Continuous change

دفتر :

هندسة معوليّة

Reliability

الاسبوع (6)

إعـــداد

بيان عادل للدكتور: رائد العثامنة

اللجنة الأكاديمية لقسم الهندسة الصناعية

2025

TurboIEG.Com

Turbo Team Youtube

Reliability Engineering

Time Dependent Failure Models (Chapter 4)

1

1

Magic → بتشكل ديناميكي الأكثر استخداماً

4.1 The Weibull Distribution ← Valid for - constant -increase -decrease

- One of the most useful probability distributions in reliability is the Weibull
- The Weibull failure distribution may be used to model both increasing and decreasing failure rates

b ← بتحدد نوع الأقتران

- The Weibull hazard rate function:

$$\lambda(t) = a\, t^{b}\, ,\ with\ \ a > 0$$

b>0 → increasing
b<0 → decreasing
b=0 → constant

Formal Formula هاي موال

- The function λ(t) is increasing for b >0 decreasing for b < 0, and constant for b =0

2

2

2parameter for weibull → θ → Scale
→ β shape

3w → θ
→ β
→ location

Normal → increasing ولا ساعدك → time in it
accuracy تجعل الطالب يقلل من ال

The Weibull Distribution

- For mathematical convenience it is better to express λ(t) in the following manner:

$$\rightarrow \lambda(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} \; ;\theta > 0, \beta > 0 ; t \geq 0 \qquad b = \beta - 1$$

$\beta = 4$
$b = 3$
increase

- **β is the shape parameter**
- **θ is the scale parameter** / characteristic life

$\beta = 1$
constant

$$R(t) = e^{-\left(\frac{t}{\theta}\right)^{\beta}} \qquad f(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} e^{-\left(\frac{t}{\theta}\right)^{\beta}}$$

