Continuous change

دفتر :

هندسة معوليّة

Reliability Engineering

الاسبوع (7)

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اللجنة الأكاديمية لقسم الهندسة الصناعية

2025

TurboIEG.Com

Turbo Team Youtube

Burn-In Screening for Weibull → area failure rate

عالية بدي أفلص منها

Conditional Reliability:

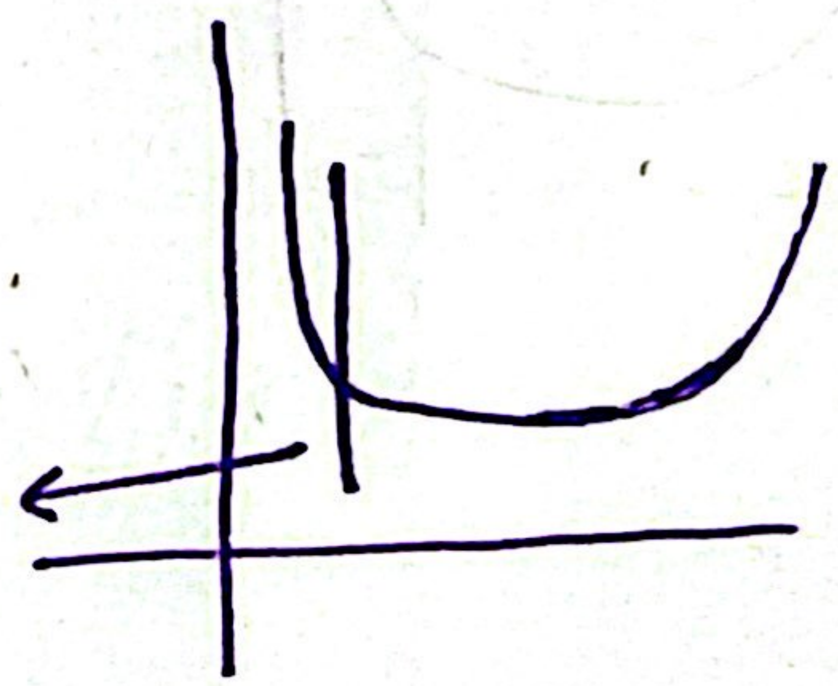
1) بالـ decrease ← بيفيد

2) increase ← بيضر

constant ← لا بيفيد ولا بيضر

طيب إذا كانوا الـ 3 زي الـ bathtub ← ال بيفيد

$$R(t|T_0) = exp\left[-\left(\frac{t+T_0}{\theta}\right)^{\beta} + \left(\frac{T_0}{\theta}\right)^{\beta}\right]$$



← الي هون ال أشيله (decrease)

فترة الاستقرار

Example:

Given a Weibull failure distribution with a shape parameter of 1/3 and a scale parameter of 16,000hrs, calculate the design life for 90% reliability if

1. No burn-in time
2. a 10-hr burn-in period is accomplished

$t_R = \theta(-\ln R)^{1/\beta}$

$16000(-\ln .90)^{3} = 18.7$

$\rightarrow .9 = \exp\left[-\left(\frac{10+T}{16000}\right)^{1/3} + \left(\frac{10}{16000}\right)^{1/3}\right]$

$-0.105 = -\left(\frac{10+T}{16000}\right)^{1/3} + \left(\frac{10}{16000}\right)^{1/3}$

$t_R = 100.6 \rightarrow 101.24$

Exercise:

Re-solve the example if the shape parameter is 1.5

اذا كانت 1.5 ← ارتفع عن ال 18.7

19

$0.9 = \exp\left(-\left(\frac{t_R+10}{16000}\right)^{1.5} + \left(\frac{10}{16000}\right)^{1.5}\right)$

$\hookrightarrow t_R = 3559.59$

$\hookrightarrow t_R = 16000 \times (-\ln .9)^{1/1.5} = 3569 > 3559$

Failure Modes

For a system comprised of n serially related components or having n independent failure modes, each having an independent Weibull failure distribution with shape parameter β and scale parameter θi, the system failure rate function can be determined:

إذا كان توالي إذا صار Failure
بوحدة بخرب كل ال system

$$\lambda_s(t) = \beta t^{\beta-1}\left[\sum_{i=1}^{n}\left(\frac{1}{\theta_i}\right)^{\beta}\right]$$

على فرض إنها ←
خربت ← راح تخرب عندي كل ال system

redundancy → Parallel

20

1.06

Failure Modes

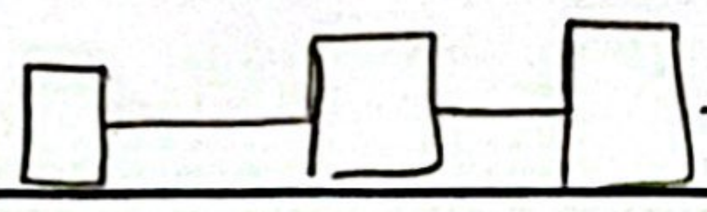
The System has a Weibull distribution with a shape parameter of β and a scale parameter θs of

لم من المعادلة ال scale ثابت ← منها يطلع كل شي من معاد لات ال weibull

$$\theta s = \left[\sum_{i=1}^{n}\left(\frac{1}{\theta_i}\right)^{\beta}\right]^{-1/\beta}$$

weibull مو شرط يكون ← ... → بعد ال[illegible] ك
واستخدم معادلة ال
بعوض بدل ال العادية Waiboll
$\theta \rightarrow \theta_s$

If the shape parameters β are different, then the system failure distribution will not be Weibull



Identical Weibull Components

- If a system of n serially related and independent components, then the system hazard rate:

function ←

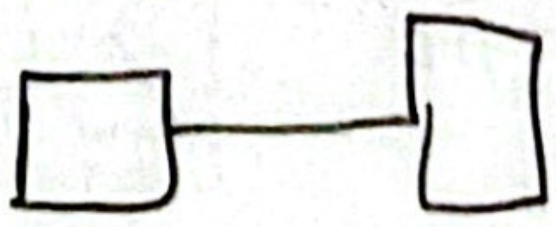
$$\lambda s(t) = \sum_{i=1}^{n}\frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} = \frac{n\beta}{\theta^{\beta}}(t)^{\beta-1}$$

