F(t) = 1 - e^{-\frac{t}{8750}}$$

- For warranty analysis, what is the probability of failure during the first year, first 2 years, first 3 years? (survey showed that the average use of the VCR is 3 hours a day)

Example 1.2

Considering 1 year warranty:

- Assuming the typical consumer will use the VCR an average of 3 hours a day, then for the first year 1095 operating hours (3 x 365) will be observed. Therefore, the probability of a unit failing is

$$F(1095) = 1 - e^{-\frac{1095}{8750}} = 1 - .8824 = .1176 \longrightarrow 11\%$$

- With over ten percent of the units sold expected to fail during the first year, the company decided to initiate a reliability growth to improve product reliability, reduce warranty costs, and increase customer satisfaction

very important

Example 1.3

- A continuous flow production line requires a product to be processed sequentially on 10 different machines
- When a machine breaks down, the entire line must be stopped until the failure is repaired – an average downtime of 12 hours → meantime to repair
- Machine specs require a 0.99 reliability for each machine over an 8 hour production run
- Assuming a constant failure rate (exponential failure distribution)
- In order to meet production quotas, the line must maintain at least a 0.92 availability

$R(t) = e^{-\lambda t}$ → reliability function for exponential distribution

MTBF → Meantime between failure

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$t = 8\,hr$

$\rightarrow MTBF = \frac{1}{\lambda}$ → $(.99)^{10} = e^{-\lambda * 8}$; λ → reliability

→ 138 ← 0.92 .99 → 80↑ عندما يكون هناك تحسين

Example 1.3

How you can decrease MTTR:

- Hiring an additional maintenance person
- Increasing machine spare parts inventory
- Relocating the inventory closer to the production line ← نقل مخزون أقرب لخط الانتاج
- Improving diagnostic procedures ← تحسين اجراءات التشخيص
- Simplifying the removal and replacement of high-failure components
- etc.....

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Reliability Specification

- **Define failure**
  - What function is performed?
  - Identify failure modes → (type of failure اصدا بوظيفي ضرب)
- **Time to failure**
  - Calendar time → يضرب بوقت معين أو لا
  - Operating hours → كم الحدة إلي ضل شغال فيها
  - Number of cycles (on/off, load reversals, missions) → كمية بحي الجهاز وبيتزل (مدة معينة)
  - Vehicle miles - incidents per 1000 vehicles (IPTV) → زي السيارات direction/magnitude
- **State normal conditions**
  - Design loads (weight, voltage, pressure, etc.)
  - Environment (temp., humidity, vibration, contaminants, etc.)
  - Operating (usage, storage, maintenance, shipment, etc.)

specification

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Reliability Specification

→ MTTF → إذا الضرب ما بحتاجه
MTBF → ضرب بعلوم حي

- **Avoid vagueness** → لا تكون غامض
  - e.g. "as reliable as possible" ←
- **Be realistic** → واقعي
  - e.g. "will not fail under any operating conditions"
- **Avoid using only the MTTF or MTBF unless failure rate is constant** → تجنب
- **Frame in terms of reliability or design life**
  - a 95 percent reliability at 10,000 operating hours
  - a design life of 10,000 operating hours with a 95 percent reliability

$R(t) = e^{-\lambda t}$

λ → Failure rate

t → Time to Failure

لو كانت ٪ 99 لازم تكون أقل

كل ما زادت النسبة تحدد الساعات اكتوفق لازم يدقه

Reliability Specification – Example

Is this reliability specification?



60 watt
Avg. lumens 870
Avg. life 1000 hours

Which average? - mean, median, mode?

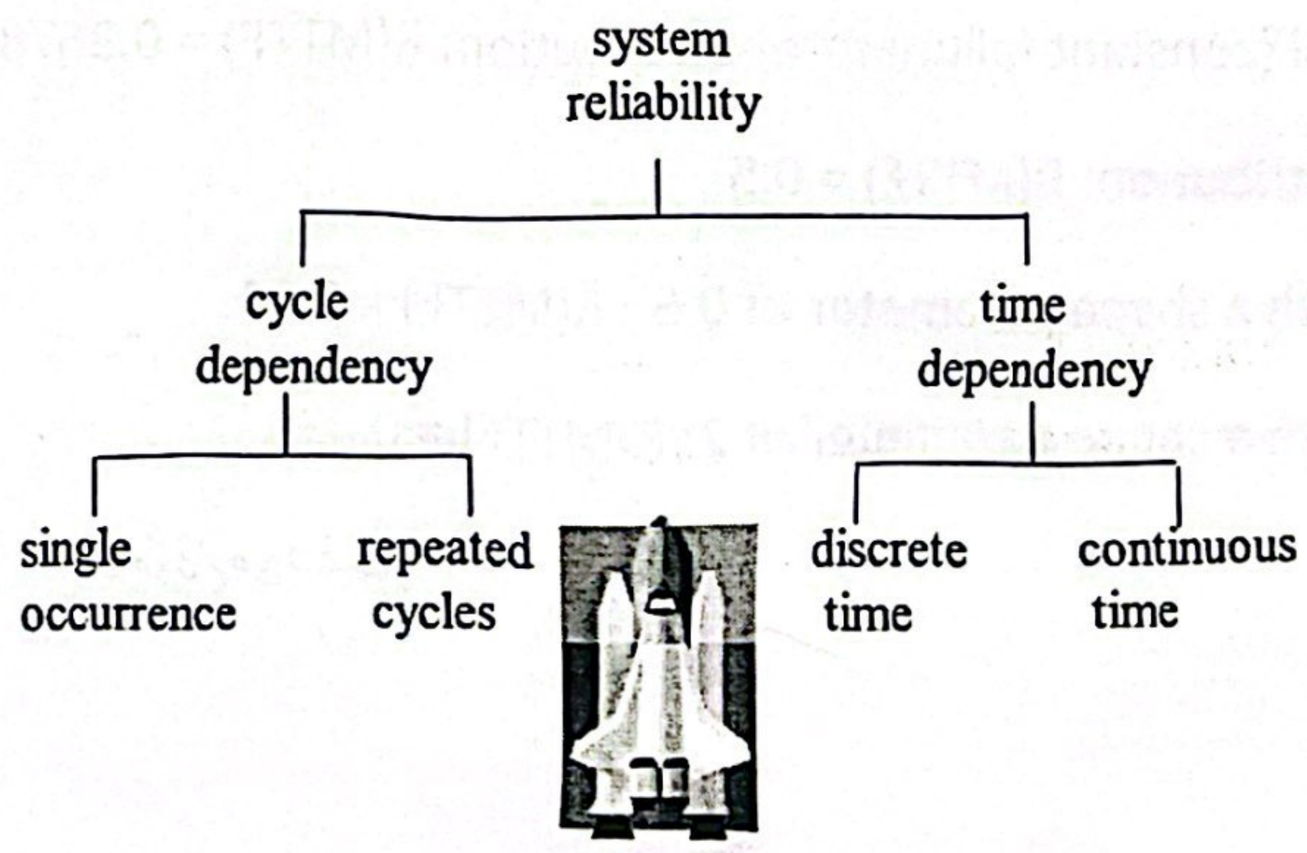
Operating hours or clock time?

What about on/off cycles?

What are the operating conditions?

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Time to failure: Cycles versus Time

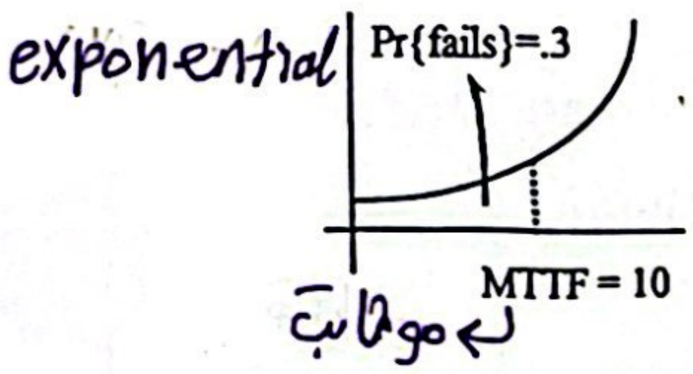


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The Failure Distribution and the MTTF

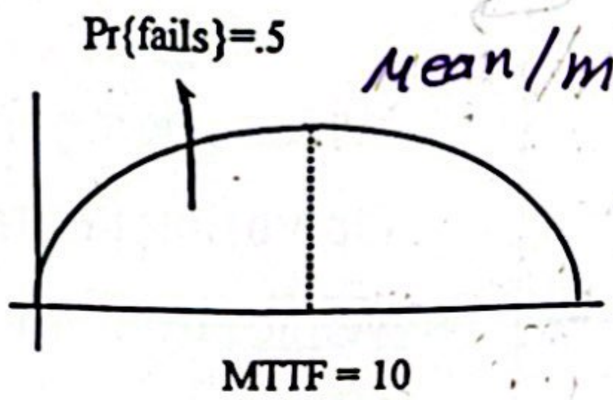
- MTTF: Mean Time To Failure

Probability MTTF

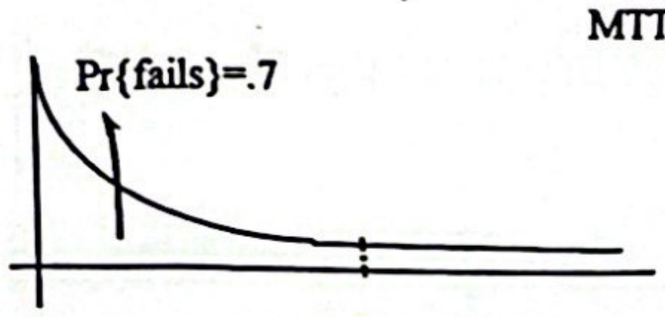


exponential

موجابت



Mean/mode = .5 (Normal)



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The Failure Distribution and the MTTF

- Exponential (constant failure rate) Distribution: R(MTTF) = 0.3678
- Normal Distribution: R(MTTF) = 0.5
- Weibull with a shape parameter of 0.5 : R(MTTF) = 0.24
- Weibull with a shape parameter of 2: R(MTTF) = 0.455

*الأرقام موحفظ

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- Reading: Chapter 1 (pages 1 -11), solve the examples.

- Pages 12 – 32: Review of probability and other information that we might use later. Do not read it now

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Reliability Engineering

The Failure Distribution (Chapter 2)

The Failure Distribution

شو رح يتضمن التشابتر ؟

- The Reliability Function وظيفة
- Mean Time To Failure
- Hazard Rate Function معدل
- Bathtub Curve
- Conditional Reliability مشروط

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3/5/2024

2.1 The Reliability Function

شو بقد رأعرفه؟

- Reliability is defined as the probability that a system (component) will function over some time period t ← احتمال يأدي عمله خلال الفترة المحددة

Let T = a random variable, the time to failure of a component → T

T → Time to Failure (أنا بتوقعها)

R(t) is the probability that the time to failure is greater than or equal to t

t → الوقت (الحقيقي)

$$R(t) = Pr\{T \geq t\}$$

0 = Time ← مستحيل يخرب فيها ← احتمالية يضرب بـ

where $R(t) \geq 0$, $R(0) = 1$, and ← لهيك R(0) = 1 ← فرضًا اجبت استخدم جهاز وكان مضمون منه ما رح أعتبرها مشكلة QC Reliability وموشكلة

$\lim_{t \to \infty} R(t) = 0$ ← مستحيل يفشل عنتي حكي شغال لـ ∞ ← لهيك R(∞) = 0 (t) أكبر أو تساوي

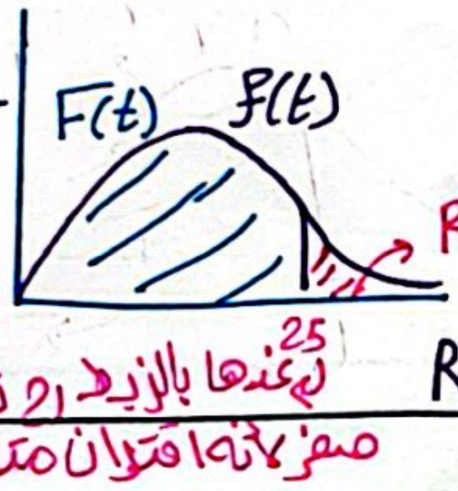
Often called the SURVIVAL FUNCTION

* الاحتمالية رح تكون بين (0, 1)

هدفي ← يفشل صالح لحدة

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3 area under = 1 ←



$F(t) = \int_0^t f(t)dt$ ← لهيك بالزبط رح تكون صفر لأنها اقترنا متصل

$R(t) = \int_1^{\infty} f(t)\,dt$ ← 25

$f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt}$

$\frac{F(t)}{dt}$ ← بدي بشتق f(t)

1 - R(t)

2.1 The Reliability Function

t = 5 ← R(t) since *

∴ T = 5, 6, ...