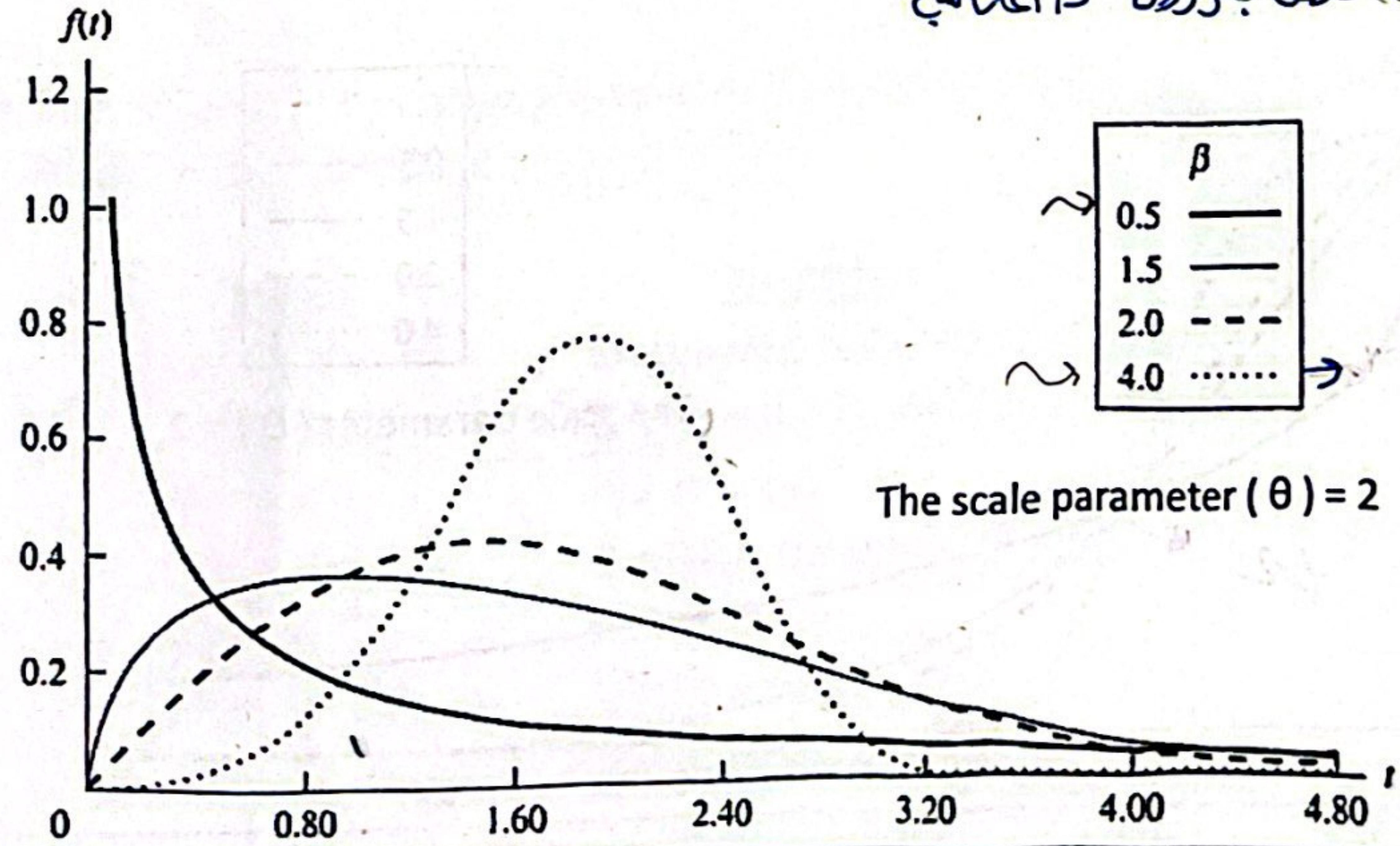
الأفضل نحفظه

$R(t) = e^{-\int_0^t \lambda(t)}$ $\qquad$ $\rightarrow \dfrac{-dR(t)}{dt}$

provide insight into the behavior or the failure process

The Shape parameter (β) → sensitivity analysis

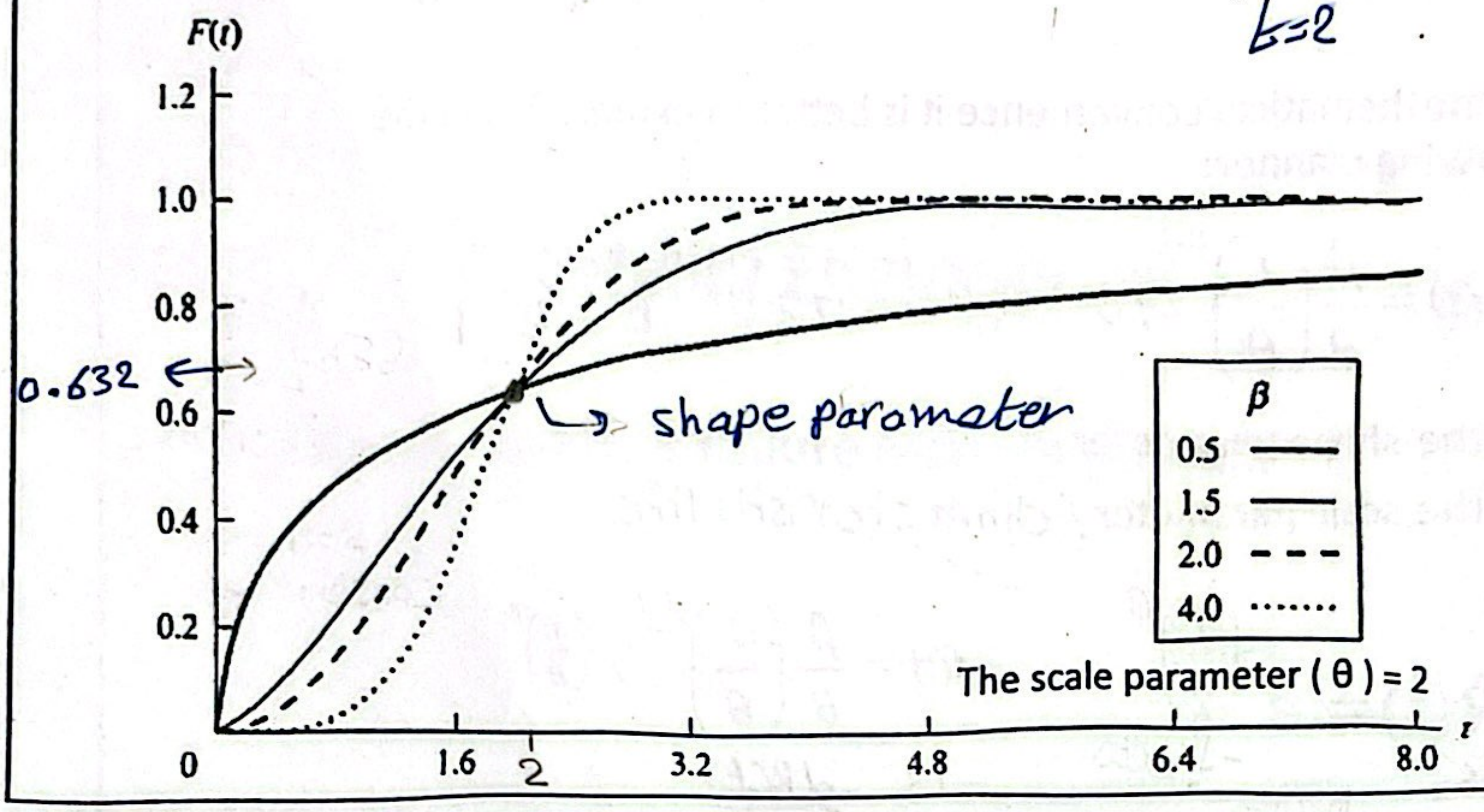
ل منها بقرر ال β المناسب