← identical

Sequence →

$$R(t) = exp\left[-n\left(\frac{t}{\theta}\right)^{\beta}\right]$$

$e^{-\left(\frac{t}{\theta}\right)^{\beta s}}$



- System shape parameter = β
- System scale parameter: $\theta s = \frac{\theta}{n^{1/\beta}}$ ←

$$R(t) = e^{-\left(\frac{t}{\theta_s}\right)^{\beta}}$$

Example 4.3:

A jet engine consists of five modules each of which was found to have a Weibull failure distribution with a shape parameter of 1.5. Their scale parameters (characteristic life) are (in operating cycles) 3600, 7200, 5850, 4780, and 9300. Find the MTTF and median time to failure of the engine

$\beta = 1.5 \quad \theta = 3600, 7200, 5850, 4780, 9300$

$MTTF = \theta \, \Gamma(1 + \frac{1}{\beta}) \quad \hookrightarrow \theta_s = \left[\sum_1^5 \left(\frac{1}{\theta_i}\right)^\beta\right]^{-1/\beta} \rightarrow 18.81$

$2 \; t_{med} = \theta(-\ln .5)^{1/\beta} \quad \left[\left(\frac{1}{3600}\right)^{1.5} + \left(\frac{1}{7200}\right)^{1.5} + \left(\frac{1}{5850}\right)^{1.5} + \left(\frac{1}{4780}\right)^{1.5} + \left(\frac{1}{9300}\right)^{1.5}\right]^*$

$\theta_{sys} = 1842.67 \quad MTTF = 1664.5 \quad -(\ln 0.9)^3 = 18.81$

$\hookrightarrow 1842.67 \, \Gamma(1 + \frac{1}{1.5})$

2) $1842.67 * (-\ln .5)^{1/1.5} = 1443.2$

Example 4.4:

An electrical system has four series connectors each having a Weibull failure law with β = 3/4 and θ = 2000 hr. Find the reliability of the system of four connectors for 150 hours.

$\beta = 3/4 \quad \theta = 2000 \quad T = 150 \quad R = ??$

$R(t) = e^{\left(-\frac{t}{\theta_s}\right)^\beta}$

$\theta_s = \frac{2000}{4^{4/3}} = 314.98$

$\hookrightarrow e^{\left(\frac{-150}{314.98}\right)^{3/4}} = 0.56$ ✓

3/24/

location → free time of failure

The Three-Parameter Weibull

- Whenever there is a minimum life t_o such that $T > t_o$, the three-parameter Weibull may be appropriate.
- The three-parameter Weibull distribution assumes that no failures will take place prior to time t_o.
- For this distribution

$$\lambda(t) = \frac{\beta}{\theta}\left(\frac{t - t_0}{\theta}\right)^{\beta - 1} \qquad t \geq t_0$$

- The parameter t_o is called the location parameter

The Three-Parameter Weibull

$$R(t) = \exp\left[-\left(\frac{t - t_0}{\theta}\right)^{\beta}\right] \qquad t \geq t_0$$

- The variance of this distribution is the same as that in the two-parameter model

$$\text{MTTF} = t_0 + \theta\Gamma\left(1 + \frac{1}{\beta}\right)$$

$$t_{\text{med}} = t_0 + \theta(0.69315)^{1/\beta}$$

$$t_R = t_0 + \theta(-\ln R)^{1/\beta}$$

Example 4.5

- The three parameter Weibull has $\beta = 4$, $t_o = 100$, and $\theta = 780$. ← Compute its MTTF, median, standard deviation, and reliability for a 500-hr mission.

$$MTTF = t_o + \theta\,\Gamma\left(1 + \frac{1}{\beta}\right)$$

$$= 100 + 780\,\Gamma\left(1 + \frac{1}{4}\right) = 806.99$$

$$\text{Median} = t_o + \theta\,(0.69315)^{1/\beta} = 811.707$$

$$R = e^{-\left(\frac{t - t_o}{\theta}\right)^{\beta}} \rightarrow e^{-\left(\frac{500-100}{780}\right)^{4}} = ~~.429~~ \rightarrow .933 \rightarrow 93\%$$

$$\sigma^2 = \theta^2\left[\Gamma\left(1+\frac{2}{\beta}\right) - \left[\Gamma\left(1+\frac{1}{\beta}\right)\right]^2\right]$$

$$\hookrightarrow 9340.6 \rightarrow \sigma = 198.3$$

مش بخرب ال system ← إذا كلهم خربوا

Redundancy with Weibull Failures

identecal توازي

- Two identical (and assumed independent) components are used to form a redundant system (both must fail for the system to fail), then the system reliability is:

$$R_S(t) = 1 - [1 - R(t)]^2 \leftarrow$$

$$R_S(t) = 1 - \left[1 - e^{-(t/\theta)^{\beta}}\right]^2 = 2e^{-(t/\theta)^{\beta}} - e^{-2(t/\theta)^{\beta}}$$

same

حتى لو كانوا ← كلهم زي بعض موشي ما يكون weibull مو زي التوالي إذا ال (Shape parameter) متساوي weibull

$$\rightarrow \text{MTTF} = \theta\Gamma\left(1 + \frac{1}{\beta}\right)\left(2 - 2^{-1/\beta}\right)$$

إذا ما كان identecal لبعضهم ببعض ← لا

$$* F(t_s) = F(t_1) \times F(t_2)$$

$$R(t_s) = 1 - F_S(t) \rightarrow (1 - R_1(t)) \times (1 - R_2(t))$$

$$\hookrightarrow 1 - (1 - R_1(t))\,(1 - R_2(t))$$

Example 4.6

- Two fuel pumps, each having a Weibull failure distribution with β =0.5 and θ = 1000 hr, are configured to provide a redundant system. Find the system reliability for a 100-hr mission and the system MTTF

→ $\beta = 0.5 \quad \theta = 1000$

$$R(t) = 1 - [1 - R(t)]^2 \rightarrow 1 - \left[1 - e^{-(t/\theta)^{\beta}}\right]^2$$

$$\rightarrow 1 - \left[1 - e^{-(100/1000)^{.5}}\right]^2$$

$$1 - [1 - .728]^2$$

92%

29

$Z(.01) \rightarrow \frac{t_R - \mu}{\sigma}$

design life → $1 - \phi(z) = .99 \quad \phi(z) = .01$

MTTF = μ Median = μ t_{mode} = μ parameter → mean
↳ s.d (variance)