F(t) is the probability that a failure occurs before time t:

$$F(t) = 1 - R(t) = Pr\{T < t\}$$ ← احتمالية

where $F(0) = 0$ and $\lim_{t \to \infty} F(t) = 1$ ← بطرح من 1

R(t) = the reliability function

F(t) = the cumulative distribution function (CDF) of the failure distribution

F(t) R(t) f(t) → كلهم احتمالية فقيمهم بين ال (1 و 0)

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The Reliability Function

- $f(t)$ = Probability Density Function (PDF): → Describe the shape of the Failure distribution

$$f(t) = \frac{d\,F(t)}{dt} = -\frac{d\,R(t)}{dt}$$

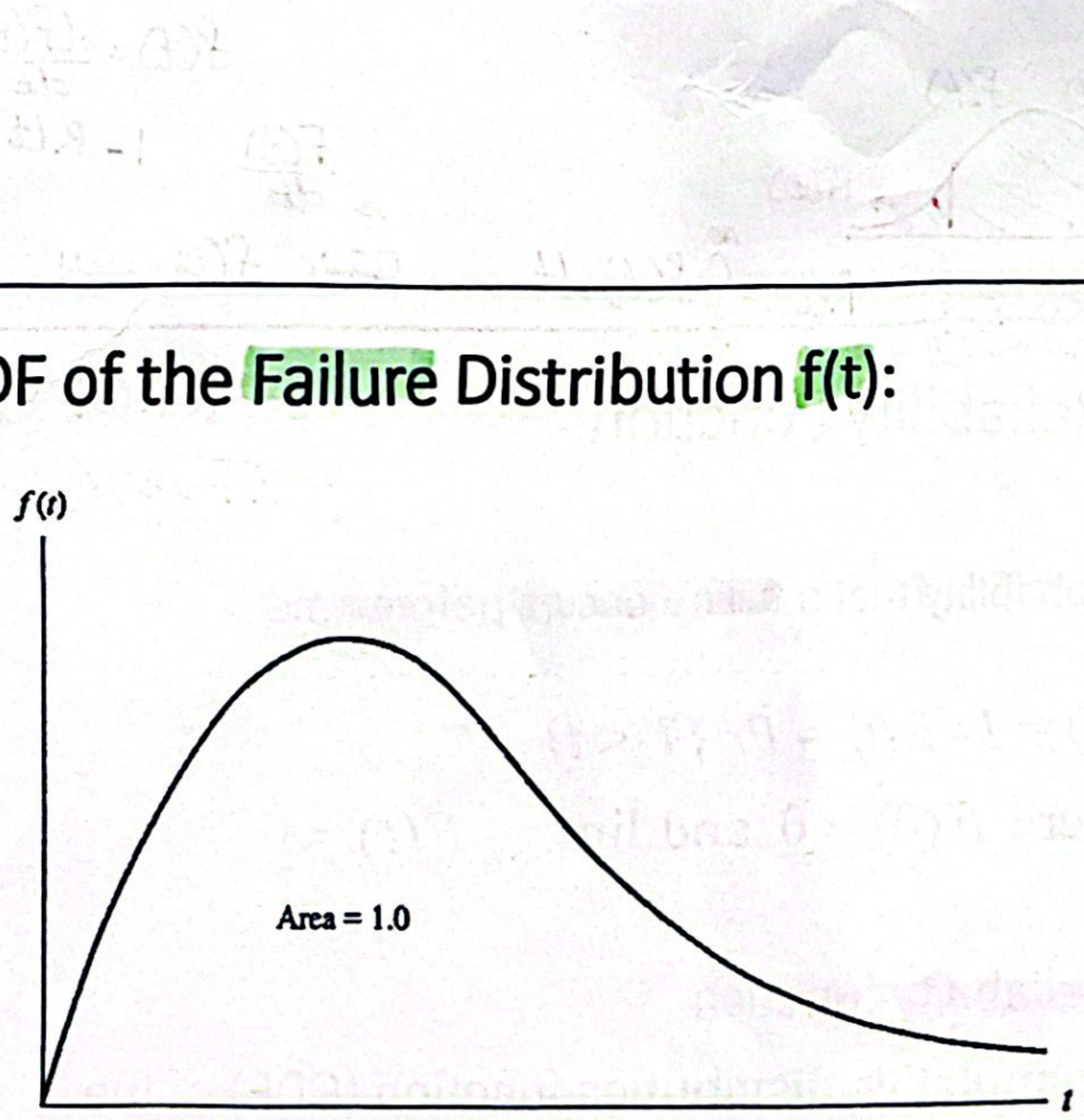
كيف بطلعه $f(t)$ بشتقها أو سالب اشتقاق $R(t)$

$$f(t) \geq 0 \quad and \quad \int_0^{\infty} f(t)dt = 1$$

↳ area under the Curve

لم كيف يعني؟
← أناص فلال الحنفي
بقدر أحدد إذا كان increase أو decrease

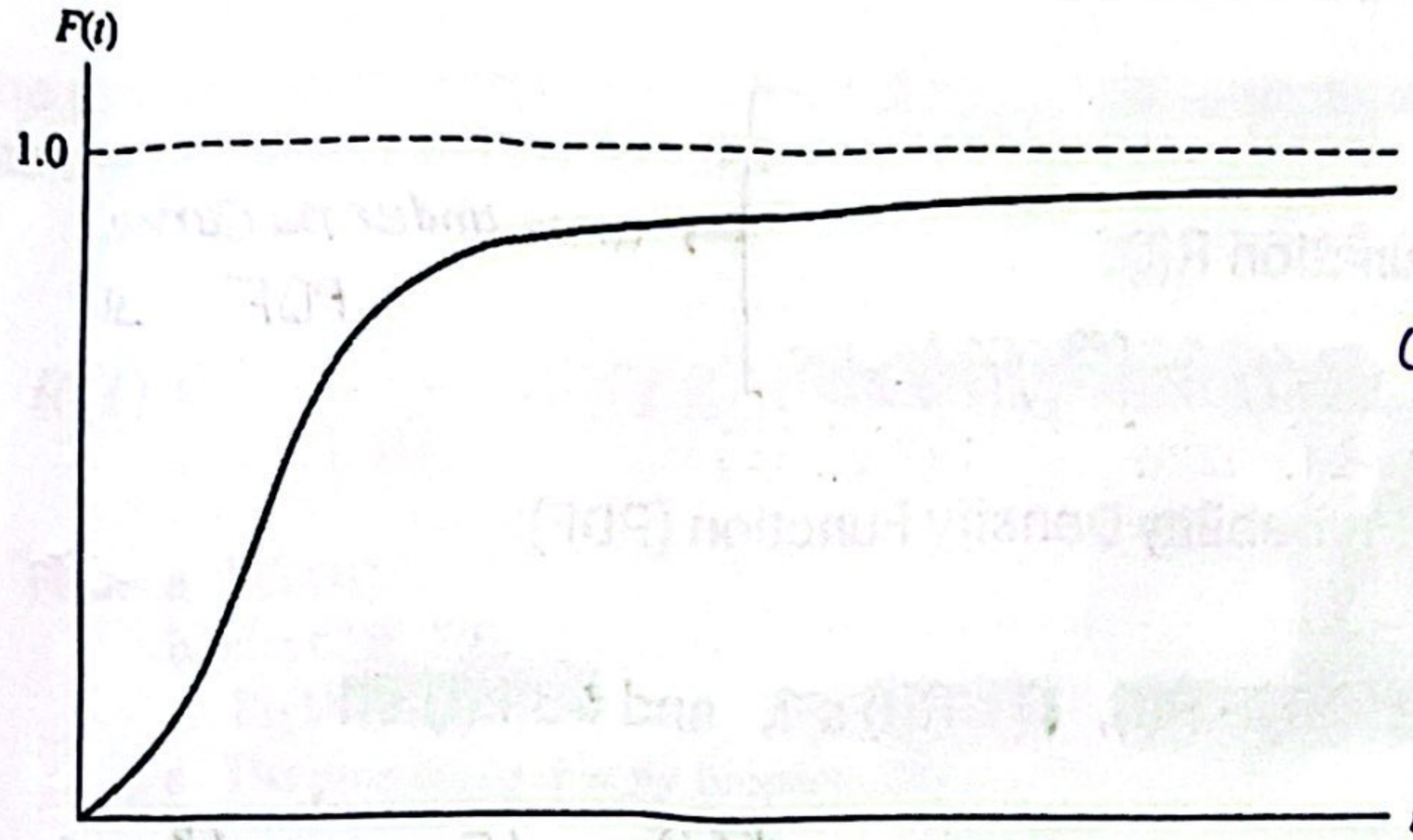
PDF of the Failure Distribution f(t):



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CDF of the Failure Distribution F(t): →

يبدأ بـ (0) وهدفه أن يزيد ليوصل الـ (1)



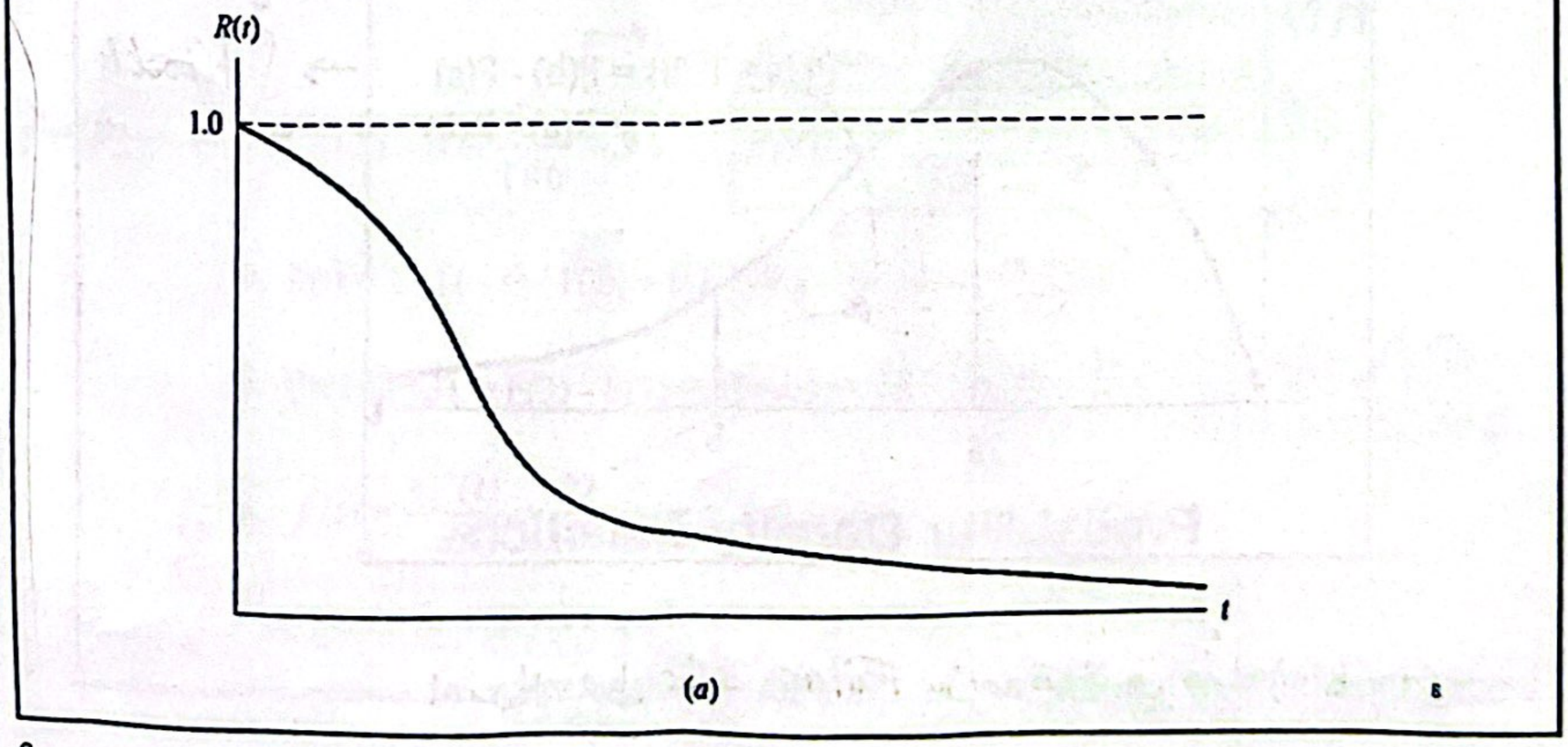
← أول ما أصنعه نسبة إنه يوصل (0) وإن تفضل تزيد لتوصل الـ (1)