بصير أقرب لل normal

The Shape parameter (β) → behavior يأثر على ال

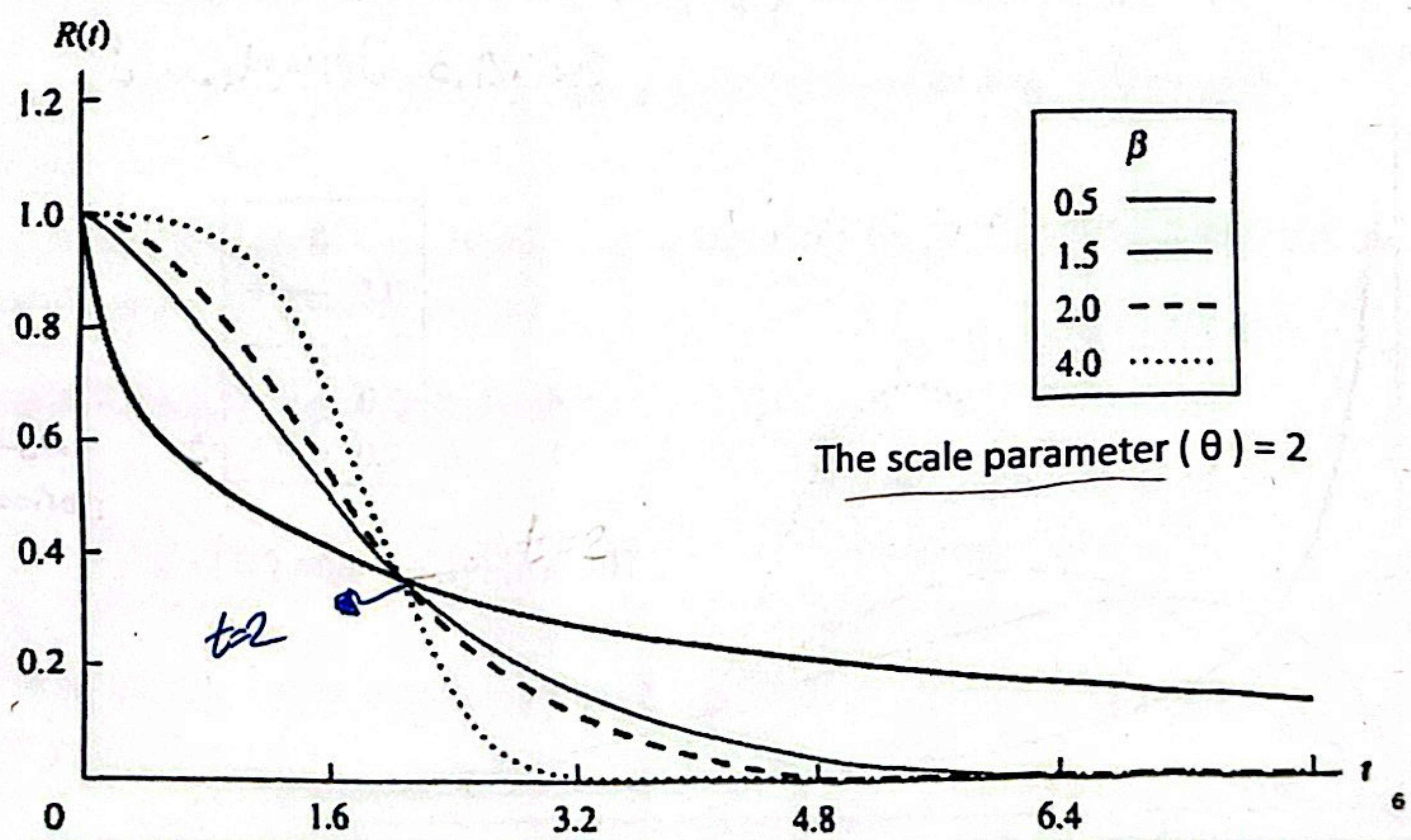
t=2



5

$R(\theta) = e^{-\left(\frac{\theta}{\theta}\right)^{\beta}} \rightarrow e^{-1^{\beta}} = 0.368 \rightarrow F(\theta) = 1 - R = 1 - 0.368 = 0.632$

The Shape parameter (β)