4.2 The Normal Distribution

standard Normal distribution
↳ mean = 0
s.d = 1
لمستخدمه عن طريق
مجموعة لا Z
جدا

30

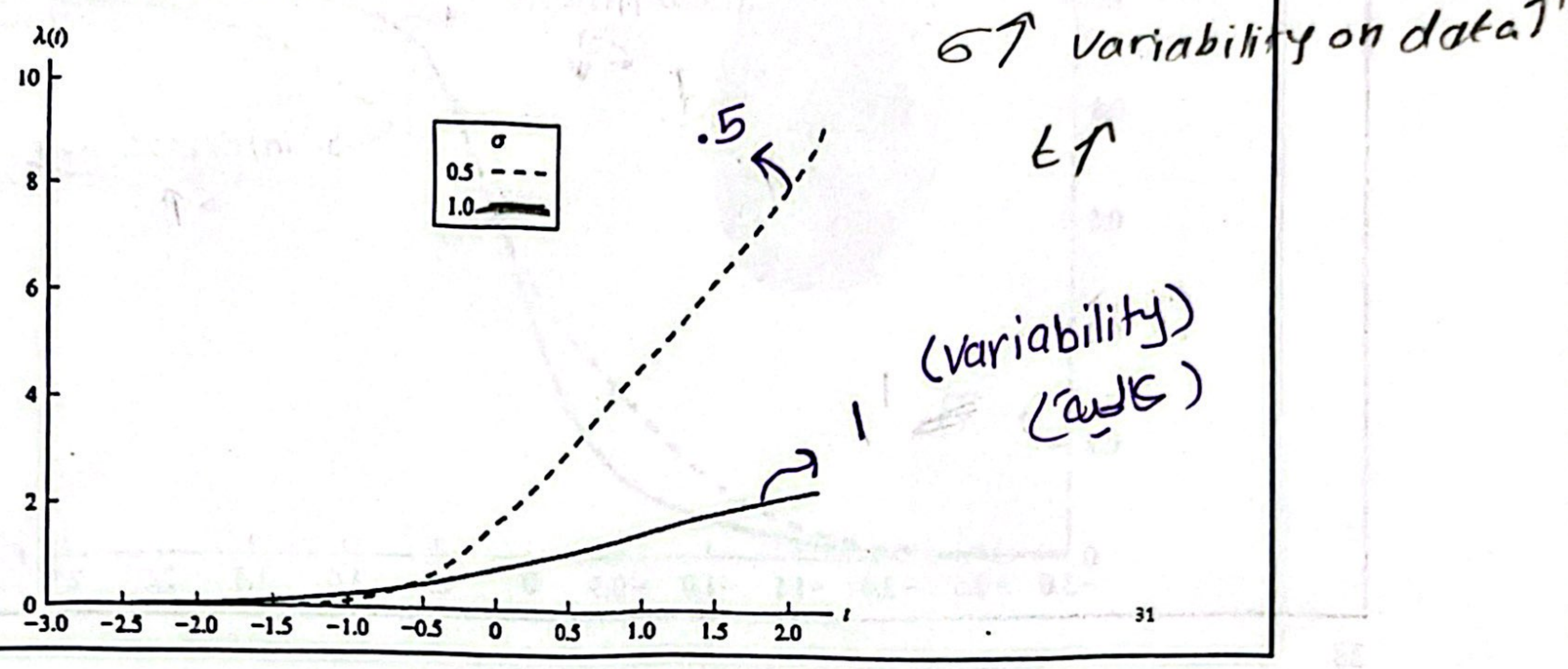
$R(t) = 1 - F(t)$
↓
$\phi(z)$

P(t) قبل
احتمالية أنها تعطل قبل

$\phi(z) = F(t)$

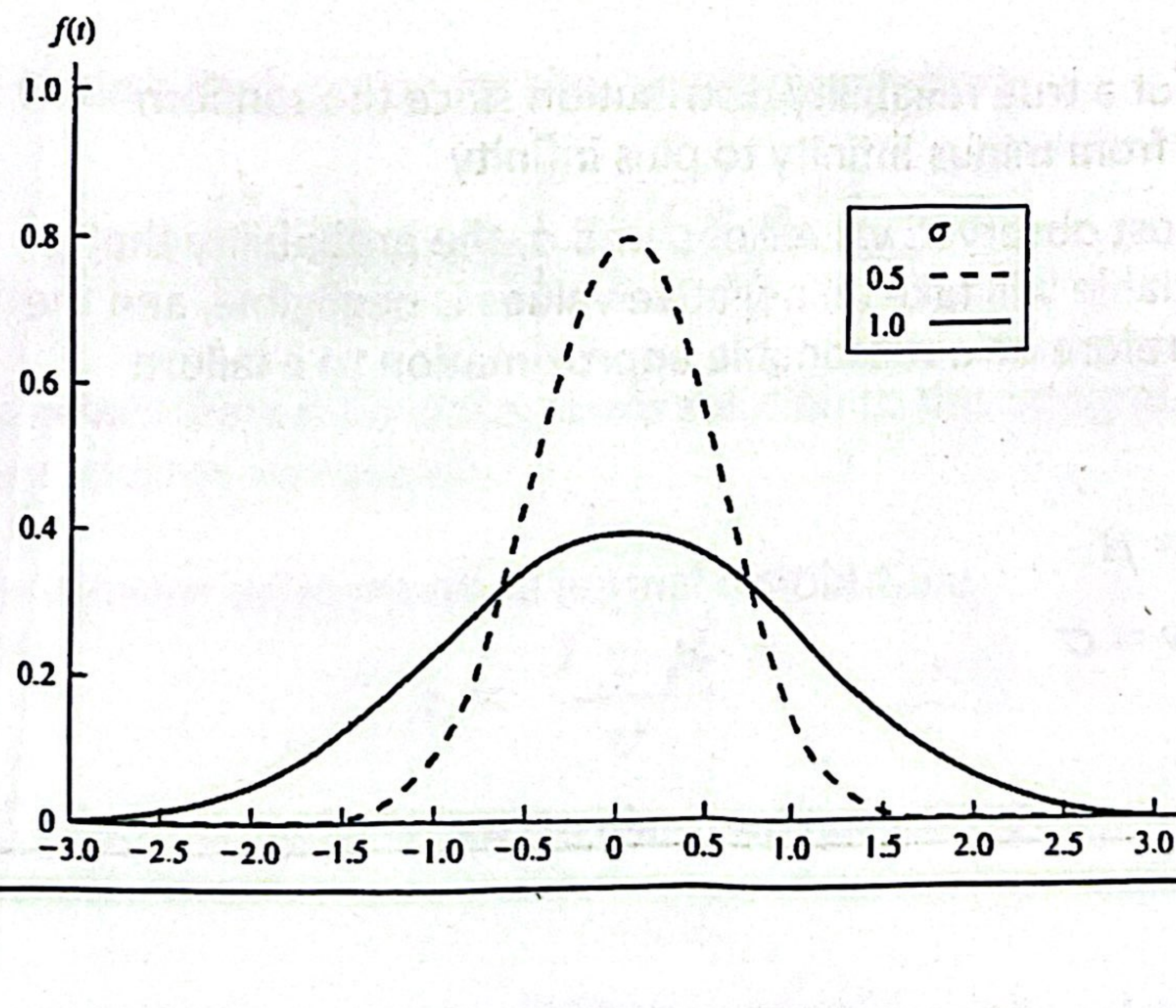
The Normal Distribution

- The normal distribution has been used successfully to model fatigue and wearout phenomena



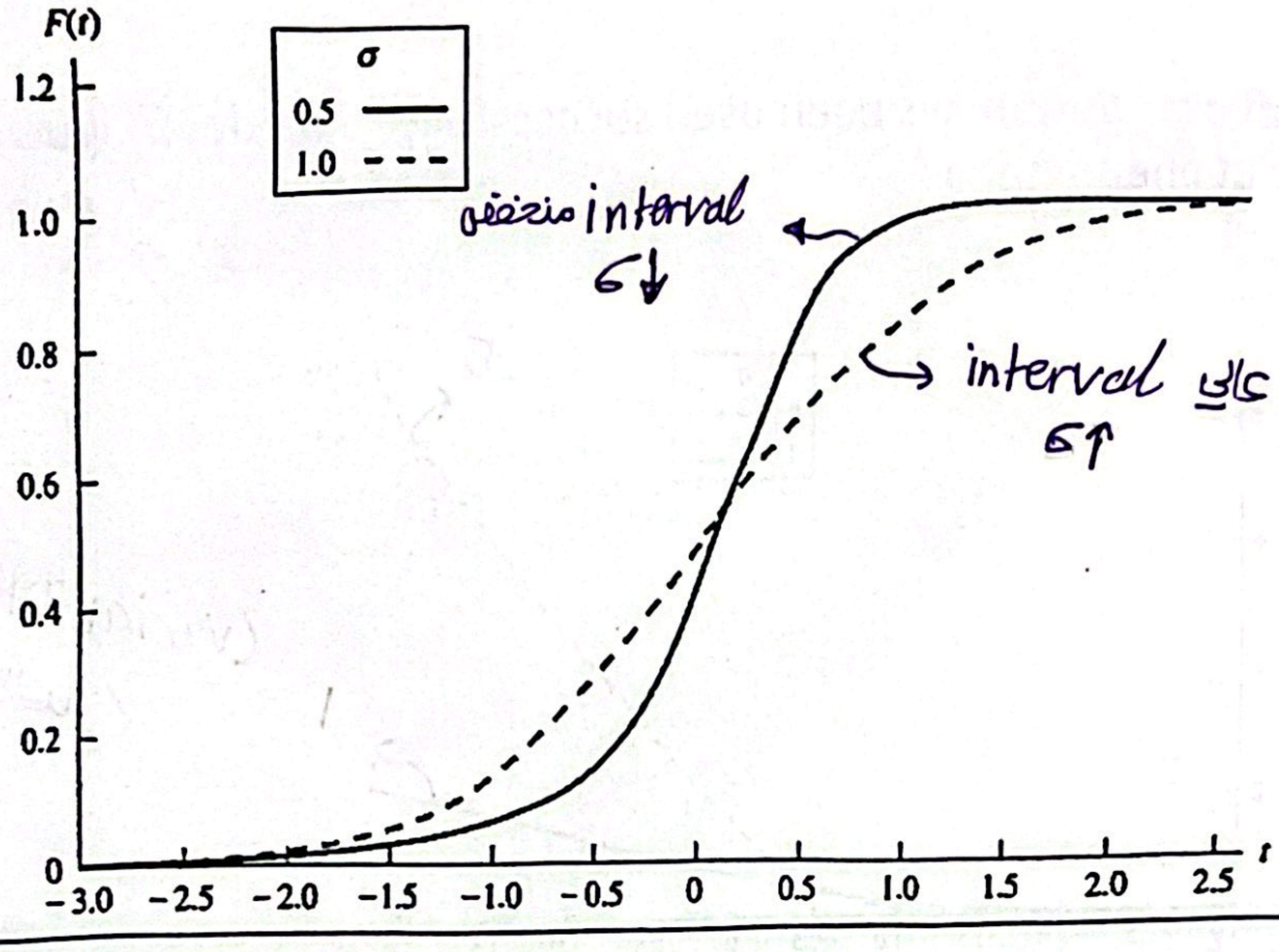
31

The Normal Distribution



32

The Normal Distribution



The Normal Distribution

- The normal is not a true reliability distribution since the random variable ranges from minus infinity to plus infinity
- However, for most observed values of μ and σ, the probability that the random variable will take on negative values is negligible, and the normal can therefore be a reasonable approximation to a failure process

$$MTTF = \mu$$
$$Std\ Dev = \sigma$$

Normal Distribution - Applications

- Tool failures
- Brake lining wear
- Tire tread wear

35

Normal Distribution

- The reliability function for the normal distribution:

$$R(t) = \int_t^{\infty} \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\frac{(t' - \mu)^2}{\sigma^2}\right] dt'$$

- However, there is no closed-form solution to this integral, and it must be evaluated numerically.