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Reliability Function R(t)

R(t) ← يبدأ بـ (1) وكلما استخدم أكثر بصير أقرب لـ (0)



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The Reliability Function

- CDF of the failure distribution F(t):

$$F(t) = \int_0^t f(t')\,dt'$$

- Reliability Function R(t):

$$R(t) = \int_t^\infty f(t')\,dt'$$

→ area under the curve PDF

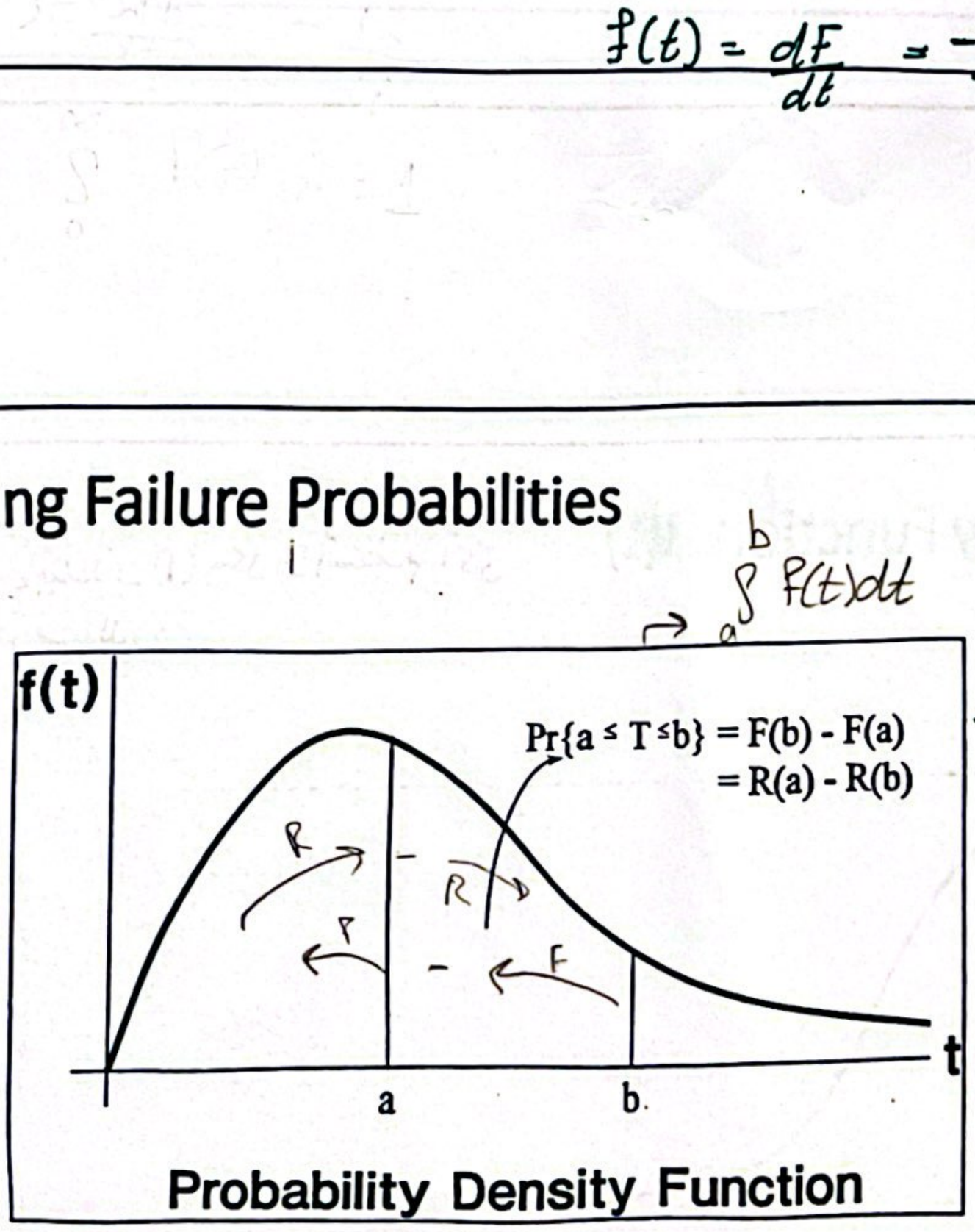
where *f(t)* is Probability Density Function (PDF):

Remember: $F(t) = 1 - R(t)$, $0 \le R(t) \le 1$, and $0 \le F(t) \le 1$

$$f(t) = \frac{dF}{dt} = -\frac{dR}{dt}$$

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Finding Failure Probabilities



Probability Density Function

$\rightarrow \int_a^b f(t)\,dt$

$\rightarrow \int_a^b f(t)\,dt$

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احتمالية حصول Failure بين وقتين

Example

The passive components of a distribution system for natural gas has the following reliability function:

$$R(t) = 1 - \frac{t^2}{100}; \quad 0 \le t \le 10 \text{ yrs}$$



Find: a. R(3 yrs)
b. The CDF, F(t)
c. Pr{1<T<3}
d. The probability density function, f(t)

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Example

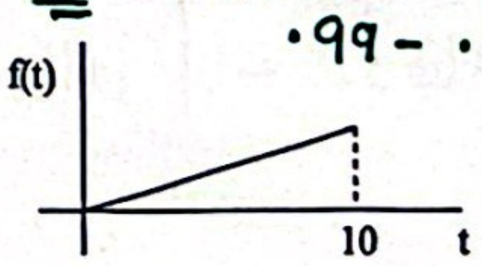
a. $R(3) = 1 - \frac{3^2}{100} = .91$

$$R(t) = 1 - \frac{t^2}{100}; \quad 0 \le t \le 10 \text{ yrs}$$

b. $F(t) = 1 - [1 - t^2/100] = t^2/100$

c. $\Pr\{1<T<3\} = F(3) - F(1) = .09 - .01 = .08$

d. $f(t) = \frac{dF(t)}{dt} = \frac{2t}{100} = \frac{t}{50}; \quad 0 \le t \le 10$



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Example 2.1

T: the time (in operating hours) to failure of a compressor,

* التكامل على الآلة الحاسبة وعلشان ما عندي ٥٥ بقدر أحط رقم كبير وبطلع مع تقريبًا نفس الحل

$$f(t) = \begin{cases} \dfrac{0.001}{(0.001t + 1)^2} & t \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

$R(t) = \dfrac{1}{0.001t+1}$

a. What is the reliability for a 100 hr operating life?

b. Find the design life if a reliability of 0.95 is desired?

A design life is defined to be the time to failure t_R that corresponds to a specified reliability R, that is $R(t_R) = R$.

$F(100) - F(0)$ or $R(0) - R(100)$

c. What is the probability that the failure occurs within 10 to 100 hrs?

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لو فرضنا السؤال الأخير هو علوي ما يكون ∞ لا يكون هاد اكبر ← لو كان 500 بحط 500 مو ∞

A) $R(t) = \int_t^{\infty} f(t)\,dt \rightarrow \int_{100}^{\infty} \dfrac{0.001}{(0.001t+1)^2}\,dt = 0.909 \rightsquigarrow R(100)$

B) $R(t) = .95 \rightsquigarrow R(t) = \int_t^{\infty} \dfrac{0.001}{(0.001t+1)^2}\,dt \rightsquigarrow \dfrac{-1}{0.001t+1}\Big|_t^{\infty}$

$\hookrightarrow \dfrac{1}{0.001t+1} \rightarrow R(t)$

$\hookrightarrow \dfrac{1}{0.001t+1} = .95 \Rightarrow 52.6$

*C) $\dfrac{1}{0.001(10)+1} - \dfrac{1}{0.001(100)+1}$

2.2 Time to Failure

Mean Time To Failure (MTTF)

- The mean of the failure distribution is only one of several measures of central tendency of the failure distribution (average)

MTTF = E(T)

$$\text{MTTF} = \int_0^{\infty} R(t)\,dt$$

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Time to Failure → بعبر عنه ب MTTF

→ Mode

→ σ^2

هو time بقسم الcomponents لقسمين

2.2 Time to Failure

لما يعني اشتغال ← خربوا

t_{med}

- **Median time to failure:**

$$R(t_{med}) = 0.5 \rightarrow$$ دايمًا

- **The median divides the distribution into two halves, with 50 percent of the failures occurring before the median time to failure and 50 percent occurring after the median**

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2.2 Time to Failure

- **Third frequently used average is the mode, or most likely observed failure time, defined by:**

القمة بال curve

$$f(t_{mode}) = \max_{0 \le t < \infty} f(t)$$

↳ continuous

R(t) → normal يكون عندي

mode = median = mean

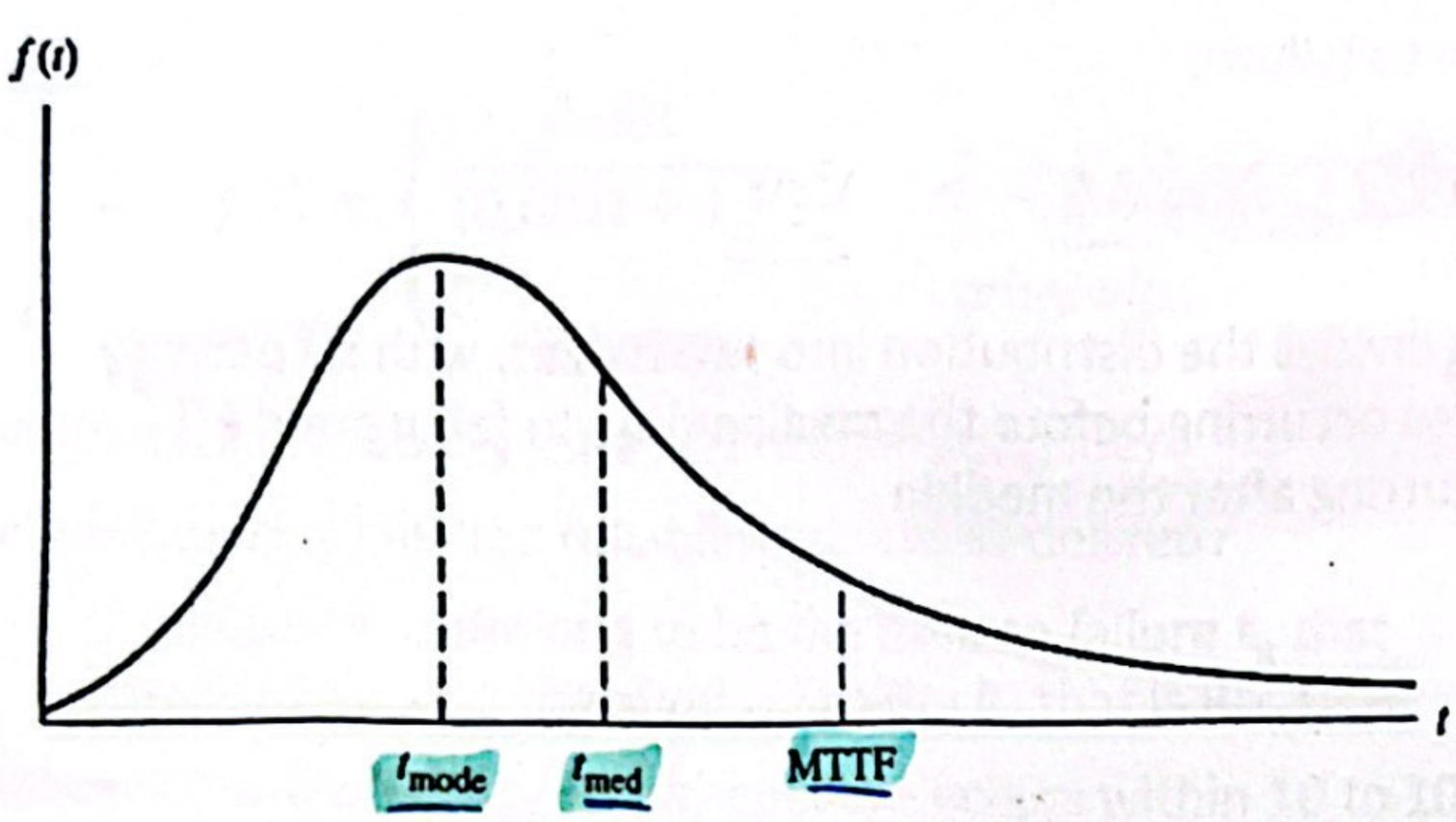
↳ = 0.5

Normal بس لما يكون

- **For a small fixed interval of time centered around the mode, the probability of failure will generally be greater than for an interval of the same size located elsewhere within the distribution**