6

بلتقي لما تكون قيمة ال t = قيمة ال θ

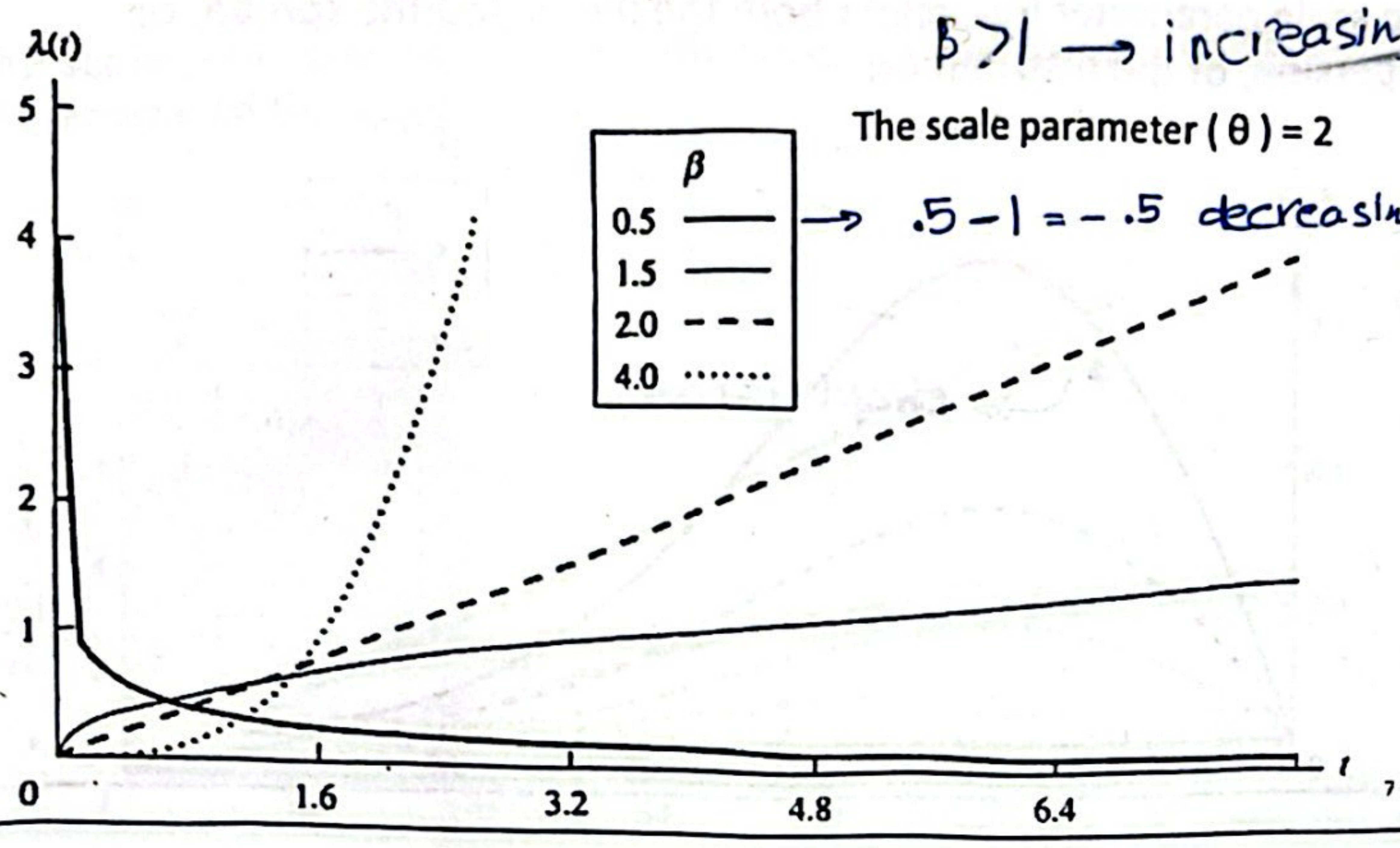
$t = 2 = \theta$

b = β - 1

The Shape parameter (β)

β<1 → decreasing

β>1 → increasing



7

The Shape parameter (β)

- The value of the shape parameter β provides insight into the behavior of the failure process + Variability

Value	Property
$0 < \beta < 1$	Decreasing Failure Rate (DFR)
← $\beta = 1$	Exponential Distribution (CFR)
$1 < \beta < 2$	IFR-concave
$\beta = 2$	Rayleigh Distribution (LFR) ←
$\beta > 2$	IFR - Convex ←
$3 <= \beta <= 4$	IFR - Approaches Normal ←
	Distribution - Symmetrical ←

b = β - 1 = 0

Scale → [location + Variability]

probability Plot → بعملها عشان أشيك وأعرف

(W)

8

63.2%

→ slope يعطي ال β

time إذا %63.2 بتكون failure

Scale Parameter كل ما زاد الـ بيدي أطول life

The Scale Parameter (θ)

- The scale parameter influences both the mean and the spread, or dispersion, of the distribution