- Transformation to standard normal distribution:

$$z = \frac{T - \mu}{\sigma}$$

z is referred to as the standardized normal variate

36

Normal Distribution ←

- The cumulative distribution function:

$$\Phi(z) = \int_{-\infty}^{z} (1/\sqrt{2\pi}) e^{-y^2/2}\, dy$$

- Table A.1 (page 624 - 631) provides cumulative probabilities of the standardized normal distribution
- The table can be used to find the cumulative probabilities of any normally distributed random variable by making use of

$$F(t) = \Pr\{T \le t\} = \Pr\left\{\frac{T-\mu}{\sigma} \le \frac{t-\mu}{\sigma}\right\}$$

$$= \Pr\left\{z \le \frac{t-\mu}{\sigma}\right\} = \Phi\left(\frac{t-\mu}{\sigma}\right)$$

TABLE A.1 Standardized normal probabilities: $\Phi(z) = \int_{-\infty}^{z} (1/\sqrt{2\pi}) e^{-y^2/2}\, dy$

z	$\Phi(z)$	$1-\Phi(z)$	z	$\Phi(z)$	$1-\Phi(z)$	z	$\Phi(z)$	$1-\Phi(z)$
−4.00000	0.00003	0.99997	−3.51000	0.00022	0.99978	−3.02000	0.00126	0.99874
−3.99000	0.00003	0.99997	−3.50000	0.00023	0.99977	−3.01000	0.00131	0.99869
−3.98000	0.00003	0.99997	−3.49000	0.00024	0.99976	−3.00000	0.00135	0.99865
−3.97000	0.00004	0.99996	−3.48000	0.00025	0.99975	−2.99000	0.00139	0.99861
−3.96000	0.00004	0.99996	−3.47000	0.00026	0.99974	−2.98000	0.00144	0.99856
−3.95000	0.00004	0.99996	−3.46000	0.00027	0.99973	−2.97000	0.00149	0.99851
−3.94000	0.00004	0.99996	−3.45000	0.00028	0.99972	−2.96000	0.00154	0.99846
−3.93000	0.00004	0.99996	−3.44000	0.00029	0.99971	−2.95000	0.00159	0.99841
−3.92000	0.00004	0.99996	−3.43000	0.00030	0.99970	−2.94000	0.00164	0.99836
−3.91000	0.00005	0.99995	−3.42000	0.00031	0.99969	−2.93000	0.00169	0.99831
−3.90000	0.00005	0.99995	−3.41000	0.00032	0.99968	−2.92000	0.00175	0.99825
−3.89000	0.00005	0.99995	−3.40000	0.00034	0.99966	−2.91000	0.00181	0.99819
−3.88000	0.00005	0.99995	−3.39000	0.00035	0.99965	−2.90000	0.00187	0.99813
−3.87000	0.00005	0.99995	−3.38000	0.00036	0.99964	−2.89000	0.00193	0.99807

Normal Distribution

- Table A.1 can be used to find the cumulative probabilities of any normally distributed random variable by making use of

$$F(t) = \Phi\left(\frac{t-\mu}{\sigma}\right)$$

$$R(t) = 1 - \Phi\left(\frac{t-\mu}{\sigma}\right)$$

MTTF = Median = Mode = μ

39

Example 4.9:

Failure عند 24%

5% آكثر من 95% Relibitity ← 5% أقل من

- Five percent of a certain grade of tires wear out before 25,000 miles, and another 5 percent of the tires exceed 35,000 miles. Determine the tire reliability at 24,000 miles if wearout is normally distributed.

Reliability ($P(X<25000) = .05$

$P(Z<Z_i) = 0.05$

$\phi(Z_i) = 0.05$

$\frac{25000-\mu}{\sigma} = -1.645$

$P(X<35000) = 0.95$

$\frac{35000-\mu}{\sigma} = +1.645$

5% ← 25K قبل ال
5% ← 35K بعد ال
بينهم : 90%
احتمال تعزز

$\mu = 30000$

$\sigma = 3029.5$

$R(24K) = 1 - \phi\left(\frac{24000-30000}{3029.5}\right) = 97.56\%$

40

$1 + 1.9805 \quad \frac{6000}{3029.5}$

Example:

The time to failure of a fan belt is normally distributed with a MTTF = 220 (in hundreds of vehicle miles) and a standard deviation of 40 (in hundreds of vehicle miles). Fine R(100), R(200), R(300), R(100|200) ??

$R(100) = 1 - \Phi[\ (100\text{-}220)/40] = 1 - \Phi(-3) = .99865$
$R(200) = 1 - \Phi[\ (200\text{-}220)/40] = 1 - \Phi(-.5) = .69146$
$R(300) = 1 - \Phi[\ (300\text{-}220)/40] = 1 - \Phi(2) = .02275$
$R(100|200) = R(300) / R(200) = .02275 / .69146 = .0329$
note: both the median and mode = MTTF = 220 miles

$\frac{R(t_0+t)}{R(t_0)}$

41

41

Design Life Example:

A new fan belt is developed from a higher grade of material. It has a time to failure distribution which is normal with a mean of 35,000 vehicle miles and a standard deviation of 7,000 vehicle miles. Find its designed life if a .97 reliability is desired.

$R(t) = 1 - \Phi[\ (t - 350)/70] = .97$; find t !
From the normal table, $1 - \Phi(-1.88) = .96995$
Therefore; $(t - 350) / 70 = -1.88$
and $t_{.97} = 350 - 1.88\ (70) = 218.4$ or 21,840 vehicle miles

42

42

Exercise:

The operating hours until failure of a halogen headlamp is normally distributed with a mean of 1200 hr. and a standard deviation of 450 hr.

Find:

a. The 5 year reliability if normal driving results in the use of the headlamp an average of 0.2 hr. a day.

b. The 0.90 design life in years.

Exercise Solution:

a. $t = .2$ hr./da. x 365 da./yr. x 5 yr. = 365 hr.

$R(365) = 1 - F[(365 - 1200)/450] = 1 - F[-1.86] = .96856$

b. $R(t_{.90}) = .90$

or $1 - F[(t_{.90} - 1200)/450] = .90$

$(t_{.90} - 1200) / 450 = -1.28$

$t_{.90} = 1200 - 1.28 (450) = 624$ hr.

or $t_{.90} = 624 / (.2 \times 365) = 8.5$ yr.

4.3 The Lognormal Distribution

- If the random variable T, the time to failure, has a lognormal distribution, the logarithm of T has a normal distribution.
- The density function for the lognormal is:

$$f(t) = \frac{1}{\sqrt{2\pi} st} \exp\left[-\frac{1}{2s^2}\left(\ln \frac{t}{t_{med}}\right)^2\right] \qquad t \geq 0$$

where the parameter s is a shape parameter and t_{med}, the location parameter, is the median time to failure.

- The distribution is defined for only positive values of t and is therefore more appropriate than the normal as a failure distribution.