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2.2 Time to Failure



2.2 Time to Failure

- One measure that is often used to further describe a failure distribution is its variance σ^2,
- Variance represents an average squared distance a failure time will be from the MTTF مربع بعد النقطة عن ال mean
- defined by:

$$\sigma^2 = \int_0^\infty (t - MTTF)^2 f(t)\, dt \quad \leftarrow$$

$$\sigma^2 = \int_0^\infty t^2 f(t)\, dt - (MTTF)^2 \quad \leftarrow \text{أسهل للحل عادةً}$$

* $R(t) = \frac{1000 - t}{1000}$ $\quad t \rightarrow (0 \rightarrow 1000)$

$\hookrightarrow \int_0^{1000} \frac{1000 - t}{1000} dt \rightarrow t - \frac{t^2}{2000} \Big|_0^{1000}$

$\hookrightarrow 1000 - \frac{10^6}{2000} \rightarrow 1000 - 500 = 500 \leftarrow$ MTTF

3/5/2024

Example

- For the distribution system, find the MTTF, Median, and Mode



$\rightsquigarrow R(t) = e^{-0.002t}$ $\quad \rightsquigarrow \sigma^2 = \int_0^{\infty} t^2 f(t)\, dt - MTTF^2 \rightarrow \int_0^{\infty} t^2 * e^{-0.002t}$

$dt - 500^2 \rightarrow 250{,}000$

Mode $\hookrightarrow t_{mode} = 0$

19

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إذا كانت محصورة نحط حدود التكامل لحدودين هي محصورة

إذا ما كانت في حد أحد حد التكامل يكون (∞) ← إذا ما كانت محصورة

Example 2.2

$$f(t) = \begin{cases} 0.002e^{-0.002t} & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Plot function: http://www.fooplot.com

$\rightsquigarrow R(t) = \int_0^{\infty} 0.002e^{-0.002t} dt$

$\hookrightarrow \frac{0.002e^{-0.002t}}{-0.002} \Big|_0^{\infty}$

20

20

$\infty * e^{-0.002t} = R(t)$

$\int_0^{\infty} e^{-0.002t} dt = \frac{e^{-0.002t}}{-0.002} \Big|_0^{\infty} = 500$

$\rightarrow$ Median $\rightarrow \ln .5 = \ln e^{-0.002 t_{med}} \rightarrow t_{med} = 346.6$

2.3 Hazard Rate Function

another name

- In addition to the probability functions defined earlier the failure rate or hazard rate function is often used in reliability
- hazard rate function provides an instantaneous (at time t) rate of failure

$$\Pr\{t \le T \le t + \Delta t\} = R(t) - R(t + \Delta t)$$

$T > t \rightarrow R(t) - R(t+\Delta t)) \quad = \frac{f(t)}{R(t)}$

$\Delta t R(t)$

- Hazard Rate Function or Failure Rate Function $\lambda(t)$:

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{-[R(t + \Delta t) - R(t)]}{\Delta t} \cdot \frac{1}{R(t)}$$

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2.3 Hazard Rate Function

- Hazard Rate Function or Failure Rate Function $\lambda(t)$:

$$\lambda(t) = \frac{f(t)}{R(t)}$$

$$R(t) = \int_t^{\infty} f(t')\, dt'$$

$$R(t) = exp\left[-\int_0^t \lambda(t')\, dt'\right] \quad R = e^{-\int_0^t \lambda(t)}$$

- Failure rates can be characterized as:
  - Increasing Failure Rate (IFR) when λ(t) increasing → Failure rate
  - Decreasing Failure Rate (DFR) when λ(t) decreasing
  - Constant Failure Rate (CFR) when λ(t) increasing constant

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Hazard Rate Function

- The cumulative failure rate over a period of time t is defined by:

$$L(t) = \int_0^t \lambda(t')\, dt' \qquad L(t) = -\ln R(t)$$

$$e^{L(t)} = 1/R(t)$$

- Average failure rate between two times t_1 and t_2:

$$AFR(t_1, t_2) = \frac{L(t_2) - L(t_1)}{t_2 - t_1} = \frac{\ln R(t_1) - \ln R(t_2)}{t_2 - t_1}$$

$$AFR(t) = \frac{\ln R(0) - \ln R(t)}{t - 0} = \frac{-\ln R(t)}{t} = \frac{L(t)}{t}$$

لما بين الصفر و t

$t_1 = 0$

AFR(t) متناولة اكادة أي شيء مع بقدر أطبع

منباع الشغلات

R(t)
F(t)
f(t)
AFR(t)
L(t)

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Hazard Rate Function

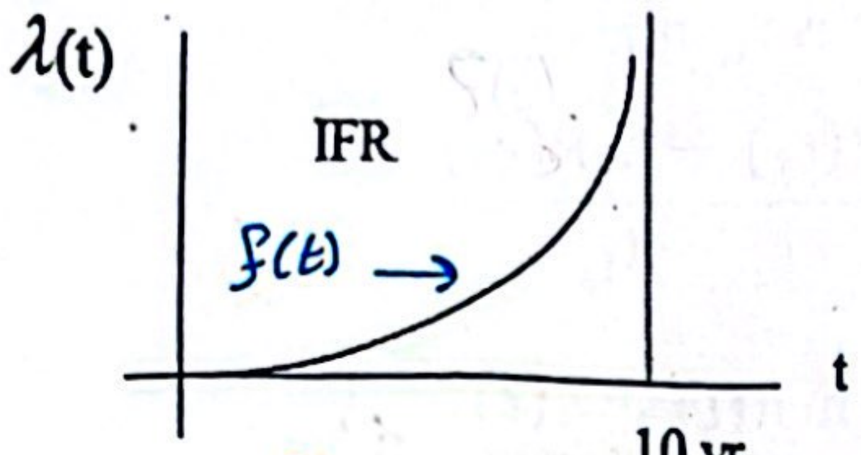
- Average Failure rates can be as:
  - Increasing Failure Rate Average (IFRA) when *AFR*(t) increasing
  - Decreasing Failure Rate (DFRA) when *AFR* (t) decreasing

Constant Flow

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Example

- For the distribution system, find failure rate function (hazard)?
  Cumulative failure rate function after t period of time?
  And average failure rate function after t period of time? $L'(t)/t$





$\int_t^\infty f(t)dt \rightarrow R(t) \rightsquigarrow \lambda(t) = f(t)/R(t)$

$\rightsquigarrow L(t) = \int_0^t \lambda(t)dt$

$\rightarrow AFR = L(t)/t$

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Example 2.5

- Given the linear hazard rate function

$$\lambda(t) = 5 \times 10^{-6} t$$

1) where t is measured in operating hours, what is the design life if a 0.98 reliability is desired?
2) What is the cumulative failure rate function?
3) What is the average failure rate function?

1) $R(t) = \exp - \int_0^t 5*10^{-6} t\,dt \Rightarrow \exp\left(\frac{-5*10^{-6}t^2}{2}\right)\Big|_0^t$

$= \exp(-2.5*10^{-6}t) \rightarrow .98 = e^{-2.5*10^{-6}t}$

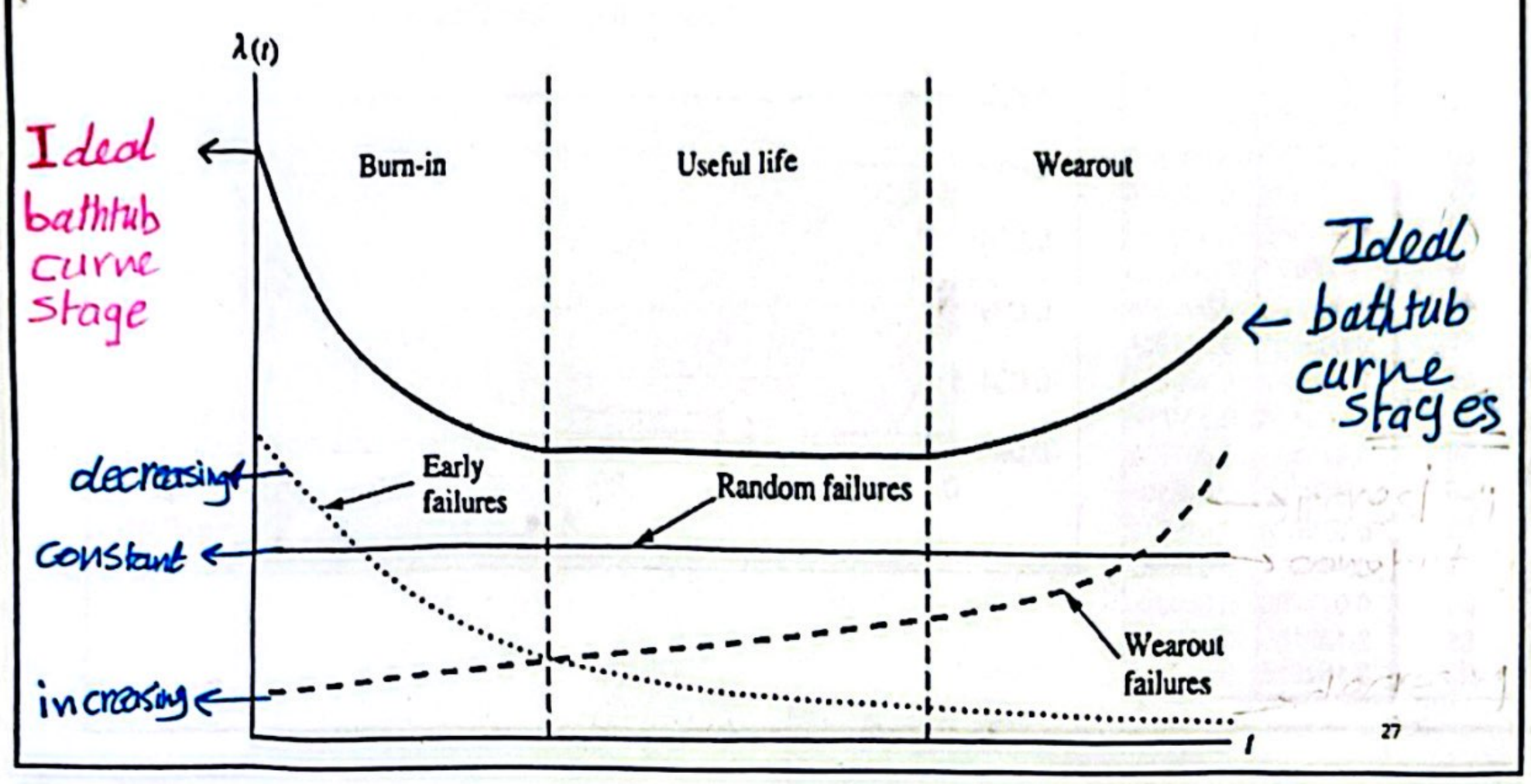
$t = 89.89$
≈ 90 hr

2) $L(t) = \int_0^t 5*10^{-6} t\,dt \rightarrow \frac{5*10^{-6}t^2}{2}\Big|_0^t \rightarrow 2.5*10^{-6}(t)^2$