β ←

قديش ←

Failure ع

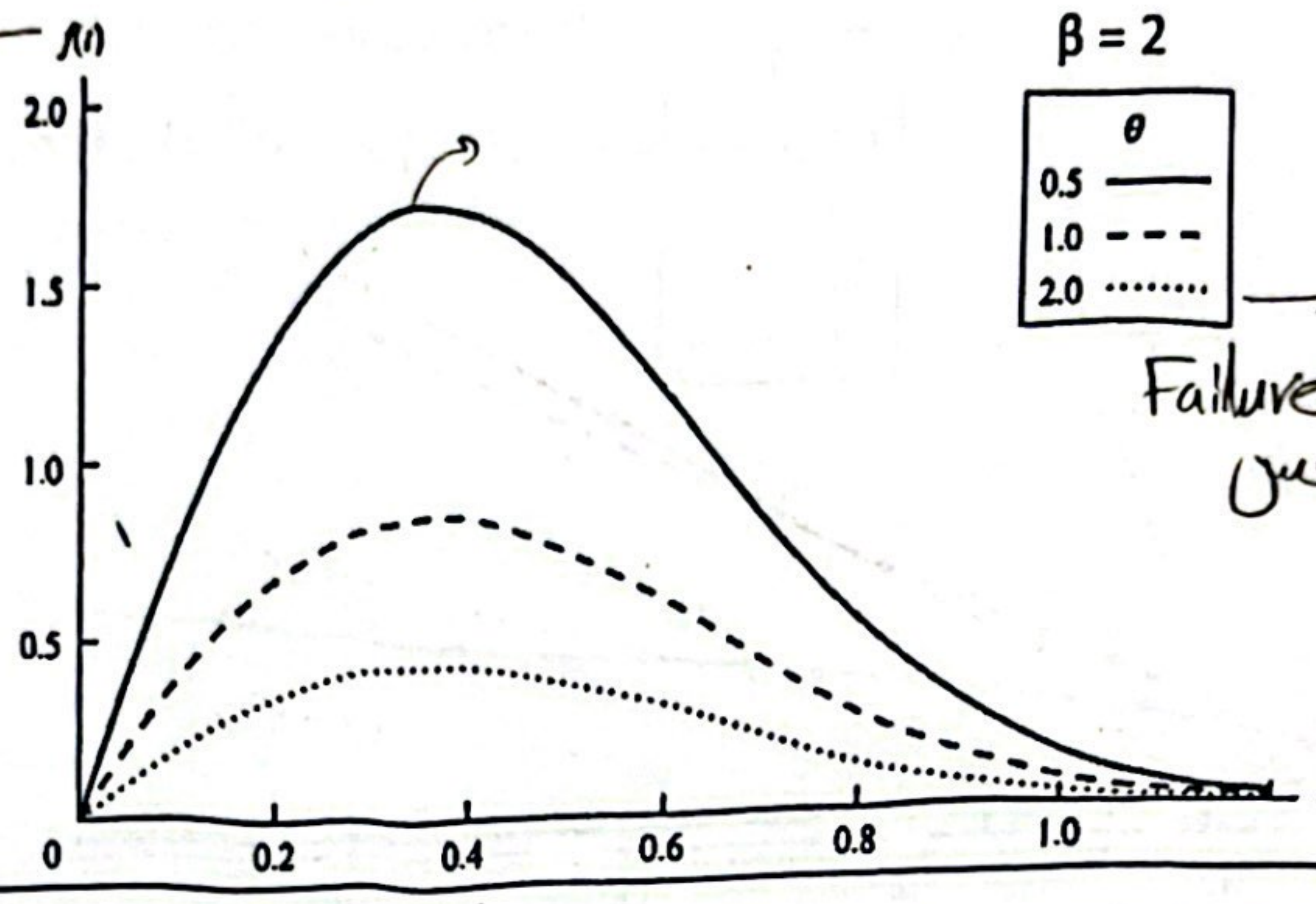
كل ما زاد

أحسن

كل ما قل

أسوأ بزيد

الFailure



قل الFailure

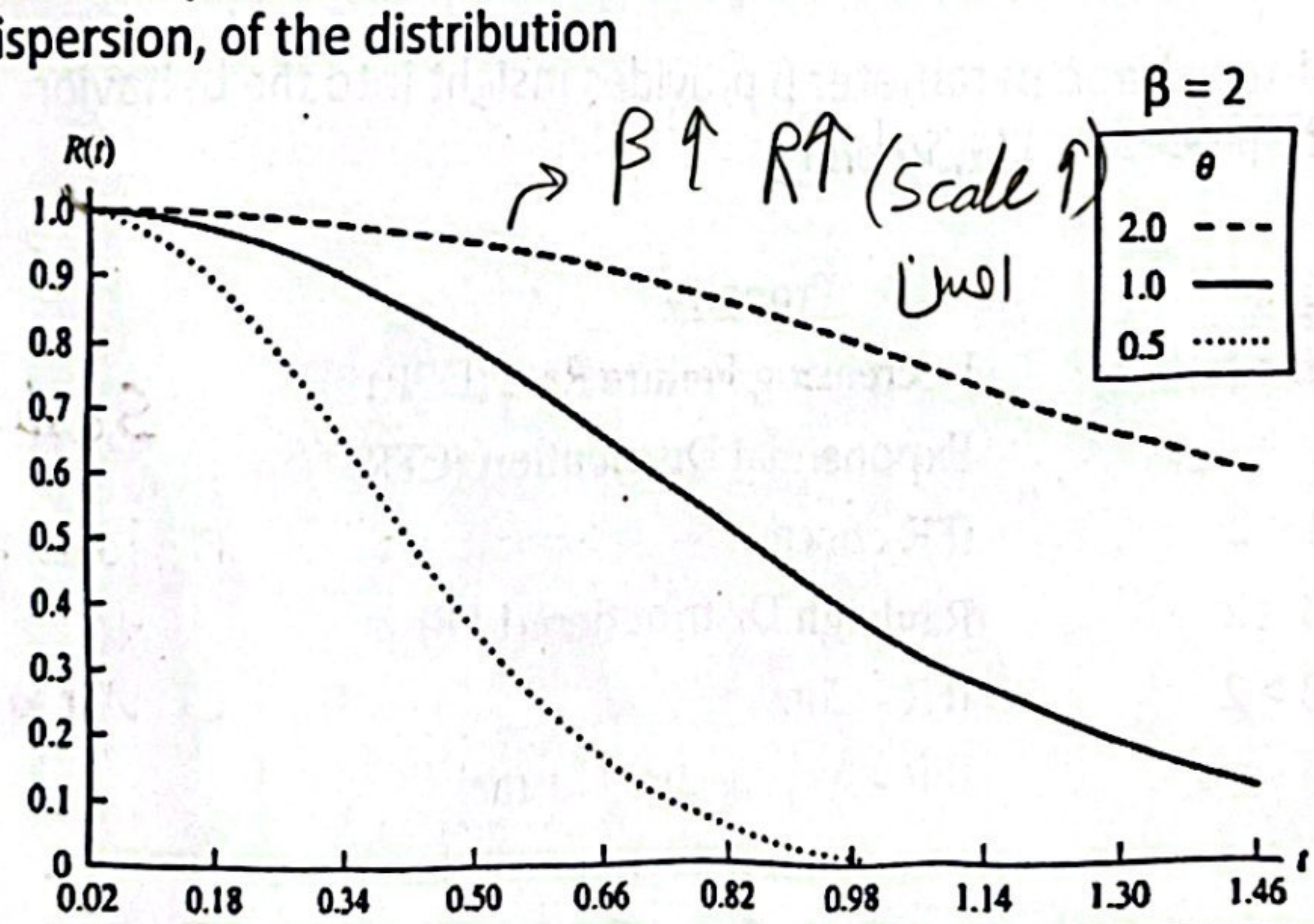
ما حسن

The Scale Parameter (θ)

- The scale parameter influences both the mean and the spread, or dispersion, of the distribution

β↑ R↑ (Scale ↑)