45

Not

Follow for normal $t_1 \rightarrow \ln t_1$, $t_2 \rightarrow \ln t_2$] → follow normal

t_3

tmed → location

$t > 0$

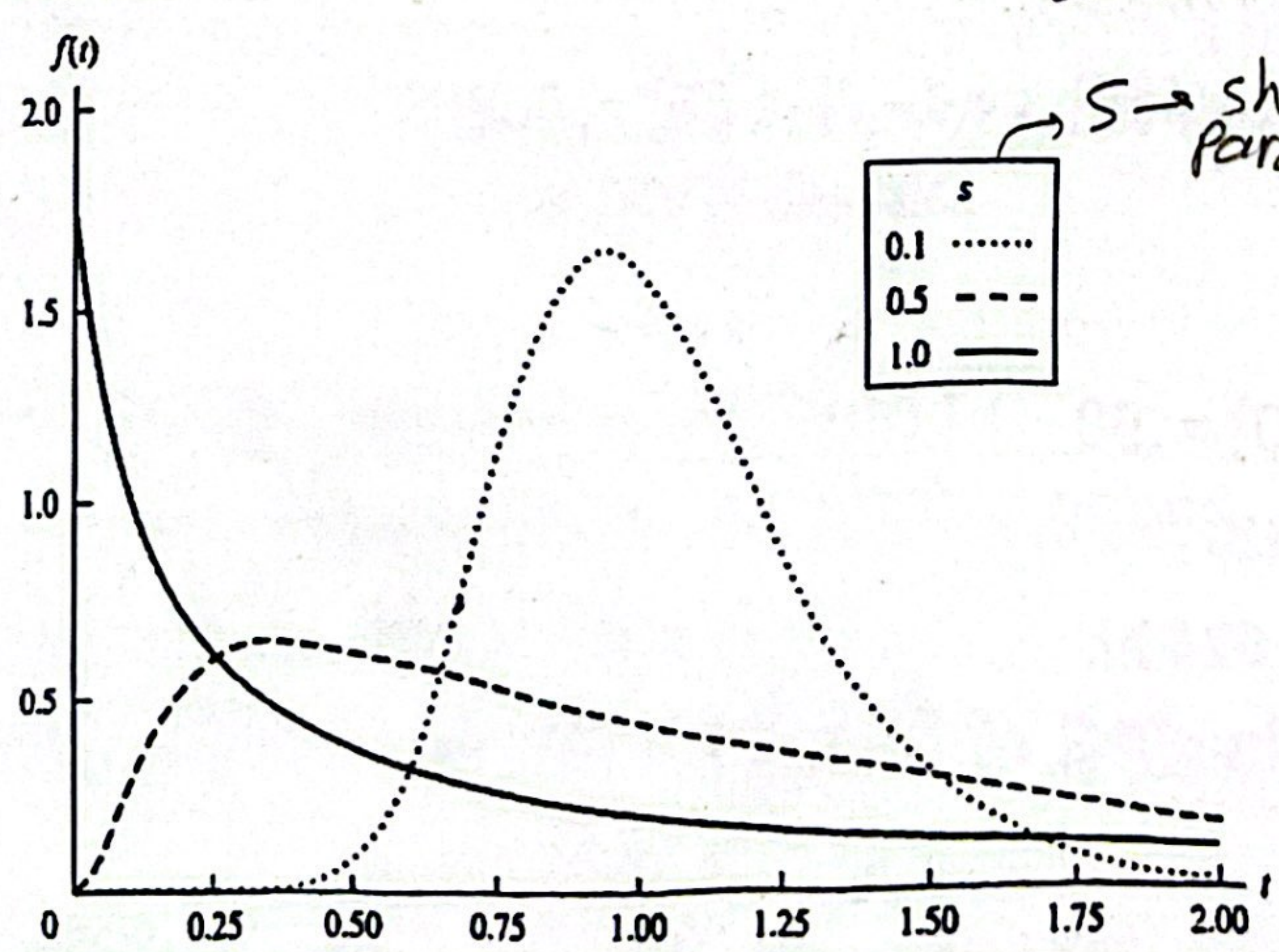
45

The Lognormal Distribution

- Like the Weibull distribution, the lognormal can take on a variety of shapes

tmed = 1

s → shape parameter] s.d ln t



It is frequently the case that data that fit a Weibull distribution will also fit a lognormal distribution

46

46

The Lognormal Distribution

- The mean, variance, and mode of the lognormal are:

$$\text{MTTF} = t_{med} \exp(s^2/2)$$

$$\sigma^2 = t_{med}^2 \exp(s^2)[\exp(s^2) - 1]$$

$$t_{mode} = \frac{t_{med}}{\exp(s^2)}$$

47

Lognormal vs. Normal Distribution

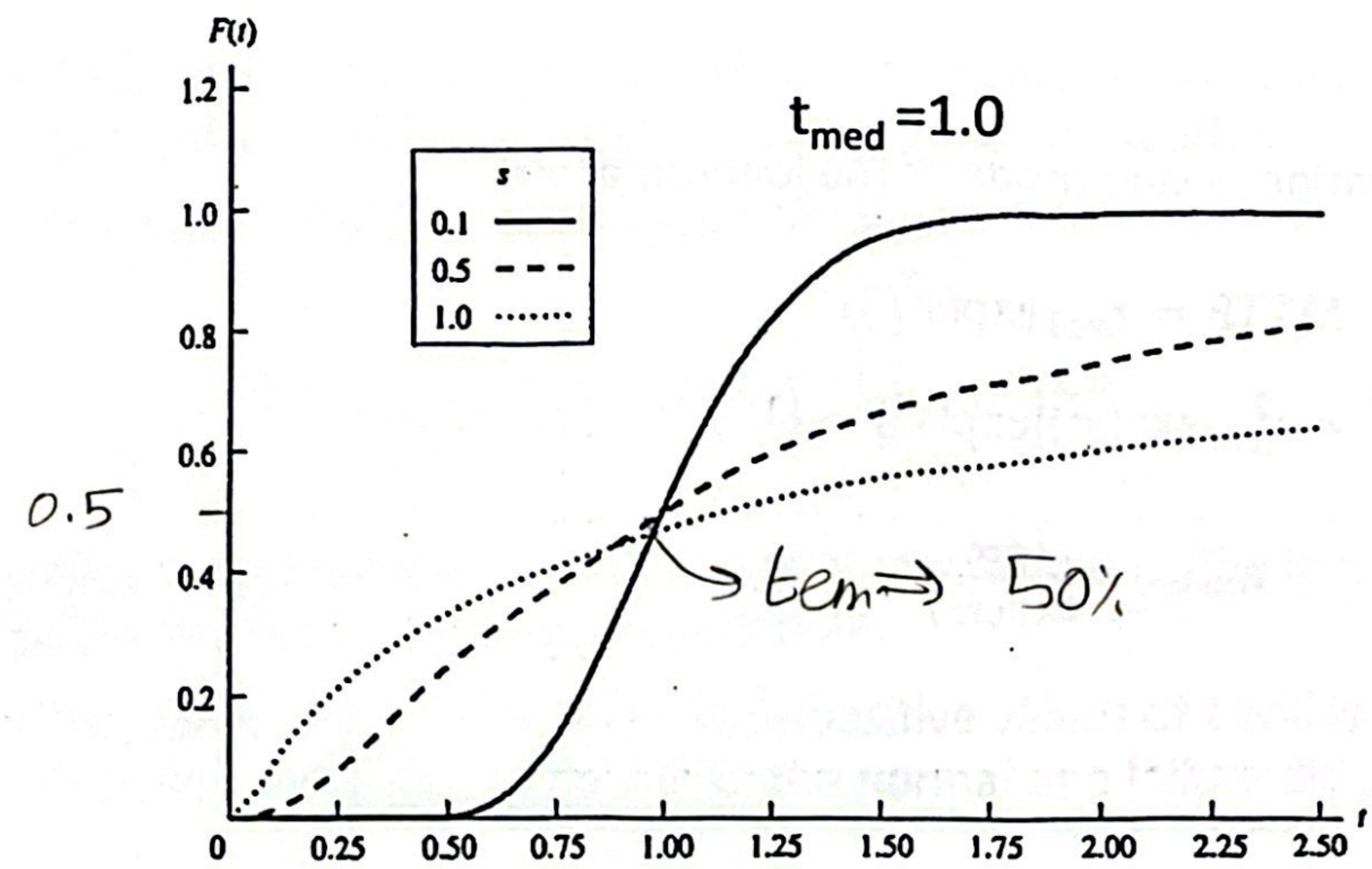
- This relationship between the lognormal and normal distributions is summarized in Table 4.2:

Relationship between lognormal and normal distributions†

Distribution	Lognormal	Normal
Mean	$t_{med} \exp\left(\frac{s^2}{2}\right)$	$\ln t_{med}$ = Mode
Variance	$t_{med}^2 \exp(s^2)[\exp(s^2) - 1]$	s^2

48

The Lognormal Distribution- CDF

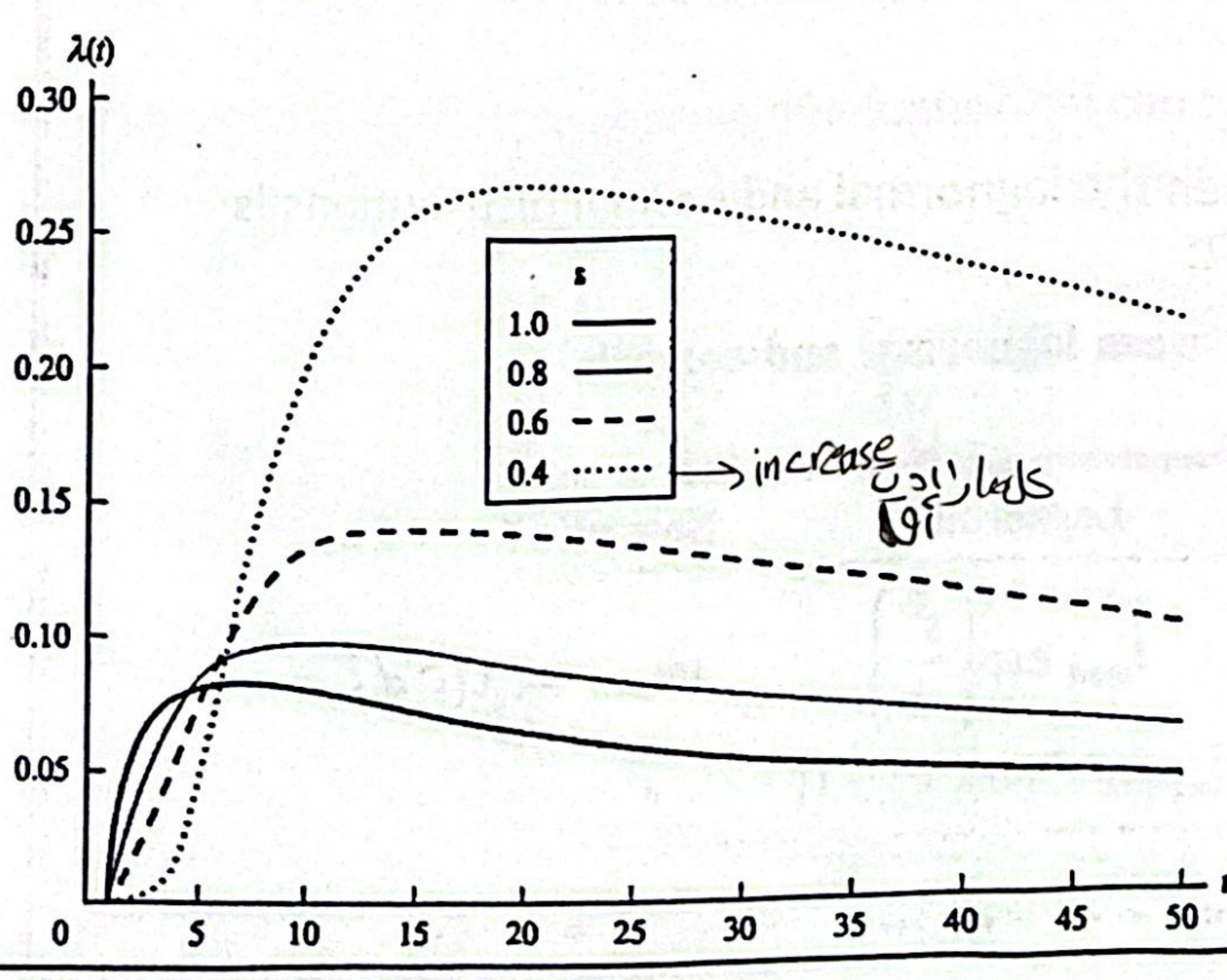


49

49

decrease increase ← bathtub curve

The Lognormal Distribution – Hazard Rate Function



The hazard rate function increases until it reaches a peak, and then it slowly decreases.