3) $AFR = L(t)/t = \frac{2.5*10^{-6}t^2}{t} = 2.5*10^{-6}t$

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2.4 Bathtub Curve



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The bathtub curve

	Characterized by	Caused by	Reduced by
Burn-in	DFR	Manufacturing defects: welding flaws, cracks, defective parts, poor quality control, contamination, poor workmanship	Burn-in testing Screening Quality control Acceptance testing
Useful life	CFR	Environment Random loads Human error "Acts of God" Chance events	Redundancy Excess strength
Wear-out	IFR	Fatigue Corrosion Aging Friction Cyclical loading	Derating Preventive maintenance Parts replacement Technology

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بشغله فترة قصيرة في أحسن العيوب الصنعية

لجعل الآلة في حال فريت حتى لو كانت تشتغل الآلة البديلة ثانياً أو ثانية أو كذا أو هكذا أجيب نفس الآلة

السعة ثابتة لظرف معين ومنع الوضع

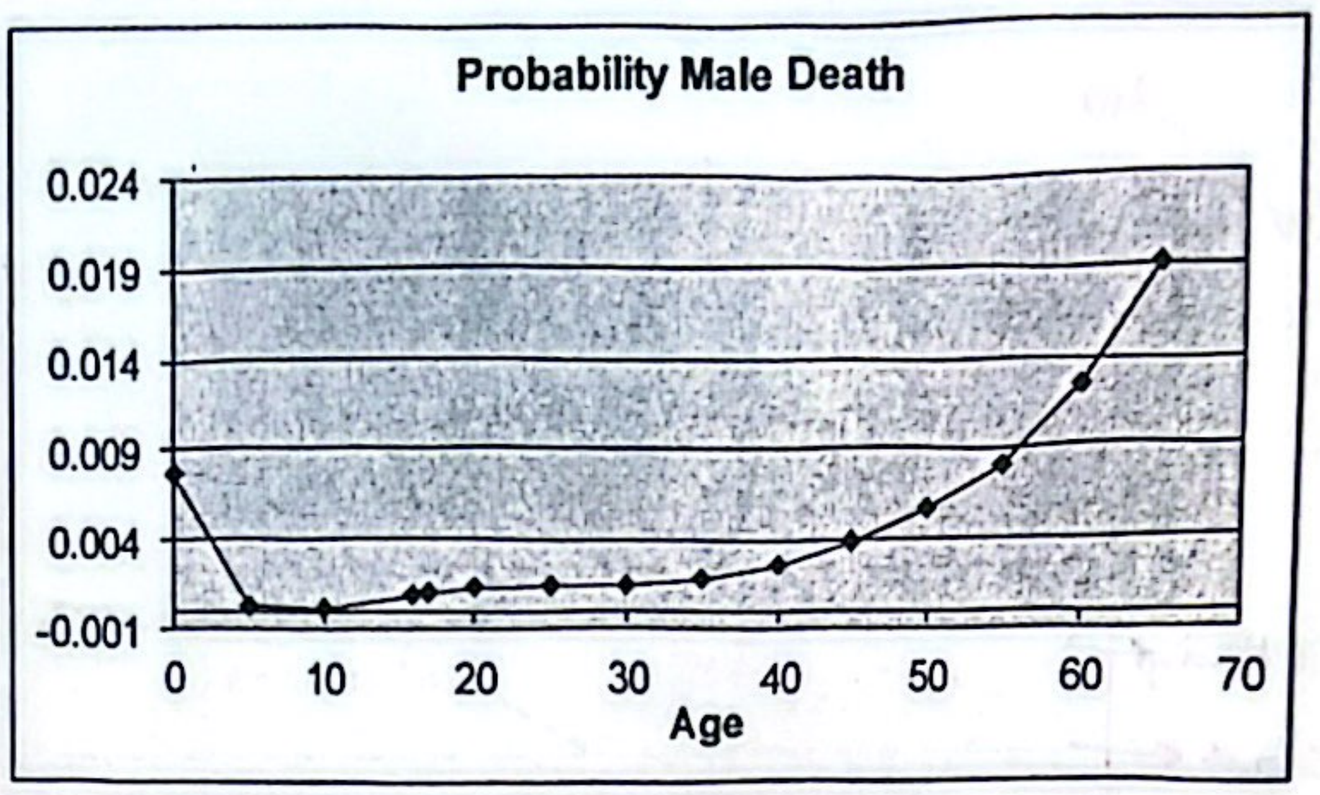
28 الصنع هكذا حتى ما يحصل أكثر من 20 بالمئة

2/ أقلل استخدامه بحيث ما أوصل أثناء الاستخدام لـ (20) بالمئة حتى أطول من ميانه بالاستخدام

3/5/2

Human Mortality Curve

age	Male	Female
0	0.007644	0.006275
5	0.000202	0.000152
10	0.00011	0.000113
16	0.00081	0.000375
17	0.000964	0.000423
20	0.00129	0.000456
25	0.001379	0.000499
30	0.001389	0.000628
35	0.00177	0.000953
40	0.002589	0.001514
45	0.003891	0.002264
50	0.005643	0.003227
55	0.008106	0.004884
60	0.012405	0.007732
65	0.019102	0.012199
70	0.029824	0.019312
75	0.046499	0.030582
80	0.073269	0.050396
85	0.120186	0.086443
90	0.192615	0.147616



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2.5 Conditional Reliability

* عاش الـ product ضعف سنين
هو احتمالية يعيش كمان (3) سنين مثلاً

- Conditional reliability describes the reliability of a component or system following a burn-in period T_0 or after a warranty period T_0
- We define conditional reliability as the reliability of a system given that it has operated for time T_0:

Reliability امتى ال
Failure حتى أقل من ال

$$R(t \mid T_0) = \Pr\{T > T_0 + t \mid T > T_0\}$$

T_0 لو عاش 5 مثلاً
t ← ع يعيش كمان
T → Failure time

$$= \frac{\Pr\{T > T_0 + t\}}{\Pr\{T > T_0\}} = \frac{R(T_0 + t)}{R(T_0)}$$

↳ R(5/1) → R(6)/R(1)

$$= \frac{\exp\left[-\int_0^{T_0+t} \lambda(t')\,dt'\right]}{\exp\left[-\int_0^{T_0} \lambda(t')\,dt'\right]} = \exp\left[-\int_{T_0}^{T_0+t} \lambda(t')\,dt'\right]$$

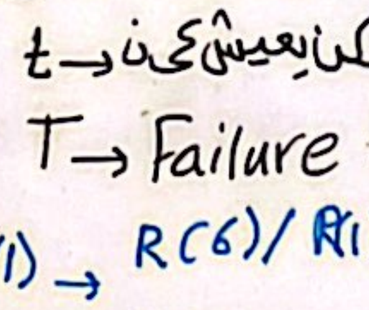
لم يعمل تشغيل تجريبي قبل طرحه في السوق
↳ لفيها test ع
بس أعمل الـ من حسابي
فترة التجريب بيفضها ويحسب بدونها

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بدل ١٥ سنين
بتصير 8 حسب ع شغلته
وهيك ع تزيد نسبة Reliability

R(10) = 90%
R(10/2) → 96%

ع هيك النسبة ←



R(5/2) → .90
R(5) = .95
R(5/1) = .93

لما ال increasing ↑ ١٠ ك ←

Residual MTTF

Residual MTTF (after T_o):

$$\text{MTTF}(T_0) = \int_0^{\infty} R(t \mid T_0)\, dt = \int_{T_0}^{\infty} \frac{R(t')}{R(T_0)}\, dt' = \boxed{\frac{1}{R(T_0)} \int_{T_0}^{\infty} R(t')\, dt'}$$

* بنحلى هادالقانون

Where $t' = t + T_o$

Example

- For the distribution system, calculate the reliability of 5 years given that the burn-in period is 1 year? And calculate the MTTF after the burn-in period?

T_0

$P(5/1)$

$$R(t) = 1 - \frac{t^2}{100};\quad 0 \le t \le 10 \text{ yrs}$$

$\frac{R(6)}{R(1)}$

$$\frac{1}{1 - \frac{1}{100}} * \left[\frac{1 - \frac{6^2}{100}}{1 - \frac{1}{100}} \right]$$

$$1 - \frac{1}{100}$$



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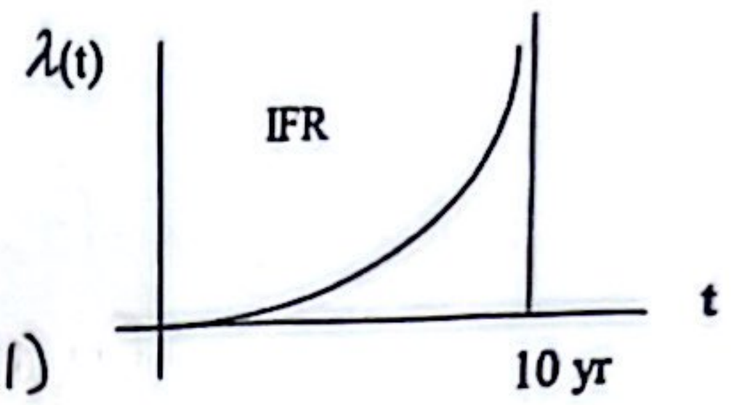
Example

$$R(t)=1-\frac{t^2}{100}$$

$$R(t|1)=\frac{R(t+1)}{R(1)}=\frac{1-(t+1)^2/100}{1-(1/100)}$$

$$=\frac{100-(t+1)^2}{100(.99)}=1.01-(t+1)^2/99$$





Therefore: R(5|1) = 1.01-36/99=.646
where R(5) = 1 - .25 = .75

$$MTTF(T_0)=\frac{1}{1-T_0^2/100}\int_{T_0}^{10}1-\frac{t^2}{100}dt=\frac{1}{.99}\left[t-\frac{t^3}{300}\right]_{T_0=1}^{10}$$

$$or\ MTTF(1)=\frac{1}{.99}\left[10-\frac{10^3}{300}-1+\frac{1}{300}\right]=\frac{5.67}{.99}=5.72yr.$$

ex 2.8

Example:

DFR:

$$\lambda(t)=\frac{0.5}{1000}\left(\frac{t}{1000}\right)^{-0.5}\ t\ in\ years$$

1) Calculate the reliability of 11 year?
2) If 1 year consumed in the burn-in process, what is the reliability of 11 years?