احسن

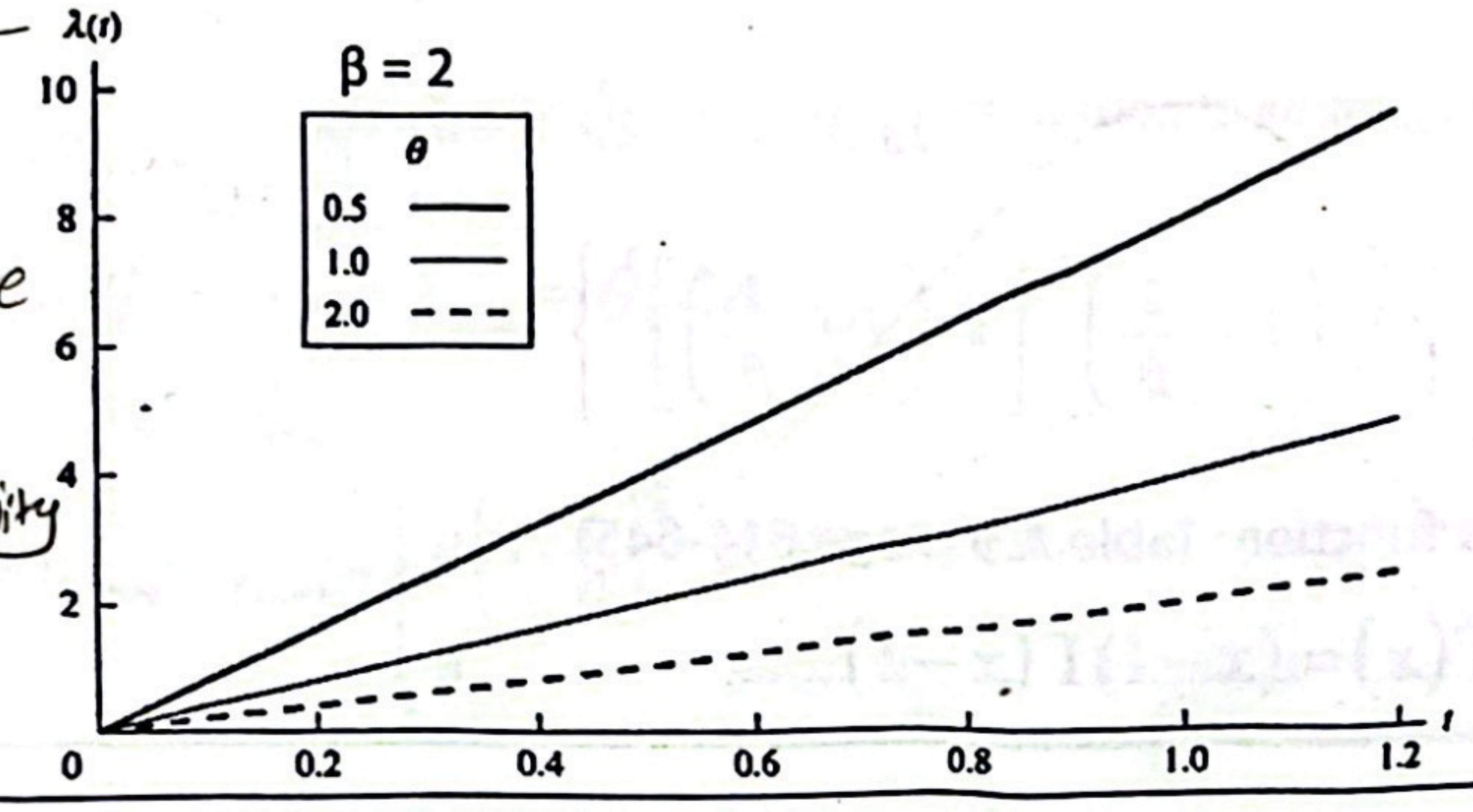


The Scale Parameter (θ)

- The scale parameter influences both the mean and the spread, or dispersion, of the distribution