This is an uncommon failure rate behavior for most components

The smaller the value of s, the greater the time before the peak is reached

50

50

Mix between increasing and decreasing

The Lognormal Distribution – Hazard Rate Function

If t_{med} =10

s	1.0	0.8	0.6	0.4
Mode	3.7	5.3	7.0	8.5
MTTF	16.5	13.8	12.0	10.8
Max $\lambda(t)$	7	10	16	20

The Lognormal Distribution

$$F(t) = \Pr\{T \le t\} = \Pr\{\ln T \le \ln t\}$$

$$= \Pr\left\{\frac{\ln T - \ln t_{med}}{s} \le \frac{\ln t - \ln t_{med}}{s}\right\}$$

$$= \Pr\left\{z \le \frac{1}{s}\ln\frac{t}{t_{med}}\right\}$$

$$= \Phi\left(\frac{1}{s}\ln\frac{t}{t_{med}}\right)$$

$$R(t) = 1 - \Phi\left(\frac{1}{s}\ln\frac{t}{t_{med}}\right)$$

TABLE A.1 Standardized normal probabilities: $\Phi(z) = \int_{-\infty}^{z} (1/\sqrt{2\pi})e^{-y^2/2}\,dy$

z	$\Phi(z)$	$1-\Phi(z)$	z	$\Phi(z)$	$1-\Phi(z)$	z	$\Phi(z)$	$1-\Phi(z)$
−4.00000	0.00003	0.99997	−3.51000	0.00022	0.99978	−3.02000	0.00126	0.99874
−3.99000	0.00003	0.99997	−3.50000	0.00023	0.99977	−3.01000	0.00131	0.99869
−3.98000	0.00003	0.99997	−3.49000	0.00024	0.99976	−3.00000	0.00135	0.99865
−3.97000	0.00004	0.99996	−3.48000	0.00025	0.99975	−2.99000	0.00139	0.99861
−3.96000	0.00004	0.99996	−3.47000	0.00026	0.99974	−2.98000	0.00144	0.99856
−3.95000	0.00004	0.99996	−3.46000	0.00027	0.99973	−2.97000	0.00149	0.99851
−3.94000	0.00004	0.99996	−3.45000	0.00028	0.99972	−2.96000	0.00154	0.99846
−3.93000	0.00004	0.99996	−3.44000	0.00029	0.99971	−2.95000	0.00159	0.99841
−3.92000	0.00004	0.99996	−3.43000	0.00030	0.99970	−2.94000	0.00164	0.99836
−3.91000	0.00005	0.99995	−3.42000	0.00031	0.99969	−2.93000	0.00169	0.99831
−3.90000	0.00005	0.99995	−3.41000	0.00032	0.99968	−2.92000	0.00175	0.99825
−3.89000	0.00005	0.99995	−3.40000	0.00034	0.99966	−2.91000	0.00181	0.99819
−3.88000	0.00005	0.99995	−3.39000	0.00035	0.99965	−2.90000	0.00187	0.99813
−3.87000	0.00005	0.99995	−3.38000	0.00036	0.99964	−2.89000	0.00193	0.99807

Example 4.10:

- Fatigue wearout of a component has a lognormal distribution with t_{med} = 5000 hr and s = 0.20. Find: MTTF, Standard Deviation, t_{mode}, Reliability for 3000 hrs?

Example 4.10:

t_{med} = 5000 hr and s = 0.20. Find: MTTF, Standard Deviation, t_{mode}, Reliability for 3000 hrs?

$$MTTF = 5000e^{(0.20)^2/2} = 5101 \text{ hr}$$

$$\sigma^2 = 5000^2 e^{(0.20)^2}\left[e^{(0.20)^2} - 1\right] = 1.0619 \times 10^6$$

$$\sigma = 1030 \text{ hr}$$

$$t_{mode} = \frac{5000}{e^{(0.20)^2}} = 4804 \text{ hr}$$

$$R(3000) = 1 - \Phi\left(\frac{1}{0.2}\ln\frac{3000}{5000}\right)$$

$$= 1 - \Phi(-2.55) = 0.99461$$

55

Design Life:

Let R represent the desired reliability

Then $$1 - \Phi\left(\frac{1}{s}\ln\frac{t_R}{t_{med}}\right) = R$$

or $$\Phi\left(\frac{1}{s}\ln\frac{t_R}{t_{med}}\right) = 1 - R$$

and $$\frac{1}{s}\ln\frac{t_R}{t_{med}} = z_{1-R}$$

where z_{1-R} is found in Table A.1 such that

$$\Phi(z_{1-R}) = 1 - R$$

Solving for t_R:

$$t_R = t_{med}\, e^{s z_{1-R}}$$

56

Example 4.11:

From the previous example, find the design life for a reliability of 0.95

$$t_{0.95} = 5000\, e^{0.20(-1.64)} = 3602 \text{ hr}$$

Z(0.05)

Exercise

Reliability testing of the new 1.6 liter automotive engine has resulted in a time to failure distribution which is lognormal with t_{med} = 100,000 mile and s = 0.70. Find:

a. R(36,000 mile)

b. MTTF and Std. Dev.

c. R(64,000|36,000)

d. $t_{.95}$

Exercise – solution

a. $R(36{,}000) = 1 - \Phi[\,(1/.7)\ln(36{,}000/100{,}000)]$
$= 1 - \Phi[-1.46] = .92786$

b. $MTTF = 100{,}000\, e^{.49/2} = 127{,}762$ mi.
$Var = 100{,}000^2\, e^{.49}\,[\,e^{.49} - 1] = 1.032 \times 10^{10}$
Std Dev = 101,594 mi.

c. $R(64{,}000|36{,}000)$
$= R(100{,}000)/R(36{,}000)$
$= 0.5 / .92786 = .539$

Exercise – solution

d. $R(t_{.95}) = .95$
$1 - \Phi[\,(1/.7)\ln(t_{.95}/100{,}000)\,] = .95$
$(1/.7)\ln(t_{.95}/100{,}000) = -1.64$
$t_{.95} = 100{,}000\, e^{-1.64 \times .7} = 31{,}727$ mi.

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