Reliability Engineering

Constant Failure Rate Model (Chapter 3)

↳ عندي 3 أنواع ولكن راح نركز بهاد الـ Chapter على constant

حيث معالجاء ممكن مغفط فجأة مثلاً

3.1 Exponential Distribution

- A failure distribution that has a constant failure rate is called an (exponential probability distribution) another name
- The exponential distribution is one of the most important reliability distributions. Many systems exhibit constant failure rates ←
- Failures due to completely random (1) or chance events (2) will follow this distribution
- It should dominate during the useful life of a system or component
- It is also one of the easiest distributions to analyze statistically

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Exponential Distribution CFR

Let $\lambda(t) = \lambda$, $t \geq 0$ $\quad \lambda(3) = \lambda(5) = \lambda(t)$

↳ ξt في جميع الأماكن constant هي

Then

$$R(t) = e^{-\int_0^t \lambda dt'} = e^{-\lambda t} , \ t \geq 0$$

$$F(t) = 1 - e^{-\lambda t}$$

اشتقاق

$$f(t) = \frac{d\,F(t)}{dt} = \lambda e^{-\lambda t}$$

3

3

$-\lambda t$

$R = e$

Exponential (constant) Distribution

لـ دائماً ثابت

$\lambda \rightarrow \frac{1}{MTTF}$

$-\frac{1}{MTTF} * MTTF$

e^{-1}

↳ e^{-1}

$$MTTF = \int_0^\infty e^{-\lambda t} dt = \frac{e^{-\lambda t}}{-\lambda}\Big|_0^\infty = \boxed{\frac{1}{\lambda}}$$

note that $R(MTTF) = e^{\frac{-MTTF}{MTTF}} = e^{-1} = .368$ → λ مهما كانت

$$\sigma^2 = \int_0^\infty \left(t - \frac{1}{\lambda}\right)^2 \lambda e^{-\lambda t} dt = \frac{1}{\lambda^2}$$

$$and \quad \sigma = \frac{1}{\lambda} = MTTF$$

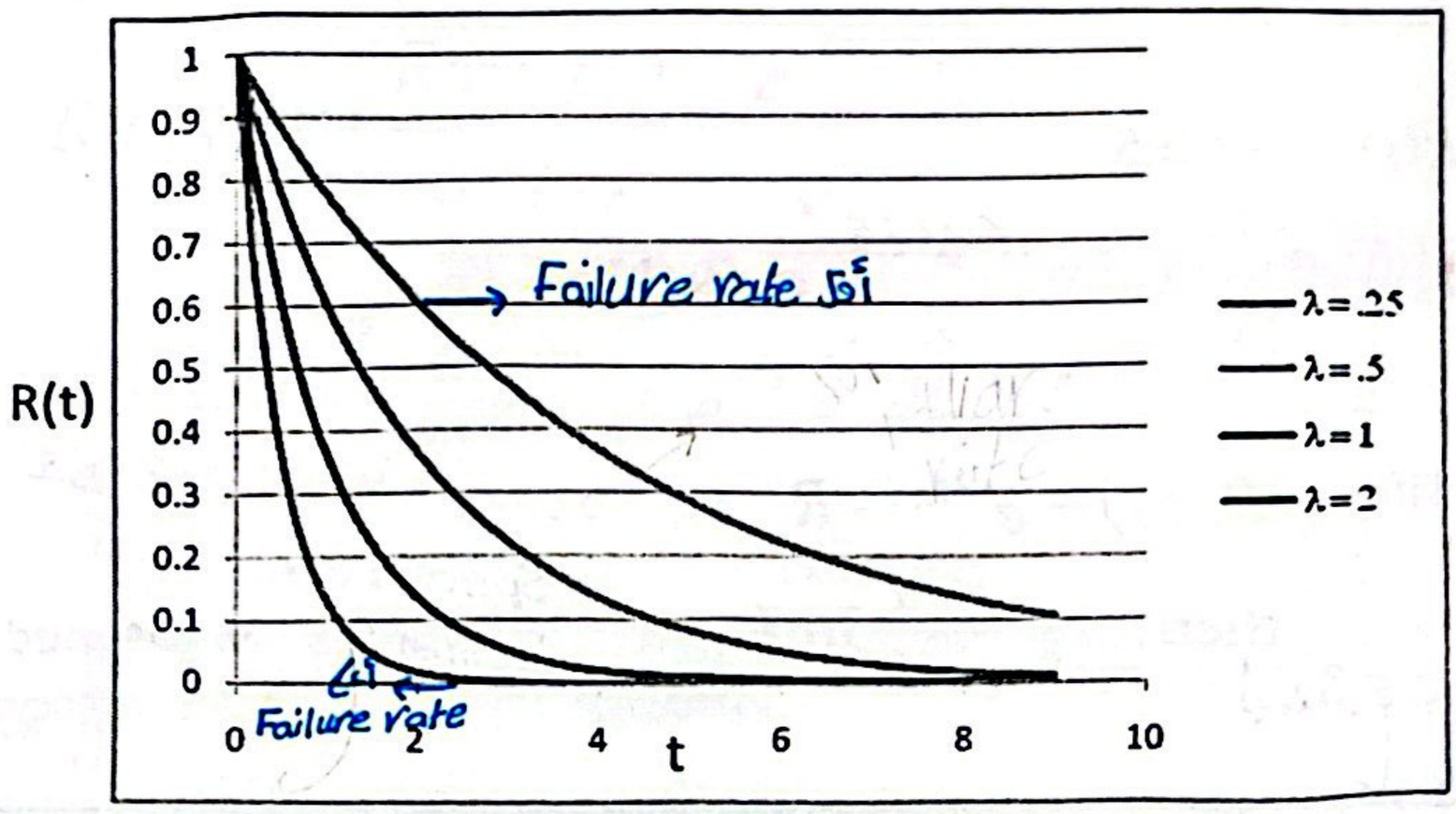
$R(MTTF)$

4

4

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The Reliability Function R(t)



علاقة عكسية بين λ و R

5

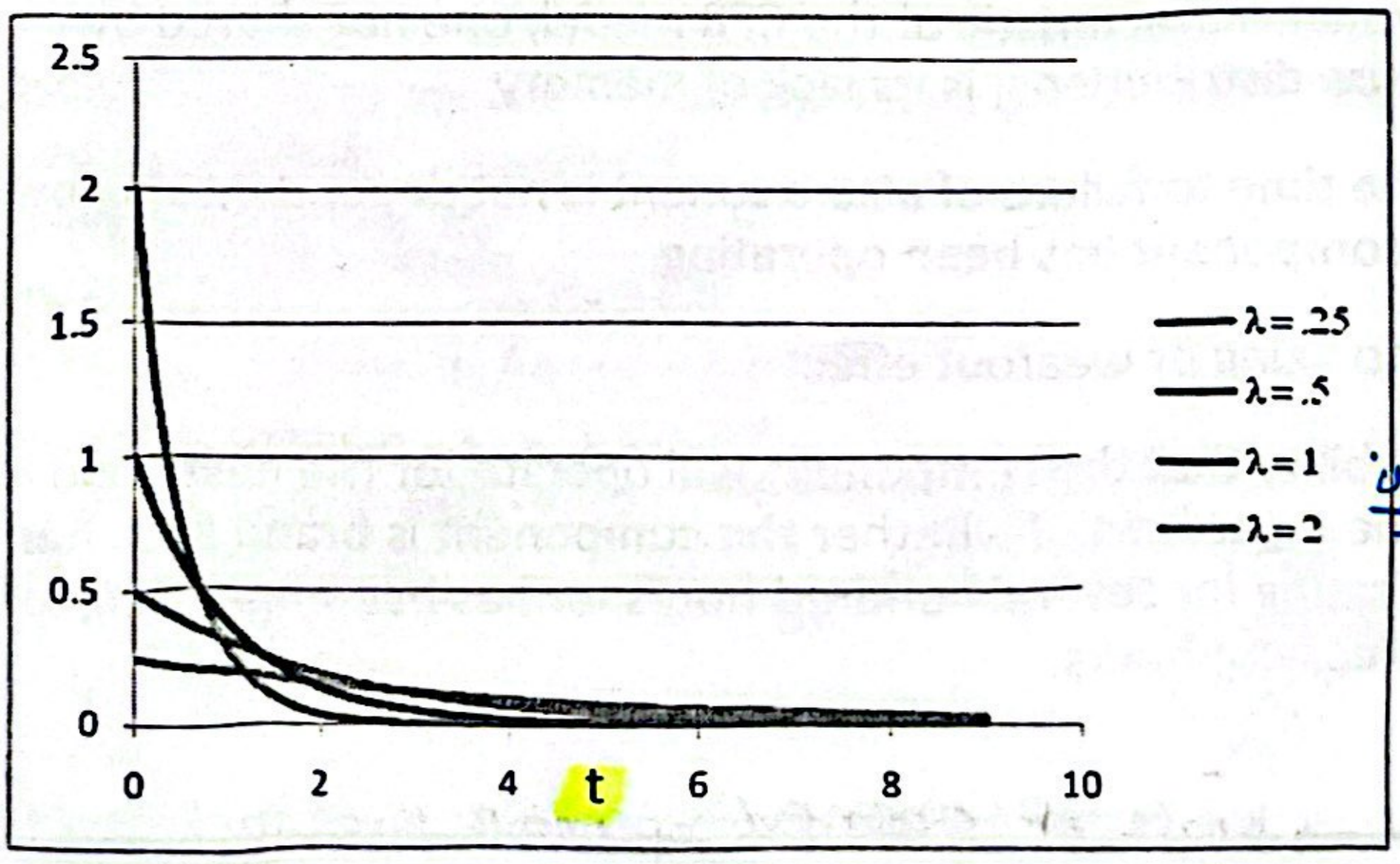
5

Probability Density Function f(t)

λ↑ early Failure↑*

لكن

late Failure قليلة



علاقة طردية بين الـFailure و λ لحد معين

6

6

نوجعنتكريانما (constant / random event)

Exponential Distribution

Median

$$R(t) = e^{-\lambda t} = .5$$

باخذ ln للطرفين

$$t_{med} = -\frac{1}{\lambda}\ln .5 = \frac{.69315}{\lambda} = .69315\,\text{MTTF}$$

←The Design Life $\quad R(t_R) = e^{-\lambda t_R} = R$

then $\quad t_R = -\frac{1}{\lambda}\ln R$

time تتبع فتاوصل لـ Reliability معينة (٪90) لمتدّع قيمة (t) إلي أنا متأكدة مارح تخرب فيها

هو لح نعرف إنه tmed هي Special case من ال design life

بدي R(100) لكن ما بعرف هو مديد / اشتغل (100) ساعة قبل ما بعرف بعرف ال (20) نفسها

Memoryless

- A well-known characteristic of the CFR model, one not shared by other failure distributions, is its lack of memory
- That is, the time to failure of a component is not dependent on how long the component has been operating
- There is no aging or wearout effect
- The probability that the component will operate for the next 1000 hr is the same regardless of whether the component is brand new, has been operating for several hundred hours, or has been operating for several thousand hours

R → length of observed operation time just من العمر الحقيقي

Memoryless

$$R(t \mid T_0) = \frac{R(t+T_0)}{R(T_0)} = \frac{e^{-\lambda(t+T_0)}}{e^{-\lambda T_0}}$$

$$= \frac{e^{-\lambda t} \cdot e^{-\lambda T_0}}{e^{-\lambda T_0}} = e^{-\lambda t} = R(t)$$

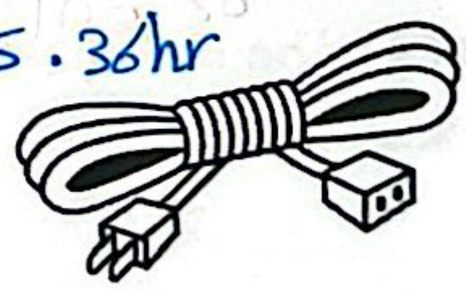
(a burn-in period T_0) has no subsequent effect on reliability and will not improve the component's reliability

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Example 1

An electrical transmitter has been tested and found to have a CFR of 0.001 defects per hour. Find:

a. R(100) $= e^{-\lambda t} \rightarrow e^{-0.001 * 100} = 0.905$
b. R(1000) $= e^{-0.001 * 1000} = 0.368$
c. MTTF $\rightarrow 1/\lambda = 1/0.001 = 1000$
d. Standard Deviation $\rightarrow = 1/\lambda = 1000$
e. $t_{med} \rightarrow -(1/\lambda)\ln .5 \rightarrow 693.197 hr$
f. 90 percent design life $t_r = \frac{1}{0.001} \ln .9 = 105.36 hr$



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3/12/2

3.2 Failure Mode → كل قطعة لحال بس تخرب بتعطل على الجهاز كامل

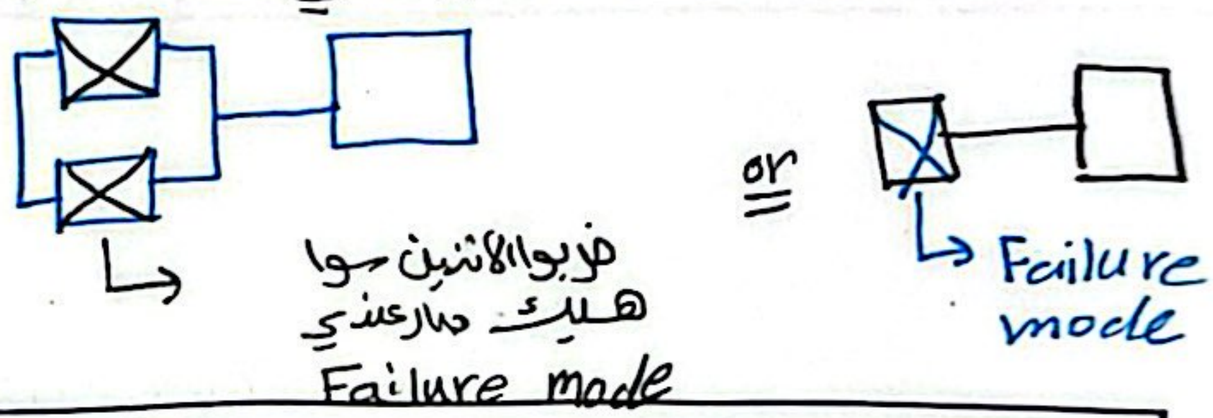
أي عطل بأي قطعة بحيث عم بايخ الجهاز

- Complex systems will fail through various means resulting from different physical phenomena or different failure characteristics of individual components