θ↑ slope λ↓

Failure rate أقل يعطي Relaiability أحسن



11

11

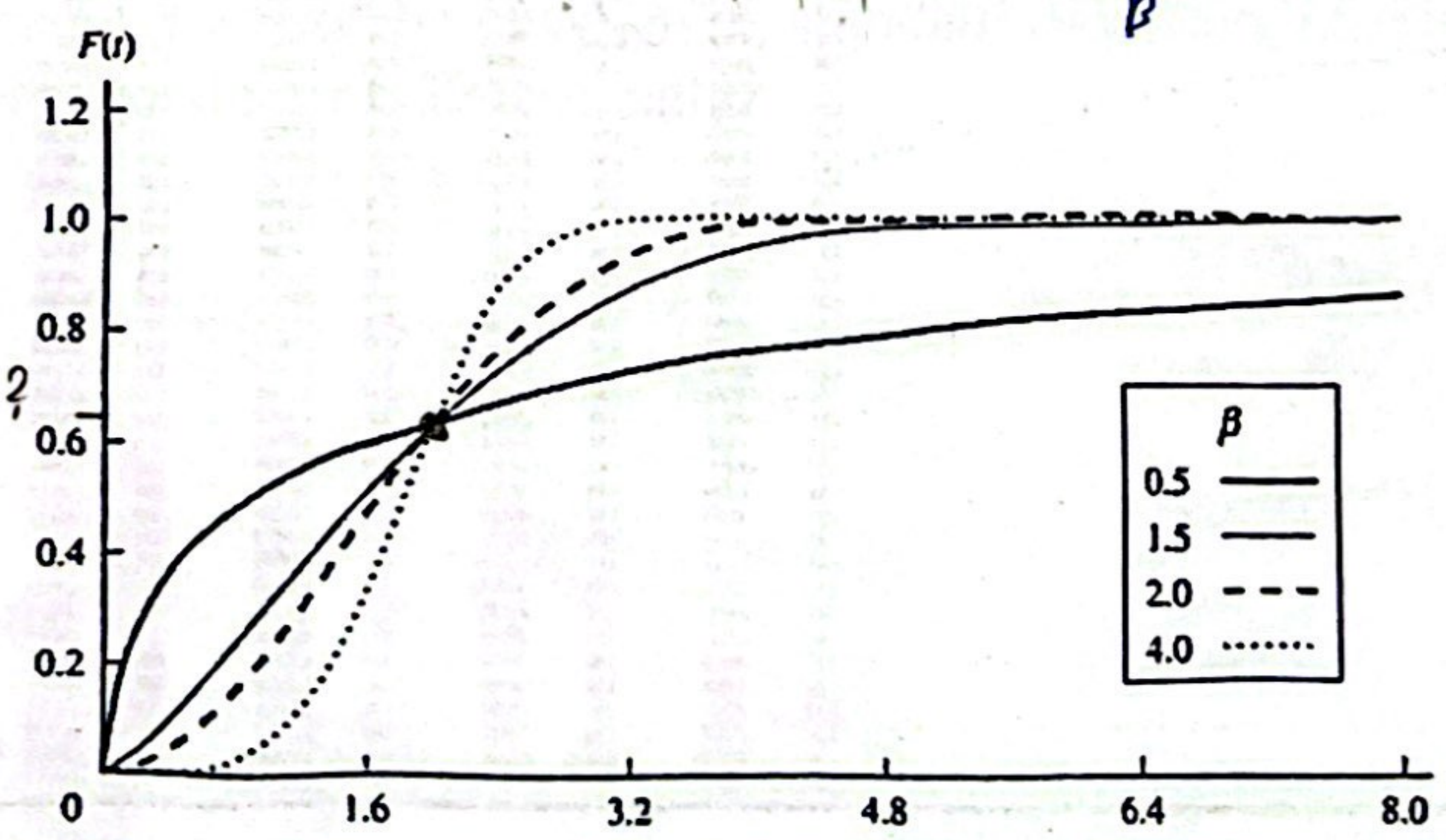
Characteristic Life θ

- 63.2 percent of all Weibull failures will occur by time t = θ (Scale parameter) regardless of the value of the shape parameter β

Scale parameter = 2

%63.2

θ → ثابتة



12

12

$\Gamma(1) = 1 \quad \Gamma(0) = 0$

MTTF, and Standard Deviation

$$MTTF = \underline{\theta\,\Gamma\left(1+\frac{1}{\beta}\right)}$$

بشتق من $0 \rightarrow \infty$

$\Gamma(x) = (x-1)!$

$\Gamma(3) = 2$

$\Gamma(4) = 6$

$\Gamma(x)$ = the gamma function $= \int_0^\infty y^{x-1} e^{-y}\,dy$ ←

$\Gamma(4.5) = \Gamma 3.5 * \Gamma 2.5 * \Gamma 1.5$

أو $\quad 3.5 * \Gamma 3.5$

$\rightarrow 3.5 * 2.5 * \Gamma 2.5$

$$\sigma^2 = \theta^2 \left\{ \Gamma\left(1+\frac{2}{\beta}\right) - \left[\Gamma\left(1+\frac{1}{\beta}\right)\right]^2 \right\}$$

The gamma function: Table A.9 (Page 644-645)

$$\Gamma(x) = (x-1)\,\Gamma(x-1) \leftarrow$$

13

Table A.9 (Page 644-645)

Gamma function

x	$\Gamma(x)$	x	$\Gamma(x)$	x	$\Gamma(x)$	x	$\Gamma(x)$
1.01	.99433	1.51	.88659	2.01	1.00427	2.51	1.33875
1.02	.98884	1.52	.88704	2.02	1.00862	2.52	1.34830
1.03	.98355	1.53	.88757	2.03	1.01306	2.53	1.35798
1.04	.97844	1.54	.88818	2.04	1.01758	2.54	1.36779
1.05	.97350	1.55	.88887	2.05	1.02218	2.55	1.37775
1.06	.96874	1.56	.88964	2.06	1.02687	2.56	1.38784
1.07	.96415	1.57	.89049	2.07	1.03164	2.57	1.39807
1.08	.95973	1.58	.89142	2.08	1.03650	2.58	1.40844
1.09	.95546	1.59	.89243	2.09	1.04145	2.59	1.41896
1.10	.95135	1.60	.89352	2.10	1.04649	2.60	1.42962
1.11	.94740	1.61	.89468	2.11	1.05161	2.61	1.44044
1.12	.94359	1.62	.89592	2.12	1.05682	2.62	1.45140
1.13	.93993	1.63	.89724	2.13	1.06212	2.63	1.46251
1.14	.93642	1.64	.89864	2.14	1.06751	2.64	1.47377
1.15	.93304	1.65	.90012	2.15	1.07300	2.65	1.48519
1.16	.92980	1.66	.90167	2.16	1.07857	2.66	1.49677
1.17	.92670	1.67	.90330	2.17	1.08424	2.67	1.50851
1.18	.92373	1.68	.90500	2.18	1.09000	2.68	1.52040
1.19	.92089	1.69	.90678	2.19	1.09585	2.69	1.53246
1.20	.91817	1.70	.90864	2.20	1.10180	2.70	1.54469
1.21	.91558	1.71	.91057	2.21	1.10785	2.71	1.55708
1.22	.91311	1.72	.91258	2.22	1.11399	2.72	1.56964
1.23	.91075	1.73	.91467	2.23	1.12023	2.73	1.58237
1.24	.90852	1.74	.91683	2.24	1.12657	2.74	1.59528
1.25	.90640	1.75	.91906	2.25	1.13300	2.75	1.60836
1.26	.90440	1.76	.92137	2.26	1.13954	2.76	1.62162
1.27	.90250	1.77	.92376	2.27	1.14618	2.77	1.63506
1.28	.90072	1.78	.92623	2.28	1.15292	2.78	1.64868
1.29	.89904	1.79	.92877	2.29	1.15976	2.79	1.66249
1.30	.89747	1.80	.93138	2.30	1.16671	2.80	1.67649
1.31	.89600	1.81	.93408	2.31	1.17377	2.81	1.69068
1.32	.89464	1.82	.93685	2.32	1.18093	2.82	1.70506
1.33	.89338	1.83	.93969	2.33	1.18819	2.83	1.71963
1.34	.89222	1.84	.94261	2.34	1.19557	2.84	1.73441
1.35	.89115	1.85	.94561	2.35	1.20305	2.85	1.74938
1.36	.89018	1.86	.94869	2.36	1.21065	2.86	1.76456
1.37	.88931	1.87	.95184	2.37	1.21836	2.87	1.77994
1.38	.88854	1.88	.95507	2.38	1.22618	2.88	1.79553
1.39	.88785	1.89	.95838	2.39	1.23412	2.89	1.81134
1.40	.88726	1.90	.96177	2.40	1.24217	2.90	1.82736
1.41	.88676	1.91	.96523	2.41	1.25034	2.91	1.84359
1.42	.88636	1.92	.96877	2.42	1.25863	2.92	1.86005
1.43	.88604	1.93	.97240	2.43	1.26703	2.93	1.87673
1.44	.88581	1.94	.97610	2.44	1.27556	2.94	1.89363
1.45	.88566	1.95	.97988	2.45	1.28421	2.95	1.91077
1.46	.88560	1.96	.98374	2.46	1.29298	2.96	1.92814
1.47	.88563	1.97	.98768	2.47	1.30188	2.97	1.94574
1.48	.88575	1.98	.99171	2.48	1.31091	2.98	1.96358
1.49	.88595	1.99	.99581	2.49	1.32006	2.99	1.98167
1.50	.88623	2.00	1	2.50	1.32934	3.00	2