- A useful analysis approach in reliability engineering is to separate these failures according to the mechanisms or components causing the failures

- These categories of failures are then referred to as failure modes

* عادةً توالي هي هال كان توازي وهمنبود التوازي سوا فهيك بتعطل كامل



هذيك اللي سوا هيك صار عندي Failure mode

Failure Mode

- If $R_i(t)$ is the reliability function for the i^{th} failure mode, then, assuming independence among the failure modes, the system reliability $R(t)$ is obtained as:

$$R(t) = \prod_{i=1}^{n} R_i(t)$$ ⇝ ما بعصبني ال distribution إلي أنا فيه دايمًا باخد منوذب

$R_1 \rightarrow exp$
$R_2 \rightarrow weibull$
$R_3 \rightarrow R_1 * R_2$

- $R(t)$ is the probability that none of the n failure modes occurs before time t

- The system hazard rate function:

  Let $\lambda_i(t)$ be the failure rate function for the i^{th} failure mode. Then:

$$\lambda(t) = \sum_{i=1}^{n} \lambda_i(t)$$

component system معين ← 0.7 / 0.5 / 0.6 ←

$R(6) \Rightarrow 0.7 * 0.5 * 0.6 \rightarrow$

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إذا طلعت عل ← بقسم أطبق قوانين exp العادية

Failure Mode with CFR Model

- If a system consists of *n* independent, serially related components each having a constant failure rate λ_i, then:

$$\lambda(t) = \lambda_s = \sum_{i=1}^{n} \lambda_i \qquad \text{MTTF} = \frac{1}{\lambda_s} = \frac{1}{\sum_{i=1}^{n} \lambda_i} = \frac{1}{\sum_{i=1}^{n} 1/\text{MTTF}_i}$$

system ال ← s

$$R(t) = \exp\left[-\int_0^t \lambda\, dt'\right] = \exp[-\lambda t] \qquad MTTF = \frac{1}{\sum \frac{1}{MTTF}}$$

← لجمع العكوس. مش لجمع بعدين يقلب

- If the components are all identical (i.e., $\lambda_i = \lambda_1$ for $i = 2,3, \ldots, n$), then

Failure ال ← نفس

$$\lambda = n\lambda_1 \qquad \text{MTTF} = \frac{1}{n\lambda_1}$$

* مع (3) بقلب الأول والثاني والثالث ويجمع بالآخر مقلوب كل واحد

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Example 2

- An engine tune-up kit consists of 3 parts each having CFRs (in failures per mile) of 0.000034 (λ_1), 0.000017 (λ_2), and 0.0000086 (λ_3).
- Find the MTTF, standard deviation, median time to failure, and reliability of the tune-up kit at 10,000 miles.

1) MTTF = 1 / λ



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Example 2

$\lambda_s = .000034 + .000017 + .0000086 = .0000596$

$MTTF = \sigma = 1/\lambda_s = 16{,}778.5\ mi.$

$t_{med} = .69315/\lambda_s = 11{,}630\ mi.$

$R(10{,}000) = e^{-.0000596(10{,}000)} = .551$

λ_s ← .69315/(MTTF)



Example 3

- An integrated circuit board consists of the following components each having a CFR.

Component	(a) Failure Rate (10^{-7})	(b) Quantity	(a) x (b)
Diodes, silicon	0.041 (λ)	10 (N)	0.41
Resistors	1.4	25	35.00
Capacitors	0.15	12	1.80
Transformer	0.20	2	0.40
Relays	0.65	6	3.90
Inductive devices	0.04	12	0.48
		total	**41.99×10^{-7}** → λ_S

Therefore $R_{sys}(t) = e^{-.000004199\,t}$ ($e^{-\lambda t}$) and MTTF = $1/(41.99 \times 10^{-7})$ ($1/\lambda$)

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أشغالات بستخدمها (on/off mode) ← عبنعمل عليها (Failure on demand)

* لو فرضنا المنو مزب بنزب و أنا بشغل نادر مابيخرب وهو شغال أساسا

Failures on Demand

- Certain components that operate on a cyclical basis may fail on demand
- For example, an air conditioning and heating system whose cycle time includes an operating time and an idle time may fail when switching from an idle to an operating mode
- Lightbulbs are more likely to fail when being turned on than while operating
- If a constant failure rate can be assumed for each failure mode and a constant probability of failure on demand can be assumed, then the component will have an effective constant failure rate (λ_i) on a clock-hour basis

17

17

* → operating ← لـ idle تو بخرب وأنا بجوه من
2) idle ← (طافي)

↓ أو ← عكن يخرب وهو هون

↓ وهو هون

Failures on Demand

ما بتخرب وهو طافي ممكن وهو شغال أو وأنا بشغلها

λ_I = the average failure rate while idle (may be zero; e.g., failures per idle hour)

λ_o = the average failure rate while operating (e.g., failures per operating hour)

p = the probability of failure on demand

t_I = average length of the idle time period per cycle

t_o = average length of the operating time per cycle

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t_I = 16 ← لو بشتغل 8 ساعات

$$\lambda_{eff} = \frac{t_I}{t_I + t_O}\lambda_I + \frac{t_O}{t_I + t_O}\lambda_O + \frac{p}{t_I + t_O}$$

$t_I + t_O$ = 24h ← t_O = 8

$t_I + t_O$ = 24h

$t_I + t_O$ = 24h

For example

Example 3.4

$t_o = 20 \quad I_I = 60 - 20 = 40$

- An air conditioning compressor operates once for an average time of 20 minutes each hour. While operating, it has experienced a failure rate of 0.01 failure per operating hour, and while idle has experienced a dormant failure rate of 0.0002 failure per idle hour. The probability that the compressor fails on demand is 0.03.

$t_o \leftarrow$ 20 minutes each hour. $\lambda_{F_o} = 0.01 \leftarrow$ rate of 0.01 failure per operating hour, λ_I 0.0002 failure per idle hour. P_I 0.03.

- Find the effective failure rate → 1) $\lambda_{eff} = \frac{t_I}{o+I}\lambda_I + \frac{t_o}{o+I}\lambda_o + \frac{P}{o+I}$
- Find Reliability at 24 hours
- Find MTTF

$$\left(= \frac{40}{60} * .0002 + \frac{20}{60} * .01 + \frac{.03}{1} \right)$$

$\lambda \rightarrow = 0.0335$

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2) $e^{-\lambda_{eff}(24)} = e^{-0.0335(24)} \doteq 0.4475$

3) $\frac{1}{\lambda_{eff}} \rightarrow \frac{1}{0.0335} \rightarrow 29.851$

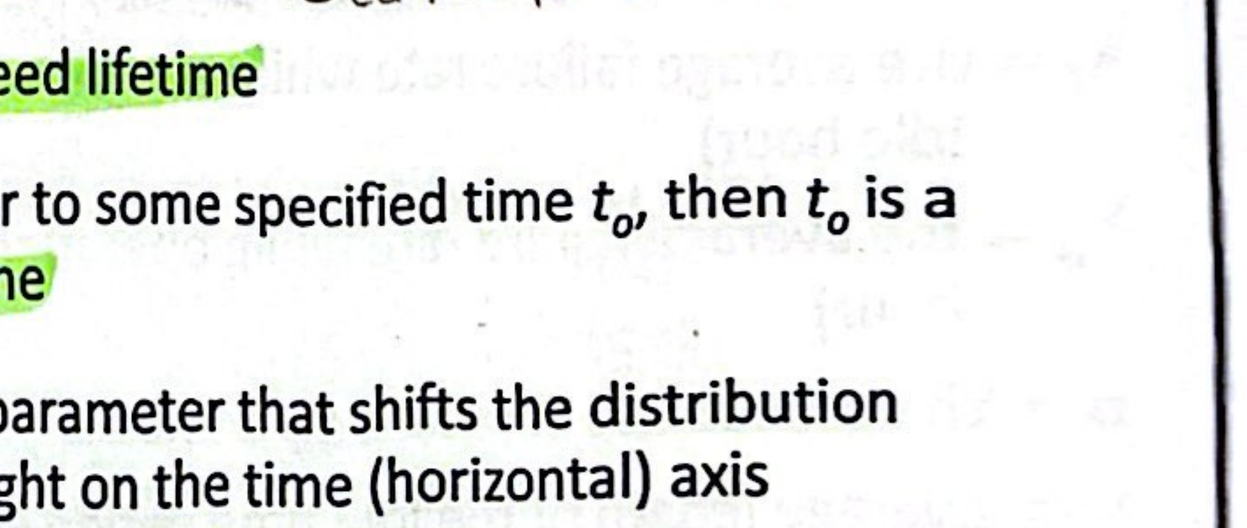
3.4 The Two-parameter Exponential Distribution

↳ $\lambda \rightarrow$ Failure rate $t_o \rightarrow$ location parameter

- It is also known as the guaranteed lifetime
- If a failure will never occur prior to some specified time t_o, then t_o is a minimum time or no-failure time
- The parameter t_o is a location parameter that shifts the distribution an amount equal to t_o to the right on the time (horizontal) axis
- This is equivalent to rewriting the density function by replacing t with $t - t_o$, where $t \geq t_o$

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← يكون بنامنة مثلاً أول همس ساعات
يكي هي (to) طبع يعني
Failure لكن بعدها ممكن
يبدأ يعس



The Two-parameter Exponential Distribution

- Rewriting the density function by replacing t with $t - t_o$, where $t \geq t_o$:

$$f(t) = -\frac{dR(t)}{dt} = \lambda e^{-\lambda(t-t_0)} \qquad 0 < t_0 \leq t < \infty$$

$$R(t) = e^{-\lambda(t-t_0)} \qquad t \geq t_0$$

shift يعني تحركت

$$\text{MTTF} = \int_{t_0}^{\infty} \lambda t e^{-\lambda(t-t_0)} dt = t_0 + \frac{1}{\lambda} \qquad \sigma = \frac{1}{\lambda}$$

النقاط العيدة عن ال mean ما تغيرت هو تغير الموقع ولكن البعد عن ال mean ثابت عشان هيك ما بتغير

$$t_{\text{med}} = t_0 + \frac{\ln 0.5}{-\lambda} = t_0 + \frac{0.69315}{\lambda}$$

$$t_R = t_0 + \frac{\ln R}{-\lambda} \rightarrow \text{design life}$$

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Example 3.8

λ

معناها أنا هنا منتظار بس 200hr ∫ Failure عشان

Let constant failure rate = 0.001 per hour, and the free failure time is 200hr, find: (t_o)

a. R(1000)= $R(t) = e^{-\lambda(t-t_0)} \rightarrow e^{-0.001(1000-200)} = 0.4493$

b. MTTF $\rightarrow 200 + \frac{1}{0.001} = 1200$

c. Median $\rightarrow 200 + \frac{0.69315}{0.001} = 893.147$

d. Standard deviation $\rightarrow 1/\lambda = 1/0.001 = 1000$

e. 95% design life

$200 + \frac{\ln .95}{-0.001} = 251.29$

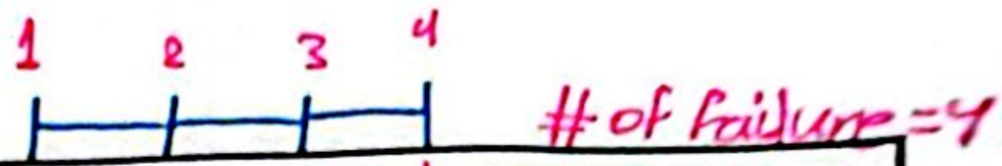
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* time between failure → exp
* Number of failure → Poisson

Parameter → λ



3.5 Poisson Process * discrete ملف كسور

- If a component having a constant failure rate λ is immediately → $t=0$ repaired or replaced upon failing, the number of failures observed over a time period t has a Poisson distribution. ↳ $\lambda t \rightarrow E(n) = \sigma^2$
- The probability of observing n failures in time t is given by the Poisson probability mass function $p_n(t)$:

$$p_n(t) = \frac{e^{-\lambda t}(\lambda t)^n}{n!} \qquad n = 0, 1, 2, \ldots \rightarrow \text{exactly (\# of Failure)}$$

$$E[n] = \sigma^2 = \lambda t \rightarrow \text{mean}$$

- The probability of observing 0 failures in time t = Reliability = ↳ = 0 Failure

$$p_0(t) = \frac{e^{-\lambda t}(\lambda t)^0}{0!} = e^{-\lambda t} = R(t)$$

↳ exp بتيجي

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exp → special case of Γ gamma

The Gamma Distribution ~ cont.