14

$x =$ عدد صحيح

$\Gamma(2) = 1$

$\Gamma(4) = (x-1)\,\Gamma(x-1)$

$3\,\Gamma(3) = 3 * 2 = 6$

$\Gamma(5) = 4\,\Gamma(4)$

$4 * 3\,\Gamma(3)$

$12 * 2 \sim 4! = 24$

$x =$ عدد كسري

$\Gamma(1.5) = 0.88623$

$\Gamma(3.5) = 2.5 * \Gamma(2.5)$

$2.5 = 1.32939$

$\Gamma(5.5) = 4.5 * \Gamma 4.5$

$4.5 = 3.5 * \Gamma 3.5$

Design Life, Median, and Mode

Design Life t_R:

$$R(t) = e^{-\left(\frac{t}{\theta}\right)^{\beta}} = R \leftarrow \ln$$

then $t_R = \theta(-\ln R)^{\frac{1}{\beta}} \leftarrow$

Median t_{med}: $t_{.50} = t_{med} = \theta(-\ln .5)^{\frac{1}{\beta}}$

Mode t_{mode}: $t_{mode} = \begin{cases} \theta\left(1-\frac{1}{\beta}\right)^{\frac{1}{\beta}} & \text{for } \beta > 1 \\ 0 & \text{for } \beta \le 1 \end{cases} \rightarrow$ بتنازل من فوق

15

Example 4.1:

A compressor experiences wearout with a linear hazard rate function. β = 2 and θ = 1000 hr. Find MTTF, standard deviation, Median, Mode, the design life for 0.99 reliability?

$MTTF = \theta\,\Gamma(1+\frac{1}{\beta})$

$\rightarrow 1000\,\Gamma(1+\frac{1}{2}) \rightarrow 1000 * \Gamma 1.5 \rightarrow 0.88623 * 1000$

886.23

Designlife =

$t_R = \theta(-\ln R)^{\frac{1}{\beta}}$

$\rightarrow 1000(-\ln R)^{\frac{1}{2}} = .99$

Median $= 1000(-\ln .5)^{\frac{1}{2}} = 832.55$

Mode $= 1000(1-\frac{1}{2})^{\frac{1}{2}} \rightarrow 1000 * \sqrt{.5}$

$= 707.1$

$\sigma = \sqrt{1000^2(\Gamma(2) - (\Gamma(1.5))^2}$

$\rightarrow \sqrt{1000^2 * (1-0.88623)^2}$

$\rightarrow t = 100.25$

$t \rightarrow 90\%$ is يكون أقل

$99.5\% \rightarrow$ أعلى

Example 4.2:

θ β

Given a Weibull failure distribution with a shape parameter of 1/3 and a scale parameter of 16,000, completely characterize the failure process (R (t), Type of failure mode: decreasing, increasing, or constant?, MTTF, Median, Mode, Standard Deviation, 90% reliability, 99% reliability)

$* \text{Median} \rightarrow \theta * (-\ln .5)^{\frac{1}{\beta}} \rightarrow 16000 * (-\ln .5)^{3} = 5328.39$

$* \text{MTTF} \rightarrow 16000\Gamma 4 \rightarrow 16000*6 = 96000$

$* \text{mode} = 0 \qquad *\sigma = \left[\sqrt{(16000)^2 * \Gamma\left(1 + \frac{2}{\frac{1}{3}}\right) - \Gamma\left(1 + \frac{1}{\frac{1}{3}}\right)^2}\right]$

~~1142~~ $\sqrt{(418454.29)}$ 17

17 $90\% = R \rightarrow t_R = \theta(-\ln .9)^3 \rightarrow 18.71 \rightarrow 646.88$

رح تقل $99\%.R \quad t_R = \theta(-\ln .99)^3 \rightarrow 0.0162$