- Let T_i = time between failure $i-1$ and failure i having an exponential distribution with parameter λ.

Y_3 → قديش الوقت لحد ما أحصل على 3 Failure

$$Y_k = \sum_{i=1}^{k} T_i \rightarrow \text{Time between failure}$$

Y_k = the time of the k^{th} failure. The sum of k independent exponential random variables has a gamma distribution with parameters λ and k.

Parameter of gamma → λ
↳ K
when K=1 → exp

* Σ exp → gamma

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The Gamma Distribution

The CDF is the probability that the k^{th} failure will occur by time t:

$$\Pr\{Y_k \le t\} = F_{Y_k}(t) = 1 - e^{-\lambda t}\sum_{i=0}^{k-1}\frac{(\lambda t)^i}{i!}$$

باخذ معادلة ال Poisson ففي عكسها فيكون 1-P

↳ (K-1 → Spare Part)

Y → 3 Failure احتمال اقل من

بـ 50 ساعة أو أقل // عكسها إنه ما أحصل ٣ أو أقل فيكون

مع احتمالية (0, 1, 2) بطرح من (1) يكون معي احتمالية أطلع ٣ أو أكثر

The mean value for Y_k is k/λ

The variance is k/λ^2

The mode is $(k-1)/\lambda$

$K=5 \rightarrow f(t)$ ~ 4 Spare Part, ما بدي يعوض أكثر من 4 مرات

ما بدي يعوض أكثر من عدد ال Spare Part اللي عندي

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Inventory Analysis

- The Poisson process is often used in inventory analysis to determine the number of spare components when the time between failures is exponential.
- If S spare components are available to support a continuous operation over a time period t, then:

$$R_S(t) = \sum_{n=0}^{S} P_n(t)$$

Poisson $P_n(t) \rightarrow \frac{e^{-\lambda t}(\lambda t)^n}{n!}$

بعوض $R_0 + R_1 + R_2 + R_3$

- $R_S(t)$ is the cumulative probability of S or fewer failures occurring during time t
- Or $R_S(t)$ is the component reliability if there are S spares available for immediate replacement when a failure occurs

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Example 3.9

A specially designed welding machine has a nonrepairable motor with a constant failure rate of 0.05 failures per year. The company has purchased two spare motors. If the design life of the welding machine is 10 yr, what is the probability that the two spares will be adequate?

$s = 2$

$$R_S(t) = \sum_{0}^{S} P_n(t)$$

$$\sum_{0}^{2} \frac{e^{0.05(10)}\ 0.5^{n}}{n!} \longrightarrow 0,1,2 = 98.5\%.$$

Example 3.9 Using Gamma

A specially designed welding machine has a nonrepairable motor with a constant failure rate of 0.05 failures per year. The company has purchased two spare motors. If the design life of the welding machine is 10 yr, what is the probability that the two spares will be adequate?

$$R(t) = e^{-\lambda t} \sum_{0}^{k-1} \frac{(\lambda t)^i}{i!}$$

$$R = e^{-0.05 \times 10} \sum_{0}^{2} \frac{(0.05 \times 10)^i}{i!} = 98.5\%$$

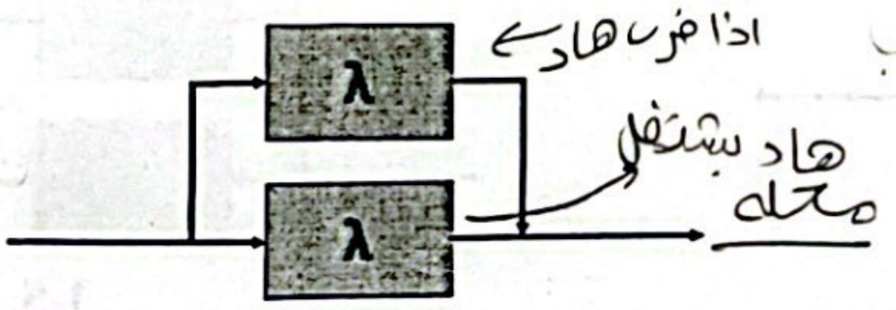
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Related Failure Distributions

Random Variable	Distribution	Parameter(s)
T, time to failure	Exponential	λ
Y_k, time of the k^{th} failure	Gamma	λ, k
N, number of failures in time t	Poisson	λ

3.6 Redundancy with Constant Failure Rate Model

- Consider the case of two independent and redundant components each having the same constant failure rate λ



$$R(t) = 1 - (1 - e^{-\lambda t})^2$$

$$= 1 - (1 - 2e^{-\lambda t} + e^{-2\lambda t})$$

$$= 2e^{-\lambda t} - e^{-2\lambda t}$$

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{2\lambda e^{-\lambda t} - 2\lambda e^{-2\lambda t}}{2e^{-\lambda t} - e^{-2\lambda t}}$$

$$= \frac{\lambda(1 - e^{-\lambda t})}{(1 - 0.5e^{-\lambda t})}$$

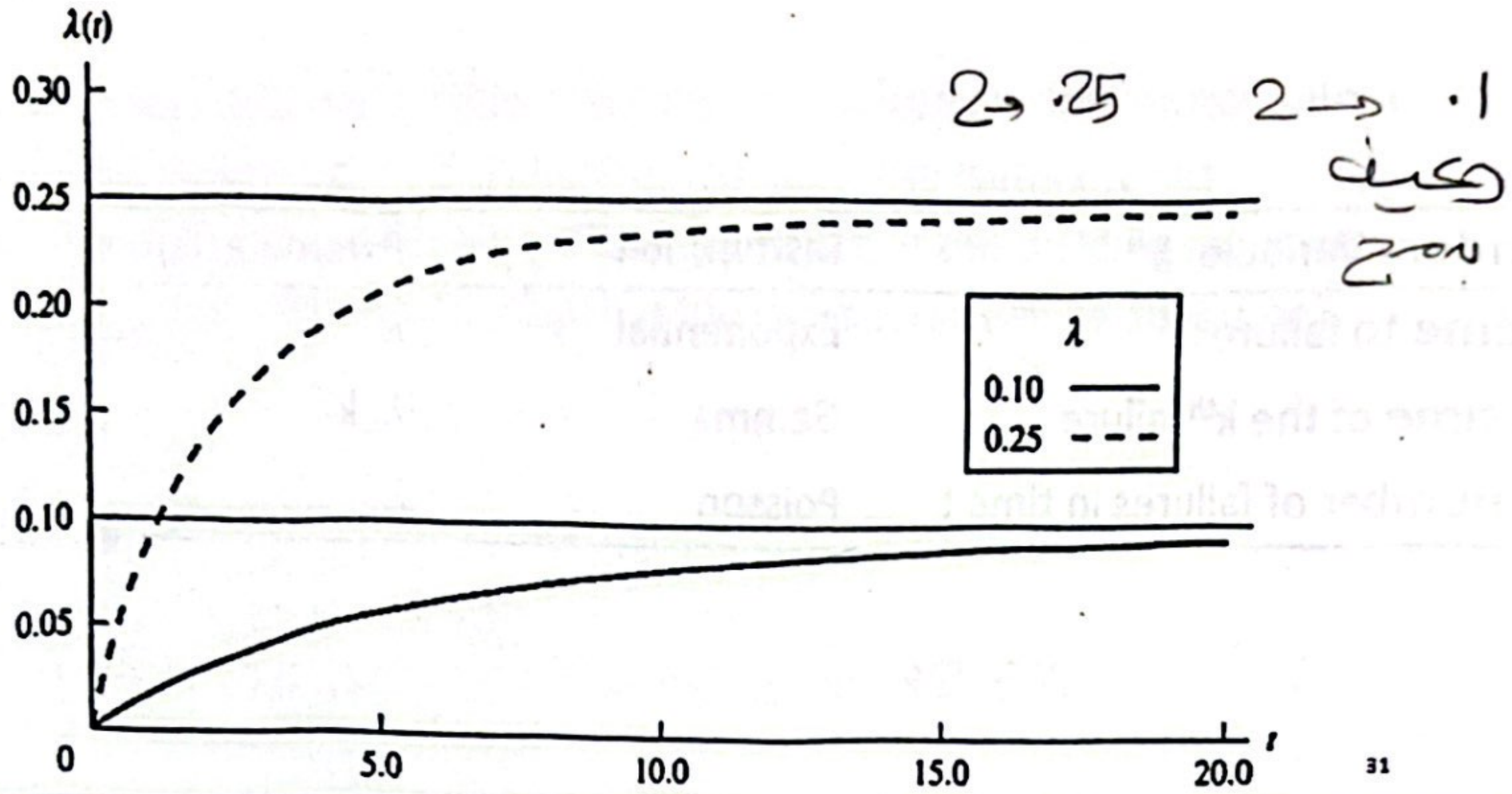
$\lambda(t)$ is not constant

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$e^{-\lambda t} - e^{-\lambda t} - e^{-\lambda t}(e^{-\lambda t})$$

$$2e^{-\lambda t} - e^{-2\lambda t}$$

Redundancy with Constant Failure Rate Model



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Redundancy with Constant Failure Rate Model

- Consider the case of two independent and redundant components each having the same constant failure rate λ

$$MTTF = \frac{1}{\lambda}$$

$$R(t) = e^{-\lambda t}$$

$$\lambda(t) = \lambda$$

$$MTTF = \frac{1.5}{\lambda}$$

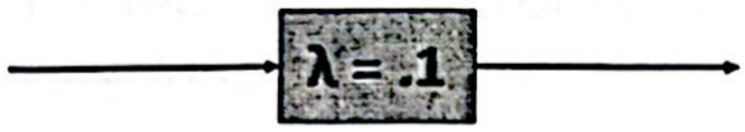
$$R(t) = 2e^{-\lambda t} - e^{-2\lambda t}$$

$$\lambda(t) = \frac{\lambda(1 - e^{-\lambda t})}{(1 - 0.5e^{-\lambda t})}$$

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من الاسم واضح بطلب (constant) صارت متزايد

Example

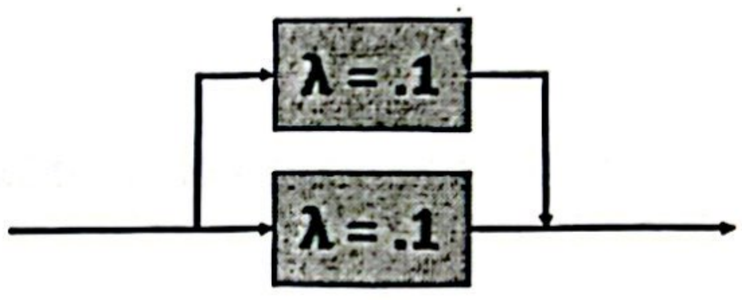


t = 5

$$MTTF = \frac{1}{0.1} = 10$$

$$R(t) = e^{-0.1*5} = 0.6065$$

$$\lambda(t) = 0.1$$



t = 5

$$MTTF = \frac{1.5}{0.1} = 15$$

$$R(t) = 2e^{-0.1*5} - e^{-2*0.1*5} = 0.8451$$

$$\lambda(t) = \frac{0.1(1 - e^{-0.1*5})}{(1 - 0.5e^{-0.1*5})} = 0.0565$$

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